Campaign Rhetoric and the Hide-and-Seek Game

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Abstract

We examine the choice of political rhetoric when one candidate's willingness to misinform the voter is checked by the other's ability to inform. In our model, a message is an argument whose correctness can be ascertained only if a counterpoint is offered, i.e. through debate. Therefore, a competition between a bad candidate and a good one reduces to a matching pennies game where the good type wants to engage in a fruitful debate and the bad type wants cross talk. However, the quality of the rival candidate can only be discovered through costly research.

We find that although negative advertising and slander go down as the political climate (prior probability of a candidate being good) improves; voter welfare in terms of probability of the good candidate winning is not monotonic in the climate. It is easiest for a good candidate to separate himself from a bad one in a very bad climate and hardest in a very good climate. Also, the good candidate invests less in search than the bad candidate. This inefficiency due to search decreases as the search cost reduces, but the overall voter welfare may not decrease.

In an extension, we consider a continuous type space and show that it breaks endogenously into a "good" set and a "bad" set, and the equilibrium from the two-type model is replicated.

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1 Introduction

Campaign rhetoric is an important source of information for voters about the candidate quality. Clearly, the candidate who is more suitable for office (i.e. the "better" candidate) has an incentive to reveal information to the electorate about his and/or his rival's quality while the "worse" candidate has an incentive to conceal such information. Therefore, the provision of political information to the voters through campaigns is a competitive process in which the candidates choose their messages strategically, trying to influence the extent of information that the voters possess while making their decision. There are several channels through which two candidates may compete to influence the perception of voters through a strategic choice of message. In this paper, we model the campaign process as a public debate and study the candidate's choice of the issue to be argued about, and we illustrate that in the more concrete context of the choice between a positive and a negative campaign message.

To capture the competitive process of information provision, we introduce two innovations to the standard models used in politics and economics. In many of the existing models of information provision by candidates, the action taken (message sent) by each candidate is treated by the voter as a signal of his private information (quality) independent of the action taken by the other candidate. We recognise that voters often make judgements about the quality of candidates by comparing the arguments and counter-arguments made during the campaign process, in effect making inferences about candidate types from the *profile* of messages rather than individual messages independently. Second, it is difficult to capture the half-truths, lies and slander commonly observed in electoral campaigns by modelling information as only hard (immediately verifiable) or only of the soft (completely non-verifiable) kind. Political information in our model stands somewhere in the middle of the two extremes in terms of "hardness": even when a statement made by a candidate is not independently verifiable, the voter can ascertain its correctness if the other candidate offers a counterpoint to it, i.e. if the issue is actually debated. If campaign statements are verifiable only when contradicted, a candidate willing to reveal information has an incentive to offer a counter-argument to his rival's statement while a candidate with an incentive to conceal would prefer to advance a fresh argument on a different issue.

We examine these incentives to reveal or conceal in the context of positive and negative advertising. Suppose there is a single quality dimension in which voters compare the candidates, and there are only two possible kinds of candidates: good and bad. In the campaign, each candidate chooses between a positive message claiming to be a good type and a negative message claiming that the rival is a bad type. Speeches are considered simultaneous. The campaign process is akin to a public debate between the two candidates where the median voter is the judge. The innovations mentioned in the previous paragraph are subsumed in the following protocol of information revelation. If one candidate employs a positive message and the other a negative one, then there is a *fruitful debate* about the former candidate's quality and his true quality is revealed to the voter. However, if both use positive messages or both use negative messages, then they are talking at cross-purposes. Since the two candidates are talking on two different issues, we assume that no new information is revealed to the voter. Therefore, if a "good" candidate faces a "bad" candidate, the former would prefer to engage in a fruitful debate and reveal information to the voter, while the latter would prefer cross talk. Since the good type tries to chase the bad type by "unmatching" messages, and the bad type tries to hide by mimicking the good type's message, we call this game a "hide-and-seek" game.

At a more applied level, we are interested in explaining the effect of political climate on the nature of campaign rhetoric. There is considerable agreement in academic and journalistic circles that as the political climate worsens, there is more negative advertising and more slander. In this paper, the political climate is indicated by the commonly known prior probability of a candidate being good. Note that if the voter is the only uninformed party, i.e. each candidate knows the other's type and so forth, then the equilibrium in the hide and seek game is independent of the prior distribution of types. In particular, if the candidates are of different types, the game reduces to the matching pennies game where both types mix the two messages equally irrespective of the prior distribution. The channel through which political climate affects candidate behaviour is one of incomplete information between candidates. While deciding on the message, a candidate takes into account the probability distribution over the rival type only if he is uncertain about exact type of the rival. In this paper, we capture this incompleteness of information by the assumption that information about the rival's quality is not free, and each candidate has the option to investigate and find out the rival's type at a given cost. A candidate would undertake search only if the benefit of conditioning his campaign message on the information acquired outweighs the cost. Incorporating search explicitly as an option allows us to capture in our framework the phenomenon we commonly understand as slander. If a candidate employs negative advertising against his rival without conducting prior research, we term the attack as slander.

In this paper, we analyse two sets of questions: one of which is positive and the other a normative one. First, we look into candidate behaviour: the content of message of each type of candidate, their respective incentives for search and how such behaviour varies with the political climate. Second, we draw some welfare conclusions in terms of the probability of selection of the good candidate (as against the bad one) and how that varies with the political climate and search cost. It is worth noting here that although the good candidate and the voter have congruent interests, the candidate has to take into account the cost of search which the voter does not internalise. Because of this phenomenon, the good candidate does not search as much as the voter would like her to, leading to a suboptimality in voter welfare.

The model confirms the conventional wisdom that the incidence of negative advertising increases as the political climate worsens. However, negative messages have a beneficial role too: we get complete separation in a very bad climate and complete pooling in a very good climate. Thus, it is easiest for a good candidate to distinguish herself when she is sufficiently rare. For the moderate range of climates, we have partial separation of types with both types engaging in search. Search is probabilistic, and the bad type always searches with a higher frequency than the good type. Since the game is biased in favour of the good type, the good type cannot commit to spending more effort in search than the bad type. This leads to a suboptimality, which we refer to as *inefficiency due to search*. It is worthy of note that although the search cost can be seen as an index of the extent of asymmetry in information between the candidates, the welfare loss due to differential incentives for search is not monotonic in the cost of search. If a policy maker could set the cost of search to maximise voter welfare, she would set it very high in a bad climate and very low in a good climate.

The remainder of the paper is organized as follows. Section 2 discusses the relationship of this work with the existing literature. Section 3 sets up the benchmark model with a binary type space and characterises the equilibrium. In section 4, we discuss how the candidate behaviour in the benchmark case changes as the political climate and the search cost change, and analyse the welfare implications. If we consider only two types, the information revealed by the debate determines the winner; rational inference by the voter does not alter the outcome. In section 5, we introduce a more general type space where voter inference matters. We examine the inference process and show that we can replicate the results of the discrete case in equilibrium. Section 6 presents a few extensions and possible directions of future reseach. Section 7 concludes. Most of the proofs are relegated to the appendix.

2 Related Literature

Our analysis has close links with the debate in political science on positive and negative advertising and their role in the democratic process. Ansolabehere and Iyengar (1995) claims that negative advertising is inherently anti-democratic, as it reduces turnout and polarizes the electorate. Others have pointed out the information-providing role of attacks (Polborn and Yi 2006). The contribution of this paper is to show that negative advertising can perform both roles (providing and concealing information) and demonstrate how these roles interact in equilibrium. Several experimental and empirical studies (see Lau, Spiegelman, Heldman and Babbitt (1999) for a detailed meta-study) seem to suggest an empirical regularity: while positive advertising increases support for a candidate and negative advertising reduces support for the opponent; negative advertising might hurt the sponsor himself. Earlier formal work on choice of message (Skaperdas and Grofman 1995, Harrington and Hess 1996) assumes an influence function with these effects. In our paper, these effects arise as features of the equilibrium. But we point out the possibility that the effect of a candidate's message on the voter may depend on the rival's message, and this has implications for experiments to be undertaken in this field.

Recent work by Polborn and Yi (2006) concentrates on voter inference of candidate quality from campaign messages that act as signals of the individual candidate types. This paper considers only hard information conveyed in advertising and finds that a candidate uses negative advertising when he either does not have too much to say about himself or he knows that his opponent is very bad. By considering "softer" information, our framework accounts for both "good" and "bad" motivations for attack, including slander and outright lies. It also shows that the full knowledge of rival type assumed in Polborn and Yi does not hold, even for very small search costs.

The term Hide and Seek game (Crawford and Iriberri (2005), others) refers to the matching pennies game where the payoffs to the players may be asymmetric. This game comes up not just in the context of political campaigns, but also in advertising, military strategy, market entry and in various other important situations. The only equilibrium in this game is playing both actions with equal probability for both players. In our model, if the two candidates are of different types and the search cost is close to zero, then the outcome approximates that of the hide-and-seek game, although we still do not have complete information. The assumption of private types and the costly search option makes our game a non-zero sum game, and allows for an equilibrium where different types exhibit different behaviour. In equilibrium, each type employs a pure message action in the debate conditional on the search action undertaken, and the mixing occurs through probabilistic search. This innovation can be thought of as one way to break the unappealing type-independent outcome in the hide and seek game.

3 Basic model: Binary types

There are two players: candidates 1 and 2 with a private quality type $\theta_i, (i = 1, 2)$ which can be either Good (G) and Bad (B).For both candidates, quality follows a commonly known Bernoulli distribution with the prior in favour of the good type $\alpha \in (0, 1)$. From here onwards, we shall refer to the good type in the feminine gender and the good type in the masculine gender. If $\alpha > \frac{1}{2}$, each candidate is more likely to be good than bad, and we call that a *good political climate*. Conversely, a setting with $\alpha < \frac{1}{2}$ is called a *bad political climate*.

There are two actions chosen by player $i = 1, 2$: the debate or message action $M_i \in \{P, N\}$ and the search action $X_i \in \{S, NS\}$. P denotes a positive campaign message and N denotes a negative message. If player i undertakes action S (search), he knows the type of the rival $-i$ with certainty. If action NS is taken, the rival's type is not known. One can think of the search action taken before the message action, so that the message action can be conditioned on the information obtained through search. But since the search action itself is private, we do not need to consider the search stage and the debate stages separately. Speeches are considered simultaneous from the strategic point of view: a candidate cannot condition his speech on the message of the other[1]. We denote the message profile by $\mathbf{M} = \{M_1, M_2\}$, the search action profile by $\mathbf{X} = \{X_1, X_2\}$ and the type profile by $\boldsymbol{\theta} = \{\theta_1, \theta_2\}$. The payoff to player i from an action and type profile $\{\mathbf{M}, \mathbf{X}, \boldsymbol{\theta}\}$ is assumed to be the sum of the payoff $u_i(\mathbf{M}, \boldsymbol{\theta})$ from debate and the cost of search, which is $c > 0$ if search is undertaken ($X_i = S$) and zero otherwise. Note that the search action affects the debate payoff only by affecting the private information available to a player.

The judge (median voter) selects the winner in the debate after listening to the messages. We consider the actual process of debate as a black box and assume a protocol which determines how information about candidate types is revealed to the judge depending on the message profile. A debate is called *fruitful* if the messages are different, i.e. if $M_1 \neq M_2$. We assume that a fruitful debate fully reveals the type of the candidate whose quality is being argued about, i.e. the candidate

[1] The reality this assumption reflects is that most of the time, the campaign strategy of each candidate is decided before the campaigning actually starts.

using P. If the debate is fruitless, i.e. if $M_1 = M_2$, nothing is revealed, presumably because the candidates do not co-ordinate on discussing an issue. The rather extreme assumption that a fruitful debate reveals the true type of the candidate under focus while cross talk reveals nothing is unnecessary for the qualitative results of the model to go through. It has been made for technical convenience so as to be able to drive the basic point home without using unnecessary parameters.

In a fruitful debate, if a candidate is found to be good, he is voted for and if he is found to be bad, he is voted against. If there is cross talk, the winner is chosen randomly with equal probability. This theory of voting behaviour allows us to keep the voter outside the benchmark model. Although simplistic, it works for a two-type case because if one candidate is revealed to be good (bad), the other candidate cannot be better (worse). We show in the appendix that this "passive" voter behaviour arises rationally when the voter uses certain natural out-of-equilibrium beliefs, and leads to the same set of equilibria.

