

External Validation by Parameter Recovery: An Application to Voter Turnout Models

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- ▶ What explains voter turnout?

Voter Turnout: Some Facts

- ▶ There is considerable (cross-section and time-series) variation in turnout both within and across countries as well as within and across types of elections.
- ▶ Turnout decreases with the size of the electorate.
- ▶ Abstention rates are correlated with demographic characteristics.
- ▶ Turnout increases with the importance of the election.

(e.g., Blais (2000), Wolfinger and Rosenstone (1980))

Voter Turnout: Basic Framework

- ▶ The “calculus of voting” framework (Downs (1957), Tullock (1967), Riker and Ordeshook (1968)): citizen i votes only if

$$p_i B_i + D_i > C_i$$

- ▶ Benefits = $p_i B_i + D_i$
 - ▶ p_i = probability the voter swings the election
 - ▶ B_i = benefit from preferred outcome realized
 - ▶ D_i = benefit from “doing one’s civic duty” or “expressing one’s views”
- ▶ Costs = C_i
 - ▶ cost of obtaining information, processing it, making a decision and going to the poll

Voter Turnout: Models

- ▶ Pivotal-voter models (e.g., Palfrey and Rosenthal (1983, 1985), Ledyard (1984), Borgers (2004)):
 - ▶ model p_i
- ▶ Ethical-voter models (e.g., Harsanyi (1980), Feddersen and Sandroni (2006), Coate and Conlin (2004)):
 - ▶ model D_i
- ▶ Uncertain-voter models (e.g., Matsusaka (1995), Feddersen and Pesendorfer (1996, 1999), Degan and Merlo (2007)):
 - ▶ model C_i
- ▶ Expressive-voter models (e.g., Brennan and Lomasky (1993))
 - ▶ different interpretation of D_i

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- ▶ Uncertain-voter models: YES
 - ▶ e.g., Degan and Merlo (2007)
- ▶ Expressive-voter models: YES and NO
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Voter Turnout: Can Models Explain Evidence?

- ▶ How to assess/compare empirical performance of models?
 - ▶ Internal validation: goodness-of-fit within sample.
 - ▶ External validation: predictions out-of-sample.
- ▶ This paper: external validation by parameter recovery using experimental data.

External Validation by Parameter Recovery

- ▶ Use experimental data (controlled laboratory environment and individual outcomes measured without error).
- ▶ Specify structural model(s).
- ▶ Perform “limited-information” estimation done under a “veil of ignorance” about some experimentally controlled parameters.
- ▶ Assess whether estimation successfully recovers the true experimental parameter(s).

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 - ▶ **In the application we focus on the distribution of voting costs, which play a critical role in voter turnout models**

Voter turnout experiments by Levine and Palfrey (2007):

- ▶ there are N voters, divided between two parties: the minority party with m voters; and the majority party with M voters; where $m + M = N$ and $M > m$.
- ▶ There is an election between two candidates, one representing each party.
- ▶ Voting is simultaneous, and each voter faces a binary decision: to vote or abstain.
- ▶ Voting is costly, and the act of voting has an idiosyncratic cost of c_i to voter i .
- ▶ Each voter receives a payoff B if the candidate from their own party wins, and receives a payoff $L < B$ if the candidate from the other party wins.

Environment (cont'd)

- ▶ The direct payoff to i , V_i depends only on the outcome of the election, whether i votes, and c_i :

$$\begin{aligned}V_i &= B \text{ if } i's \text{ party wins and } i \text{ did not vote} \\ &= B - c_i \text{ if } i's \text{ party wins and } i \text{ did vote} \\ &= L \text{ if } i's \text{ party loses and } i \text{ did not vote} \\ &= L - c_i \text{ if } i's \text{ party loses and } i \text{ did vote} \\ &= \frac{B + L}{2} \text{ if there is a tie and } i \text{ did not vote} \\ &= \frac{B + L}{2} - c_i \text{ if there is a tie and } i \text{ did vote}\end{aligned}$$

