King of the hill

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Abstract

The fight for power is not only over immediate rents, but also over advantageous positions in future power struggles. Such an incumbency edge may stem from the control of the army, the police and other instruments reserved for the government. When incumbency yields an extra fighting edge, current struggles may involve high stakes as a victory today increase the chance of a victory also tomorrow. The conclusions drawn from static conflict models are turned on its head when the fight is also over the incumbency edge. For example: A sharper incumbency edge increases the implicit prizes of winning. The fighting intensity may therefore rise when the strength of the incumbent increases. This is in contrast to the standard result that equal strengths give the most intense fighting.

1 Introduction

By knocking down the present "king of the hill" the winner can take the king's place and enjoy both the prestige and the fighting advantage that standing on the top of the hill entail. Many real power struggles are positional just like this - incumbents versus challengers, insiders versus outsiders, market leaders vs followers - but the structure is often less symmetric than the children game suggests. Contestants may have highly different ability to take advantage of incumbency. Some winners in real life contests -be it in battlefields, politics, or market places - receive an even stronger position than their predecessors. They become the king, not just of a hill, but maybe of a whole mountain – raising the prestige of the position and making it even more difficult to overthrow them as the competition continues. Other winners hardly get a mound to start out from in the next period.

Below we explore the positional dynamics where fighting for power is both over rents and over positions and fighting edge in the subsequent fights. Like Esteban and Ray (1999) we focus on a situation where there is no collective decision rule and where groups with opposed interests are willing to spend resources to increase the chance of obtaining their preferred outcome. Unlike most other papers, however, we focus on alternating powers in lasting conflicts. While the immediate rent of a contestant is the difference in utilities between the implementation of his own favored policy and that of his opponents, the advantages in future battles are captured by differences in the unit costs of influence. The effective prize of winning a battle contains both an immediate rents and the value of the incumbency cost advantage and will generally be different for the two groups.

The unequal stakes may reflect differences in the supporting groups and in the access to resources. The immediate rents to a party in power may vary with ideological orientations, political capabilities and economic opportunities. Advantages in future contests may reflect varying capabilities to utilize incumbency to raise campaign contributions or, in more violent struggles, varying sympathies from the army, the police and from other parts of state apparatus.

Thus in a conflict different things are obviously at stake for the two opposing sides. We claim that these disparities play an essential role in lasting power struggles. They shape the dynamics of the conflicts. A specific incumbency advantage implies that a victory today may guarantee the victory also tomorrow, and as the expected outcome today depends on who is the incumbent, the victory of tomorrow actually depends on who was yesterday's winner. The past thus affects present fighting efforts, which again affect the future path of the struggle. Without unequal stakes this path dependence would disappear.

Building on such mechanisms we explore three related issues:

(1) Inequalities among contestants: Does levelling of the battlefield maximize conflict efforts?

(2) Fighting efforts and the turnover of incumbents: Do circumstances that lead to higher fighting intensities imply lower regime stability?

(3) Power and payoffs: Is more power always to the advantage of the powerful?

Applying traditional static contest models one would be lead to insist on an unambiguous yes to all three questions. (For an excellent review of models of static rent-seeking contests, see Shmuel Nitzan 1994). Firstly, in static contest models an edge to one player raises his chances of winning and induces the opponent to fight less. This reduction in the efforts of the opponent lead the more powerful to spend less in the conflict as well. Thus in static models inequality of power dampens the struggling. The level of resources wasted in the fighting relative to the total prizes at stake is highest when the contestants are equally strong and fight for the same prize. Secondly, the average turnover must be highest when the chances of winning is fifty fifty for each side of the conflict implying that a levelling of the battlefield raises the rate of regime turnover. Thus, more fighting go together with lower regime stability. Thirdly, as long as the prizes are given independently of fighting efforts the powerful is bound to gain as he is more likely to obtain the prize for less effort.

All three answers can be contested both empirically and theoretically. If the predictions from static contests were generally relevant, a third party that wishes to reduce fighting and stabilize a regime, should provide resources to the incumbent to lower his costs of influence and to deter the opposition from fighting. Such interventions may work well in some, but far from all, cases. Recent experiences, such as Ethiopa's intervention in Somalia in 2006, and Nato's intervention in Afghanistan after 2001, demonstrate that a strengthening of the incumbent can be met by more fighting by the challengers – the Union of Islamic Courts in Somalia and the Taliban in Afghanistan.

Somalia is a particularly instructive case of lasting power struggles. Since the disintegration of the central government in 1991 the country has been wracked by internal clan warfare and a struggle with the Islamic movement. Since 2000 there have been attempts to reestablish central authority in the country by a transitional government placed in Kenya for security reasons. In June 2006, after months of fighting between Mogadishu's US-backed militia leaders and the Union of Islamic Courts, the Transitional Federal Government lost its power as the islamists took control of the capital city and appointed a hard-line Islamic leader to head its new legislature. With a significant Ethiopian support the Transitional Federal Government retook Mogadishu on 28 December 2006. Since then the fighting has just escalated. The extra edge that the Ethiopian support provided, did far from deterring the fighting by the Islamic movement. In contrast to what static conflict models would predict,

the extra incumbency edge induced harder opposition.

An approach that incorporates unequal stakes and advantages with endogenous turnover of power positions would lead to conclusions that are more in line with these observations. A sharper incumbency edge for one side may not only imply that this incumbency becomes more valuable, but also that the position of the challenger becomes worse. Thus, since the gain of winning is the value of not loosing, the stakes may go up for both the incumbent and the challenger, implying that the fighting intensity may go up when the strength of each side becomes more unequal.

More generally, exploring the causes and consequences of unequal stakes we learn that the difference principle – *the gain of winning is the value of not loosing* – has richer implication in dynamic contests than in static ones. Obviously each contestant has reason to care not only about his own policies when in power, but also about those of the opponents when they are in power. How long they can hold on to their power is equally important as how long he can hold on to his. The stakes of a contestant may be high because his insider benefits are high, or because his outsider benefits are low. The stakes may also be high because incumbency is expected to last for a long period implying that the contestant can either enjoy the insider benefits for long, or be squeezed as an outsider for a long period. A weak challenger can therefore fight hard for an advantageous incumbency position in future battles, or he can fight hard to eliminate the harmful consequences of a strong incumbent of today. In other words, a contestant may put in a lot of effort either because winning enables him to remain in power for long, or because winning prevents the opponents from an advantageous incumbency that allows them to remain in power for a long period. To prevent the opponent from getting the insider edge can be equally important as to obtain it for oneself.