Assume that the winner of the debate gets a prize $w > 0$. The utility $u_i(\mathbf{M}, \boldsymbol{\theta})$ from debate is thus given by:

$$\begin{aligned} u_i(P, P, \boldsymbol{\theta}) = u_i(N, N, \boldsymbol{\theta}) &= \tfrac{w}{2}, \\ u_i(P, N, G, \theta_{-i}) = u_i(N, P, \theta_i, B) &= w, \\ u_i(P, N, B, \theta_{-i}) = u_i(N, P, \theta_i, G) &= 0, \end{aligned}$$

We look for symmetric Bayesian Nash equilibirum in weakly undominated strategies.

The strategy set for player i is the set of probabilities $p_i(\theta), q_i(\theta)$ and $r_i(\theta, \theta')$, which are defined as:

$$\begin{aligned} &p_i(\theta) \equiv \Pr(X(\theta) = S) \\ &q_i(\theta) \equiv \Pr(M_i(\theta) = P \mid X_i(G) = NS) \\ &r_i(\theta, \theta') \equiv \Pr(M_i(\theta) = P \mid X_i(\theta) = S, \theta_{-i} = \theta') \end{aligned}$$

Here, $p_i(\theta)$ is the probability with which type θ performs the search. If a type θ does not search, then he plays the action P with probability $q_i(\theta)$. If a type θ searches and discovers that the rival is of type θ', then he plays the action P with probability $r_i(\theta, \theta')$. Denote the debate action taken by type θ of candidate i as $M_i(\theta)$ and the search action by $X_i(\theta)$.

If a candidate is of type G and knows that the rival is also of type G, then it is strictly dominant for her to use a positive message. Irrespective of what the rival does, she can improve her payoff by $\frac{w}{2}$ by discussing her own qualities. By a similar logic, if a candidate of type B knows that his opponent is also of type B, it is strictly dominant for him to turn the focus away from himself by employing a negative message. Thus, we must have $r_i(G, G) = 1$ and $r_i(B, B) = 0$ for $i = 1, 2$.

Since we are using symmetric strategies, we shall often suppress the subscripts denoting the player, and index actions by types. The remaining elements in the strategy set of a player is the set of probability numbers $p_G,\ p_B, q_G, q_B, r_G$ and r_B where p_θ is $p_i(\theta)$, q_θ is $q_i(\theta)$, r_G is $r_i(G, B)$ and r_B is $r_B(B, G)$. We solve for these quantities in equilibrium.

Next, we define a composite event which will be very useful in analysing the game. Define $\theta P\theta'$ as the probability of the event that when the two candidates facing each other are of types θ and θ' respectively, the one with type θ uses message P. Therefore:

$$\begin{aligned}\theta P\theta' &= \Pr(M_i(\theta) = P|\theta_{-i} = \theta') \\ &= \begin{cases} \Pr(M_i(\theta) = P|\theta_{-i} = \theta', X_i(\theta) = S) \cdot \Pr(X_i(\theta) = S) + \\ \Pr(M_i(\theta) = P|\theta_{-i} = \theta', X_i(\theta) = NS) \cdot \Pr(X_i(\theta) = NS) \end{cases} \\ &= p(\theta)r(\theta, \theta') + (1 - p(\theta))q(\theta), \text{ where } \theta, \theta' = G, B \end{aligned} \tag{1}$$

Expanding on equation (1), we define:

$$\left.\begin{aligned} GPG &= p_G + (1 - p_G)q_G \\ BPG &= p_B r_B + (1 - p_B)q_B \\ GPB &= p_G r_G + (1 - p_G)q_G \\ BPB &= (1 - p_B)q_B \end{aligned}\right\} \tag{2}$$

3.1 Bayesian Equilibrium Strategies

We start with a pair of lemmata, one for each type, that specify what determines the probabilities of mixing the positive and negative messages in the debate conditional on information available from search.

Lemma 1 *The probablity r_G of the good type using a positive message conditional on searching and finding the rival to be a bad type is given by:*

$$r_G \begin{cases} = 0 \ if \ BPG > \frac{1}{2} \\ = 1 \ if \ BPG < \frac{1}{2} \\ \in [0, 1] \ if \ BPG = \frac{1}{2} \end{cases}$$

The probablity q_G of the good type using a positive message conditional on not searching is given by:

$$q_G \left\{ \begin{aligned} &= 0 \ if \ \Pr(B) \cdot BPG > \tfrac{1}{2} \\ &= 1 \ if \ \Pr(B) \cdot BPG < \tfrac{1}{2} \\ &\in [0, 1] \ if \ \Pr(B) \cdot BPG = \tfrac{1}{2} \end{aligned} \right\}, \ where \ \Pr(B) = 1 - \alpha$$

Proof. If the good type knows that the rival is of a bad type, then by using P instead of N, she (the good type) gains $\frac{w}{2}$ when the rival is playing N and loses $\frac{w}{2}$ when the rival is playing P. Since the probability of the rival (type B) using message P when facing type G is BPG, she uses P if $BPG > \frac{1}{2}$ and N if $BPG < \frac{1}{2}$. This determines r_G.If the rival type is unknown, by using P instead of N, the good type loses $\frac{w}{2}$ only in the case when the rival is bad and is using P,which happens with probability $\Pr(B) \cdot BPG$, and gains $\frac{w}{2}$ otherwise. Therefore, she strictly prefers to employ a positive message conditional on not searching if and only if $\Pr(B) \cdot BPG > \frac{1}{2}$, thus determining q_G. ■

The lemma says that the message choice of a good type depends only on BPG, the probability with which she expects a bad type of the rival to employ the positive message against her. This lemma brings out the hide-and-seek nature of the game: if the bad type goes positive with a high probability, the good type prefers to go negative and expose the bad type, and if the bad type goes negative with a high probability, the good type prefers to go positive and reveal her true type.

In the same way, for the decision making of a bad type, we have just as before:

Lemma 2 *The probablity r_G of the good type using a positive message conditional on searching and finding the rival to be a bad type is given by:*

$$r_B \begin{cases} = 0 \ if \ GPB < \frac{1}{2} \\ = 1 \ if \ GPB > \frac{1}{2} \\ \in [0,1] \ if \ GPB = \frac{1}{2} \end{cases}$$

The probablity q_G of the good type using a positive message conditional on not searching is given by:

$$q_G \left\{ \begin{array}{c} = 0 \ if \ \Pr(G) \cdot GPB < \frac{1}{2} \\ = 1 \ if \ \Pr(G) \cdot GPB > \frac{1}{2} \\ \in [0,1] \ if \ \Pr(G) \cdot GPB = \frac{1}{2} \end{array} \right\}, \ where \ \Pr(G) = \alpha$$

Proof. As before. ■

This lemma works in exactly the same way as the earlier one, and the equilibrium message of the bad type depends only on GPB, the probability that the good type of the rival uses a positive message against him. The bad type always prefers to avert a confrontation with the good type, and therefore tries to mimic her message. If the good type plays the positive message with a high probability, the bad type goes positive too, and if the good type goes negative with a high probability, the bad type goes negative.

Note that, as an implication of Lemma 1, conditional on not searching, the good type has a strictly dominant message of P in a good climate, irrespective of how the bad type behaves. Similarly, Lemma 2 implies that conditional on not searching, the bad type finds it strictly dominant to play N.in a bad climate. Therefore,

$$\left.\begin{array}{l}\alpha > \frac{1}{2} \Rightarrow q_G = 1 \\ \alpha < \frac{1}{2} \Rightarrow q_B = 0\end{array}\right\} \tag{3}$$

Before specifying equilibrium behaviour, we define the parameters on which such behaviour depends. The political climate is one such parameter, and another is the effective cost of search. If search is effective, it changes a draw to a win, or a loss to a draw. Thus, the possible gain obtainable from spending a cost c is $\frac{w}{2}$.The cost-benefit analysis of search is captured by the parameter $k = \frac{c}{w/2}$. Note that what matters is the ratio of c and w, not their absolute values. An increase in cost of search with the prize from winning office remaining unchanged has the same effect as an equivalent decrease in size of the prize with the cost of search remaining fixed. We classify the situation with $k > \frac{1}{2}$ as one of *high effecticve cost,* and that with $k < \frac{1}{2}$ as one of *low effective cost.* We shall see that search occurs only when the effective cost is low and the political climate is not too extreme.

Proposition 1 demonstrates the equilibrium behaviour of candidates under high effective cost

Proposition 1 *When effective cost is high ($k > \frac{1}{2}$), there is a fully separating equilibrium in a bad political climate and a fully pooling equilibrium in a good political climate.*

(i) If $\alpha < \frac{1}{2}$, the unique equilibrium is with the bad type employing nagative message and the good type using positive message, i.e. $p_G = p_B = 0$, $q_G = 1, q_B = 0$, and (off equilibrium), $r_G = 0$, $r_B = 1$.

(ii) If $\alpha > \frac{1}{2}$, the unique equilibrium is with both types going positive, i.e. $p_G = p_B = 0$, $q_G = q_B = 1$, and (off equilibrium), $r_G = 0$, $r_B = 1$.

Proof. In appendix. ■

A high effective cost prohibits search for both types. By equation (3), when not searching, the good type has a strictly dominant strategy to use P if the political climate is good. Lemma 2 dictates that then the bad type will use P too. Similarly, when the political climate is bad, the bad type has a strictly dominant strategy to use N. By Lemma 1, the good type will use P.

This proposition can be thought of a benchmark with candidate quality being completely private information. We find that in such an extreme case, the payoffs are entirely determined by the political climate.

With no knowledge of the rival type, each candidate will determine its debate message expecting to meet the more common type. Thus, in a bad climate, the bad type has a dominant message (N), and in a good climate, a good type has a dominant message (P). Given a political climate, since the action of the more common type can be predicted,.the rarer type has an advantage in the hide and seek game. In a good climate the bad type can effectively hide by playing P, and in a bad climate, the good type can effectively chase by playing P too. The good climate thus turns out to be very conservative with everyone pooling on the defensive message, and it becomes impossible to seperate the good type from the bad. On the other hand, the bad climate has a lot of attack advertising by the bad type, but it is a competitive one: the good type can always seperate itself from the bad type by defending.

When the effective cost of search is low, both types search in equilibrium, but only of the climate is not too extreme. We describe the unique equilibrium strategies for a low effective cost in Proposition 2.

Proposition 2 *When effective cost is high ($k < \frac{1}{2}$), the following are the equilibrium strategies for different political climates*[2] :

(i) *If $\alpha < k$, there is a fully separating equilibrium with no type searching, the good type using positive message and the bad type attacking (slander) i.e. $p_G = p_B = 0$, $q_G = 1$, $q_B = 0$, and (off equilibrium), $r_G = r_B = 1$*

(ii) *If $\alpha \in (k, 1-k)$, there is a partially separating equilibrium with both types indulging in search with positive probability, i.e. $p_G = \frac{1}{2}(1-\frac{k}{\alpha})$, $p_B = \frac{1}{2}(1+\frac{k}{1-\alpha})$. The good type employs a positive message when she does not search and a negative message when she searches and finds the rival to be a bad type, i.e. $q_G = 1$, and $r_G = 0$. The bad type employs a negative message when he does not search and a positive message when he searches and finds the rival to be a goo type, i.e. $q_B = 0$ and $r_B = 1$.*

(iii) *If $\alpha > 1-k$, there is a fully pooling equilibrium with both types employing the positive message with no search, i.e. $p_G = p_B = 0$, $q_G = q_B = 1$, and (off equilibrium), $r_G = 0$, $r_B = 1$*

Proof. See appendix. ■

[2]We do not consider the case $\alpha = k$ since it is non-generic. In this case, we can have a continuum of equilibria. However, the equilibria discussed in the proposition extended to $k \to \alpha$ still exist in the limit $k = \alpha$. The case of $\alpha = 1-k$ is not considered due to the same reasons.

