

Collective and Private Provision of Public Goods in Democracy

Christoph Lülfesmann*

Simon Fraser University

June 2008

Abstract

This paper analyzes the provision of goods with consumption externalities in hybrid institutional settings: the provision level (the ‘policy’) is chosen in democratic fashion by majority vote, but individual agents are free to contribute additional amounts before or after the collective decision is made. Prominent examples include policy making in federal states, charities, education, and health care. A key result establishes that regardless of the timing of private and public actions, the results of the median voter theorem apply. Moving from a public to a dual system with ex-ante private contributions finds unanimous support in society, while adopting an ex-post contribution regime may be opposed by a minority of high-preference individuals. Importantly, a system with endogenous timing is shown to dominate both the other dual regimes, and induces a larger consumption level of public goods. Finally, we show that with externalities, dual regimes are also majority preferred over a market-based private system.

JEL Classification: D02, D78, H11, H40, P16.

Keywords: Public goods, Majority voting, private provision, dual provision, federalism, charities, health care.

*Department of Economics, Simon Fraser University, Vancouver B.C. email: cluelfes@sfu.ca. Thanks to Richard Rogerson, Berthold Herrendorf, Anke Kessler, and seminar participants at Arizona State, Bonn, Munich, Simon Fraser, Vienna, Cologne and Berlin for helpful comments and suggestions.

1 Introduction

Democratic policy decisions are often supplemented by private actions. Charities step in where, in the view of the donors, tax dollars do not guarantee a sufficient scale of services. Countries with a tax-financed public health care system often see individuals seek enhanced services from private doctors or hospitals. Various spending decisions or regulatory measures in federal states are taken at a central level of government, but member regions are allowed to augment these policies at their own discretion. While some of these ‘private contributions’ have the characteristic of a private good whose benefits accrue only to the contributor, in many cases there are positive consumption externalities from one agent, or one federal region, to the other. A new or upgraded road or airport delights visitors from other countries, in the same way as helping the City’s poor may reduce crime rates, positively affecting more affluent citizens in the community. When a state government tightens federal regulation on vehicle or industry pollution, neighboring states are likely to benefit from the adoption of this local policy measure.

The aim of the present paper is to explore situations of this type in a stylized model. A group of agents makes a decision to provide goods or services in a democratic fashion, using majority voting to determine the level of spending. Each agent can also enhance the consumption of this good, by privately buying additional amounts. Public and private provision may generate consumption externalities across all agents in society. This framework follows a body of research (for a brief review, see below) that combines two baseline models often used in the literature: the democratic choice of policymaking under majority rule on the one hand, and voluntary private contributions to public goods on the other. While merging these two topics in a unified setting is of significant interest, overcoming the technical hurdles may be seen as challenging: since the voting and contribution decisions of all agents are interrelated and subject to strategic behavior, it is a priori unclear whether the median voter theorem as a convenient tool to characterize political outcomes remains applicable.¹

¹Stiglitz (1974) and Glomm-Ravikumar (1998) show that single-peakedness usually does not prevail in models where individuals can choose whether to consume a publicly provided private good such as

A main contribution of the paper is to show that these difficulties can be overcome in various versions of the basic model which is analyzed here. We first explore a setting where private contributions are made simultaneously and non-cooperatively in a first stage, before a political decision on the uniform financing of the public good is taken in a second stage. In this regime with ex-ante contributions (or for short ‘ex-ante regime’), individual preferences at the policy stage are single peaked for any arbitrary profile of initial contributions. The median voter theorem then applies and the individual with median preferences determines the policy outcome. In contrast to the familiar majority voting scenario, though, the identity of the median voter is endogenous: it depends on the vector of initial private contributions, specifically because an individual’s private buy decision in stage one changes her subsequent policy preferences relative to those of other individuals. Nevertheless, we establish a ‘rank preservation’ property under which in equilibrium, the ‘natural’ median individual (the median voter under pure public provision without private contributions) always remains the pivotal decision maker. With this result, one can then immediately show that only high-preference individuals voluntarily contribute ex ante. These individuals are well aware of the crowding out effect of their contributions. As a consequence, ex ante equilibrium contributions will be smaller than those in a private market, and smaller than the best responses to the equilibrium policy. Also, the equilibrium policy is shown to be smaller and each agent’s public goods consumption to be larger than in a pure public system where private topping up is infeasible.

The paper then investigates an economic environment with reverse timing, an ‘ex-post regime’ where democratic decisions occur before individual contributions can be made. This setting allows for a policy commitment vis-a-vis individual agents, and is the focus of much of the existing literature. Unfortunately, this scenario suffers from the fact that in presence of externalities, individual preferences over policies are not necessarily single peaked. This is because for different policies, the profile of subsequent private contributions and, potentially, the set of contributors changes. An individual may be a contributor for a small but not for a large policy, making the shape of his utility function difficult to predict. A key finding of our paper is that notwithstanding this schooling, or to consume a private alternative instead.

problem, the results of the median voter theorem continue to hold. To establish this result, we show that all individuals with preferences larger than the median prefer the policy that is globally preferred by the median individual over any smaller policy. In addition, all lower-preference agents prefer the median voter's bliss point over any larger policy. As a consequence, the policy preferred by the natural median cannot be defeated in majority voting, which makes her the democratic decision maker in society. This result then allows a precise characterization of equilibrium contributions and equilibrium policies. Among other things, equilibrium policies are shown to be smaller than in the ex-ante regime, while private contributions are larger.

An interesting aspect is the economic comparison of different institutional regimes. Both ex ante and ex post regime have in common that only a subset of high-preference agents make private contributions, while all other individuals including the median and all people with lower preferences do not.² However, private contributions in the ex-post setting are larger than in the ex-ante setting, while the reverse pattern characterizes the respective equilibrium policies. These findings can easily be understood in terms of the commitment capabilities assigned to the relevant agents. In the ex-post setting, the median voter as the pivotal player knows that implementing a relatively small policy will trigger large contributions from high-preference individuals. Reducing public provision boosts the median voter's private-goods consumption, while the associated loss in public good consumption is at least partially mitigated through enhanced private contributions of high-preference individuals. Conversely, in the ex-ante setting where private contributions are made first, each potential contributor knows that lowering his private contribution induces a larger public provision. Hence, high-preference individuals can partially free ride on public provision.³ These different commitment capabilities induce strictly larger equilibrium policies in the ex-ante regime as compared

²Hence, a majority of the population never makes a private contribution. In general (and in presence of externalities), even a subset of agents with larger-than-median-preferences will not privately contribute because in contrast to the preferences revealed in the political process, a private contribution does not involve cost sharing with other agents.

³For the polar case of pure public goods, it is also relatively easy to compare total public goods consumption in either regime: while it is larger in the ex-post regime when income effects are absent and if the set of contributors is non-empty in each regime, the opposite can happen otherwise.

to the ex-post regime. Moreover, equilibrium policies in both dual regimes are smaller than those under purely public provision: with normal preferences, the median voter's private goods consumption cannot be lower in a dual system where higher-preference individuals make additional contributions into public goods consumption. Accordingly, in each dual regime, the median voter will scale back public funding.

From a political economy perspective, it is important to scrutinize the relative support of different regimes by the citizens in society. For a regime change to happen, a well defined majority or super-majority of agents must prefer some alternative institution over the status quo. Again, our model allows to derive interesting and clear cut results. A majority of citizens, comprising all individuals with low preferences for the public good, prefers the ex-post regime over both the ex-ante regime and pure public provision. However, a minority of high-demand agents may oppose a transition from the pure public to the system with ex-post contributions, in fear of exploitation by the majority that controls political decisions. Strikingly, no such resistance occurs with a system change from the pure public to the ex-ante contributions regime. In fact, we show that all agents unanimously supports this transition. This result may provide some guidance for the direction of policy reforms in areas where the ex ante regime is likely to apply. Adopting a dual provision system can be a Pareto improvement if designed in the right way: after the public decision is taken, no additional private contributions should be allowed, while these contributions are encouraged prior to the collective decision.

As a final step, we endogenize the timing and allow agents to make private contributions before and after the policy is chosen. This institutional regime is the most natural in environments where governments are unable to control the amount and timing of private contributions. Immediate intuition suggests that economic outcomes must coincide with those for the ex-post setting; after all, the median voter can adopt the same policy, forcing high-demand citizens to privately contribute at the final stage. Perhaps surprisingly, this argument is misleading. In fact, we show that the endogenous timing regime will often dominate the ex-post setting at least for a majority of the population, including all high-demand agents. The explanation is based on a reciprocal commitment argument. The ex-ante contribution stage allows high preference individuals to

provide contributions at a level which convince the median voter not to expect additional ex-post contributions. Ex-ante contributions thus offset the median agent's incentive for strategic underprovision, leading her to adopt a larger policy than in the ex-post setting.⁴ Since in addition, a majority of citizens prefers the 'general' system over the ex-ante regime, this institutional design emerges as the only stable institution under majority vote.