Experimental Data

- ▶ The data set consists of a series of 2350 elections conducted through computers in the CASSEL laboratory at UCLA.
- ▶ In each election, n subjects were randomly divided into two parties of size m and M , with $0 < m < M < n$.
- ▶ Each voter was independently assigned a voting cost for that election, c_i from a uniform distribution on the interval $[0, C]$.
- ▶ All of this was common knowledge among the participants of the election. Each voter's voting cost and party assignment was private information.
- ▶ All voters made independent decisions simultaneously to either vote for their party's candidate, or abstain.
- ▶ After everyone voted, the number of votes cast for each party was reported to each participant, and payoffs accrued according to V_j .

Experimental Data (cont'd)

- ▶ Subjects were recruited from the UCLA campus.
- ▶ A total of 342 subjects participated in the study, and data was gathered in 19 independent sessions.
- ▶ Each subject participated in exactly one session.
- ▶ The total number of voters, n , in an election were fixed during each session.
- ▶ In each session 100 elections were conducted, half using a landslide treatment ($M = 2m$) and half using a close-election treatment ($M = m + 1$).
- ▶ This allowed multiple (100) observations of each subject in two different treatments, and in both roles (m -voter and M -voter).

Experiment Parameters

- ▶ The experimental design had 6 different treatments, where n , m , and M were systematically varied:

n	m	M	$\#Elections$
3	1	2	850
9	3	6	450
9	4	5	450
27	9	18	200
27	13	14	200
51	17	34	100
51	25	26	100

- ▶ Additional parameters: $B = 1$, $L = 0$, $C = 0.55$.
- ▶ We treat C as the *concealed parameter*.

Data Summary

- ▶ Turnout rates:

n	m	M	p_m	p_M
3	1	2	0.54	0.57
9	3	6	0.44	0.40
9	4	5	0.48	0.45
27	9	18	0.38	0.28
27	13	14	0.39	0.36
51	17	34	0.33	0.27
51	25	26	0.39	0.36

- ▶ Lower turnout in landslide elections than close elections.
- ▶ Lower turnout by M -voters than m -voters.
- ▶ Lower turnout in larger elections.

Structural Models of Voter Turnout

All models include taste shocks to allow for stochastic choice, and the equilibrium concept is Quantal Response Equilibrium (QRE):

- ▶ Pivotal (self-interested) QRE model
- ▶ Expressive QRE model
- ▶ Group-altruism QRE model
- ▶ Group-ethical QRE model
- ▶ Mixed (Pivotal + Group-Ethical) QRE model

We also consider versions of each of these models that allow for one dimension of unobserved heterogeneity across the voters.

Pivotal QRE Model

- ▶ Individuals are self-interested and maximize their own expected payoffs.
- ▶ Restrict attention to quasi-symmetric equilibria (all voters in the same party use the same strategy).
- ▶ Let p_m and p_M denote the ex ante probability that a randomly selected voter from party m or M turns out to vote.
- ▶ Given (p_m, p_M) we can compute i 's pivot probability, which is the sum of the probability of a tie and the probability the other party is ahead by one vote, if i does not vote (which differs depending on i 's party): for example, $Q^m =$

$$\sum_{k=0}^{m-1} \binom{m-1}{k} \binom{M}{k} (p_m)^k (1-p_m)^{m-k-1} (p_M)^k (1-p_M)^{M-k} +$$

$$\sum_{k=0}^{m-1} \binom{m-1}{k} \binom{M}{k+1} (p_m)^k (1-p_m)^{m-k-1} (p_M)^{k+1} (1-p_M)^{M-k-1}$$

Pivotal QRE Model (cont'd)