Proposition 2 demonstrates the behaviour of candidates in equilibrium when search is feasible. The good type tries to keep the focus on herself while the bad type tries to avert the focus. The good candidate always proffers arguments supporting herself (positive message) unless she is sure that the rival is a bad type, and in that case she tries to expose the rival by going negative. The bad candidate on the other hand has a default message which is negative, but when he is sure that the rival is a good type, he tries to ensure cross talk by defending himself (positive message), hoping that the rival has not searched and is going to employ a positive message too.

As demonstrated by the above two propositions, irrespective of whether search occurs or not, when two good candidates are in competition, there cross talk with both candidates arguing in support of themselves (positive message). Similarly, when two bad candidates face each other, we again have cross talk, but with both players attacking each other in the debate (negative message). A fruitful debate can occur only between a good and a bad type. Thus, when a type is revealed to be good (bad) through the process of debate, the other one is automatically inferred to be bad (good). However, when two different types face each other, we might also have cross talk with both types either going positive or both going negative.

In what follows we provide an intuitive, step-by-step explanation of how the model works. For convenience of exposition, we first define some terms.

When a candidate of type θ knows that the rival is also of type θ, then the candidate has a strictly dominant message action. Denote this message by $D(\theta)$. Formally,

$$D(G) = P, \; D(B) = N$$

Denote by $\overline{D(\theta)}$ the other available action in the message space, i.e. $\overline{D(\theta)} = \{P, N\} \backslash D(\theta)$. In the same way, denote by $\overline{\theta}$ the type other than θ, i.e. $\overline{\theta} = \{G, B\} \backslash \theta$.

3.2 Message Choice

In this model, for there to be search with positive probability by any type θ, the following must be true:

First, θ must find it strictly optimal to play different messages with different rival types after search. $D(\theta)$ being strictly dominant against rival type θ, $\overline{D(\theta)}$ must be optimal with rival type $\overline{\theta}$. If not, θ could play $D(\theta)$ against both types of the rival and save the search cost. Therefore:

$$\left. \begin{array}{l} p_G > 0 \Rightarrow r_G = 0 \\ p_B > 0 \Rightarrow r_B = 1 \end{array} \right\} \qquad (4)$$

Second, $D(\theta)$ must be the strictly optimal action to play for type θ conditional on not searching. Note that this is already true of G for a good political climate and of B for a bad climate by equations 3.A little algebra establishes that this is true irrespective of the political climate. Hence,

$$\left.\begin{array}{l} p_G > 0 \Rightarrow q_G = 1 \\ p_B > 0 \Rightarrow q_B = 0 \end{array}\right\} \tag{5}$$

From (4) and (5), we see that conditional on B and G facing each other, B goes positive only when he searches and G goes positive only when she does not search. Therefore, using equation (2), we can say that:

$$BPG = p_B \text{ and } GPB = 1 - p_G. \tag{6}$$

3.3 Search behaviour

Equations (4) and (5) imply that both with and without search, θ plays $D(\theta)$ with the rival candidate of type θ.Therefore, for type θ, the difference in expected payoff from search comes from playing the action $\overline{D(\theta)}$ with rival type $\overline{\theta}$. By playing $\overline{D(\theta)}$ instead of $D(\theta)$, the payoff to θ increases by $\frac{w}{2}$ when $\overline{\theta}$ plays P (probability $\overline{\theta}P\theta$) and decreases by $\frac{w}{2}$ when $\overline{\theta}$ plays N (probability $1-\overline{\theta}P\theta$). Hence, the marginal expected benefit from search is given by $\Pr(\overline{\theta})(\overline{\theta}P\theta-(1-\overline{\theta}P\theta))\frac{w}{2} = \Pr(\overline{\theta})(2\overline{\theta}P\theta-1)\frac{w}{2}$.This is weighed against the cost c. Hence, type θ searches with positive probability if and only if

$$\Pr(\overline{\theta})(2\overline{\theta}P\theta - 1) \geq k \tag{7}$$

Equation (7) is the key inequality in this paper. If the left hand side (benefit) is higher than k, type θ searches with certainty. If the benefit is less than k, θ does not search. And p_θ can take any value if it is satisfied with an equality. Based on the values of parameters k and α, we have either both types searching (equilibrium with search) or neither type searching (no-search equilibrium).

3.3.1 Equilibrium with search

By inequality (7), search is too expensive for type θ if $\Pr(\overline{\theta}) < k$, i.e. if the other type is too rare. Assume for now that neither type is too rare, or:

$$\min(\Pr(G), \Pr(B)) > k \tag{8}$$

Since $\Pr(G) = \alpha$, and $\Pr(B) = 1-\alpha$, condition (8) can also be written as $\alpha \in (k, 1-k)$. Note that this condition cannot be satisfied if the effective cost is high ($k > \frac{1}{2}$). If this condition is satisfied, in equilibrium each type chooses p_θ such that $\theta P\overline{\theta}$ satisfies (7) with equality, and keeps the type $\overline{\theta}$ indifferent between searching and not searching. This mixed strategy equilibrium is very similar to the equilibrium in the matching pennies game where both actions are played with positive probabilities, except that the mixing is achived by probabilistic search. Therefore with $k < \frac{1}{2}$ and $\alpha \in (k, 1-k)$, we have:

$$p_B = BPG = \frac{1}{2}(1 + \frac{k}{\alpha})$$
$$p_G = 1 - GPB = \frac{1}{2}(1 - \frac{k}{1-\alpha})$$

This completes an intuitive demonstration of part (*ii*) of Proposition 2. We call it the *equilibrium with search*.

3.3.2 Equilibrium without search

If $k > \frac{1}{2}$ or when $\alpha < k$ or $\alpha > 1-k$, condition (8) is not satisfied. Suppose, then, that $\min(\Pr(G), \Pr(B)) = \Pr(\theta) < k$. Then, by inequality (7), type $\overline{\theta}$ does not search. Also, since[3] $\Pr(\theta) < \frac{1}{2}$, by equation (3), type $\overline{\theta}$ has a strictly dominant message in the debate. Type θ, the rarer type, has an advantage in the hide and seek game and has no incentive to search either. This is the situation in Proposition 1 and parts (*i*) and (*iii*) of Proposition 2. Search is not worthwhile here since either the effective cost of search is too high, or because the benefits of search accrue from a type that is very rare. Since no type searches in equilibrium with this set of parameter values, we call it a *no-search equilibrium*. In a no-search equilibrium, we have full separation in a bad climate and full pooling in a good climate.

3.3.3 Properties of search

A few properties of search are worthy of mention here:

1. No candidate searches with certainty, even if the search cost is very low. If one candidate searches with certainty, the rival can nullify the advantage of costlty search by mixing messages suitably. The result that search must be probabilistic casts doubt over the assumption of full information between candidates which is common in the related literature.

2. Search is *reciprocal*, i.e. if one type searches with a positive probability, the other type does so too.

[3]We ignore the "neutral political climate" here.

3. If search occurs, the bad type searches with a higher frequency than the good type. In the equilibrium with search, we must have $p_B > \frac{1}{2} > p_G$. Since the two types have different strictly dominant actions against rivals of similar type, the type G which prefers the two types playing different messages has a "natural" advantage in this game. It is this advantage that depresses the good type's incentive to search, and raises the bad type's motivation for the same. This inefficiency due to different incentives for search is present only when search is worthwhile, and we shall call it the *inefficiency due to search.* For a given political climate, the size of this inefficiency (measured as the difference between p_G and p_B) increases with the effective cost of search as long as $k < \frac{1}{2}$ and $\alpha \in (k, 1-k)$. Thus, the cost of search drives a wedge between the incentives to search of the two types. To drive the point home further, note that as $k \to 0$, the inefficiency due to search vanishes in the limit.

4. As the cost of search goes to zero, the outcome of a debate between a good and a bad type approaches that of a matching pennies game, which is a game of complete information. As $k \to 0$, for any political climate, both GPB and BPG approach $\frac{1}{2}$. Thus, both types mix the positive and negative message almost equally when they face each other. It is as if the good type ignores the uncertainty and takes into consideration only the bad type of the rival and conversely. Even though in equilibrium there is a residual incompleteness of information between the two candidates (since search probabilities are close to half), we have the same outcome that would have come about if the only asymmetric information in the model were between the voter and the candidates.

3.4 Voter Inference

So far in this model we have assumed a naive voter behaviour. While taking the voting decision, the voters go by the information revealed by the debate alone, and do not update their beliefs from equilibrium strategies about the candidates; in particular, about the one whose quality is not being debated. This simplification, however, does not make any substantive difference. We show in Appendix A that if the voter makes rational inferences about candidate quality from messages observed in equilibrium, we get exactly the same set of equilibrium strrategies and outcomes under a very natural specification of out of equilibrium beliefs. In other words, a rational voter would use the naive voting strategies in equilibrium. Therefore, if we are interested in voter inference of candidate types from messages spoken, we can use the equilibrium strategies derived earlier in this section.

It has already been pointed out that whenever there is a fruitful debate, the candidates are of different types, i.e. it is either a good candidate "exposing" a bad one, or a good candidate successfully defending herself against "slander" by a bad one. Thus, if in a debate a candidate is revealed to be good (bad), the other type must be bad (good). In case of a cross talk, the candidates mimic each other's message and since the type of both candidates is drawn from a common prior distribution, the voter cannot distinguish between the two, and ascribes to each candidate a common, updated posterior distribution. When the two candidates are of similar types, there is always cross talk; with positive messages if the rival candidates are good and with negative messages if the candidates are bad. On the other hand, an enghagement between two different types can lead either to a fruitful debate or to cross talk. This implies that when there is cross talk with a positive message, the voters would adjust their quality assesment of candidates upwards from the prior and when there is cross talk with negative messages, voters would adjust their assessment downwards. Let, in case of a cross talk, the inferred probability of the candidates being good be $\widehat{\alpha}(M)$, where M is the message employed by both candidates. A comparison of the posterior $\widehat{\alpha}(M)$ with the prior α allows us to draw several conclusions about the "meaning" of messages to the electorate.

1. If there is search with positive probability and both candidates use negative messages, then $\widehat{\alpha}(N) < \alpha$, i.e. the assessment of candidate quality goes down when the voters observe both candidates attacking each other in the debate. In fact, the voters believe that the candidates are more likely to be bad than good, i.e. $\widehat{\alpha}(N) < \frac{1}{2}$.

2. If there is search with positive probability and both candidates use positive messages, then $\widehat{\alpha}(P) > \alpha$, i.e. the assessment of candidate quality goes up when the voters observe positive messages from both candidates. In fact, the voters believe that the candidates are more likely to be good than bad, i.e. $\widehat{\alpha}(P) > \frac{1}{2}$.

3. If there is no search, then in a good climate there is full pooling on the positive message. The message becomes completely uninformative about quality, and thus $\widehat{\alpha}(P) = \alpha$ On the other hand, if there is no search in a bad climate we have full separation. Then, anyone speaking a positive message distinguishes herself as a good type while anyone attacking the rival in debate reveals himself to be a bad type. Formally, in this case, $\widehat{\alpha}(N) = 0$.

4 Comparative statics

In this section, we discuss the comparative static properties of the equilibrium for different levels of the effective cost of search and political climate, and discuss the implications of such properties.

4.1 Welfare analysis: Candidate selection

First, we examine how "efficient" the debate is as a form of competition, i.e. how often the right candidate is selected. Since the selection problem is trivial when the two candidates are of the same type, we look at the equilibrium selection properties of a debate when one candidate is good and one is bad. There is an inherent inefficiency in the debate mechanism in that it can reveal the quality of at most one candidate. However, in a two type case, revelation of one candidate's type is enough for correct selection. Voters can make a mistake, i.e. select the wrong candidate with probability $\frac{1}{2}$ if the debate is fruitless, i.e. the type of no candidate is revealed. Hence we look at the equilibrium probability of a fruitful debate conditional on candidates being of different types. Denote this probability $\beta(\alpha, k)$. Proposition 3 shows how β changes with the parameters of the model[4].

Proposition 3 *In a no-search equilibrium, when the political climate is good, different types never engage in a fruitful debate, i.e.* $\beta(\alpha, k) = 0$*; and when the political climate is bad, different types always engage in a fruitful debate, i.e.* $\beta(\alpha, k) = 1$*. In an equilibrium with search, we have* $\beta(\alpha, k) = \frac{1}{2} - \frac{k^2}{2\alpha(1-\alpha)}$.