2 Brief Review of the Literature

Our paper is part of the growing literature on the allocative and political economy implications of a dual private-public provision of goods and services. Most existing papers confine attention to what is here called the ex-post system: first the political decision is made by majority rule, before individuals can privately contribute in a second stage. Epple and Romano (1996) pioneered the analysis of this setup. They explore a dual health care system, with a public-provision element being funded through linear income taxes, chosen in the democratic process. The equilibrium provision of public services in the hybrid system is shown to be larger than in a purely public system, or in a market based private system. In addition, a majority of citizens strictly prefers the dual regime. Combining theoretical analysis with empirical testing, Fernandez and Rogerson (2003) compare a variety of school funding systems in a dual provision setting. For institutional setups with private, public, or dual ex-post regime, their theoretical results largely coincide with those in Epple and Romano.⁵

In both of these papers, the relevant good is a private good which imposes no consumption externalities across agents in society. For this reason, the political choice is not implicated by strategic behavior on the voters' parts, which is shown to guarantee

⁴The equilibrium policy is then a best response to the (and only the) ex-ante private contributions. In contrast, in the ex-post setting, the equilibrium policy choice is strictly lower than the best response to the equilibrium ex-post contributions: for strategic reasons, policy underprovision arises in order to stimulate subsequent private contributions.

⁵Their paper also examines several other possible regimes, and provides a thorough empirical analysis.

the single peakedness of individual preferences and the validity of the median voter theorem.⁶ Recognizing the prevalence of externalities, Epple and Romano (2003) generalize their previous research, and analyze the case of a pure public good. In order to make the strategic effects of voting tractable, individuals are assumed to behave ‘myopically’ in the sense of disregarding the effect of their first-stage political votes on second-stage private contributions. In this setting with not fully rational agents, equilibrium does not necessarily exist but if it does, the natural median voter is the pivotal individual. In a related framework and analyzing policymaking in federations, Hafer and Landa (2005) find that the lack of preference single peakedness generally prevents the applicability of the median voter theorem. Assuming Cobb-Douglas preferences, they derive sufficient conditions for equilibrium existence, and provide a variety of additional characterization results.

Closest to the present paper are those by Cremer and Palfrey (2000, 2006), and by Alesina et al. (2005). In line with our assumptions, public provision is financed through uniform lump sum taxation. All these papers are set in the context of federal systems. Cremer and Palfrey investigate ‘federal mandates’, implemented by a central government via majority vote among individual regions. Essentially, a federal mandate imposes a minimum or maximum standard on a policy whose costs are incurred by individual states. These standards can subsequently be tightened by state legislation, which in our setting corresponds to a private contribution. In Cremer and Palfrey (2000), policy choices and private contributions do not yield external effects across regions, and the median voter theorem applies. It is shown that equilibrium mandates are tighter than desired by a majority, and some high-demand states augment the mandate by imposing even more rigid standards. Cremer and Palfrey (2006) allow policies to cause positive externalities. According to their main finding, majority vote equilibria cannot be guaranteed to exist. To circumvent this problem, the paper characterize ‘local’ policy equilibria, defined as the set of policies which is majority

⁶Note, though, that even in a setting where voters have identical references, the median voter need not coincide with the median income individual. Under income taxation, rich individuals may in fact favor lower taxes because they face a larger absolute tax burden than poorer agents.

preferred to other policies in the vicinity.⁷ Alesina et al. (2005) consider a framework similar to Cremer and Palfrey, but with homogenous agents within districts. Like our paper, their work explores not only the ex-post regime, but also the ex-ante regime. While they find that in general only a majority of agents prefers the ex-ante system over public provision, we show that in fact, the ex-ante system is preferred unanimously. Moreover, their paper acknowledges that all results require the validity of the median voter theorem, but do not prove its validity.⁸

Our paper extends existing results in showing that in dual institutional systems, the median voter theorem often holds even in presence of consumption externalities, and with fully rational voters. In particular, this is true if policies are financed through uniform funding.⁹ With regard to empirical relevance, uniform taxation and redistributive taxation are both used in a variety of circumstances. Central funding in federations is sometimes (not always) based on the per capita wealth of individual member states; contributions to a state funded health care system are made lump sum in Canada, but through progressive taxes in many European countries. Federal mandates are usually imposed uniformly of all member states within a federation. Unless there are reasons to believe that implementation costs systematically differ across states, uniform taxation appears as a good first pass on this scenario.

The remainder of the paper proceeds as follows. Section 3 below describes the baseline model. Section 4 explores some benchmark scenarios. Section 5 provides a general analysis of the systems with ex-ante and ex-post contributions, and Section 6 highlights some special cases. Section 7 focuses on the endogenous timing of contributions, and

⁷There is one significant difference between the settings analyzed in Cremer and Palfrey, and our paper. While in Cremer and Palfrey (2006) each voter in each region participates in federal policy decisionmaking, our setting can be interpreted as one where only the district medians cast their federal votes. Our results suggest that this seemingly minor modification restores the existence of global majority vote equilibrium.

⁸Alesina et al. acknowledge that the theorem may not hold (See p. 614 of their paper).

⁹Under income taxation, high-income individuals face a tradeoff between their higher demands and the redistributive aspects of higher taxes. In contrast to our setting, it is therefore possible for higher-income (and thus higher-preference) individuals to prefer smaller policies, making it more difficult to obtain transparent results. Note also that in absence of income heterogeneity, lump sum taxation and income taxation are the same.

Section 8 briefly compares dual regimes and a market system. Section 9 concludes.

3 The Model

Consider an economy with $N \geq 3$ agents, where N is odd. Individual i derives his utility $U^i(x_i, G_i)$ from the consumption of a private good x_i , and a public good G_i which will be defined more precisely below. The utility function is quasiconcave, and both x and G are normal goods according to i 's preferences. Moreover, we impose the standard Inada conditions and normalize the prices of both public and private good to one. Agents may differ in their exogenous incomes, y_i , and in their tastes for the public good.¹⁰

All agents play a version of the following general game. In a stage 1, they can simultaneously and non-cooperatively make private 'ex-ante' contributions \hat{g}_i towards the public good. These contributions become public information of all agents, and they can alternatively be viewed as individual i 's private purchase and consumption of the commodity, with spillover effects to other agents.¹¹ In a subsequent stage 2, all individuals decide in a political decisionmaking process on a mandatory uniform lump-sum contribution g that is imposed on each agent. For concreteness, we follow the positive literature on public goods supply and assume that g is chosen by majority vote. Finally, in stage 3, individuals can simultaneously expand public-goods consumption by making additional 'ex-post' contributions, \tilde{g}_i . Agent i 's consumption of the private good is then given as $c_i = y_i - g - \hat{g}_i - \tilde{g}_i$. As we have already emphasized in the Introduction, an analysis of private ex-ante and ex-post contributions allows us to explore the commitment effects of policy choices and of private consumption decisions, respectively.

Regarding the characteristics of commodity G , we will allow for pure and impure

¹⁰Throughout the paper, we assume incomes to be sufficiently large that each individual is able to pay his tax contribution. This assumption is common to all papers in the literature.

¹¹In both interpretations, the 'public good' may be offered by suppliers in a competitive market, or it may be produced in home production where all individuals have access to the same technology.

forms of public goods or services. The public goods consumption of each individual is composed of a uniform amount G^U which is provided to everyone through the political process, of the private contributions of the individual himself, and of the contributions of other individuals. In particular, $G^U = g(1 + \beta(N - 1))$ where $\beta \in (0, 1)$ indicates the degree of consumption externalities. For $\beta = 0$, there are no spillovers and G^U becomes a private good with uniform consumption requirement. For $\beta = 1$, the commodity in question is a pure public good, while all interior specifications of β capture intermediate characteristics. The overall public good consumption of a citizen i is then

$$G_i = G^U + [\hat{g}_i + \tilde{g}_i + \beta \sum_{j \neq i} (\hat{g}_j + \tilde{g}_j)],$$

where private contributions of individuals $j \neq i$ again cause spillovers at a rate β . Note that unless $\beta = 1$ where $G_i = G$ for all i , individuals who contribute more towards the public good have a higher overall consumption.

Let g be a public policy and \hat{g} and \tilde{g} be the vectors of ex-ante and ex-post private contributions, respectively.¹² For subsequent reference it is useful to define

$$\Delta^i(g, \hat{g}, \tilde{g}) = \frac{U_G^i(c_i, G_i)}{U_C^i(c_i, G_i)}$$

as individual i 's marginal rate of substitution between the private and the public good. We impose

Assumption 1. (Single crossing property) *For any g and \hat{g}, \tilde{g} with the property $\hat{g}_i = \hat{g}_j, \tilde{g}_i = \tilde{g}_j$ for all i, j , the rank order of the vector $\Delta(g, \hat{g}, \tilde{g}) = \{\Delta^i\}_{i=1, \dots, n}$ remains unchanged. We say that an individual $i = 1, \dots, n$ with a higher index exhibits 'larger preferences' for the public good.*

The single crossing assumption allows us to order individuals according to their 'natural' rank in terms of preferences towards the public good: for any arbitrary public-good

¹²In what follows, we assume that $g \in [0, \bar{g}]$ where the maximum policy \bar{g} is smaller than the income of the poorest individual in society. Alternatively, we could assume that an agent's policy contribution is $\min\{y_i, g\}$, and close the model by imposing a sufficiently harsh punishment in case that the individual pays less than y_i . This latter specification would not alter any qualitative results but significantly complicate the exposition.

level generated by identical funding from each individual, the ranking of the marginal rates of substitution across individuals remains the same. Notice that the definition is flexible enough to accommodate not only taste, but also income differences. For example, consider two individuals with identical homogenous preferences but different incomes. For identical individual contributions to the public good, their private consumption differs, and the lower-income individual displays a smaller $\Delta(\cdot)$ and is considered a lower-preference individual. Another important class of preferences consistent with Assumption 1 are quasilinear utilities, where agent preferences and incomes are heterogenous: $U^i = c_i + \alpha_i H(G_i)$, with $\alpha_i > 0$ being a preference parameter for the public good. In this representation, an individual with larger α_i is characterized by a larger index i . In what follows, we will say that individual m is the ‘natural’ median individual according to the ordering described above.