- ▶ In a QRE, depending on the voting cost, voter i 's turnout decision, $q_i(c_i)$ is a continuous and strictly decreasing probability, which is exactly equal to .5 at the indifference point (i.e., when the cost of voting is equal to the utility benefit of breaking a tie, $\frac{1}{2}$ times the probability of being pivotal),

$$q^m(c_i) = \frac{1}{1 + e^{\lambda(c_i - \frac{1}{2}Q^m)}}$$

$$q^M(c_i) = \frac{1}{1 + e^{\lambda(c_i - \frac{1}{2}Q^M)}}$$

and

$$p_m = \int_0^C \frac{q^m(c)}{C} dc$$

$$p_M = \int_0^C \frac{q^M(c)}{C} dc$$

Expressive QRE Model

- ▶ Inject a direct benefit d from expressing one's opinion into the voting calculus of self-interested voters, which partially offsets voting costs: a voter compares the expected benefits of voting to $c_i - d$.
- ▶ The equilibrium analysis is identical to the one in the previous model, with the single exception that the support of the distribution of voting costs is shifted down by d .

Group-Altruism QRE Model

- ▶ Utility to i is given by a weighted sum of i 's own payoff and the total payoff to other members of the same party:

$$\begin{aligned}U_i &= (1 - \alpha)V_i + \alpha V^m \text{ if } i \in m \\ &= (1 - \alpha)V_i + \alpha V^M \text{ if } i \in M,\end{aligned}$$

where the total group payoffs are given by:

$$\begin{aligned}V^m &= \sum_{i \in m} V_i \\ V^M &= \sum_{i \in M} V_i\end{aligned}$$

Group-Altruism QRE Model (cont'd)

- ▶ Characterization of a QRE is similar to the one in the pivotal model except that:

$$q^{am}(c_i, \alpha) = \frac{1}{1 + e^{\lambda(c_i - \frac{1}{2}(1 - \alpha + \alpha m)Q^m)}}$$
$$q^{aM}(c_i, \alpha) = \frac{1}{1 + e^{\lambda(c_i - \frac{1}{2}(1 - \alpha + \alpha M)Q^M)}}$$

Group-Ethical QRE Model

- ▶ Each individual is a group-utility maximizer: the ethical choice is defined by the strategy that would maximize the expected total benefits to the members of one's own group, assuming all other voters in one's own party followed it, taking as given the strategy used by all voters of the other party (who are also ethical, but with respect to a different group).
- ▶ Given p_M , which characterizes the expected turnout rate of the M -voters, the utility to an ethical m -voter from choosing ethical rule c^{em} is:

$$\begin{aligned} V^m(c^{em}|p_M) &= m \left(W^m(c^{em}, p_M) - \int_0^{c^{em}} \frac{c}{C} dc \right) \\ &= m \left(W^m(c^{em}, p_M) - \frac{(c^{em})^2}{2C} \right) \end{aligned}$$

where $V^m(c^{em}|p_M)$ is the probability that the minority candidate wins plus $\frac{1}{2}$ times the probability of a tie.

Group-Ethical QRE Model (cont'd)

$$V^m(c^{em}|p_M) =$$

$$\sum_{k=0}^m \binom{m}{k} \binom{M}{k} \left(\frac{c^{em}}{C}\right)^k \left(1 - \frac{c^{em}}{C}\right)^{m-k} (p_M)^k (1 - p_M)^{M-k} +$$
$$\sum_{k=1}^m \sum_{j=0}^{k-1} \binom{m}{k} \binom{M}{k} \left(\frac{c^{em}}{C}\right)^k \left(1 - \frac{c^{em}}{C}\right)^{m-k} (p_M)^j (1 - p_M)^{M-j}$$

Group-Ethical QRE Model (cont'd)

- ▶ Given the taste shocks associated with each possible ethical rule, in a QRE, the probability that an m -planner chooses $c^{em} = \hat{c}$ equals:

$$\pi^m(\hat{c}) = \frac{e^{\lambda(W^m(\hat{c}, p_M) - \frac{\hat{c}^2}{2C})}}{\int_0^C e^{\lambda(W^m(x, p_M) - \frac{x^2}{2C})} dx}$$

- ▶ It follows that the probability an m -voter with cost c will vote is equal to the probability the m -planner chooses $\hat{c} \geq c$

$$q^{em}(c) = \int_c^1 \pi^m(\hat{c}) d\hat{c}$$

and

$$p_m = \int_0^C \frac{q^{em}(c)}{C} dc$$

Mixed QRE Model

- ▶ A known fraction of the electorate $\eta \in [0, 1]$ consists of ethical voters, and the remaining fraction consists of self-interested voters.
- ▶ The problems solved by each type of voters are modified accordingly.