Proof. In appendix ■

When there is no search because of high cost or extreme priors, the equilibrium is determined entirely by the political climate: we have full separation in the bad climate and no separation in the good climate. Thus, a good type has the maximum advantage when she is sufficiently rare and the minimum when she is sufficiently common.

When the effective cost of search is low and the prior is moderate, both types search in equilibrium. Search being a randomisation device in the hide and seek game, each message profile occurs with positive probability. In the polar case when the search cost is almost nonexistent, i.e. $k \to 0$, the debate between the good and bad types reduces to the

[4]We could have used an alternative definition of voter welfare, which is the total probability of a good candidate winning, given a prior. We can show that for low cost, that the probability is a piecewise continuous function of α which is increasing except two drops at k and $1-k$. The size of the discontinuities increases with k.Similar results hold for the high cost too, except that there is only one discontinuity at $\alpha = \frac{1}{2}$.

matching pennies game in which there is no advantage to either type: hence ach message profile occurs with equal probability. Then we have $\beta(\alpha, k) \to \frac{1}{2}$, and this is true for almost all values of the prior. As the effective search cost increases, we have a further inefficiency due to differential incentives for search that the two types have, and that confers an advantage to the bad type. This inefficiency due to search leads to a further welfare loss of $\frac{k^2}{2\alpha(1-\alpha)}$. For a given political climate, this loss increases with the effective search cost. For a given value of k, this loss increases as the prior becomes skewed away from $\frac{1}{2}$, in either direction. In Figure 1, we show how $\beta(\alpha, k)$ changes for different values of α, when the search cost is low. The dashed line shows β when $k \to 0$.

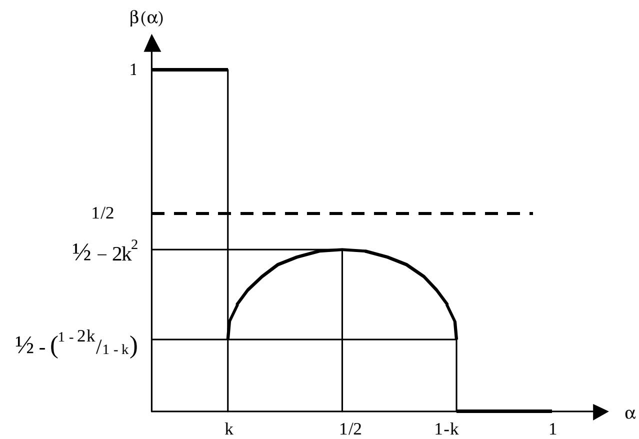


Figure 1: Probability of correct selection for a low effective search cost.

The relationship between effective search cost and correct selection probability helps us answer the main welfare question that this paper is concerned with. If the uninformed voters have to base their decision solely on the messages from self-interested candidates, what should be the optimal cost of information? In other words, if in principle, a social planner could choose k to maximise voter welfare, what level would she choose?

Note that if there is no search, given a political climate, the rarer type has an advantage in the hide and seek game. We utilise this fact to choose that level of k that favours the good type the most. In a good political climate, we unambiguously improve welfare by reducing k. For a bad political climate, there are two opposing effects. When $k < \alpha$, a marginal reduction in k reduces the inefficiency due to search and thus improves the selection probability of the good type. But in an equilibrium with search, β is always bounded below $\frac{1}{2}$. But, for any given

$\alpha < \frac{1}{2}$, if we set $k > \alpha$, we get the no-search equilibrium and thus full separation of types. Hence, a social planner can achive full separation in a bad political climate by setting the effective search cost high enough. On the other hand, the best choice of k in a good climate would be as close to zero as possible, and thus achieve β close to $\frac{1}{2}$.

4.2 Commitment

While the voter always prefers that the good type wins the debate, the voter does not internalise the candidate's investment in search. The good type trades off the cost of search with the possible gain from search in terms of the probability of winning the debate. This leads to welfare loss for the voter, and the problem is worsened by the fact that the good type economises on search cost more than the bad type does. We claim that if the good type could somehow commit to search with certainty, not only the voter would be strictly better off for all political climates, even the good type herself would be better off than she is in the Nash equilibrium in a good political climate.

To examine the commitment problem, suppose the good type is forced to search in the game studies in the previous section. In other words, consider the new game $\mathcal{C}(\alpha, k)$ which is the same game as in the previous section except that the strategy space of the good type consists only the functions q_G and $r(G, \theta)$, and p_G is restricted to be equal to 1. In this game, when the effective search cost is low, the inefficiency due to search is eliminated.

Proposition 4 *If effective cost is low ($k < \frac{1}{2}$) and climate is moderate ($k < \alpha < 1 - k$), the unique equilibrium of the game $\mathcal{C}(\alpha, k)$ has $q_G = 1$, $r_G = \frac{1}{2}(1 + \frac{k}{\alpha})$, $q_B = 0$ and $r_B = 1$..The bad type searches with probability $\frac{1}{2}$, and the good type mixes both messages with a positive probability against the bad type. For all values of α in this range, we have $\beta(\alpha, k) = \frac{1}{2}$.*

Proof. In appendix. ■

Since the good type has full information of her rival's type, the best response of the bad type is to mix the two messages equally against the good type. As a result, we get back the outcome in the matching pennies game, and $\beta = \frac{1}{2}$ for all political climates. Note that here the good type is indifferent between the two messages to play with the bad type. If she were not forced to search, her best response would have been to save on search cost and play P against both types. This is precisely why the good type cannot commit to searching with certainty in the original game.

Thus, if the good type could commit to searching, for all values of $\alpha > k$, the voter would be strictly better off. We next show that the good type herself would be better off too, but only in a good climate.

To compare the utlities of the good type across the commitment and no-commitment situation, we need to consider only the payoff against the bad type, because the payoff to the good type against the good type is always the same. Denote the expected utility of the good type conditional on the rival being bad by $U_N(k, \alpha)$ in the no-commitment case and $U_C(k, \alpha)$ in the commitment case. Since we are examining the inefficiency due to search, we only look at the case with low effective search cost, i.e. $k < \frac{1}{2}$. From proposition 2, we can show that as the good type becomes more and more common, her advantage over the bad type reduces: $U_N(k, \alpha)$ is weakly decreasing in everywhere α, and strictly decreasing in $(k, 1-k)$. In particular, $U_N(k, \alpha) = \frac{3w}{4} - c$ at $\alpha = \frac{1}{2}$.

On the other hand, when $\beta = \frac{1}{2}$, the good type always gets $\frac{3w}{4}$ in expected payoff from debate against the bad type. Thus, $U_C(k, \alpha) = \frac{3w}{4} - c$ for all α. From a simple comparison, we can arrive at the following result:

Remark 1 *For the good type, the Nash equilibrium payoff $U_N(k, \alpha)$ is strictly less than the commitment payoff $U_C(k, \alpha)$ in a good political climate and is strictly greater in a bad political climate.*

4.3 Negative advertising

Next, we look at the type of messages traded in the debate as the prior varies. As the climate improves, candidates become more and more conservative and do not risk going negative in the fear of meeting a good type. Therefore, the ex-ante probability that a candidate will play a negative message, denoted by $\gamma(\alpha, k)$, is a falling function of α. The following lemma demonstrates how $\gamma(\alpha, k)$ changes with α and k.

Lemma 3 *In a no-search equilibium, we have $\gamma(\alpha, k) = 1 - \alpha$ in a bad climate and $\gamma(\alpha, k) = 0$ in a good climate. In an equilibrium with search, we have $\gamma(\alpha, k) = (1 - \alpha) - \frac{k}{2}(1 - 2\alpha)$*

Proof. Follows from Propositions 1 and 2. ■

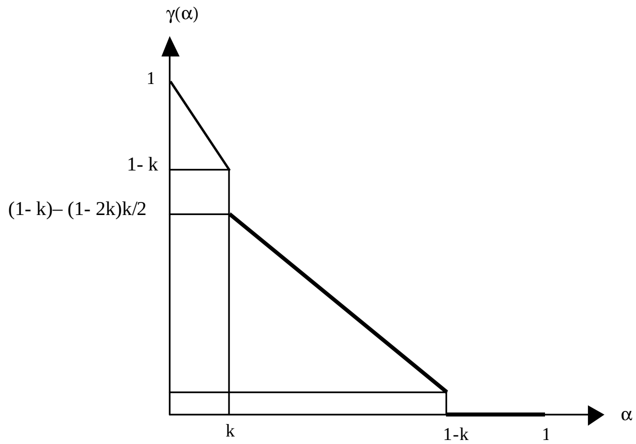


Figure 3: Probability of a negative message with low cost

Figure 3 traces the values of $\gamma(\alpha, k)$ for a low cost. In a no-search equilibrium we have complete pooling on the positive message in the good political climate and complete separation in the bad, with the bad type going negative. Notice here the role of negative advertising in ensuring candidate selection. For moderate climates, the volume of negative messages decreases because the good type's increasing aggressiveness (in going negative) is more than compensated by the bad type's increasing conservatism (in going positive). Therefore,

Remark 2 $\gamma(\alpha, k)$ *is weakly decreasing in* α. *If effective search cost is high,* $\gamma(\alpha, k)$ *is strictly decreasing in the bad political climate, and if the effective cost is low,* $\gamma(\alpha, k)$ *is strictly decreasing in* $\alpha < 1 - k$.

Proof. Evident, from Lemma 3. ■

Corollary 1 *Negative advertising is more probable than positive advertising* ($\gamma > \frac{1}{2}$) *in a bad political climate, and less probable* ($\gamma < \frac{1}{2}$) *in a good political climate.*

The remark and its corollary are a formal evidence of something that is often conjectured by political scientists. The remark says that as the expectation of the rival candidate being of a bad type increases, the debate becomes more competitive, i.e. the proportion of attack advertising increases. The corollary says that in particular, attack dominates defesive messages in a bad climate and vice versa in a good climate.

4.4 Slander

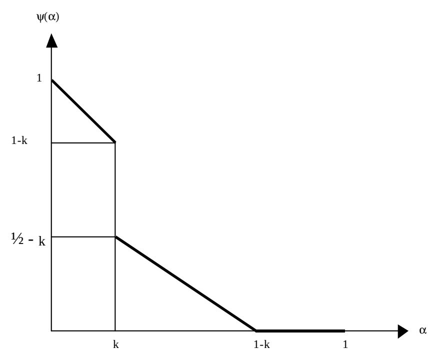
We define slander as attacking the rival in the debate without finding out information about the rival, i.e. without pior search[5]. Slander may save the cost of searching, but if by mistake one attacks a good type who successfully defends herself, then one loses the game. One major finding of the model is that this kind of risk is only taken by the bad type. Furthermore, all negative messages employed by the bad type are slanderous. The probability of a candidate engaging in slander, say $\psi(\alpha, k)$, is given by the following remark:

Remark 3 *In a no-search equilibium, we have $\psi(\alpha, k) = 1 - \alpha$ in a bad climate and $\gamma(\alpha, k) = 0$ in a good climate. In an equilibrium with search, we have $\psi(\alpha, k) = \frac{1}{2}(1 - \alpha) - \frac{k}{2}$.*

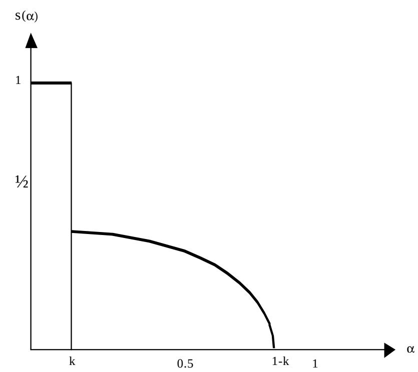
Proof. Follows from Propositions 1 and 2. ∎

In a very bad climate, since search is too expensive to undertake, all messages of the bad type are slanderous, and in moderate climates, only $(1 - p_B)$ share of messages of the bad type are slanderous. The probability of slander decreases with the improvement in climate because the bad type himself becomes less common, and he also starts searching more. In a very good political climate, there is no negative advertising and therefore, no slander. Note that a "clean" electorate is this sense is very inefficient. Note that as the climate improves, a growing share of negative advertising is informed attacks by the good type. Thus, not only the absolute volume of slander, but also the share of slander in all negative messages exhibits a decreasing trend with the prior. Figure 4a depicts, for a low cost, the share of slander $\psi(\alpha, k)$ in *all* advertising, and Figure 4b depicts the proportion of slander in negative advertising alone.