4 Benchmark cases

Before providing a general analysis, we briefly investigate the two simple benchmark cases of pure public provision, and of a situation where G is a private good. In the first of these frameworks, the consumption of the public good is determined exclusively through collective decision-making. In the second benchmark, no externality problem exists and, as a consequence, a private provision system that imposes no collective force would implement a Pareto optimum.

4.1 Pure public Provision

Suppose public goods are provided exclusively through the political process, that is, no private consumption decisions can be made. Each individual i prefers a policy outcome $g_i^{pp} = \arg \max_g U^i(y_i - g, g(1 + \beta(N - 1)))$, which is implicitly defined by the first-order condition $\Delta_i(\cdot) = U_G^i(c_i, G_i)/U_c^i(c_i, G_i) = 1/(1 + \beta(N - 1)) \equiv 1/z \leq 1$. Under our previous assumptions, the single-peakedness requirement on individual preferences is satisfied so that the median voter theorem applies. Moreover, since Δ^i and thus g_i^{pp}

are increasing in the index i , the preferences of individual m with median bliss point represent the unique outcome of majority voting. We have

Proposition 1. *Consider pure public provision of G . The unique equilibrium policy g_m^{pp} is determined by the preferences of the natural median voter m , and is implicitly given by*

$$\Delta^m = \frac{U_G^m(y_m - g_m^{pp}, g_m^{pp}(1 + \beta(N - 1)))}{U_c^m(y_m - g_m^{pp}, g_m^{pp}(1 + \beta(N - 1)))} = \frac{1}{1 + \beta(N - 1)} = \frac{1}{z}. \quad (1)$$

Under the Inada conditions, the selected policy g_m^{pp} is interior. Moreover, note that in the special case where individuals differ only in their incomes, normality ensures the median-income individual to be the median voter m in the community.

4.2 Purely private goods

Consider now the special case of a purely private good, $\beta = 0$. This case has received significant attention in the existing literature on dual institutions. Since consumption externalities are absent, voting is not subject to strategic behavior, a feature that considerably simplifies the analysis.

Before presenting results for dual institutional regimes, suppose individuals privately take their consumption decision and there is no collective provision of the ‘public’ good in question. In such a pure private market system, in unique equilibrium each individual buys and consumes the ‘public’ good at a level G_i^* where his marginal rate of substitution Δ_i equals the marginal costs. Since there are no externalities, this outcome is clearly Pareto efficient. Next, suppose good G is provided through a pure public system. As found in the previous subsection, the median voter then determines the uniform provision and funding level g_m , given by $\Delta_m(\cdot) = 1$. In this allocation, any individual rather than m does worse compared to a private market system: lower-preference agents consume too much, and higher-preference agents consume too little.

With this pretext, let us now find out the consequences of dual private-public provision regimes. Our analysis yields

Proposition 2. *Suppose $\beta = 0$. In both the ex-ante and the ex-post regime, there exists a continuum of equilibria, characterized by equilibrium policies $g^* \in [0, g_m^{pp}]$. In addition,*

- a) *Each high-preference agent with rank $i \geq m$ consumes the individually efficient amount of ‘public’ goods G_i^* , as given by $\Delta^i(G_i^*) = 1$.*
- b) *For any equilibrium policy g^* satisfying $g > g_1^{pp}$, a subset of low-preference agents with ranks $j < m$ is forced to overconsume, $\Delta^j(G_j^*) < 1$. The size of this subset strictly increases in g^* , and it comprises all individuals $j < m$ if $g^* = g_m^{pp}$.*

Proof: Consider the ex-post regime and a policy $g \leq g_m^{pp}$. For any such stage-1 policy outcome, each individual k finds it optimal to privately contribute $\tilde{g}_k = \max\{g_k^{pp} - g, 0\}$ in stage 2. Hence, agent k overconsumes relative to the privately efficient level G_k^* if $g > g_m^{pp}$, while k is able to achieve his privately optimal consumption G_k^* otherwise. For this reason, any $g \in [0, g_m^{pp}]$ can be supported as an equilibrium policy: each individual $i \geq m$ and thus a majority of agents is indifferent among policies from this set but strictly prefers each of them over any $g > g_m^{pp}$. Conversely, no $g > g_m^{pp}$ is a candidate for policy equilibrium because it would be majority rejected.

Next, consider the regime with ex-ante contributions. In this setting, no $g > g_m^{pp}$ can be the chosen equilibrium policy in stage 2. This is because first, in absence of private stage-1 contributions, any such policy is dominated by majority. Second, each individual’s preferred policy is single-peaked, decreasing in his own contribution, and (since $\beta = 0$) unaffected by the contributions of other agents. To the contrary, any policy $g \leq g_m^{pp}$ can be supported as a majority voting equilibrium.

To see this, suppose individual $k \geq m$ believes the date-1 contributions of all other individuals to be given by some vector g_{-k} . Specifically, g_{-k} is such that if k contributes $\hat{g}_k = G_k^* - g^*$ (for any arbitrary $g^* \leq g_m^{pp}$), the associated stage-2 majority equilibrium policy will be g^* . Note that for any such g^* , the assumed vector \hat{g}_{-k} always exists. In particular, individuals j with $G_j^* \leq g^*$ contribute nothing, whereas all other individuals i contribute $\hat{g}_i = G_i^* - g^*$. (In stage 2, the contributors then form the majority and have identical preferences regarding the optimal policy, supporting g^* .) Next, verify

that given the expectations on \hat{g}_{-k} , agent k 's best stage-1 response is indeed to provide $g_k = G_k^* - g^*$. Similarly, no other individual has an incentive to deviate from the assumed stage-1 behavior, and in equilibrium a majority of the population prefers policy g^* over any alternative policy. In any such equilibrium, a majority of agents including any $i \geq m$ achieves G_i^* . Conversely, a subset of minority individuals (with size increasing in g^*) does not privately contribute but is forced to overconsume relative to the first best whenever $g^* > g_1^{pp}$. \square

The Proposition conveys that in the private-goods case, there exists a continuum of equilibrium outcomes. In addition, regardless of the timing of private contributions, these equilibria have the same characteristics regarding contribution vectors and policies. Each of these equilibria allows individuals with preferences larger than the median to replicate the efficient consumption patterns that prevail in a private market system, by equating marginal rate of substitution and marginal rate of transformation. In contrast, a subset of low-demand agents consumes and pays more of the publicly provided good than desired. Not surprisingly, the private market system thus Pareto dominates any system with collective provision if consumption spillovers play no role.¹³

Notice also that the upper bound of equilibrium policies, policy g_m^{pp} , coincides with the unique equilibrium under pure public provision. But this immediately implies both dual-provision regimes to Pareto dominate pure public provision. Agents with rank larger than m are strictly better off because they can enhance their consumption of the 'public' good. At the same time and depending on the specific equilibrium, some lower preference individuals increase their utilities by reducing (or even avoiding) overconsumption.

Before concluding, it is worth emphasizing that in contrast to our setting where taxes are uniform, multiplicity of equilibria does not arise in a setting with income-dependent funding.¹⁴ For example, with linear income taxes as analyzed in Epple and Romano

¹³Of course, this is strictly true for equilibrium policies larger than g_1^{pp} . For smaller policies, the dual provision regimes and the market system of course have identical welfare consequences.

¹⁴Cremer and Palfrey (2000) consider uniform taxation. They circumvent the multiplicity issue by assuming that regional populations are heterogenous, but no individual knows his place in the

(1996) and Fernandez and Rogerson (2003), individuals have strict preferences over tax rates.¹⁵ The next Sections will show that in a more general framework that allows for externalities, unique equilibria also prevail in the present model.

5 General Analysis

5.1 Dual System with Ex-ante contributions

Our analysis of dual public-private institutions starts with a setting where only ex-ante private contributions \hat{g}_i are feasible or relevant. For example, the public good may be a new hospital, opera, or other public construction project whose technical specifications (design, size etc.) are irreversibly determined in the stage-2 political process.¹⁶ The ex-ante regime also captures situations in which individual agents - such as states in a federation, or charities - act as ‘leaders’ who commit to a contribution level before political decisions on additional funding are made. In the context of charity work, for instance, the extraordinary capital endowment of the Bill and Melinda Gates foundation make this organization a strategic leader regarding health aid to developing countries. Similarly, in the context of the EU, large states such as Germany or France are sometimes said to preempt collective decisions at the federal level in a variety of policy fields of common interest.