It is these long lasting consequences of present struggling that distinguishes dynamic contests from static ones. Fighting in the shadow of future conflicts imply that the relationships between resources spent and expected turnover of incumbents, and between power and payoff can be less straight forward than the static models predict. Incorporating a simple but essential dynamic element shed new light on how conflicts affect political instability and how strength affects payoffs.

Our stylized set-up may have several interpretations. The struggling can be over the control of natural resources (a diamond mine); over political representation; over religion (sharia laws); or over technological leadership. The issues can be political, economic or institutional: colonial rule versus independence; democratic versus one party rule; The simple dynamics we incorporate does not exclude that the victory of one side can become an absorbing state while the struggling continues as long as the present incumbent is undefeated. In the bitter fighting struggles in South-Africa, for instance, the stakes for the apartheid regime were to

prevent a permanent loss of their privileges. For the ANC, however, winning implied that the hated system was gone forever.[1]

One old but prominent example of the kind of conflicts that have all the ingredients we have in mind is the struggle between the Gulphs and the Ghibellines, the two opposing factions in German and Italian politics in the 13th and 14th centuries. The feuds took place as a long series of battles over the control of cities like Bergamo, Ferrara, Florence, Lucca, Milan, Padua, Parma, Piacenza, Treviso, Verona and Vicenza. In each battle the faction in power had a clear advantage over the challenger and the losing party was often exiled from the city. The two parties had diverging economic interests. The Gulphs were middle class merchants, artisans and burgers, the Ghibellines landed aristocrats - implying that their opportunity costs of fighting were quite unequal. Once in power each party favored different policies that the opposing party strongly disliked. The Gulphs favored mercantile liberties of urban communes and fought for the pope and against the Emperor's encroachment. The Ghibellines were traditionalists who fought against the temporal power of the pope,

More recently Latin-American has similarly experienced repeated incumbency shifts in lasting power struggles between non-autocratic and autocratic rule with rather asymmetric powers. Argentina, for instance, had four episodes of moving into autocracy plus four episodes of moving out of autocracy from 1950 to 1990, Honduras and Bolivia had three plus three, while several others had two plus two (Przeworski et al. 1990). Clearly, the incumbency advantages that the two regimes enjoyed was dramatically unequal. While authoritarian regimes had more support from the military, less authoritarian regimes had more legitimacy and more popular support.

We build on an expanding literature where conflicts are seen as rent seeking contests, see Haavelmo (1954), Tullock (1980), Hirschleifer (1991) , Grossman (1994), Skaperdas (1992), Konrad and Skaperdas (1998). The two papers closest to ours are the seminal contributions by Joan Esteban and Debraj Ray (1999) and by William Rogerson (1982). While Esteban and Ray construct a general model of multi group conflicts with heterogenous prizes, the somewhat overlooked contribution by Rogerson focus on insiders and outsiders in a symmetric lobbying game over an homogenous prize where a winning outsider becomes the new insider. (to be con't)

[1]The main mechanism of our approach may also be relevant for some aspects of market competition, for instance the contests over mobile phone designs between Nokia and Ericsson, where winning the consumers over in one round may imply a technological leadership that can help in future contests as well.

2 A model of conflicts with unequal incumbency edge

An incumbent and a challenger compete for power and spend resources to improve their chances of winning. Incumbency entails both an immediate rent (political or economic control) and an advantage in subsequent contests. There are two possible fighting constellations: the first one is with a as the incumbent and b as the challenger; the second one is with b as the incumbent and a as the challenger. The simple dynamics that we incorporate is the endogenous switches between the two fighting constellations: when a loses in the first fighting constellation we move to the second constellation where b is the incumbent, and so on. Future fighting constellations have implications for how the contestants behave in the present constellation. We first focus on stationary solutions, before we discuss possible non-stationary solutions.

When a is the incumbent and b is the the challenger the present value of a's pay-off can be written as

$$V_a = \rho_a \left[U_a + \delta V_a\right] + (1 - \rho_a) \left[u_a + \delta v_a\right] - Y_a \tag{1}$$

With probability ρ_a the incumbent a stays in power and with probability $(1 - \rho_a)$ he loses power. If a wins he obtains the utility U_a and starts the next period also as the incumbent, which has the present value δV_a, where $\delta < 1$ is the discount factor. If a loses he gets the utility u_a and starts next period as the challenger, which has the present value δv_a. Finally, Y_a is the amount of resources invested by a as the incumbent in the present struggle.

The expression for V_a can be rearranged as

$$V_a - \left[u_a + \delta v_a\right] = \rho_a F_a - Y_a \tag{2}$$

Here $[u_a + \delta v_a]$ is what contestant a can guarantee himself if he loses and $V_a - [u_a + \delta v_a]$ is the expected *excess return* from participating in the conflict. F_a is the prize of winning

$$F_a = \left[U_a - u_a\right] + \delta \left[V_a - v_a\right] \tag{3}$$

defined as the sum of the immediate incumbency rent $[U_a - u_a]$ and the present value of starting the next period as the incumbent rather than the challenger $[V_a - v_a]$. Here the values of V_i, v_i are endogenously determined.

The values of U_i, u_i should be thought of as outcomes of an optimal incumbency behavior without commitment. So, if a as his best choice chooses a certain policy vector as incumbent it is valued as U_a for a and as u_b for b. Thus the value of u_a in (3) reflects a's evaluation of b's best

choice as incumbent and $[U_a - u_a]$ is the immediate gain of assuming power - the incumbency rent. In a multi-group context Esteban and Ray (1999) use utility differences like these as an indication of intergroup distance. The value of $[U_a - u_a]$ measures the distance from group a to group b, the value of $[U_b - u_b]$ the distance from b to a. The larger these measures the more antagonism there is between groups and the more polarized are the preferences. Symmetric antagonism is a special case where $[U_a - u_a] = [U_b - u_b]$.