[5] Slander may also be defined in this model as simply an attack on a good type. With that definition, all qualitative statements about slander made in the remark are true. However, since the is no pre-defined "good" type in a continuous type model, we cannot carry over this definition to the next section.



4a: Slander share in all advertising



4b: Slander share in negative messages

Figures 4a and 4b: Share of slander

5 Extension: Continuous Type Space

In this section, we demonstrate that all the results we proved in the case of binary type space extend naturally to a case where quality can vary over a continuum. Moreover, we show here that the continuum breaks into a "good" set and a "bad" set endogenously, and thus the political climate can be thought to be determined in equilibrium, based on the relative sizes of the two sets.

The continuous type extension is not of mere technical interest. It provides a comparison with the existing literature on adverse selection with one principal (the median voter) and two competing agents (the candidates). Banks (1990) examines analyses a game where candidates make false announcements about their preferred positions, but lying has an exogenous cost that increases in the distance between their preferred and announced positions. While Banks finds pooling of candidate types over an interval containing the median voter's best point, Callander and Wilkie (2005) show that if there is a cheap talking type in the model, pooling happens at two disjoint intervals on either side of the median voter's best point. Thus, the "best" types (those that are preferred most by the median voter) pool in the former paper while the "moderate" types pool in the latter, and all the other types separate. The continous type space in our model can be interpreted as a space of preferred points of candidates, with higher quality implying a location closer to the median voter's ideal point. In the equilibrium in our model there are two clusters of pooling - one for the "good" types and the other for the "bad" types. Our message is that if competition reveals information about only one agent, it is possible to separate the good types from the bad types (where good and bad types are defined endogenously), but one cannot separate within the good or bad set of types.

Normalise the type space Θ and consider it to be the unit interval $[0,1]$. Suppose $F(\cdot)$ is the nonatomic prior distribution from which θ is drawn. Assume that $F(\cdot)$ has full support over Θ. Note that voter inference in this case is non-trivial. If some candidate is revealed to be of type $\theta \in (0,1)$, then the attacker potentially can be of a type that is strictly better, equal or strictly worse. In the two-type model, both types were extreme - and therefore revelation of one type was enough for the voter to decide. What is interesting is that in the continuous model too, there is a class of equilibria in which rational voting looks very much like passive voting.

The strategy functions $p(\theta), q(\theta)$ and $r(\theta, \theta')$ are defined in the same way as in Section 3. In the continous case, these functions are probability density functions - we assume them to be continuous except at a finite number of points.

Next, we define $h(\theta, \theta')$ as the probability density of the event that conditional on candidates of types θ and θ' respectively, the one with type θ employs message P. This is the equivalent of $\theta P \theta'$ as defined in equation (1) in the discrete set-up. Formally,

$$h(\theta, \theta') = (1 - p(\theta))q(\theta) + p(\theta)r(\theta, \theta'), where \theta, \theta' \in \Theta \times \Theta \tag{9}$$

Using, (9), define $g(\theta, \theta')$ as the probability density that type θ is revealed through a fruitful debate against type θ'. In other words, $g(\theta, \theta')$ as the probability density of the event that conditional on type θ facing type θ', type θ plays message action P and type θ' plays message N.

$$g(\theta, \theta') = h(\theta, \theta') \left(1 - h(\theta, \theta')\right), \text{ where } \theta, \theta' \in \Theta \times \Theta \tag{10}$$

Next, define $e(\theta)$ as the expected type of candidate $-i$ when candidate i has been revealed to be of type θ.

$$e(\theta) = \mu(\theta_{-i} | \theta_i = \theta)$$

When a type θ is revealed in equilibrium with positive probability, using (10), we can calculate $e(\theta)$ as:

$$e(\theta) = \frac{\int_0^1 \theta' g(\theta, \theta') dF(\theta')}{\int_0^1 g(\theta, \theta') dF(\theta')}, \; when \int_0^1 g(\theta, \theta') dF(\theta') > 0 \tag{11}$$

If some type θ is not revealed in equilibrium with a positive probability, then $e(\theta)$ has to be determined by an appropriate specification of out of equilibrium beliefs.

Signaling games with continuous types often admit multiple equilibria. Both Banks (1990) and Callander and Wilkie (2005) use the refinement of universal divinity to select equilibria. Here, we are interested in the link between passive and rational beliefs, and specifically in the existence of equilibria that can be supported by passive beliefs. Since we do not want equilibria that are too dependent on beliefs off the equilibrium path, we assume the following restriction on the set of equilibria that we consider:

In equilibrium, all types $\theta \in \Theta$ should be revealed with positive probability, i.e.

$$\int_0^1 g(\theta, \theta')dF(\theta') > 0 \text{ for all } \theta \in \Theta \tag{12}$$

This guarantees that $e(\theta)$ is defined by (11) for all θ.

Next, separate the type space into disjoint sets G, M and B such that

$$\begin{aligned} G &= \{\theta : e(\theta) < \theta\} \\ B &= \{\theta : e(\theta) > \theta\} \\ M &= \{\theta : e(\theta) = \theta\} \end{aligned}$$

Since we are looking for equilibria that look like those in Section 5.2, we consider equilibria where M is a collection of a finite number of points, and thus has measure zero. Henceforth, we will ignore the set M and look only at G and B.

If candidate i is revealed to be of type $\theta \in G$, $\mu(\theta_{-i}|\theta \in G) = e(\theta) < \theta$, implying that the voter will vote in favour of candidate i. Conversely, if candidate i is revealed to have type $\theta \in B$, he is voted against. Hence, if there is an equilibrium with restriction (12), it would look like the equilibrium with binary types and passive strategies. Note however that in Section 3, the prior in favour of the good type was exogenous, and in this section the measure of the set G is determined endogenously. Formally, call

$$\int_{\theta \in G} dF(\theta) = \alpha$$

We need to show that in equilibrium, we must have $\alpha \in (0, 1)$.

To put more structure on the set of equilibria, we consider another restriction. This restriction is not necessary for the existence of binary equilibria with passive voting strategies, however it selects equilibria among those with such strategies that lead to a "natural" interpretation of good and bad. This is a weaker form of the monotonicity restriction in Polborn and Yi (2006)[6]. Define the expected utility in equilibrium for type θ of player i as $U_i(\theta)$. We stipulate that in equilibrium a type must not have a strictly greater expected utility than a higher type, i.e.

$$\text{For } \varphi, \varphi' \in \Theta^2,\ \varphi < \varphi' \Rightarrow U_i(\varphi) \leq U_i(\varphi') \tag{13}$$

In the candidate equilibrium, all $\theta \in G$ and all $\theta \in B$ play the same strategy, and therefore we have for $\varphi, \varphi' \in \Theta^2$

$$\left.\begin{array}{l} \varphi \in G, \varphi' \in G \Rightarrow U_i(\varphi) = U_i(\varphi') \\ \varphi \in B, \varphi' \in B \Rightarrow U_i(\varphi) = U_i(\varphi') \\ \varphi \in G, \varphi' \in B \Rightarrow U_i(\varphi) \geq U_i(\varphi') \end{array}\right\} \tag{14}$$

Also, unless there is complete pooling in equilibrium, for any $\varphi \in G$ and $\varphi' \in B$, we must have $U_i(\varphi) > U_i(\varphi')$.Therefore, the restriction (13) and conditions (14) imply that for such a candidate equilibrium, if all types do not pool on the same strategy, there has to exist some $\theta^* \in (0,1)$ such that

$$\varphi \in G, \varphi' \in B \Leftrightarrow \varphi' < \theta^* < \varphi \tag{15}$$

In other words, $G = \{\theta : \theta > \theta^*\}$ and $B = \{\theta : \theta < \theta^*\}$. With this definition of the sets G and B, there indeed exists a class of equilibria with the requisite properties for the low cost case. Moreover, any number in the range $(k, 1-k)$ can serve as θ^*.

Proposition 5 is a formal statement of the existence and characterisation of equilibria in the continuous type case. It demonstrates that the type space endogenously breaks into to sets which conform to the "natural" definition of good and bad, and that the equilibrium with the binary type space is replicated.

Proposition 5 *Consider $k < \frac{1}{2}$ and any $\theta^* \in (k, 1-k)$. Define sets G and B as in* (15). *Then the following is an equilibrium obeying restrictions* (12) *and* (13) :

$$p(\theta) = \begin{cases} p_G \ \textit{if}\ \theta \in G \\ p_B \ \textit{if}\ \theta \in B \end{cases}$$

$$q(\theta) = \begin{cases} q_G \ \textit{if}\ \theta \in G \\ q_B \ \textit{if}\ \theta \in B \end{cases}$$

$$r(\theta, \theta') = \begin{cases} 1 \ \textit{if}\ \theta \in G, \theta' \in G \\ r_G \ \textit{if}\ \theta \in G, \theta' \in B \\ r_B \ \textit{if}\ \theta \in B, \theta' \in G \\ 0 \ \textit{if}\ \theta \in B, \theta' \in B \end{cases}$$

[6] Polborn and Yi (2006) assumes that expected utility strictly increases in type - we assume weak monotonicity.

where, if $\alpha = 1 - F(\theta^*)$,*the quantities* p_G, p_B, q_G, q_B, r_G *and* r_B *are given by Proposition* $2(ii)$

The fact that the strategies mentioned in the proposition constiute an equilibrium conforming to the aforementioned restrictions can be easily checked.

This proposition states that for low effective cost of search, there is always a monotonic equilibrium with rational voting that is supported by passive voting strategies. We have a cutoff type above which all types are deemed to be "good" and below which all types are deemed to be "bad". All good types behave like type G in the discrete case, and all bad types behave like type B. The voter simply votes in favour of a candidate is he is revealed to be a good type and against him if he is revealed to be a bad type. Note however that this cutoff type is not unique. In fact, any number in the range $(k, 1-k)$ can serve as the cut-off type Thus, it is possible that in two electorates where the prior distribution from which the two candidates are drawn are exactly the same, the definition of a good candidate and a bad candidate are different. Comparative static conclusions are difficult to draw because of the multiplicity of equilibria, but we can say that the supportable set of cut-off types expands as the effective search cost decreases.

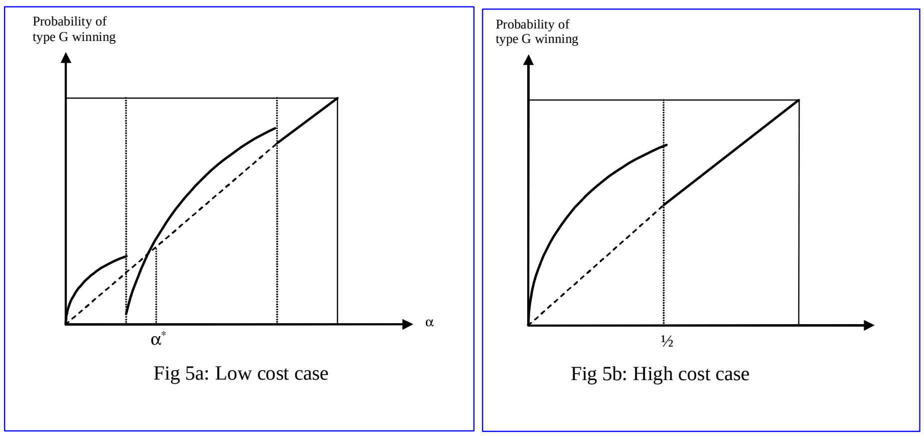
It is mentionworthy that for high effective cost of search, we can still have an equivalence between the passive and rational strategies, but since not all types are revealed in such an equilibrium, it would not satisfy the restriction (11). Specifically, in the exogenous prior case, under the good political climate, neither type is revealed and under the bad political climate, the bad type is not revealed. These outcomes can still be supported in equilibrium of the continuous type case, but we have to make additional assumptions on what the voter believes when she observes a type revealed out of equilibrium.