Using subgame perfection as the appropriate equilibrium concept, we first analyze the stage-2 political equilibrium, before going back to the initial private-contributions stage. Two crucial issues need to be addressed. First, it has to be shown that individual regional preference distribution. Voting in favor of the individually preferred consumption level is then dominant strategy at the policy stage.

¹⁵While individuals with higher-than-average incomes strictly prefer zero public provision, lower-income individuals benefit from the tax contributions of richer individuals, and each of them favors a type specific positive tax level. Note, however, that the preferences of below-average income agents are not necessarily monotone in income: while more affluent individuals within this group *ceteris paribus* favor larger consumption, poorer individuals pay a smaller unit tax price.

¹⁶The political decision on project design then essentially determines the scale of the public good, and additional private contributions would leave consumption levels of this good unaffected.

stage-2 preferences are single-peaked so that majority voting equilibrium exists and the preferences of the (endogenously chosen) median voter prevail in political equilibrium. Second, upon establishing single-peakedness, one must ask whether in equilibrium, the ‘natural’ median m or some other individual actually becomes the chosen median voter. Since each individual’s policy preferences in stage 2 depend on the voluntary contribution profile of all agents, the identity of the median voter is endogenous and the answer to this latter question is not immediate.

We start with the following lemma.

Lemma 1. *For any arbitrary stage-1 contribution profile $\hat{g} = \{\hat{g}_i\}_{i=1,\dots,n}$, individual preferences in stage 2 over g are single-peaked, with bliss point g^{i*} . Hence, the choice of the individual M with stage-2 median preferences, $g^{M*}(\hat{g})$, prevails in majority vote.*

Proof: Consider an arbitrary contribution profile \hat{g} . For any such profile, an individual i ’s preferences over policies g are described by the utility function

$$U^i(y_i - \hat{g}_i - g, g(1 + \beta(N - 1)) + \beta \sum_{j \neq i} (\hat{g}_j + \hat{g}_i)).$$

Since preferences are quasiconcave by assumption, they are single-peaked in g and (if positive) the maximizer g^{i*} for individual i is described by the first-order condition

$$\Delta^i(\hat{g}_i, \hat{g}_{-i}, g^{i*}) \equiv \frac{U_G^i(\cdot)}{U_c^i(\cdot)} = 1/(1 + \beta(N - 1)) = 1/z.$$

The individual $i = M$ with ex-post median preferences then determines the policy outcome in pairwise majority vote.¹⁷ \square

To be more specific about the equilibrium outcome, we now investigate the private-contributions game in stage 1. The analysis first yields

Lemma 2. *There exists an equilibrium in which the natural median m becomes the median voter. No individual j with index $j \leq m$ contributes in stage 1, while some of the higher-demand individuals $j > m$ may contribute.*

¹⁷For sufficiently large initial contributions \hat{g} , M may prefer the corner solution $g = 0$. In this case, she shares her preferences with more than half of the population and remains decisive in stage 2.

Proof: Consider the stage-2 median voter as induced by stage-1 contribution vector \hat{g} . Call this individual M (again, remember she is not necessarily the ‘natural’ median m).

The proof is by contradiction. Suppose an equilibrium with $M = m$ exists, and no $j \leq m$ contributes. Since the lowest-preference $(n + 1)/2$ individuals do not invest in stage 1, their ranking of policies g corresponds to the natural order of their preferences, and individual m by definition exhibits the maximum bliss point $g^{m*}(\hat{g})$ within this group. Clearly, individual m then is the median individual in stage 2 unless some agent $i > m$ contributes \hat{g}_i in a way that $g^{i*}(\hat{g}) < g^{m*}(\hat{g})$.

We show that indeed no $i > m$ will make such a contribution. To prove this, note first that because of normality the preferred second-stage policy g^{i*} of any individual i is strictly decreasing in \hat{g}_i . For $\hat{g}_i = 0$, this implies $g^{i*} > g^{m*}$, irrespective of the contribution profile $\hat{g}_{-i} = \{\hat{g}_1, \dots, \hat{g}_{i-1}, \hat{g}_{i+1}, \dots, \hat{g}_n\}$ of all other individuals. Suppose one individual $i > m$ contributes in a way that $g^{i*} < g^{m*}$ and $g^{i*} \leq g^{M*}$ (note that in order to generate $m \neq M$ there must be at least one such individual i). The stage-2 optimality condition for i then implies

$$\Delta^i(\hat{g}_i, \hat{g}_{-i}, g^{M*}(\hat{g})) \leq \frac{1}{z}, \quad (2)$$

and holds with equality if $M = i$. We establish that for $\beta > 0$, this condition violates the optimality condition for i 's stage-1 investments. For contributions \hat{g}_i inducing $g^{i*} < g^{m*}$, agent i 's stage-1 first-order condition reads

$$\Delta^i(\hat{g}_i, \hat{g}_{-i}, g^{M*}) = 1 + \frac{1}{U_c^i(\cdot)} [U_c^i(\cdot) - U_G^i(\cdot)z] \frac{dg^{M*}}{d\hat{g}_i}. \quad (3)$$

The second term on the right-hand side reflects the effect of \hat{g}_i on the policy chosen by (and possibly, on the identity of) the median individual. Note that $dg^{M*}/d\hat{g}_i < 0$ whenever $\beta > 0$: with normal preferences, raising i 's contribution lowers the preferred stage-2 policy of any individual, and thus, of the induced median voter.¹⁸ In addition, $dg^{M*}/d\hat{g}_i > -1$ because otherwise, a raise in \hat{g}_i would induce a lower public

¹⁸This is true because raising \hat{g}_i can alter the identity of the median voter in only one direction: an individual k with a smaller stage-2 Δ^k might become M . This indirect effect reinforces the direct effect of increasing \hat{g}_i .

goods consumption for M , violating preference normality. From the stage-2 optimality condition (2), the term in square brackets in (3) $U_c^i(\cdot) - U_G^i(\cdot)z$ is zero if $M = i$, yielding an immediate contradiction since (2) and (3) become incompatible. Otherwise, when $i < M$ according to induced stage-2 preferences, $dg^{M^*}/d\hat{g}_i > -1$ implies $\Delta^i(\cdot) > 1 - \frac{1}{U_c^i(\cdot)}[U_c^i(\cdot) - U_G^i(\cdot)z]$. Since $\Delta^i(\cdot) = \frac{U_G^i(\cdot)}{U_c^i(\cdot)}$, this condition simplifies to $1 > z$ which is impossible. Taken together, the assumed strategy cannot be optimal for agent i , implying individual $i > m$ will never contribute in a way as to switch stage-2 preferences with m .¹⁹ To complete the argument, note that $m = M$ if she does not contribute. Hence, m will not invest since she can always consume the same amount of public goods spending a smaller amount of private resources (i.e., her stage-1 optimization yields a corner solution). Finally, since $g^{j^*} < g^{m^*}$ for any $j < m$, none of these individuals will be a contributor which proves the lemma. \square

Lemma 3. *The equilibrium as described in Lemma 2 is unique if $\beta > 0$.*

Proof: The proof of Lemma 2 implies that as long as $\hat{g}_m = 0$ in an equilibrium, the natural median is the induced median individual in stage 2, $m = M$. Specifically, for any individual $i > m$, contributions with the property $g^{i^*}(\hat{g}) \leq g^{M^*}(\hat{g})$ were shown to be incompatible with utility maximization. Suppose now there exists an additional equilibrium in which $\hat{g}_m > 0$. Two cases need to be distinguished. First, suppose that $g^{m^*} > g^{M^*}$ in this equilibrium. For this to be true, at least one individual $i > m$ has contributed in a way that $g^{i^*} < g^{m^*}$ (and $g^{i^*} \leq g^{M^*}$). But invoking the arguments of Lemma 2, contributions with this property cannot be the best response to profile \hat{g}_{-i} for any individual $i > m$. Second, suppose that $g^{m^*} < g^{M^*}$ in the assumed equilibrium. We first establish that regardless of the contribution profile \hat{g}_{-m} , individual m can always generate stage-2 references $g^{m^*} \geq g^{M^*}$: for $\hat{g}_m = 0$, $m = M$ unless one individual $i > m$ invests in a way that $g^{i^*} \leq g^{m^*}$. By choosing $\hat{g}_m = 0$, M 's stage-2 policy preferences in the assumed equilibrium are therefore smaller than those of individual m . But contributing $g_m > 0$ in a way that $g^{m^*} \leq g^{M^*}$ cannot be m 's

¹⁹For $\beta = 0$, the last Section has shown that each individual $i > m$ will invest in such a way as to share stage-2 preferences with the median-income individual. Again, Lemma 2 continues to apply because m remains pivotal in stage 2. See our discussion below.

best response to any contribution profile \hat{g}_{-m} of other agents. In analogy to the arguments in Lemma 2, for $g^{m*} \leq g^{M*}$ to apply, m 's stage-2 references would have to satisfy $\Delta^m(\hat{g}_m, \hat{g}_{-m}, g^{M*}(\hat{g})) \leq 1/z (< 1)$. But contribution \hat{g}_m then violates m 's the stage-1 optimality condition (3), yielding a contradiction. In summary, there cannot exist equilibria in which individual m contributes $\hat{g}_m > 0$; as a consequence, $M = m$ and the equilibrium characteristics given in Lemma 2 are unique. \square

We have established that the natural median individual m will be pivotal for the political decision in equilibrium, and she will provide no private ex-ante contribution on her own. The intuitive logic behind this result is that no individual with larger natural preferences has an incentive to switch preference ranks with m . Voluntarily moving ‘to the left of the median’ means that an agent’s private contribution has been too large. This is true because achieving larger consumption through private contributions is more costly for the individual than provision through the political process, in which all individuals collectively pay for the public good. But if no (high preference) individual has an incentive to switch ranks, m has no incentive to privately contribute because she becomes the pivotal voter anyway and thus, can implement her preferred public goods level in the political process without spending a large amount of private resources.