Throughout we use the convention that capital letters U, V, Y reflect incumbency position and lower case letters u, v, y reflect a challenger position. Using this convention v_b denotes the present value for b when starting out as a challenger against a. By analogy to (2), it can be written as

$$v_b - [u_b + \delta v_b] = (1 - \rho_a) F_b - y_b \tag{4}$$

where b's prize of winning is

$$F_b = [U_b - u_b] + \delta [V_b - v_b] \tag{5}$$

Recall that ρ_a denote the probability that a remains the incumbent, while $(1 - \rho_a)$ is the probability that a is replaced by b as the new incumbent. The winning probabilities are determined by a combination of *luck* and *relative strength*. Let S_a be the measure of relative strength of a as incumbent, and $s_b$the relative strength of b as challenger. With a unit cost of influence equal to C_a the resource use of a as incumbent Y_a translates into Y_a/C_a units of fighting effort. Similarly, with an unit cost of influence equal to c_b the resource use of b as challenger y_b translate into y_b/c_b units of fighting effort.[2] Then relative strengths can be defined by

$$S_a = 1 - s_b = \frac{Y_a/C_a}{Y_a/C_a + y_b/c_b} \tag{6}$$

To incorporate the role of luck we assume that winning requires a relative strength larger than an uncertain threshold - analogous to probabilistic voting models. Hence, for a as the incumbent we can write the probability of winning (the probability that S_a is higher than the threshold) as

$$\rho_a = \Psi (S_a) \tag{7}$$

We have three requirement for the relationship $\Psi(.)$ between relative strength and probability of winning.

[2]For convinience we use S_a insted of relative effort $(Y_a/C_a) / (y_b/c_b)$. Note,however, that there is a one two one mapping between the two as $S_a = (1 + (Y_a/C_a) / (y_b/c_b))^{-1}$

1. Anonymity (i.e. symmetry): For all S we require that $\Psi(S) = 1 - \Psi(1-S)$

2. Strength pays: $\Psi'(S) > 0$.

3. No strength implies losing for sure: $\Psi(0) = 0$.

It follows from the first requirement that $\Psi'(S) = \Psi'(1-S)$ and $\Psi''(S) = -\Psi''(1-S)$. Clearly, the much used Tullock contest success function, $\Psi(S) = S$, is a special case of our set-up.[3]

How sensitive the winning probabilities are to relative strength (spending) may reflect institutional arrangements. In a pure unrestricted power struggle the Ψ-function is more sensitive to changes in relative strength than it would be with more checks and balances. In elections, for instance, where the opposing blocks are not allowed to buy political advertisements in TV and instead share equally the time on public television to present their political programs (as is the case in Scandinavia), the Ψ- function is rather flat around $S = 1/2$.1

The first order conditions for the choice of fighting efforts follows from (2) and (4) as

$$\frac{\partial V_a}{\partial Y_a} = 0 => S_a(1-S_a)F_a\Psi'(S_a) = Y_a \tag{8}$$

$$\frac{\partial v_b}{\partial y_b} = 0 => S_a(1-S_a)F_b\Psi'(S_a) = y_b \tag{9}$$

The second order conditions can be written

$$-\frac{2}{S_a} < \frac{\Psi''(S_a)}{\Psi'(S_a)} < \frac{2}{1-S_a} \tag{10}$$

In addition we impose participation constraints, that the excess returns are positive, assuring equilibrium in pure strategies:

$$\rho_a F_a - Y_a > 0 \text{ and } (1-\rho_a)F_b - y_b > 0 \tag{11}$$

Using (6) it follows that in a Nash equilibrium where both (8) and (9) are satisfied then relative strength is

$$S_a = \frac{F_a/C_a}{F_a/C_a + F_b/c_b} \tag{12}$$

Thus each contestant's prize relative to the unit costs of influence determines the the strengths of the two sides. A contestant with either a high stake, or a low costs, or both, has a high equilibrium strength.

[3]See Skaperdas (1996) for a structured discussion of success functions in the M-player case.

From (8) and (9) and from the symmetry of Ψ it follows that the expected excess return for each player is determined by the same function $h()$ such that

$$\frac{\rho_a F_a - Y_a}{F_a} = \Psi(S_a) - S_a(1-S_a)\Psi'(S_a) \equiv h(S_a) \tag{13}$$
$$\frac{(1-\rho_a)F_a - y_b}{F_b} = 1 - \Psi(S_a) - S_a(1-S_a)\Psi'(S_a) \equiv h(s_b) = h(1-S_a)$$

Here $h(S_a)$, $h(1-S_a)$ capture the excess return relative to the prize for a and b, when a is the incumbent.[4] Combining with (11) it follows that the participation constraints can be written

$$h(S_a) > 0 \text{ and } h(1-S_a) > 0 \tag{15}$$

One illustration of the relationship between S, Ψ, and h is given in Figure 1. By definition the h function is everywhere below the Ψ. The h function reaches unity when S is one as then the prize comes with certainty and for free. The h function is zero when S is zero as then the prize cannot be won and no efforts are wasted.

When b is the incumbent and a is the the challenger the present value for b is

$$V_b = \rho_b F_b + [u_b + \delta v_b] - Y_b \tag{16}$$

while the present value for a when starting out as challenger is

$$v_a = (1-\rho_b)F_a + [u_a + \delta v_a] - y_a \tag{17}$$

where the prizes F_a and F_b are defined by (3) and (5). The probability that the incumbent b wins is

$$\rho_b = \Psi(S_b) \tag{18}$$
$$\text{where } S_b = \frac{Y_b/C_b}{Y_b/C_b + y_a/c_a} \tag{19}$$