6 Discussion and extensions

Looking at political campaigns as debates between candidates with partial information about each other helps us understand a few important issues about the efficiency of the campaign process, and how that relates to the political climate. The major finding of the model is that as the political climate improves, the probability that a god candidate is selected through the electoral process may actually *go down*. To state that in formal terms, we examine the function $f_k(\alpha)$: the total probability that a good candidate is selected, given a certain cost of search[7]. Figure 5a demonstrates this function when the effective cost is high and

[7] The functional form for $f_k(\alpha)$ can be decuced from Propositions 1 and 2 as:

Figure 5b shows the function when the effective search cost is high. In both cases, there exist downward jumps in otherwise piecewise continuous and monotonically increasing graphs. These downward jumps are larger as the cost of information increases.



Figures 5a and 5b: Total probability of selection of a good candidate

The above analysis has an interesting long term implication for the average quality of the candidate pool competing in the electoral process. Imagine that everytime a good candidate is selected, the public assessment of the political climate goes up, and everytime a bad candidate is selected, the assessment goes down. If this adjustment process occurs for sufficient number of times, the assessment of α will settle at that level where the probability of selection of te good candidate is equal to her probability of occurence in the candidate pool. In other words, the long term assessment of the political climate is expected to settle at a fixed point of the function $f_k(\alpha)$. In the fugures 5a and 5b, the fixed points are those points where the 45^0 line intersects the function $f_k(\alpha)$. Note that there are multiple fixed points for the function f_k. In particular, all values of $\alpha > 1-k$ for the low cost and all $\alpha > \frac{1}{2}$ for high cost can serve as fixed points. Thus, the political system can settle at a "good" political climate in the long term. However, it is also possible that the system settles at a "bad" climate, which has been indicated by α^*. This bad long term equilibrium climate always exists for a low effective search[8] cost. Thus, the political system can be stuck at a cycle

$$f_k(\alpha) = \begin{cases} 2\alpha - \alpha^2 \text{ if } \alpha < \min(\frac{1}{2}, k) \\ \frac{3}{2}\alpha - \frac{1}{2}\alpha^2 - \frac{1}{2}k^2 \text{ if } k < \alpha < 1-k \text{ and } k < \frac{1}{2} \\ 2\alpha - \alpha^2 \text{ if } \alpha > \max(\frac{1}{2}, 1-k) \end{cases}$$

[8]The existence of α^* in the range $(1-k, k)$ is guaranteed by the fact that in the range, the function starts below the 45^0 line, and ends above it. Examining

of low expectations. We can also show that α^* is increasing in k, implying that the political climate will worsen in the long term if information becomes cheaper. Note that there is a fixed point at $\alpha = 0$ too. Note however that none of the equilibria are stable: a small perturbation can lead to a path moving away from the fixed point.

The debates framework can be extended to examine the incumbency advantage. According to this model, the source of the advantage is the ability to alter the voters' perceprions. Suppose the incumbent has successfully been able to raise the prior probability of being good to α_I, which is higher than the prior α_C from which the challenger is drawn. If we have $0 < \alpha_C < \alpha_I < 1$ and $k < \frac{1}{2}$, then in equilibrium, the good type of the incubment always employs a positive message, the bad type of the challenger always goes negative. The bad type of incumbent mixes through search, imitiating the good type partially. Similarly, the good type of the challenger mixes, unable to separate himself from the bad type fully. The good type of the incumbent always wins. Even the bad type of the incumbent sometimes wins against the good type of challenger. In the converse situation, if the incumbent is assessed to be worse than the average challenger ($\alpha_C > \alpha_I$), then the incumbent always loses to the good type of the challenger. Even the good type of the incumbent may lose to the bad type of the challenger. Thus, the voter rewards good performance and punishes bad performance of the incumbent. Note that this result hinges on observability and not on risk aversion of the voter. If the voter is risk averse, then the incumbent can have an advantage over the challenger even if he performs somewhat badly, since his quality would be known with more certainty than that of the challenger. This analysis indicates that the whole objective of political activity is to alter voters' expectation of candidate quality before the actual campaign begins.

A feature of the debates model as discussed in the main body of the paper is that the voters have no independent channel to verify a candidate's claims other than what his rival says. This assumption has been made to isolate the effect of the check that one candidate can exercise over the other, and it clearly underestimates the role of the media in defining the debate or in verifying the claims made by each candidate. We treat the actual process of debate as a black box. The rather extreme assumption that a fruitful debate reveals the true type of the candidate under focus with certainty while cross talk reveals nothing is unnecessary for the qualitative results of the model to go through. To enrich the model, we can incorporate the role of media or that of good

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the functional form, $\alpha^* = \frac{1}{2} - \frac{\sqrt{1-4k^2}}{2} < \frac{1}{2}$. Also, the concave graph shifts upwards everywhere in this range if k goes down, hence α^* goes down.

and bad arguments by including probabilities of revelation of the true type even when there is cross talk. Furthermore, the probabilites can be made to be contingent on the quality of the candidates. In the current paper, we have not included such straightforward extensions to drive the basic point home while economising on the number of parameters.

We have made a few more substantive assumptions that limit the score of the model. First, the model being static, we cannot look at the dynamic choice of arguments and counter-arguments. We assume, with some support from what is observed in real electoral races, that the public campaign strategy is fixed in advance of the time the messages are actually spoken. Second, a candidate chooses exactly one of a positive and negative message. The action space does not permit a combination of the two. This assumption, although standard in the literature, is an abstraction from reality where we observe candidates using a combination of both positive and negative messages. Third, candidate quality is one-dimensional in this model. There are only two relevant issues: the quality of each candidate. Talking about two different issues is equivalent to talking about two different persons. If candidate quality were multidimensional, a candidate could dodge a fruitful debate by focussing on a different dimension than on a different person. Thus, we could still have a fruitless debate with one candidate going positive and the other going negative. This assumption of unidimensional quality simplifies the analysis considerably and demonstrates some of the basic tradeoffs involved in competitive information provision at the cost of a certain loss of richness. Each of these assumptions could be relaxed and the model can thereby be extended in what we consider promising directions for future research.

7 Conclusion

In this paper we have examined the choice between contradicting the opponent (revealing information through fruitful debate) and mimicking his argument (concealing information through cross talk). We have studied the case of choice between positive and negative advertising as a particular case of this more general decision problem. Instead of modelling this as a signalling game, we find that the hide and seek game is a better tool to analyse such a process of competitve information provision where the good type tries to reveal information and the bad type tries to hide information. While hard-information models preclude the analysis of lies, cheap talk models lack the ability to capture genuine information provision through rhetoric. There is a lot of information that is conveyed through arguments and counter-arguments that cannot be captured by modelling information as either of these two extremes.

Our claim is that, to analyse provision of information, we must model information as *semi-soft.* Using this model, we confirm most of the existing results in the literature on positive and negative advertising, and additionally, explain issues like lies and slander. We point out that a lie or slander, while useless to the voter, might be useful to the candidate in "muddling" the debate and thus distracting the attention of the electorate. Besides, we demonstrate several interesting features of information search by candidates, the most important of which is that the bad type searches more frequently than the good type. This arises from a commitment problem of the good type since the good type has a higher incentive to economise on search cost than the bad type. This divergence of interest between the good type of candidate and the voter is an additional source of welfare loss for the electorate, and we can capture this welfare loss only if we allow for the option of investment in information. We show that, contrary to popular perception, voter welfare is not monotonic in the political climate because of the hide-and-seek nature of political campaigns.

We believe that this model is interesting due to several reasons apart from the analysis of electoral rhetoric. It demonstrates an interesting interplay of two levels of information asymmetry: one, between senders and the receiver, and two, between the two senders. It also suggests a solution to the unappealing type independent mixed strategy outcome in the matching pennies game by including uncertainty of types. However, it is to be noted that the solution, though suited to this particular case, makes several strong assumptions which may not be applicable in a general sense. We also suggest a solution to the question why and to what extent advertisement messages are believed. On the one hand, much of the existing literature on advertising avoids the question by modelling advertisement messages as verifiable On the other hand, another major strand deems the content of advertisement irrelevant by considering advertisements as an exercise in money burning. We point out that since each candidate's willingness to misinform is potentially checked by the ability of his rival to inform the electorate, advertisements can be treated as partially credible signals of the type of the sponsor. These contributions, we believe, would make the paper independently interesting to those not actively engaged with the literature on positive and negative advertising.

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9 Appendix A : Rational voter model

In the main body of the paper, the voter was passive, i.e. the role of the voter as a player was not explicitly considered. In this section, we solve the game with a rationally updating voter and a more general space of types.

Call the game with two types and passive voter considered in the main paper $\mathcal{H}(\alpha, w, c)$ where α, w and c are the respective parameter values. Now, consider a game with a rational voter, type space Θ and common prior $F(\cdot)$ over types of both players. Call this game $\mathcal{G}(\Theta, F, w, c)$.

Our objective in this section is to solve this game $\mathcal{G}$, which we now specify in detail.

There is a voter and two candidates $i = 1, 2$. Each candidate has a private quality type θ_i that is drawn from a common type space $\Theta \subset R$ according to a common distribution $F(\cdot)$. The action space of candidates $i = 1, 2$ is exactly the same as specified in Section 3. We now formally introduce the debate protocol as a revelation function $R : M^2 \times \Theta^2 \to \Theta \cup \phi$.

$$
\begin{aligned}
R(P, P, \boldsymbol{\theta}) &= R(N, N, \boldsymbol{\theta}) = \phi \\
R(P, N, \boldsymbol{\theta}) &= \theta_1 \\
R(N, P, \boldsymbol{\theta}) &= \theta_2
\end{aligned}
$$

Formally, this is a three stage game. In stage 1, the candidates observe their own realised type and choose search action X privately. In stage 2, the candidates play debate action M. In stage 3, the voter observes $R(\mathbf{M}, \boldsymbol{\theta})$ and votes for either candidate, and the utilities are realised. Since action in stage 1 is private, we consider stages 1 and 2 as simultaneous.

Assume that if the winning candidate's type is θ, the vote receives a utility of θ. The voter's action based on information $R(\mathbf{M}, \boldsymbol{\theta})$ is $v_i(R)$, which is the probability of voting for candidate i, $i = 1, 2$.

Obviously, there is a restriction $v_1(R(\mathbf{M}, \boldsymbol{\theta})) + v_2(R(\mathbf{M}, \boldsymbol{\theta})) = 1$. Assuming that the voter votes for both candidates with the same probability whenever he is indifferent, we have $v_i \in \{0, \frac{1}{2}, 1\}$.

The utility of candidate i from debate is

$$u_i(\mathbf{M}, \boldsymbol{\theta}) = w \cdot v_i, \; w > 0, \; i = 1, 2.$$

Note that the above implies that $u_1(\mathbf{M}, \boldsymbol{\theta}) + u_2(\mathbf{M}, \boldsymbol{\theta}) = w$. The total utility of the candidate, as before, is the utility from winning the debate less the search cost.

Our equilibrium concept is symmetric perfect Bayesian Nash equilibrium with undominated strategies, which in this case, is the same as sequential equilibrium (Kreps and Wilson 1982).

9.1 Voter and candidate strategies

In our initial game $\mathcal{H}$, the voter did not infer the type of the attacker when the type of the defending candidate was revealed. In the above set up, these *passive voting strategies* can be specified as:

$$v_i\left(R(\mathbf{M},\boldsymbol{\theta})=\theta_i\right)=\begin{cases}1 \; if \; \theta_i=G\\ 0 \; if \; \theta_i=B\end{cases},$$
$$v_i\left(R(\mathbf{M},\boldsymbol{\theta})=\phi\right)=\frac{1}{2}$$

On the other hand, in the game $\mathcal{G}$, the voter rationally forms beliefs about candidate types based on information R. Suppose the expected type of candidate i is $\mu(\theta_i|R)$. Note that this function will be the same for both $i=1,2$ since the candidates are drawn from a common prior distribution and play symmetric strategies. Since this is also the expected utility from a candidate when he is elected, the voter compares $\mu(\theta_1|R)$ and $\mu(\theta_2|R)$ and votes for whichever is higher. If they are equal, the voter is indifferent and randomises with equal probability.