We are now prepared to summarize the equilibrium outcomes.

Proposition 3. *Suppose individuals can make ex-ante contributions \hat{g}_i into public policies. In equilibrium, the ‘natural’ median-preference individual m becomes the median voter and chooses the equilibrium policy $g^{m*}(\hat{g})$. Moreover,*

- 1) *No individual j with natural preferences $j \leq m$ contributes in stage 1. These individuals consume identical amounts of of the public good, $G_j^* = G_m^* = g^{m*}[1 + \beta(N - 1)] + \beta \sum_i \hat{g}_i$.*
- 2) *Some highest-preference individuals $i > m$ may make a private contribution in stage 1. Equilibrium contributions are rank-ordered, i.e., $\hat{g}_n \geq \hat{g}_{n-1} \geq \dots \geq \hat{g}_{m+1}$, with strict inequality for positive contributions. Accordingly, the public goods consumption of individual i is $G_i^* = G_j^* + \hat{g}_i(1 - \beta)$, which yields $G_n^* \geq G_{n-1}^* \geq$*

... $\geq G_{m+1}^*$ (with strict inequality whenever $\beta < 1$ and $\hat{g}_i > 0$ for at least one of two adjacent individuals).

While most of these results have been established above, the monotonicity characteristics of equilibrium contributions are proved in the Appendix. In fact, monotonicity of contributions in type is very intuitive. Higher-preference individuals make larger private contributions, and therefore consume larger quantities of the public good in equilibrium.

It is instructive to compare the economic outcomes achieved in the hybrid regime with ex-ante contributions, to those of pure public system where no private contributions are allowed. This is done in

Proposition 4. *Consider the dual ex-ante system (superscript ea). In comparison to a public system (superscript pp), we obtain the following results:*

- 1) *public provision is characterized by equilibrium policies $g^{ea} \leq g^{pp}$, with strict inequality if $\beta > 0$ and if the set of contributors is non-empty.*
- 2) *public-goods consumption for any non-contributor j (including the median) is $G_j^{ea} \geq G_j^{pp}$, with strict inequality if $\beta > 0$, if the set of contributors is non-empty, and if consumption of the public good is strictly normal. Contributors consume even more of the public good whenever $\beta < 1$.*

Proof: To establish part 1), recall that individual m chooses public policy g in both institutional settings, and in a way that $\Delta^m = 1/z$. Let $\beta > 0$ and suppose the set of contributors in the dual provision regime is non-empty. If public goods consumption is only weakly normal (e.g., in a quasi-linear environment), m 's optimization induces $G_m = gz + \beta \sum_{i \in C} \hat{g}_i$ to be constant across regimes. Accordingly, $g^{ea} < g^{pp}$. If consumption of both goods is strictly normal, the median voter's private consumption c_m in the dual regime must be strictly larger, which again implies a smaller equilibrium policy. Part 2) follows immediately: with strictly normal preferences and because c_m is larger in the dual-provision setting, G_m will also be larger in this regime. \square

From a policy perspective, it is important to know which individuals support the

hybrid over the pure public system, or vice versa. Our findings are unambiguous in this respect.

Proposition 5. *All individuals unanimously prefer the hybrid ex-ante system over the public system. This preference is strict if $\beta > 0$ and the equilibrium set of private contributors is non-empty.*

Proof: Consider an individual j who does not contribute in the dual system. Any such individual, consumes $G_j^r = G_m^r$ in regime $r \in \{ea, pp\}$. By Proposition 3, $G_j^{ea} \geq G_j^{pp}$, $g_m^{ea} \leq g_m^{pp}$ (with strict inequality if $\beta > 0$ and if the set of contributors is non-empty). Hence, $c_j^{ea} \geq c_j^{pp}$ so that individual j must prefer the ex-ante system. Consider now an individual i who contributes $\hat{g}_i > 0$. If this agent had made no private contribution, he would have received a utility not smaller than in the public system: if he is the only contributor, a decision not to contribute triggers identical equilibrium policies and consumption levels in either regime. Second, if there are other contributors, $\hat{g}_i = 0$ still induces $G_i^{ea} \geq G_i^{pp}$, and $g_m^{ea} \leq g_m^{pp}$ (with strict inequality if $\beta > 0$). Hence, i would prefer the ex-ante system, and strictly so if $\beta > 0$. By revealed preference, making a contribution must raise i 's utility even further. Hence, each equilibrium contributor i also prefers the hybrid ex-ante system which completes the proof. \square

This result is quite intriguing. According to the Proposition, each agent prefers the flexible public-private system over public provision, and strictly so in the most plausible and relevant scenarios of positive spillovers and a non-empty set of private contributors. Hence, a policy reform from a pure public system to a dual private-public provision regime is strictly preferred by every individual in society. We should emphasize that this finding remarkably differs from results in the existing literature on institutional. While in the literature some form of hybrid system is usually preferred by a simple majority, constitutional changes in reality often demand the approval of some more demanding super majority of citizens. Proposition 4 shows that for the assumed timing of private contributions, even a Pareto improving institutional change is feasible.²⁰

²⁰To our knowledge, the only exception is Alesina et al. (2005) who investigate the same setting (with the restriction to quasi-linear preferences), but arrive at a different result. While according to Alesina et al. in general only a majority of voters prefers the dual ex-ante system, we show that

5.2 Ex-post Contributions: Policy Commitment

This Section investigates a setting where the order of moves in a dual public-private system is reversed. Suppose the policy g is determined in a first stage by democratic majority voting. In a subsequent second stage, each individual can boost the provision of the public good by making a private contribution \tilde{g}_i . As before, these ex-post contributions are made simultaneously and in a non-cooperative fashion. In this scenario, collective political decisions shape the private behavior of individuals.²¹ Since this is seen as reasonable in many empirically relevant situations, the ex-post contributions system has received most of the attention in the existing literature.

As has been emphasized in previous contributions, one cannot generally take the existence of a political equilibrium for granted if there are externalities, for the following reason. The set of second-stage contributors, as well as the size of their respective contributions, depend on the policy g that was implemented in the first stage. Single-peakedness of preferences at the policy stage is then elusive. In particular, a change in policies may induce an individual to become a contributor or a non-contributor, respectively, altering not only his individual utility representation but also the best responses of other members of society.

A main result of the present paper is to show that even though the single-peakedness requirement may not be satisfied, the result of the median voter theorem still applies in the present setting. The only tax alternative which cannot be beaten in majority vote is the alternative proposed by the natural median individual, m . In addition, we will offer a precise characterization of this equilibrium, and provide a detailed comparison to the model with ex-ante contributions.

The central issue is the outcome of the policy choice stage. It is first established that

the stronger statement of Proposition 4 can be made. Notice that in an extended setting with intra-regional preference heterogeneity (as analyzed in Cremer and Palfrey's work), the result would have to be rephrased as saying that all regional median voters favor the dual system. Conversely, a minority of low-preference individuals in high-preference districts may oppose it.

²¹In a federalism context, for example, it imposes the strategic 'leader' role on the central government, and the 'follower' role on regions or lower-tier policy makers within a federation.

each individual $j < m$ with preferences smaller than the natural median will prefer the policy which is preferred by m , over any larger public provision level. We indicate the policy preferred by m as g_m^{ep} , where ep stands for ex post contributions. Second, we show that any individual $i > m$ with preferences larger than the natural median, prefers the tax rate favored by m over any smaller tax rate. Combining these results allows to show that the median voter theorem must hold, the potential non-regularity of preferences notwithstanding. For subsequent reference, it is useful to define

- $g_i^T = g + \tilde{g}_i(g)$ as the total contribution of individual i towards the supply of public goods;
- $G^{-i} = G_i - g_i^T = g\beta(N-1) + \beta \sum_{j \neq i} \tilde{g}_j$ as the portion of i 's public goods consumption derived from contributions of all other individuals.

In what follows, let g_m^{ep} indicate the policy preferred by m , where ep stands for ex post contributions. We now develop our main result in a series of lemmas.

Lemma 4: Each individual $j < m$ prefers g_m^{ep} over any other policy $g > g_m^{ep}$.