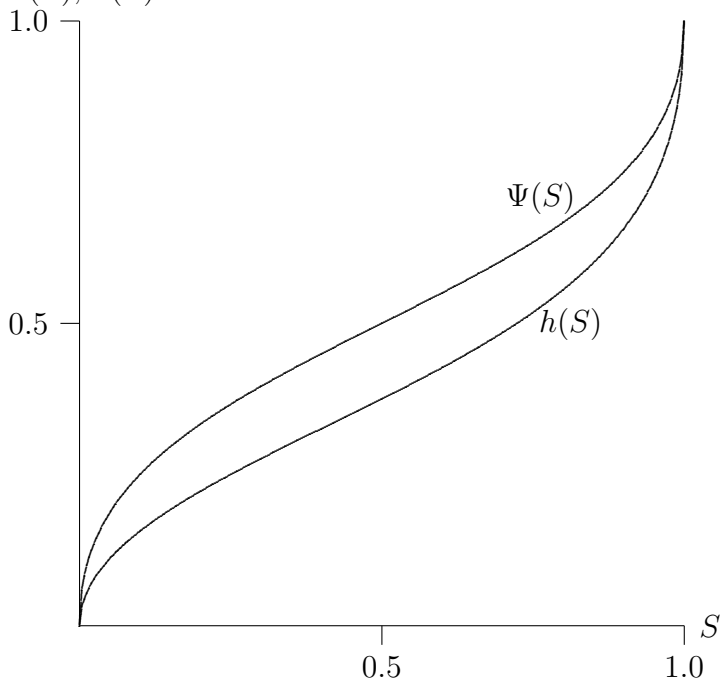
[4]In the case where the prizes are fixed and equal to $F_a = F_b = F$, the sum of $h(S)$ and $h(1-S)$ is negatively associated with the social waste of the fight $Y_a + y_b$. We define the total waste ratio as total resources spent on the conflict relative to the prize. The waste ratio function $\omega(S)$ is given by

$$\omega(S) = 1 - (h(S) + h(1-S)) = 2S(1-S)\Psi'(S) > 0 \text{ when } S \in \langle 0,1\rangle \tag{14}$$

When Ψ'' is not too large the waste ratio ω has its maximum at $S = 1/2$.

$\omega(S_a)$ has local extrema when $\Psi''/\Psi' = (2S_a - 1)/\left(S_a - S_a^2\right)$.Given the symmetry of the Ψ-function $S_a = 1/2$ obviously satisfies the first order condition and if Ψ is S-shaped ($\Psi'' < 0$ to the right of $S_a = 1/2$ i.e. a strong decreasing returns to effort) $S_a = 1/2$ is a unique and global maximum. As it happens, the function $\Psi(S) = 1/2 - k/4(\ln(1-S) - \ln(S))$ (where $\Psi'(1/2) = k$) has exactly the curvature needed for the social waste to be independent of S. as in this case $\omega(S) = k/2$ for all S.

Figure 1: Relationship between S, $\Psi(S)$, and $h(S)$



Solving the problem as above it follows that in Nash equilibrium as before

$$S_b = \frac{F_b/C_b}{F_b/C_b + F_a/c_a} \tag{20}$$

$$\frac{\rho_b F_b - Y_b}{F_b} = h\left(S_b\right) \tag{21}$$

$$\frac{\left(1-\rho_b\right) F_a - y_a}{F_a} = h\left(1 - S_b\right) \tag{22}$$

where h is the same function as defined in (13).

2.1 Static results

Before we consider the dynamic of the contest, with switches between fighting constellations, it is useful to establish some static results for the case where the prizes F are fixed. These static results may reflect a situation where the future does no matter, $\delta = 0$.

First when the $F-$s are fixed (12), (20), and (13) give a simple recursive solution of the model. Any change in costs or prizes will affect the S that in turn determines the payoffs for the parties via $h\left(\right)$. We can show using (10) and (15), the following two properties for h

$$\frac{\partial h\left(S\right)}{\partial S} > 0,\ h\left(0\right) = 0,\ h\left(1\right) = 1 \tag{23}$$

$$0 < \frac{\partial V_i}{\partial F_i} = h_i + \frac{\partial h\left(S_i\right)}{\partial S_i}\frac{\partial S_i}{\partial F_i}F_i < 1 \quad \text{for} \;\; i = a, b \tag{24}$$

These two static results are confirmed by Figure 1 and they can be summarized in the following proposition

Proposition 1 *Assume that the prizes F_a and F_b were exogenously fixed, then a higher relative strength to a contestant is unambiguously to that contestant's benefit. If the higher strength is due to a higher prize then the net gain is always less than the prize increase.*

The proposition may appear trivial, but below we show that these results are not generally true when considering the dynamics of the game. As stated the dynamics we consider implies that the struggle can alternate between the two fighting constellations with different incumbents.

2.2 The value of power

By incumbency advantage for i we mean a situation where $C_i < c_i$.[5] It follows directly from (12) and (20) that when no group has incumbency advantage then

$$C_a = c_a \text{ and } C_b = c_b \iff S_a + S_b = 1 \iff \rho_a + \rho_b = 1 \tag{25}$$

If at least one group has incumbency advantage then

$$C_a < c_a \text{ and/or } C_b < c_b \iff \begin{cases} S_a + S_b > 1 \text{ and} \\ \quad S_a > s_a \quad \text{and} \\ \quad S_b > s_b \quad \text{and} \\ \rho_a + \rho_b > 1 \end{cases} \tag{26}$$

A stationary Markov equilibrium of the game is characterized by F_a and F_b such that (3) and (5) holds. Hence,

$$F_a = \left(U_a - u_a\right) + \delta\left(h\left(S_a\right) - h\left(s_a\right)\right)F_a \tag{27}$$

$$F_b = \left(U_b - u_b\right) + \delta\left(h\left(S_b\right) - h\left(s_b\right)\right)F_b \tag{28}$$

Solving (27) and (28) it follows that

$$F_a = \frac{\left[U_a - u_a\right]}{1 - \delta\left(h\left(S_a\right) - h\left(s_a\right)\right)} \geq \left[U_a - u_a\right] \tag{29}$$

[5]Absence of incumbency advantage for i implies $C_i = c_i$. Hence, we do not consider incumbency *disadvantage* ($C_i > c_i$). That would, however, be a straight forward extension of our framework..

$$F_b = \frac{[U_b - u_b]}{1 - \delta\left(h\left(S_b\right) - h\left(s_b\right)\right)} \geq [U_b - u_b] \tag{30}$$

Firstly, it should be observed that, as long as $\left(h\left(S_i\right) - h\left(s_i\right)\right) > 0$, the prizes become higher the higher is their discount factor δ. Since fighting today may have long run run implications, more are at stake when contestants are more patient or equivalently are more certain that the fighting will continue in future periods. Thus a longer horizon intensifies present power struggles.