If $R=\phi$, both candidates are playing the same action. Since they are drawn from a common prior, the voter cannot distinguish between the two. Hence, $\mu(\theta_1|\phi)=\mu(\theta_2|\phi)$, and therefore $v_i(\phi)=\frac{1}{2}$. If $R=\theta_i$, $\mu(\theta_i|\theta_i)=\theta_i$ while $\mu(\theta_{-i}|\theta_i)$ is inferred by the voter either by Bayes Rule from equilibrium strategies or by out-of-equilibrium beliefs. Hence:

$$v_i\left(\theta_i\right)=\begin{cases}1 \; if \; \theta_i>\mu(\theta_{-i}|\theta_i)\\ 0 \; if \; \theta_i<\mu(\theta_{-i}|\theta_i)\\ \frac{1}{2} \; if \; \theta_i=\mu(\theta_{-i}|\theta_i)\end{cases}$$
$$and \; v_i\left(\phi\right)=\frac{1}{2}$$

These are called *rational voting strategies*.

The strategy space for player $i=1,2$ is defined by the following functions:

$$p_i(\theta)\equiv\Pr(X(\theta)=S)$$
$$q_i(\theta)\equiv\Pr(M_i(\theta)=P\;|X_i(G)=NS)$$
$$r_i(\theta,\theta')\equiv\Pr(M_i(\theta)=P\;|X_i(\theta)=S,\theta_{-i}=\theta')$$

Note that this set up can handle any type space of reasonable generailty. For now, we consider a binary tye space. In the main body of the paper we handle the continuous type space in section 5 of the main body of the paper. Suppose that there are only two types: Good (G) and bad (B). In formal terms, $\Theta=\{G,B\}$, where G and B are two real numbers with $G>B$. The distribution $F(\cdot)$ now becomes Bernoulli, with the prior $\Pr(\theta_i=G)=\alpha\in(0,1)$ for $i=1,2$. Note that if the voter uses passive strategies, the game reduces to $\mathcal{H}$. We show that for a natural specification of out-of-equilibrium beliefs, the game $\mathcal{G}$ has the same equilibrium as the game $\mathcal{H}$[9]. To show that, we show that for game $\mathcal{G}$, rational voting strategies are the same as passive voting strategies, and therefore lead to the same outcomes.

We assume that if $\theta_i = G, B$ is revealed out of equilibrium in a debate, then the voter assumes that both types G and B of candidate $-i$ have small positive probabilities η_G and η_B of having deviated and played N. This implies that off the equilibrium path we have

$$B < \mu(\theta_{-i}|\theta_i) < G, \;\; \theta_i = G, B \tag{16}$$

Next, note that in game $\mathcal{H}$, for the equilibrium with any parameter values, whenever any type θ of candidate i is revealed, there is a positive probability of both types G and B of candidate $-i$ to have attacked candidate i. Hence, along the equilibrium path too, by Bayes rule:

$$B < \mu(\theta_{-i}|\theta_i) < G, \;\; \theta_i = G, B \tag{17}$$

(16) and (17) imply that for any equilibrium of $\mathcal{H}$, when $R(\mathbf{M}, \boldsymbol{\theta}) = \theta_i$

$$\begin{aligned} \mu(\theta_{-i}|\theta_i) &> \theta_i \; if \;\; \theta_i = B \\ \mu(\theta_{-i}|\theta_i) &< \theta_i \; if \;\; \theta_i = G \end{aligned}$$

Hence the rational voting strategy supporting this equilibrium, on and off the equilibrium path, is:

$$v_i(R) = \begin{cases} 1 \; if \;\; R = \theta_i = G \\ 0 \; if \;\; R = \theta_i = B \\ \frac{1}{2} \; if \;\; R = \phi = 0 \end{cases}$$

Hence, passive voting is rational here. Thus, one equilibrium of the game $\mathcal{G}$ is the equilibrium of the game $\mathcal{H}$. Using the next result, we show that we get no other equilibria using rational voting for this game.

Lemma 4 *Using rational voting, there is no equilibrium where for some type $\theta_i \in \{G, B\}$, we have $\mu(\theta_{-i}|\theta_i) = \theta_i$*

Proof. In Appendix B ■

What the above lemma implies that we cannot have an equilibrium with rational voting where a given type is revealed (on or off the equilibrium path) only by the same type of the rival, and not the other type. Thus, with rational voting, in any equilibrium, we must always have:

[9] To be exact, these games are specifed differently, so the equilibria can never be same. But what we mean to say is that in equilibrium, the candidate and voter behaviour is the same in both games.

$$B < \mu(\theta_{-i}|\theta_i) < G,\ \theta_i = G, B$$

This implies that in a discrete type set up, rational voting always lead to passive voting. Therefore, we have the following proposition:

Proposition 6 *If the type space in game $\mathcal{G}$ has two types with positive priors, given the out-of-equilibrium beliefs stated in* (16), *the solution to $\mathcal{G}(\Theta, \alpha, w, c)$ is the same is the solution to the game $\mathcal{H}(\Theta, \alpha, w, c)$.*

10 Appendix B: Proofs of Propositions

To define the equilibrium strategies in this setting, we first introduce some basic notation.

Define $Eu_i^\theta(M|p,q,r,I)$ as the expected utility from debate to type $\theta \in \{G, B\}$ of player i from playing message $M \in \{P, N\}$ when player $-i$ is using strategies $p(\cdot), q(\cdot)$ and $r(\cdot,\cdot)$, and the informaion available to the player is $I \in \{G, B, \phi\}$. If there is search and the type of the rival is known then $I = G$ or $I = B$, else, $I = \phi$. This expected utility is constructed from the debate payoff $u_i(\mathbf{M}, \boldsymbol{\theta})$, taking expectation over the possible messages of the rival (from the strategies) and if the rival type is not known, then over possible types of the rival too.

Define as $Eu_i^\theta(m|p,q,r,I)$.the expected utlity when type θ of player i plays message P with a probability $m \in [0,1]$.

Define $EU_i^\theta(S|p,q,r) = E_{(\theta')}\left[\underset{m\in[0,1]}{\arg\max} Eu_i^\theta(m|p,q,r,\theta')\right] - c$ as the extected utlity from search, taking into account the optimal message choice post search, and taking expectation over rival types and $EU_i^\theta(NS, m|p,q,r) = Eu_i^\theta(m|p,q,r,\phi)$ as the expected utility from not searching and playing a mix m of messages.

Equilibrium strategies is a triad of functions $\{p_i^*(\cdot), q_i^*(\cdot), r_i^*(\cdot,\cdot)\}$ for $i = 1, 2$, $\theta \in \{G, B\}$ and $\theta' \in \{G, B\}$ such that:

1. $r_i^*(\theta, \theta') = \underset{m\in[0,1]}{\arg\max} Eu_i^\theta(m|p^*, q^*, r^*, \theta')$,

2. $q_i^*(\theta) = \underset{m\in[0,1]}{\arg\max} Eu_i^\theta(m|p^*, q^*, r^*, \phi)$, $\theta \in \{G, B\}$, and

3. $p_i^*(\theta) = \underset{p\in[0,1]}{\arg\max} \left\{pEU_i^\theta(S|p^*, q^*, r^*) + (1-p)EU_i^\theta(NS, q^*|p^*, q^*, r^*)\right\}$.

We shall first prove a few results in the form of claims and use those results to find the equilibrium strategies for different parameter values. From here onwards, define

Claim 1 $Eu_i^\theta(D(\theta)|p,q,r,\theta) - Eu_i^\theta(\overline{D(\theta)}|p,q,r,\theta) = \frac{w}{2} > 0$

Proof. For $\theta = G,\ D(\theta) = P$, and $\overline{D(\theta)} = N$

$Eu_i^G(P|p,q,r,G) - Eu_i^G(N|p,q,r,G) = \left\{\frac{w}{2}GPG_{-i} + w(1-GPG_{-i})\right\} - \left\{\frac{w}{2}(1-GPG_{-i})\right\} = \frac{w}{2}$

For $\theta = B,\ D(\theta) = N$, and $\overline{D(\theta)} = P$

$Eu_i^B(N|p,q,r,B) - Eu_i^B(P|p,q,r,B) = \left\{\frac{w}{2}(1-BPB_{-i}) + wBPB_{-i}\right\} - \left\{\frac{w}{2}BPB_{-i}\right\} = \frac{w}{2}$

The above claim establishes that if type θ finds it strictly dominant to use message $D(\theta)$ when he knows that the rival is of type θ. ■

Claim 2 *If* $p_i(\theta) > 0$, *we must have* $Eu_i^\theta(D(\theta)|p,q,r,\overline{\theta}) < Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta})$

Proof. Suppose not. Hence, $Eu_i^\theta(D(\theta)|p,q,r,\overline{\theta}) \geq Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta})$.

Since $p_i(\theta) > 0$, we must have

$$EU_i^\theta(S|p,q,r) \geq \max\left[EU_i^\theta(NS,P|p,q,r), EU_i^\theta(NS,N|p,q,r)\right]$$

Using claim 1, we can rewrite this as:

$$\Pr(\theta)Eu_i^\theta(D(\theta)|p,q,r,\theta) + \Pr(\overline{\theta})\max\left[Eu_i^\theta(D(\theta)|p,q,r,\overline{\theta}), Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta})\right] - c$$
$$> \max\left[EU_i(NS,D(\theta)|p,q,r), EU_i^\theta(NS,\overline{D(\theta)}|p,q,r)\right]$$

From our supposition,

$$\begin{aligned} LHS &= \Pr(\theta)Eu_i^\theta(D(\theta)|p,q,r,\theta) + \Pr(\overline{\theta})Eu_i^\theta(D(\theta)|p,q,r,\overline{\theta}) - c \\ &= EU_i^\theta(NS,D(\theta)|p,q,r) - c \end{aligned}$$

Now, $RHS = \max\left\{\begin{array}{c} \Pr(\theta)Eu_i^\theta(D(\theta)|p,q,r,\theta) + \Pr(\overline{\theta})Eu_i^\theta(D(\theta)|p,q,r,\overline{\theta}), \\ \Pr(\theta)Eu_i^\theta(\overline{D(\theta)}|p,q,r,\theta) + \Pr(\overline{\theta})Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta}) \end{array}\right\}$

By claim 1 and our supposition

$$RHS = EU_i^\theta(NS,D(\theta)|p,q,r). > EU_i^\theta(NS,D(\theta)|p,q,r) - c = LHS,$$

which is a contradiction. ■

The above claim establishes that whenever there is search with a positive probability, type θ uses message $\overline{D(\theta)}$ when he knows that the rival is of type $\overline{\theta}$.Claims 1 and 2 determine what actions will be played by a type when the rival type is known. Note that the choice of message post search is independent of the strategy of the rival type. Thus:

$$EU_i^\theta(S|p,q,r) = \Pr(\theta)Eu_i^\theta(D(\theta)|p,q,r,\theta) + \Pr(\overline{\theta})Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta}) - c \tag{18}$$

Claim 3 $EU_i^\theta(S|p,q,r) - EU_i^\theta(NS, \overline{D(\theta)}|p,q,r) = \frac{w}{2}\left[\Pr(\theta) - k\right]$

Proof. By equation 18, $EU_i^\theta(S|p,q,r) - EU_i^\theta(NS, \overline{D(\theta)}|p,q,r)$ equals:

$$\Pr(\theta)\left[Eu_i^\theta(D(\theta)|p,q,r,\theta) - Eu_i^\theta(\overline{D(\theta)}|p,q,r,\theta)\right]$$
$$+\Pr(\overline{\theta})\left[Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta}) - Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta})\right] - c$$

By claim 1, the above expression equals $\Pr(\theta)\frac{w}{2} - c = \frac{w}{2}\left[\Pr(\theta) - k\right]$. ■

Note that we do not consider $k = \alpha$ or $k = 1 - \alpha$ in our range of parameter values Thus, between and no search with $\overline{D(\theta)}$, one always strictly dominates the other, based on the va;ues of the parameter. Most importantly, if type θ searches with positive probability, he will not play the message $\overline{D(\theta)}$ conditional on not searching. If the probability of search is strictly between 0 and 1, then $D(\theta)$ will be played by θ conditional on playing action $X = NS$.