Proof: Suppose to the contrary that individual j strictly prefers some larger policy, say g_j^* , over g_m^{ep} . We first show that if this is true, the total contributions of all other individuals under policy g_j^* exceed those under policy g_m^{ep} , that is, $G^{-j}(g_j^*) > G^{-j}(g_m^{ep})$. Suppose not. Then, j could under policy g_m^{ep} always replicate his public-goods consumption under policy g_j^* , by choosing $\tilde{g}_j(g_m^{ep}) > 0$ in a way that $G_j(g_m^{ep}) = G_j(g_j^*)$.²² At the same time, his total public goods contribution g_j^T would be lower and his private consumption c_j would be higher under policy g_m^{ep} , yielding a contradiction. Hence, if j 's preferences are strict, $G^{-j}(g_j^*) > G^{-j}(g_m^{ep})$, and j 's total consumption of public goods under both policies regimes is characterized by $G_j(g_j^*) \geq G_j(g_m^{ep})$ due to normality of preferences.

Consider now individual m who by definition prefers g_m^{ep} over g_j^* . Two cases need to be

²²Note this is always feasible; the required stage-2 contribution (if positive) would be $\tilde{g}_j = \tilde{g}_j(g_j^*) + (g_j^* - g_m^{ep}) - [G^{-j}(g_m^{ep}) - G^{-j}(g_j^*)]$, and imply $g_j^T(g_m^{ep}) < g_j^T(g_j^*)$. Note also that \tilde{g}_j will in general not be j 's best response to policy g_m^{ep} and contributions $G^{-j}(g_m^{ep})$ of other individuals. However, \tilde{g}_j defines j 's lower utility bound by revealed preference.

distinguished. First, $G^{-m}(g_m^{ep}) > G^{-m}(g_j^*)$ may hold. If in addition $G_m(g_m^{ep}) \geq G_m(g_j^*)$ (this needs not be true, see below), j can strictly prefer g_j^* and m strictly prefer g_m^{ep} only if the conditions $G^{-j}(g_j^*) > G^{-j}(g_m^{ep})$ and $G^{-m}(g_j^*) < G^{-m}(g_m^{ep})$ are simultaneously satisfied. Note that $G_m(\cdot) = G_j(\cdot) + (1-\beta)[\tilde{g}_m(\cdot) - \tilde{g}_j(\cdot)] = G_j(\cdot) + (1-\beta)[(g_m^T(\cdot) - g_j^T(\cdot))]$. Using this fact and the definition of $G^{-i}(\cdot)$, a necessary condition for the validity of these conditions is

$$g_m^T(g_j^*) - g_m^T(g_m^{ep}) > g_j^T(g_j^*) - g_j^T(g_m^{ep}).$$

A second set of conditions is $G_m(g_m^{ep}) \geq G_m(g_j^*)$ and $G_j(g_j^*) \geq G_j(g_m^{ep})$. Again using the above definition of $G_m(\cdot)$, the validity of these two inequalities requires $g_m^T(g_j^*) - g_m^T(g_m^{ep}) \leq g_j^T(g_j^*) - g_j^T(g_m^{ep})$, yielding an immediate contradiction.²³

Next, suppose again that $G^{-m}(g_m^{ep}) > G^{-m}(g_j^*)$, but now let $G_m(g_m^{ep}) < G_m(g_j^*)$. Notice that this case can occur only if policy g_j^* forces m to make an excessive total contribution from his point of view, implying a stage-2 corner solution $\tilde{g}_m(g_j^*) = 0$. But then, agent $j < m$ must also be a non-contributor (by definition, it is impossible to have $\Delta^j(\cdot) > \Delta^m(\cdot)$ at $\tilde{g}_j = \tilde{g}_m = 0$) and as a consequence, $g_m^T(g_j^*) = g_j^T(g_j^*)$, which implies $G^{-m}(g_j^*) = G^{-j}(g_j^*)$. Note now that under the rank-order assumption, $g_j^T(g) \leq g_m^T(g)$ for any g . Hence, $G^{-j}(g_m^{ep}) \geq G^{-m}(g_m^{ep})$. But since we consider the case $G^{-m}(g_m^{ep}) > G^{-m}(g_j^*)$, and since $G^{-m}(g_j^*) = G^{-j}(g_j^*)$, this immediately implies $G^{-j}(g_j^*) \leq G^{-j}(g_m^{ep})$, a contradiction to our previous result.

Next, suppose to the contrary that $G_m^{-m}(g_j^*) \geq G_m^{-m}(g_m^{ep})$. In this case, m 's preferences are consistent only if policy g_j^* forces her to make a contribution g_j^* that m perceives as excessive. Accordingly, her private contribution is $\tilde{g}_m(g_j^*) = 0$. But if m does not contribute under policy g_j^* , individual $j < m$ will not make a private contribution either by Assumption 1, quasiconcavity, and normality of preferences. Hence, we have $G_m(g_j^*) = G_j(g_j^*)$ and $g_m^T(g_j^*) = g_j^T(g_j^*)$. Consider now policy g_m^{ep} and suppose j 's

²³This is true for any $\beta < 1$. For $\beta = 1$ so that $G_m(\cdot) = G_j(\cdot) \equiv G(\cdot)$, satisfying conditions $G_m(g_m^{ep}) \geq G_m(g_j^*)$ and $G_j(g_j^*) \geq G_j(g_m^{ep})$ requires public goods consumption under both policies to be identical, $G(g_j^*) = G(g_m^{ep})$. To prefer g_j^* over g_m^{ep} , j 's total contribution must then be characterized by $g_j^T(g_j^*) < g_j^T(g_m^{ep})$ (note that as an implication, j makes a positive stage-2 contribution under policy g_m^{ep}). But this is inconsistent: because of preference normality, j would reduce $g_j^T(g_m^{ep})$ to a level where $G_j(g_m^{ep}) < G_j(g_j^*)$, yielding a contradiction.

total contribution satisfies $g_j^T(g_m^{ep}) = g_m^T(g_m^{ep})$, so that both agents consume the same amount $G_m(g_m^{ep}) = G_j(g_m^{ep})$. Since $G^{-m}(g_j^*) \geq G^{-m}(g_m^{ep})$ by assumption, normality then (generically) implies $G_m(g_m^{ep}) < G_m(g_j^*)$. Also, since $\Delta^j(\cdot) < \Delta^m(\cdot)$ for identical total contributions of j and m (Assumption 1), j must prefer g_m^{ep} over g_j^* , given this is m 's preference ranking.²⁴ Moreover, by revealed preference, choosing $g_j^T(g_m^{ep}) \neq g_m^T(g_m^{ep})$ can only raise j 's utility under policy g_m^{ep} , reinforcing this preference. This yields a contradiction and completes the proof. \square

Lemma 5: Any individual $h > m$ prefers g_m^{ep} over any policy $g < g_m^{ep}$.

Proof: The proof mirrors the proof of Lemma 4, and is therefore omitted.

Combining the statements in these lemmas leads to the following important result.

Proposition 6. *In the dual regime with ex-post contributions, the preferred policy of the median individual m , g_m^{ep} , beats all other policy alternatives g in pairwise comparison.*

Proof: By Lemma 4, m and all individuals $j < m$ with smaller preferences for public goods than m prefer g_m^{ep} over any some larger policy $g > g_m^{ep}$. By Lemma 5, m and all individuals $h > m$ prefer g_m^{ep} over any smaller policy $g < g_m^{ep}$. Combined, no other policy can beat g_m^{ep} under majority vote, which proves the result. \square

Notwithstanding the potential lack of single-peakedness of agent preferences, the outcome of the median voter theorem applies. The next step is to characterize the stage-2 contribution levels of all individuals. We can show that again, neither the median individual nor any individual with lower rank, will make a private contribution. While this sounds intuitive, a rigorous proof is quite intricate. The core of the argument is to show that for the range of policies for which the median voter subsequently becomes a contributor, m 's first-period utility is strictly increasing in the policy level g . For this reason, m 's optimal stage-1 choice cannot be such that she is a contributor in the stage-2 Nash equilibrium.

²⁴This is clearly true if both j and m have identical incomes. Suppose not and notice that total contributions under policy g_m^{ep} must be characterized by $g_m^T(\cdot) \geq g_j^T(\cdot)$ under Assumption 1. Since their individual contributions under the alternative policy g_j^* are identical, switching from g_m^{ep} to g_j^* causes j 's private consumption to decline more. Invoking the single-crossing assumption then establishes the result.

Proposition 7. *Neither the median individual m , nor any smaller preference individual $j < m$ will contribute in stage 2.*

Proof: see the Appendix.

Proposition 7 is perhaps best understood by the following heuristic argument. Consider a policy level g_m sufficiently low so that m contributes in the stage-2 continuation equilibrium. The median voter knows that raising g_m further forces some low-preference individuals (who are non-contributors in stage 2) to provide additional resources for the supply of the public good. Hence, raising g_m transfers income from low-preference individuals to high-preference individuals. This allows the median voter m and all other contributors to reduce their private contributions not only by the amount of their increased policy payment but also by the amount of this transfer, while leaving their public goods consumption unaffected. As a result, m as a contributor can raise her private consumption, and at the same time increase her utility level because naturally, non-contributors cannot reduce their private supplements. In this light, it does not surprise that the median voter will want to expand g at least to the level where she ceases to contribute subsequently.