Secondly, the two expressions of the prizes also reflect that contestants may fight for different incumbency rents $U_i - u_i$ that can be enjoyed for different expected length. In the expressions the magnitudes $\left(h\left(S_i\right) - h\left(s_i\right)\right)$ capture the long run valuation of incumbency for contestant i. If at least one group has incumbency advantage it follows from (26) and (23) that $\left(h\left(S_i\right) - h\left(s_i\right)\right) > 0$ for both groups. That the effective prizes are higher than the corresponding incumbency rents reflects that the contest is not only over the rent but also over the relative power in later contests.

Consider contestant a: The prize F_a is at its highest level when $\left(h\left(S_a\right) - h\left(s_a\right)\right)$ is close to unity. In this case the prize is simply equal to the present value of all future rents $\left[U_a - u_a\right]/\left(1 - \delta\right)$. The requirement for $\left(h\left(S_a\right) - h\left(s_a\right)\right)$ to be close to unity is that incumbency is secure and cheap $\left(h\left(S_a\right) \approx 1\right)$ while the challenger position is highly dismal $h\left(s_a\right) \approx 0$. The prize F_a is at its lowest level when $\left(h\left(S_a\right) - h\left(s_a\right)\right)$ is close to zero which is the case only when neither group has an incumbency advantage.

An example of the relationship between the relative effort and the value of incumbency is provided in Figure 2. Consider the case where in equilibrium $S_b = S'$ and $S_a = S'$. This is a case of symmetric incumbency advantage. The valuation of incumbency for a and b is

$$h\left(S_b\right) - h\left(s_b\right) = h\left(S_a\right) - h\left(s_a\right) = h\left(S'\right) - h\left(1 - S'\right) = \text{A-B}$$

where A-B refers to the figure. Now if a becomes even stronger as incumbent the situation becomes asymmetric. On example is the case where $S_b = S'$ and $S_a = S'' > S'$ as illustrated in the figure. In this case the valuation of incumbency for a is

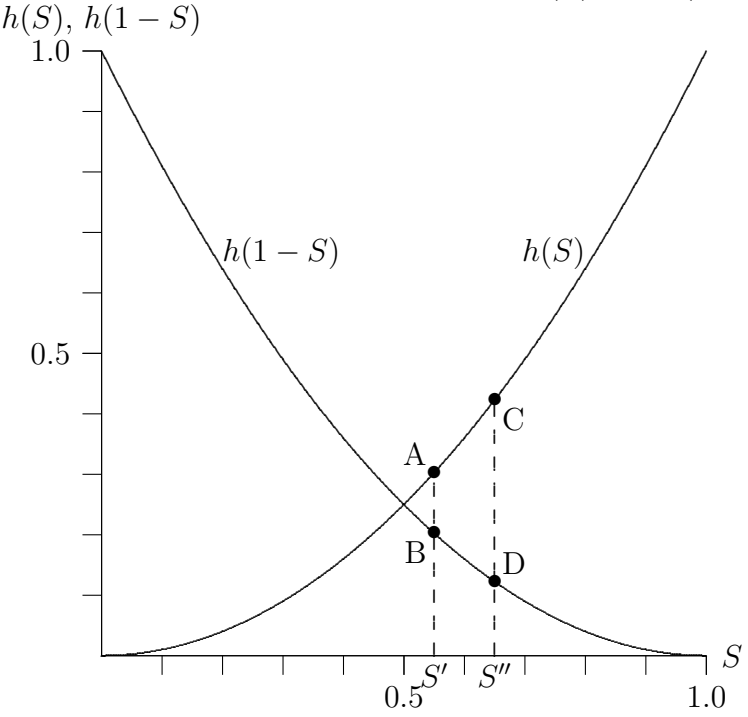
$$h\left(S_a\right) - h\left(s_a\right) = h\left(S''\right) - h\left(1 - S'\right) = \text{C-B}$$

while the incumbency valuation for b is

$$h\left(S_b\right) - h\left(s_b\right) = h\left(S'\right) - h\left(1 - S''\right) = \text{A-D}$$

This illustrates a case where the valuation of incumbency goes up for both when one party

Figure 2: Relationship between states and $h(S)$ and $h(1-S)$



gets a stronger advantage.

Next consider a case where a becomes stronger also as challenger. One example is the case where $S_a = S''$ and $s_a = 1 - S_b = S'$. This is a case where a's chance of winning is above 50 percent in both states but highest as incumbent. In this case the valuation of incumbency for a is

$$h\left(S_a\right) - h\left(s_a\right) = h\left(S''\right) - h\left(S'\right) = \text{C-A}$$

while the incumbency valuation for b is

$$h\left(S_b\right) - h\left(s_b\right) = h\left(1 - S'\right) - h\left(1 - S''\right) = \text{B-D}$$

In this case, compared to the first two, the valuation of incumbency is low. It illustrates the general tendency that high valuation of incumbency for one gives high valuation of incumbency for the other.

The case with no incumbency edge is an interesting reference point. No incumbency edge implies that $C_a = c_a$ and $C_b = c_b$. From this reference point, introducing an incumbency advantage to either party raises the stakes for both. For the contestant who gets the edge the incumbency position increases in value, for the party that does not get the edge the challenger position decreases in value.

By differentiating (29), (30), (12), and (20), using (25) it follows that

$$\frac{\mathrm{d}F_i}{F_i} = -S_a S_b \delta h'(S_i)\left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b}\right) \tag{31}$$

$$\frac{\mathrm{d}S_i}{S_i} = -S_a^2 S_b^2 \delta (h'(S_i) - h'(1-S_i))\left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b}\right) - S_a S_b \frac{\mathrm{d}C_i}{C_i} \tag{32}$$

hence we have the following proposition:

Proposition 2 *Compared to the case without incumbency advantage an incumbency edge to one contestant raises the prize to both sides of the struggle. The effects on prizes are the same irrespective who gets the incumbency advantage.*

Even an incumbency edge that favors one party only, makes the stakes higher for both. The reason is that the prize of winning is high both when the payoff of becoming the incumbent is high and when the payoff of becoming the challenger is low. The winning contestant obtains the *difference* between these as part of the prize. In Figure 2 a case without incumbency edge is for example captured as a case where $S_a = (1 - S_b) = S'$. For a the proposition states that, starting out in A with $S_a = s_a = S'$, the difference $h(S_a) - h(s_a)$ increases by the same rate whether S_a increases or s_a goes down.