Claim 4 $EU_i^\theta(S|p,q,r) - EU_i^\theta(NS, D(\theta)|p,q,r) = \frac{w}{2}\left[\Pr(\overline{\theta})(2\overline{\theta}P\theta_{-i} - 1) - k\right]$

Proof. By equation 18, $EU_i^\theta(S|p,q,r) - EU_i^\theta(NS, \overline{D(\theta)}|p,q,r)$ equals:

$$\Pr(\theta)\left[Eu_i^\theta(D(\theta)|p,q,r,\theta) - Eu_i^\theta(D(\theta)|p,q,r,\theta)\right]$$
$$+\Pr(\overline{\theta})\left[Eu_i^\theta(\overline{D(\theta)}|p,q,r,\overline{\theta}) - Eu_i^\theta(D(\theta)|p,q,r,\overline{\theta})\right] - c$$

By simple algebra, the above expression equals

$$\Pr(\overline{\theta})(2\overline{\theta}P\theta_{-i} - 1)\frac{w}{2} - c = \frac{w}{2}\left[\Pr(\overline{\theta})(2\overline{\theta}P\theta_{-i} - 1) - k\right]$$

■

This claim, along with claim 3 establishes that when there is search, either by type θ of player i, we must have $\frac{w}{2}\left[\Pr(\overline{\theta})(2\overline{\theta}P\theta_{-i} - 1) - k\right] \geq 0$. If search probability is strictly less than unity, i.e. there is indifference between search and no search, then the inequality must be satisfied as an equality. Note that this depends on the strategy of the rival canddate. With these four basic claims, we can prove the propositions.

10.1 Proof of proposition 1: High cost ($k > \frac{1}{2}$)

10.1.1 Part *(i)*: Bad political climate ($\alpha < \frac{1}{2}$)

$\Pr(G) = \alpha < \frac{1}{2} < k$

By claim 3, $EU_i^G(S|p,q,r) - EU_i^G(NS, N|p,q,r) < 0 \Rightarrow p_G = 0$

Also, by claim 4, $EU_i^B(S|p,q,r) - EU_i^B(NS, N|p,q,r) < 0 \Rightarrow p_B = 0$

From Lemma 2, $q_B = 0$ (dominant strategy)

$\Rightarrow BPG = 0 \Rightarrow r_G = q_G = 1$ (from Lemma 1)

10.1.2 Part *(ii)*: Good political climate ($\alpha > \frac{1}{2}$)

$\Pr(B) = 1 - \alpha < \frac{1}{2} < k$

By claim 3, $EU_i^B(S|p,q,r) - EU_i^B(NS,P|p,q,r) < 0 \Rightarrow p_B = 0$

Also, by claim 4, $EU_i^G(S|p,q,r) - EU_i^G(NS,P|p,q,r) < 0 \Rightarrow p_G = 0$

From Lemma 1, $q_G = 1$ (dominant strategy)

$\Rightarrow GPB = 1 \Rightarrow r_B = q_B = 1$ (from Lemma 2)

10.2 Proof of proposition 2: Low cost ($k < \frac{1}{2}$)

10.2.1 Part *(i)*: $\alpha < k < \frac{1}{2}$

By claim 4, $\alpha < k \Rightarrow EU_i^B(S|p,q,r) < EU_i^B(NS,N|p,q,r)$

By claim 3, $1 - \alpha > k \Rightarrow EU_i^B(S|p,q,r) > EU_i^B(NS,P|p,q,r)$

Thus, for type B of either player, using the negative message and not searching strictly dominates other strategies.

Hence, $p_B = q_B = 0 \Rightarrow BPG = 0$. By Lemma 2, $q_G = r_G = 1$

Also, $BPG = 0$ implies

$$EU_i^G(S|p,q,r) - EU_i^G(NS,P|p,q,r) = -\frac{w}{2}k < 0 \Rightarrow p_G = 0.$$

10.2.2 Part *(iii)*: $\alpha > 1 - k > \frac{1}{2}$

By claim 4, $1 - \alpha < k \Rightarrow EU_i^G(S|p,q,r) < EU_i^G(NS,P|p,q,r)$

By claim 3, $\alpha > k \Rightarrow$

Thus, for type G of either player, using the positive message while not searching strictly dominates other strategies.

Hence, $p_G = 0$ and $q_G = 1 \Rightarrow GPB = 1$. By Lemma 1, $q_B = r_B = 1$

By claim 2, $r_B = 1 \Rightarrow p_B = 0$

10.2.3 Part *(ii)*: $k < \alpha < 1 - k$

Here, $\min(\alpha, 1-\alpha) > k$. Therefore, by claim 3, for $\theta \in \{G, B\}$,

$EU_i^\theta(S|p,q,r) - EU_i^\theta(NS,\overline{D(\theta)}|p,q,r) > 0$.

Thus, whenever the action NS is played by type θ with positive probability, $D(\theta)$ is the message employed, or:

$$p_G < 1 \Rightarrow q_G = 1 \text{ and } p_B < 1 \Rightarrow q_B = 0 \tag{19}$$

Next, we claim that we cannot have search with certainty for either type.

Claim 5 *We cannot have $p_\theta = 1$ in equilibrium for $\theta \in \{G, B\}$.*

Proof. We have already shown that $p_\theta = 0$ in equilibrium for $\theta \in \{G, B\}$ as long as $\alpha < \min(k, \frac{1}{2})$ or $\alpha > \max(1-k, \frac{1}{2})$. We need to prove this claim only for the case $k < \alpha < 1 - k$. Suppose $k < \alpha < 1 - k$ and $p_G = 1$ for i. Therefore, $GPB_i = r_G = 0$. This implies by Lemma 1 that for $-i$, $r_B = q_B = 0 \Rightarrow BPG_{-i} = 0$ (by Lemma 2). Using this in claim 3, we have a contradiction, since

$$EU_i^G(S|p,q,r) - EU_i^G(NS,P|p,q,r) = -\frac{w}{2}k < 0 \Rightarrow p_G = 0.$$

Now, suppose $k < \alpha < 1-k$ and $p_B = 1$ for i. Therefore, $BPG_i = r_B = 1$. Using this in claim 3, we have a contradiction, since

$$EU_i^G(S|p,q,r) - EU_i^G(NS,P|p,q,r) = \frac{w}{2}\left[(1-\alpha)-k\right] > 0 \Rightarrow p_G = 1.$$

and we have just shown that we cannot have $p_G = 1$ in equilibrium. ■

Equation 19 and claim 5 together establish that in equilibrium with $k < \alpha < 1-k$, we must have $q_G = 1$ and $q_B = 0$. Using this in claim 4 and Lemma 2, we can show similarly that neither type will have $p_\theta = 0$. Therefore, we must have $p_\theta \in (0,1)$ for both types and for both players. By claim 2, $r_B = 1$ and $r_G = 0$.

Also, from claim 4, we must have for indifference between search and no search for type θ,

$$\Pr(\overline{\theta})(2\overline{\theta}P\theta - 1) = k \Rightarrow \overline{\theta}P\theta = \frac{1}{2}\left(1 + \frac{k}{\Pr(\overline{\theta})}\right)$$

Since $BPG = p_B$ and $GPB = 1 - p_G$, we are done.

10.3 Proof of Proposition 3

By definition, $\beta(\alpha,k) = \Pr(G \text{ plays } P)\Pr(B \text{ plays } N) + \Pr(G \text{ plays } N)\Pr(B \text{ plays } P)$

For a no-search equilibrium in a bad climate, i.e. if $\alpha < \min(k, \frac{1}{2})$, we have from Proposition 1(i) and 2(i), $\beta(\alpha,k) = 1.1 + 0.0 = 1$.

For a no-search equilibrium in a good climate, i.e. if $\alpha > \max(1-k, \frac{1}{2})$, we have from Proposition 1(ii) and 2(iii), we have $\beta(\alpha,k) = 1.0 + 0.1 = 0$

For equilibrium with search, i.e. for $k < \alpha < 1-k$, we have from Proposition 2(ii) :

$$\begin{aligned}\beta(\alpha,k) &= (1-p_G)(1-p_B) + p_G p_B \\ &= \frac{1}{4}\left[\left(1+\frac{k}{1-\alpha}\right)\left(1-\frac{k}{\alpha}\right) + \left(1-\frac{k}{1-\alpha}\right)\left(1+\frac{k}{\alpha}\right)\right] \\ &= \frac{1}{2}\left(1-\frac{k^2}{\alpha(1-\alpha)}\right)\end{aligned}$$

10.4 Proof of Proposition 4

Since $p_G = 1$, $GPB = r_G$. From claim 3,

$$EU_i^B(S|p,q,r) - EU_i^B(NS,P|p,q,r) = \frac{w}{2}\left[(1-\alpha)-k\right] > 0 \qquad (20)$$

Thus, along the equilibrium path, we must have $q_B = 0$, and hence, $BPG = p_B r_B$. If $BPG > \frac{1}{2}$, $r_G = 0$ by Lemma 1. Therefore, $GPB = 0 \Rightarrow r_B = 0$ by Lemma 2. As this implies $p_B r_B = 0$, it is a contradiction.

On the other hand, if $BPG < \frac{1}{2}$, $r_G = 1$ by Lemma 1. Therefore, $GPB = 1 \Rightarrow r_B = 1$ by Lemma 2. Also, by claim 4,

$$EU_i^B(S|p,q,r) - EU_i^B(NS,N|p,q,r) = \frac{w}{2}\left[\alpha - k\right] > 0. \qquad (21)$$

From (20) and (21), we must have $p_B = 1$. Since $r_B = 1$, we have $p_B r_B = 1$, which is again a contradiction.

Therefore, we must have $p_B r_B = \frac{1}{2}$. Since $p_B > 0$, we must have $r_B = 1$, by claim 2. Therefore, $p_B = \frac{1}{2}$. Also, $p_B \in (0,1)$ implies that $EU_i^G(S|p,q,r) = EU_i^G(NS,P|p,q,r)$, which, by claim 4, implies that $r_G = \frac{1}{2}\left(1+\frac{k}{\alpha}\right)$

10.5 Proof of Lemma 4

Suppose the assertion is false, and for some $\theta \in \{G,B\}$, we have $\mu(\theta_{-i}|\theta_i = \theta) = \theta$. This cannot be off the equilibrium path due to assumption (16) on off-equilibrium beliefs. Hence, if the condition is true for some type θ, then it has to be on the equilibrium path. Note that $v_i(\theta) = \frac{1}{2}$.

Suppose $\mu(\theta_{-i}|\theta_i = G) = G$.For type G to reveal rival type G, in a symmetric equilibrium, we muct have $GPG \in (0,1)$. For the above case to hold, we also need type B never to reveal type G, which happens when either $BPG = 1$ or $GPB = 0$.

Also note that it is no longer dominant for type G to play P if rival type is known to be G.

Assume that $x_G = \Pr(M_i(G) = P \mid X_i(G) = S, \theta_{-i} = G) = 1$

Thus, $GPG = p_G x_G + (1-p_G)q_G$

Case 1 $BPG = 1 \Rightarrow p_B r_B + (1-p_B)q_B = 1$.

Hence, either $p_B = r_B = 1$, or $p_B = 0$ and $q_B = 1$.

If $p_B = r_B = 1$, we must have $p_G = 0,\ q_G = 0 \Rightarrow GPG = 0$. Contradiction.

If $p_B = 0$ and $q_B = 1$, we still have $p_G = 0,\ q_G = 0 \Rightarrow GPG = 0$. Contradiction.

Case 2 $GPB = 0 \Rightarrow GPB = p_G r_G + (1-p_G)q_G$.

This has three possibilities, which are discussed in the three subcases below.

1. $r_G = 0$, $q_G = 0$. $\Rightarrow GPG = p_G \cdot x_G$.Since $GPG > 0$, we need $p_G > 0$, $x_G > 0$.However, $r_G = 0$ and $q_G = 0 \Rightarrow p_G = 0$. Contradiction.
2. $r_G = 0$, $p_G = 1 \Rightarrow q_G > 0$ For $p_G = 1$, we need from claim 4, $BPG > \frac{1}{2}$. But if $GPB = 0$, type B responds with $q_G = r_G = 0$, implying $BPG = 0$. Contradiction.
3. $p_G = 0$, $q_G = 0$. $\Rightarrow GPG = 0$. Contradiction.

Thus, we have established that we cannot have $\mu(\theta_{-i}|\theta_i = G) = G$ in equilibrium. In the same fashion, we can show that we cannot have $\mu(\theta_{-i}|\theta_i = B) = B$ in any equilibrium with rational voting.