The median voter m and all lower-preference individuals do not make a private contribution. What about higher-preference individuals? Similar to the model with private ex-ante contributions, a subset of highest-preference individuals may want to enhance their public goods consumption. The Proposition below compares the ex-post regime and the ex-ante regime regarding private contributions and equilibrium policies g_m^{ep} and g_m^{ea} , respectively. To simplify the exposition, we focus on a situation where the set of contributors in both scenarios is non-empty.²⁵

Proposition 8. *With ex-post contributions, the equilibrium policy is strictly smaller than in the model with ex-ante contributions, $g_m^{ep} < g_m^{ea}$. Moreover,*

a) each non-contributor (including every individual $j \leq m$) prefers the ex-post

²⁵If this set is empty in the ex-post regime, it will be empty in the ex-ante regime as well. Without private contributors, equilibrium policies can be identical, and private goods consumption can be larger in the ex-ante setting. See the example below.

regime over the ex-ante regime (and over the public regime); this is true even though their public goods consumption in the former regime may be lower.

- b) Private contributions in the ex-post regime are strictly larger, and the set of contributors may be larger. Contributors strictly prefer the ex-ante over the ex-post regime, while their relative preference between ex post regime and public provision regime is ambiguous.*

Proof: see the Appendix.

The Proposition conveys a variety of interesting results. Equilibrium policies in all three institutional regimes are ranked as $g_m^{ep} \leq g_m^{ea} \leq g^{pp}$, with strict inequalities as long as some agents privately contribute. In particular, the natural median agent (the pivotal agent in all settings) chooses a lower collective funding contribution in the ex-post regime, as compared to the ex-ante regime. The intuitive reason for this result is the presence of a commitment effect: by adopting a smaller policy in the ex-post regime, the median voter can induce high-preference individuals to raise their private contributions. It is the natural that all low-preference agents (including all agents $j \leq m$) welcome this commitment device and hence, a majority of individuals prefers the ex-post setting over both alternative regimes.²⁶ Conversely, contributing high-preference agents prefer the ex-post setting, because it causes low-preference individuals to provide less input into public good provision. In fact, the collective funding cut in post regime can be so severe that for these individuals, even pure public provision may be preferable over the dual ex-post setting.

Private contributions are clearly larger in a regime with ex-post contributions, for two reasons. First, with a smaller policy private contributions become more tempting for high demand individuals. Second, there is no strategic incentive for these agents to

²⁶This is due to revealed preferences: the median voter could always choose the same policy than in the ex-ante regime, and raise his utility (and the utilities of all lower-preference agents) because private contributions go up in the ex-post setting. While these contributions are best responses to a given policy in the ex-post setting, they are smaller than the best responses to the same policy level in the ex-ante setting.

moderate their contributions, in the hope of a larger policy response. The second reason clearly suggests that at least if income effects are absent, equilibrium contributors consume strictly more of the public good in the ex-post regime

Finally, regarding total public goods consumption, regime comparisons are generally ambiguous. One can show that if income effects are absent, equilibrium contributors consume strictly more of the policy good in the ex-post regime, while non-contributors may consume less. For the special case of a pure public good, each agent consumes identical amounts in the ex-post setting and with public provision, and more than in a institutional regime with ex-ante contributions.²⁷

6 Polar Cases: Public Goods and Private Goods

At this point, it is useful to compare the two boundary scenario in which the service in question is either a private or a purely public good. Consider first an ‘almost’ private good G for which externalities disappear in the limit, $\beta \rightarrow 0$.²⁸ In contrast to the $\beta = 0$ (analyzed in Section 4.2) which featured multiple equilibria, our previous results show that the equilibrium policy now becomes unique, which is clearly desirable. Moreover, this policy is identical in both dual-provision regimes, as well as under pure public provision. This is true for two reasons. First, we have seen that regardless of the regime, the natural median voter is pivotal and will never make a private contribution. Second, since externalities disappear in the limit, strategic voting motives become absent for the median individual or anybody else in society. Regardless of the institutional setup, this leads the median voter m to select a policy such that $\Delta^m(c_m, G_m) = 1$ holds in the limit. This policy is $g^* = g_m^{pp}$. Each individual $i > m$ privately contributes to satisfy

²⁷Suppose there is a positive number of equilibrium contributors. The public goods consumption of any contributor h is characterized by $\Delta^h(\cdot) = 1$ in the ex-post setting, but by $\Delta^h(\cdot) > 1$ in the ex-ante setting where lower contributions triggers a larger stage-2 policy. Without income effects, this directly translates into a larger public goods consumption in the public regime. However, it does not readily extend to a framework with income effects, where because the larger equilibrium policy in the ex-ante regime expands h ’s consumption set, and may cause him to demand a larger total public goods consumption.

²⁸The case $\beta = 0$ has been explored in Section 4.2 above, and was shown to yield multiple equilibria.

$\Delta^i(c_i, G_i) = 1$, and *all* agents $j < m$ are forced into an excessive consumption of the public good.²⁹

This indifference regarding equilibrium policies does not extend to a scenario with public goods. For the polar case $\beta = 1$, standard results (Bergstrom et al., 1986) suggest that in absence of income effects, only the highest-preference individual will privately contribute. As we show now, his private contributions can be very large in an ex-post regime, while under otherwise identical circumstances they can be even zero in the ex-ante regime.

To see this, consider quasi-linear preferences, $U^i = c_i + \alpha_i H(G_i)$, where α_i is a preferences parameter and $H(\cdot)$ some increasing and concave function. Under pure public provision, the equilibrium policy $g_m^{pp} > 0$ yields a public goods consumption G^{pp} implicitly given by $H'(G^{pp}) = 1/N\alpha_m$, where N represents the number of agents and α_m the median individual's taste parameter. To highlight our argument, assume now that $N\alpha_m$ is 'sufficiently close' to the preference parameter of the highest-demand agent, α_h .³⁰

In the ex-post regime, the equilibrium policy g^{ep} is then zero. The median voter knows that without collective provision, only h will privately contribute in the quasi-linear setting, and he will do so in a way that $H'(G^{ep}) = 1/\alpha_h$. But since α_h and $N\alpha_m$ are similar in size, this yields a public goods provision almost as large as (or even larger than) G^{pp} at no cost to any agent other than h : the capability to commit to a policy makes it optimal for m (and a majority of citizens) to free ride on h 's contributions. For comparison, consider now the ex-ante setting. Here, h is aware that his private contributions will crowd out public provision. When $N\alpha_m$ is close to α_h , this renders it optimal for him to deny any private contribution, thereby inducing a public policy g_m^{pp} and associated public goods consumption G^{pp} in the second stage. For obvious reasons, all individuals other than h are made worse off compared to the ex-ante system, and everybody in society achieves an equilibrium utility identical to the level under pure

²⁹Again, a simple majority of higher-preference agents are indifferent between both two dual systems and a private market system, and strictly prefer any of those institutions over a regime with pure public provision.

³⁰For example, let $N = 3$, $\alpha_1 = 1$, $\alpha_2 = \alpha_m = 2$, and $\alpha_3 = \alpha_h = 6$.

public provision.³¹ Overall, these arguments show that while public good consumption levels may be very similar or even identical in both dual regimes, those regimes can display extreme differences regarding the amount of private contributions, equilibrium policies, and the overall utility distribution when significant spillovers exist.

7 Endogenous Timing of Contributions

How do these previous results play out in a general framework, where both ex-ante and ex-post private contributions are feasible, and subject to each individual's discretion? At first glance, intuition suggests that the outcome of this general model must coincide with the results for the ex-post system. Since public decisions still precede possible private contributions, adding an ex-ante contributions stage seems strategically irrelevant. Perhaps surprisingly, though, this intuition is incorrect. In fact, endogenizing the timing of private contribution generates equilibrium outcomes that differ significantly from those in all previous settings. Importantly, the general system will be shown to dominate both the ex ante and ex post system at least for a majority of members in society.

A full analysis of the general scenario is beyond the scope of our paper. However, interesting findings can already be obtained in a restricted setting. Specifically, we provide a characterization for the case of quasi-linear preferences, and a setting where the good in question is a pure public good, $\beta = 1$. Consider preferences of the form $U^i = c_i + \alpha_i \ln G_i$ with preference parameter α_i increasing in i . Then, $c_i = y_i - g - \tilde{g}_i - \hat{g}_i$ is the private consumption of individual i with income y_i , who provides funding g in the political process, makes a voluntary contribution of \hat{g}_i in an ex-ante stage, and another ex-post contribution \tilde{g}_i after the policy g has been implemented. Accordingly, $G_i = gN + \sum_i(\tilde{g}_i + \hat{g}_i)$ is i 's consumption level of public services.