That the contestants get more to fight for, as the costs are changed, in isolation induce them to fight harder. If the utility structure is not too uneven an incumbency advantage to one group may increase efforts by both groups at the same time as regime stability increase:

Proposition 3 *If both sides are equally strong ($S_i \approx 1/2$) resource use go up for both sides in each fighting constellation while regime stability goes up as one group gets an incumbency advantage. If one side (a) is particularly strong as incumbent ($S_a \approx 1$), resource use goes down for both parties as a gets a further incumbency advantage while it goes up for both parties if b gets an incumbency advantage.*

Proof. Starting out in a situation with even starting positions ($S_a = S_b = 1/2$) it follows that $\Psi'' = 0$ it then follows from the first order conditions (8) and (9) that

$$\frac{\mathrm{d}Y_i}{Y_i} = \frac{\mathrm{d}y_i}{y_i} = \frac{\mathrm{d}F_i}{F_i}$$

Combined with the results from the previous proposition we know that resource use goes up. That stability goes up follows directly from (32) as $h'(S_i) - h'(1-S_i) = 0$ when $S_a = S_b = 1/2$.

With uneven positions these results are altered. When $S_a \to 1$ it follows from (31) and (32)

that the F-s and the S-s are unaffected by a change in the C-s. It thus follows from (12) and (20) that

$$\frac{\mathrm{d}Y_a}{Y_a} = \frac{\mathrm{d}y_b}{y_b} = \frac{\mathrm{d}C_a}{C_a} \text{ and } \frac{\mathrm{d}Y_b}{Y_b} = \frac{\mathrm{d}y_a}{y_a} = -\frac{\mathrm{d}C_b}{C_b}$$

■

The proposition establishes that more fighting from both sides can go together with higher regime stability. As both contestants raise their conflict spending equally much, the winning probability of the group that obtains the edge increases and the average political stability goes up. The main result is that an incumbency edge to one group raises the stakes for both groups since it is equally important to obtain the incumbency edge as it is to prevent the opponent from getting it. The prizes of winning goes up for both groups. For the advantaged group it goes up as the pay-off of winning increases. For the disadvantaged group it goes up as the pay-off of losing declines. With higher stakes both fight harder either to win the edge for future battles, or to prevent the opponent from winning it. As a result the amount of resources wasted in the conflict increases implying that rational behavior does not exclude that more unequal strengths imply that more resources can on average be spoiled in the struggle even though the incumbent who obtains the edge wins the battle more often than the challenger.

The result that an incumbency advantage raises the prize for both contestants is not only a local phenomenon around $S_a = S_b = 1/2$. Let us define an absolute incumbency advantage where the unit costs of influence approaches zero. Clearly, when a contestant with an absolute advantage becomes the incumbent, he stays forever. When one contestant gets an absolute incumbency advantage we get the following result:

Proposition 4 *Compared to the case without incumbency advantage the introduction of an absolute incumbency advantage to one contestant raises the prizes to both sides, but most for the contestant who gets the advantage. In the limit where both groups have absolute advantage the value of incumbency is the same for both sides and equal to* $h(S_i) - h(s_i) = 1$.

Proof. Consider the case where a gets an absolute incumbency advantage ($C_a \to 0$). It follows from (12) and (20) that $S_a = 1$ and $S_b < 1$. From (23) and (14) it in turn follows that

$$h(S_a) - h(1 - S_b) = 1 - h(1 - S_b) > h(S_b) - h(1 - S_a) = h(S_b)$$

When both gets absolute incumbency power ($C_a \to 0$ and $C_b \to 0$) It follows from (12), (20) that $S_a = S_b = 1$ and that $\Psi(S_a) = \Psi(S_b) = 1$. From (23) it in turn follows that

$$h(S_a) - h(1 - S_b) = h(S_b) - h(1 - S_a) = 1 - 0$$

■

Clearly, as long as only one contestant have an absolute incumbency advantage and he remains the challenger, fighting is hard as the stakes are high for both.

We have shown that (i) introducing a minor incumbency advantage to one contestant, and (ii) introducing an absolute incumbency advantage to one side or both, raise the prizes for both sides in the conflict. But the prizes do not always increase as the unit costs of influence decreases for one contestant. To see this consider the case where $C_a = C_b$, and $U_a - u_a = U_b - u_b$ when $\Psi(S) = S$: Then, we can show that a reduction in the cost of influence for a as incumbent would lower the prize to b if the discount factor is high enough, since

$$\frac{\partial F_b}{\partial C_a} > 0 \iff C_a = C_b < 2\sqrt{\delta/(1+\delta)} - 1 \tag{33}$$

Hence, if both parties has a strong incumbency advantage initially a further advantage for one group will lower the prize for the other. This result is a combination of two effects. First when both sides have strong incumbency advantages the challenger positions is dismal for both. Hence, both v_a and v_b are low and cannot be much affected by a further reduction in the influence costs of the incumbents. Now, if party a gets an even stronger incumbency advantage, implying that C_a goes down, contestant a will fight harder as challenger lowering V_b. If the future matters sufficiently for party a (δ high) the value of V_b will go so much down that F_b also declines. From condition (33) it is clear that $\delta > 1/3$ is an absolute requirement for this to be possible for any positive C.