Models Summary

Model	Parameters	Random eff.	Random eff. distr.
Selfish	C, λ	-	-
Expressive*	C, λ, d	-	-
Altruism	C, λ, α	-	-
Ethical	C, λ	-	-
Mixed	C, λ, η	-	-
H-Selfish	C, λ, a, b, L	λ_i	<i>Beta</i> $[0, L]$
H-Expressive	λ, a, b, d	d_i	<i>Beta</i> $[0, D]$
H-Altruism	C, λ, a, b	α_i	<i>Beta</i> $[0, 1]$
H-Ethical	C, λ, a, b, L	λ_i	<i>Beta</i> $[0, L]$
H-Mixed	C, λ, η, a, b	η_i	<i>Beta</i> $[0, 1]$; $\frac{a}{a+b} = \eta$

Estimation

- ▶ Each model is estimated by maximum likelihood.
- ▶ We try to establish a “level playing field” for all the models, giving each model its best chance to fit.
- ▶ Ideally, we would like the various models to fit the data equally well.

Model	-loglik	C (SE)	λ (SE)	Other Parameter (SE)	-loglik when C=0.55
Selfish	21879	0.5764 (0.0135)	4.0015 (0.2269)	--	21881
Altruism	21878	0.5826 (0.0145)	4.1759 (0.2296)	$\alpha = 0.0052$ (0.0039)	21881
Ethical	21945	6.4192 (0.3387)	0.0892 (0.0102)	--	23186
Mixed	21587	0.6958 (0.0070)	Selfish: 5.4882 (0.1367) Ethical: 0.1331 (0.0124)	$\eta = 0.1854$ (0.0156)	21616

Model	-loglik	C (SE)	λ (SE)	Mean of rand. eff.	Other Parameters	-loglik C=0.55
H-Selfish	21001	1.8147 (0.1354)	--	1.122	$\alpha = 0.4788$ (0.0481) $\beta = 3.7869$ (0.8216)	21152
H-Expressive	20666	1.6310 (0.0829)	19.2202 (15.0610)	0.484	$\alpha = 1.7519$ (0.1864) $\beta = 2.5718$ (0.4020) D = 1.1948 (0.0966)	21025
H-Altruism	20942	0.8385 (0.0087)	13.1518 (1.1139)	0.103	$\alpha = 0.5506$ (0.0532) $\beta = 4.8102$ (0.3999)	21174
H-Ethical	20982	37.9997 (4.3947)	--	0.023	$\alpha = 0.5140$ (0.0599) $\beta = 22.1065$ (4.0627)	23185
H-Mixed	20846	1.3901 (0.0153)	Sel: 21.3185 (1.7937) Eth: 8.0075 (0.6412)	0.734	$\alpha = 0.8397$ (0.0458) $\beta = 0.3038$ (0.0092)	21233

Results

- ▶ Result 1: The Selfish model without heterogeneity is the only one that successfully recovers the cost distribution parameter C .
- ▶ Result 2: All other models over-estimate C , generating estimates that are at 10 or more standard deviations too high.
- ▶ Result 3: The Altruism model without heterogeneity is clearly rejected in favor of the Selfish model (the two models are nested).
- ▶ Result 4: The Ethical model is much worse than the other models at recovering C .
- ▶ Result 5: Adding heterogeneity leads to better fits, but worse parameter recovery.

Conclusions

- ▶ Assess the policy implications.