2.3 Self-defeating power

Above we have seen that incumbency advantage can explain higher fighting efforts and higher conflict spending for the contestants. One would think that for the contestant who gets the incumbency advantage the benefits always outweigh the costs. This, however, is not generally the case. If the initial position for contestant a is sufficiently weak, he may lose by getting an incumbency advantage. More precisely:

Proposition 5 *Power can be self-defeating: A contestant's incumbency advantage may induce so much fighting that the incumbency advantage reduces the pay-off to both contestants – also to the contestant who gets the advantage.*

Proof. *Consider a marginal incumbency advantage for group a. From the definition of*

$h()$ *it follows that*

$$(1-\delta)\frac{dv_a}{F_a} = h(S_a)\frac{dF_a}{F_a} - h'(S_a)\,dS_b$$
$$\frac{dV_a}{F_a} - \delta\frac{dv_a}{F_a} = h(S_a)\frac{dF_a}{F_a} + h'(S_a)\,dS_a$$

It follows by combining with (31) and (32) that

$$dv_a = -K\delta\left(\left(h'(S_a) - h'(S_b)\right)S_aS_b + h(S_a)\right)\frac{dC_a}{C_a}$$
$$dV_a = dv_a - K\frac{dC_a}{C_a} \text{ where } K = F_a h'(S_a)\,S_aS_b/(1-\delta) > 0$$

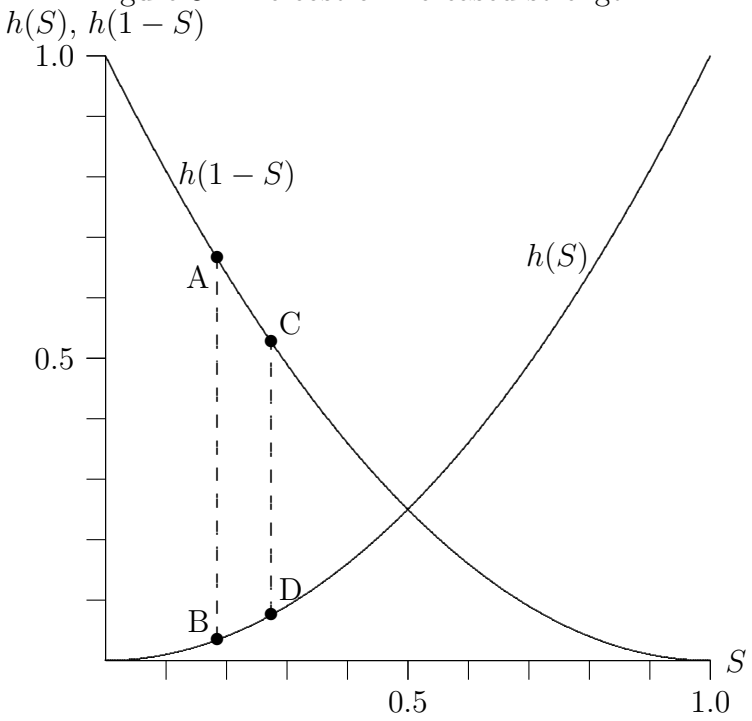
Using (13) it follows that when S_a is close to zero the last expression can be approximated by

$$dV_a = K\left[-(1-\delta) + 2\delta\Psi'(0)\,S_a\right]\frac{dC_a}{C_a}$$

where the square bracket becomes positive as δ approaches unity. ■

An illustration of the effect on the valuation of incumbency underlying the results are given in Figure 2, where it is assumed simply that $\Psi(S) = S$. The equilibrium before

Figure 3: The cost of increased strength



introducing the incumbency advantage is $S_a = (1 - S_b) = 0.2$. It is obvious that in this equilibrium contestant b is much better off than contestant a. First b has a higher prize to fight for and second he wins much more often that a. Consider now a case where a gets an incumbency advantage. The cost advantage increases S_a somewhat. Because the slope of $h(1 - S_a)$ is steep when S_a is low, b is considerably worse off as challenger. This induces b to fight much harder in order not to be knocked down to the position as challenger. As a result a is pushed on the defensive as challenger. In the new equilibrium the valuation of incumbency for a ends up at the modest D-B while the valuation of incumbency for b ends up at the larger A-C. Group b that had the most to fight for gets even more to fight for when a get an incumbency advantage and as a result group a loses from the incumbency advantage. This result is in stark contrast to the result in static contests where the return unambiguously goes up when costs go down (as seen from (23)).

2.4 Commitments

In our set-up the conflict is over political or economic rents measured by $[U_i - u_i]$. In societies with larger intergroup differences $[U_a - u_a]$ and $[U_b - u_b]$ the prizes of winning become higher and the conflict spending goes up. High rents can be viewed as a mixture of antagonistic preferences - the groups like different things - and of the extent to which one group can exclude the other from benefits that everybody likes. If commitment were possible, it would be in the interests of the two sides to commit to "comprise" or "redistributive" policies in future power positions in order to save on fighting expenditures today and in the future. Right-based policies that set a floor to how low each group can fall – implying that $u_a \geq u^*$ and $u_b \geq u^*$ at the costs of lower values of U_b and U_a – can lead to so much less fighting that both V_a and V_b go up. Rules can therefore lead to less fighting and higher payoffs than discretion. Also static conflict models may have such properties, but the dynamics of the power struggle makes the effects stronger. From (29) and (30) we know that $F_i = (U_i - u_i) m_i$ where higher rents are magnified into higher prizes by the multiplier $m_i = 1/(1 - \delta(h(S_i) - h(s_i))) > 1$.

Even an unilateral commitment by a to increase u_b via redistribution or a suitable compromise, can benefit a as incumbent in the sense that $dV_a/dU_a < 0$ taking into account that $\mathrm{d}u_b + \mathrm{d}U_a = 0$. This is the case if the group distance $U_a - u_a$ is much higher than the group distance $U_b - u_b$ and b's cost of influence c_b is low. When the rents of contestant b are lowered by raising u_b, he becomes less aggressive. This would again induce a to reduce his fighting so much that his expected pay-off V_a goes up. In the simple case where costs are constant over time $C_a = c_a$ and $C_b = c_b$ we can show that a credible promise of redistribution will have the

following effect on the welfare of group a

$$\mathrm{d}V_a = \frac{S_a^2}{1-\delta}\left(h\left(S_a\right) - h'\left(S_a\right) S_a S_b K\right)\mathrm{d}U_a\ ,\ \mathrm{d}U_a = -\mathrm{d}u_b < 0$$
$$\text{where } K = \frac{U_a + u_b}{U_b + u_a} - 1$$

A necessary condition for the redistribution to benefit a is that the state with a as incumbent has more to share, $U_a + u_b$, than the state with b as incumbent $U_b + u_a$. Hence, $K > 0$. If the power struggle is particularly wasteful ($h\left(S_a\right)$ small) at the same time as $K > 1$, the value of V_a may very well go up as U_a decreases as u_b is raised.

3 Nonstationary solutions

In our analysis we have assumed that we are in a stationary Markov equilibrium where (27) and (28) holds. In this section we will prove that there always exists at least one stationary Markov equilibrium. We will also discuss the possibility of nonstationary equilibria and the possibility of more than one stationary equilibrium.

A Markov equilibrium (not necessarily stationary) of the game is defined as time paths of F_a and F_b such that (3) and (5) holds hence

$$\begin{aligned} F_{a,t-1} &= F_a\left(F_{a,t}, F_{b,t}\right) \equiv \left(U_a - u_a\right) + \delta\left(h\left(S_a\right) - h\left(s_a\right)\right) F_{a,t} \\ F_{b,t-1} &= F_b\left(F_{a,t}, F_{b,t}\right) \equiv \left(U_b - u_b\right) + \delta\left(h\left(S_b\right) - h\left(s_b\right)\right) F_{b,t} \end{aligned} \tag{34}$$

where

$$S_a = \frac{F_{a,t} c_b}{F_{a,t} c_b + F_{b,t} C_a} \text{ and } S_b = \frac{F_{b,t} c_a}{F_{b,t} c_a + F_{a,t} C_b} \tag{35}$$

A fix point of the system (34) is defined as (F_a^{**}, F_b^{**}) such that

$$F_a^{**} = F_a\left(F_a^{**}, F_b^{**}\right) \text{ and } F_b^{**} = F_b\left(F_a^{**}, F_b^{**}\right) \tag{36}$$

In order to find the fix points we first find the fix point, F_a^*, for F_a given F_b and vise versa such that

$$\begin{aligned} F_a^* &= F_a\left(F_a^*, F_{b,t}\right) \\ F_b^* &= F_b\left(F_{a,t}, F_b^*\right) \end{aligned} \tag{37}$$

From (23) it follows that

$$1 \geq \left(h\left(S_i\right) - h\left(s_i\right)\right) \geq 0$$

therefore a fix point for F_i has to be larger than $(U_i - u_i)$ and less than $(U_i - u_i)/(1-\delta)$. From (24) it follows that

$$\frac{\partial F_{i,t-1}}{\partial F_{i,t}} = \delta\left(\left(h_i + \frac{\partial h(S_i)}{\partial S_i}\frac{\partial S_i}{\partial F_i}F_i\right) - \left(h_i + \frac{\partial h(s_i)}{\partial s_i}\frac{\partial s_i}{\partial F_i}F_i\right)\right) < 1$$

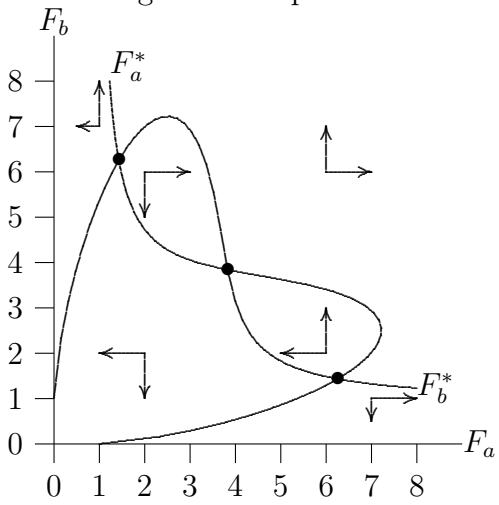
It therefore follows that (37) have unique solutions.

$$F_a^* \in S_a \equiv \left[(U_a - u_a), \frac{(U_a - u_a)}{(1-\delta)}\right]$$
$$F_b^* \in S_b \equiv \left[(U_b - u_b), \frac{(U_b - u_b)}{(1-\delta)}\right]$$

The combined problem (36) therefore always has at least one solution, moreover all solutions are found within the rectangle S_a, S_b. A simple case is the one where δ is low or $c_i \approx C_i$ for both parties then F_i^* is close to $(U_i - u_i)$ and F_a^* and F_b^* will only cross once.

If there are strong incumbency advantages combined with a high discount factor δ, however, the fix point curves may get sufficient curvature to generate multiple stationary equilibria. One illustration of this possibility is provided in Figure (4) . Here, the parameter

Figure 4: Fixpoints



configurations are symmetric ($C_a = C_b \ll 1$ and $(U_a - u_a) = (U_b - u_b) = 1$). We see that one of the equilibria is symmetric, reflecting a situation where each party inherits the others behavior when they change status. The upper left equilibrium is a case where group a , caused by a low F_a takes on a more passive role as incumbent causing a low F_a. Party b however, caused by the high F_b takes on a more aggressive role as incumbent causing a high F_b. A similar skewed equilibrium exists down and to the right. These three points all satisfies

the conditions for a stationary equilibrium.

The arrows in the diagram captures the dynamics of (34) outside of the stationary equilibria. Most paths explodes beyond bounds or bump into the axes. At each side of the symmetric equilibrium there are, however, two equilibrium paths going from the two asymmetric stationary equilibrium towards the symmetric one. Note also that when F_a^* and F_b^* only cross once there are no equilibrium paths apart from the fix point itself. In the analysis above we have focused on the fix points of the problem. In the cases where there are several equilibria we have chosen to focus on the middle one, which is both a fix point and the long run equilibrium of the equilibrium paths.

4 Further issues and concluding remarks

As long as the severity of the conflicts is proportional to total fighting efforts, poor societies and hetrogenous societies are more conflict prone than richer and more homogenous societies. In poor societies the average productivity is low, the opportunity cost of fighting is thus also low and the extent of fighting is therefore high. The impact of group inequality is different in societies with and without a clear state authority. With a clear state authority increasing inequality between the ruler and the opposition may increase both the intensity of the fight and the social waste. In this case the control of the state apparatus makes the incumbent stronger, but a stronger incumbent makes the control of the state apparatus more valuable. As a result the struggle for state control is intensified and the amount of resources wasted in the struggle goes up. Dictatorships may therefore be long lasting in spite of continuous and wasteful fighting by opposing groups. (To be cont)

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Some of the references are at this stage superfluous

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