³¹For completeness, suppose $N\alpha_m$ and α_h are not of similar size. Then, results in both dual regimes coincide. For $N\alpha_m$ sufficiently smaller than α_h , the equilibrium policy is zero and h 's equilibrium contribution yields a public goods consumption given by $H'(G) = 1/\alpha_H$, which is larger than in the public provision regime. Conversely, for $N\alpha_m$ sufficiently larger than α_h , h does not contribute and m implements a policy g^{pp} .

Using backwards induction, we first analyze the ex-post contributions at stage 3. With a pure public good and quasi-linearity, only the agent with with largest preferences will possibly contribute. In principle, the identity of this individual is now endogenous because it depends on the profile of ex-ante contributions. However, this is not the case here where wealth effect are absent and all individuals consume the same amount of the public good by definition. Since any vector \hat{g} of private ex-ante contributions leaves the rank order of all agents intact, only agent h with largest ‘natural’ preferences may possibly contribute at stage 3.

Define $\hat{G} = \sum_i \hat{g}_i$ as the sum of all individuals’ ex-ante contributions. In stage 3, individual h chooses

$$\tilde{g}_h^* = \arg \max_{\tilde{g}_h} U^h(\hat{g}, g, \tilde{g}_h) = y_h - g - \hat{g}_h - \tilde{g}_h + \alpha_h \ln (gN + \tilde{g}_h + \hat{G}) = \max \{ \alpha_h - gN - \hat{G}, 0 \}. \quad (4)$$

Notice that agent h ’s optimal ex-post contribution (if positive) decreases in both the policy g , and in total ex-ante contributions \hat{G} . Moreover, from the agent’s perspective, the timing of his own private contributions is irrelevant as long as his stage-3 choice is interior.

Consider now the policy decision at stage 2. For given first-period contribution vector \hat{g} , the natural median individual m decides on g . Her goal is to maximize

$$U^m(g; \hat{g}) = y_m - g - \hat{g}_m + \alpha_m \ln [gN + \hat{G} + \tilde{g}_h^*(g, \hat{G})]. \quad (5)$$

Taking individual h ’s response into account, the median voter’s utility function is piecewise defined. We first consider the range of policies for which h in fact makes a positive ex-post contribution. Using (4), the respective range is³²

$$g \leq \bar{g}(\hat{G}) \equiv \max \left\{ \frac{\alpha_h - \hat{G}}{N}, 0 \right\}. \quad (6)$$

For policies g from this interval, $U^m(\cdot)$ is monotonically decreasing: since $d\tilde{g}_h^*/dg = -N$, any dollar of additional individual tax payments causes a total crowding out

³²Note that $\bar{g}(\hat{G})$ is always non-negative because individual h will never invest more than α_h , no matter how much the other individuals contribute.

of private ex-post contributions.³³ This makes any public funding unattractive for (almost) the entire population, and the median voter's local optimum is $g^*(g \leq \bar{g}) = 0$.

Next, consider the reverse range of policies, $g > \bar{g}$. Now, $d\tilde{g}_h^*/dg = 0$ and the first-order condition to (5) reads

$$\frac{\alpha_m N}{gN + \hat{g}_h} - 1 \leq 0. \quad (8)$$

Using the definition of $\bar{g}(\cdot)$, a local interior optimum exists if and only if $\alpha_m N > \alpha_h$. Intuitively, for $\alpha_m N \leq \alpha_h$, agent h 's private contribution in absence of any public funding exceeds the median voter's preferred policy in absence of private contributions. In this latter case, the median voter relies on h to provide the public good. Formally, since $U^m(\cdot)$ is continuous in g and strictly decreasing over the range $g \leq \bar{g}(\cdot)$, m 's preferred policy is $g = 0$ if $\alpha_m N \leq \alpha_h$.

Let us now consider a situation with $\alpha_m N > \alpha_h$. Here, an interior solution to m 's local choice problem exists and is described as

$$g^+(\hat{g}) = \max\left\{\alpha_m - \frac{\hat{G}}{N}, 0\right\}. \quad (9)$$

Overall, the median voter's preferred policy in the general system (subscript gs) is now $g_m^{gs} \in \{0, g^+\}$ and further investigation is needed to find out the global optimum (for positive g^+). To do so, we must compare the associated stage-2 utilities for the median voter. These are

$$U^m(g = 0, \hat{g}) = y_m - \hat{g}_m + \alpha_m \ln \max\{\alpha_h, \hat{G}\},$$

and

$$U_m(g = g^+, \hat{g}) = y_m - \hat{g}_m + \alpha_m \ln N\alpha_m - \left(\alpha_m - \frac{\hat{G}}{N}\right).$$

Inspection reveals policy g^+ to dominate if

$$U_m(g = g^+, \hat{g}) > U_m(g = 0, \hat{g}) \iff \alpha_m \ln N\alpha_m - \ln \max\{\alpha_h, \hat{G}\} > \alpha_m - \frac{\hat{G}}{N}. \quad (10)$$

³³Formally, the first-order derivative to (5) reads

$$\frac{\alpha_m}{gN + \tilde{g}_h + \hat{G}} \left[N + \frac{d\tilde{g}_h^*}{dg} \right] - 1. \quad (7)$$

Since $d\tilde{g}_3^*/dg = -N$, (5) is concave and the first term in (7) representing the marginal benefit of raising g is zero.

Suppose that $\hat{G} \geq \alpha_h$ (we will later verify the validity). Then, $U^m(0, \hat{g})$ is unaffected by the size of \hat{G} while $U_m(g^+, \hat{g})$ is increasing. Note also that for \hat{G} converging to α_h , \bar{g} converges to zero, while g^+ remains strictly positive. Since $U^m(g, \hat{g})$ increases over the policy interval $[\bar{g}, g^+]$, these arguments establish g^+ to be m 's preferred policy choice if \hat{G} exceeds some threshold \hat{G}^* , with the property $\hat{G}^* < \alpha_h$.³⁴ For $\hat{G} \geq \hat{G}^*$, the median voter thus implements a positive policy g^+ , and agent h makes no ex-post contribution.³⁵ Conversely, if $\hat{G} < \hat{G}^*$, the median voter implements $g = 0$, and agent h provides an ex-post contribution of size $\tilde{g}_h^*(0, \hat{G})$.

We are now prepared to analyze the ex-ante contributions, made in stage 1. To start with, consider a situation where $\alpha_m N \leq \alpha_h$. In this case, m will always choose $g_m^{gs} = 0$ in stage 2, regardless of the private-contributions profile \hat{g} . Moreover, no agent other than h will make an ex-ante contribution. To verify this, suppose some agent $i \neq h$ does. If the stage-1 contribution profile \hat{g} satisfies $\hat{G} \geq \alpha_h$, there will be no additional ex-post contributions, and agent i 's contribution implies $\Delta^i < 1$ which cannot be optimal. Conversely, suppose the contribution profile is characterized by $\hat{G} < \alpha_h$. If i does not contribute, agent h will in stage 3 make a positive contribution \tilde{g}^* , which satisfies $\Delta^h = 1$ and thus, $\Delta^i < 1$. Again, lowering his contribution raises i 's overall utility. Taken together, no agent other than h may possibly contribute ex ante, implying $\hat{G} = \hat{g}_h$ in equilibrium. Note that since $g = 0$ and $\hat{G} - \hat{g}_h = 0$, the timing of agent h 's contribution is irrelevant, and his total private ex-ante and ex-post contributions satisfy $\hat{g}_h^* + \tilde{g}_h^* = \alpha_h$. Overall, for $\alpha_m N \leq \alpha_h$, the outcome is the same as in a framework in which only private contributions can be made: the equilibrium policy is zero, the highest-preference agent finances the public good, and provides it at a level that he finds privately optimal.³⁶

Consider $\alpha_m N > \alpha_h$. This case represents an economic environment with many agents,

³⁴Intuitively, for \hat{G} converging to α_h , agent h will not make an ex-post contribution even if $g = 0$. Increasing g above zero (and to the optimal level g^+) must boost the median voter's utility.

³⁵The threshold may even be zero, and therefore, non-binding. It will be zero if the difference $N\alpha_m - \alpha_h$ is sufficiently large that m prefers public provision with policy $g = N\alpha_m$ over a zero policy combined with h 's private contribution α_h .

³⁶Accordingly, there is always underprovision relative to the social optimum, as characterized by the Samuelson rule.

or a scenario where the spread in tastes across society is less pronounced. If the initial contribution vector satisfies $\hat{G} < \hat{G}^*$, the median voter implements a zero policy and h 's private ex-ante and ex-post contributions satisfy $\Delta^h = 1$. Again, no agent other than h contributes. More interestingly, consider now ex-ante contributions with the property $\hat{G} \geq \hat{G}^*$. As shown above, any such profile of stage-1 strategies triggers a positive policy g^+ , and zero additional ex-post contributions.³⁷ Since ex-post contributions are zero and g^* is positive and decreasing in \hat{G} at a rate $1/z$, total public goods consumption does not depend on \hat{G} . Hence, any contribution profile $\hat{G} > \hat{G}^*$ cannot constitute an equilibrium because each individual would benefit from lowering its ex ante contribution. The only remaining candidate equilibrium is such that total ex-ante contributions satisfy $\hat{G} = \hat{G}^*$, the threshold level at which g^+ becomes the stage-2 equilibrium policy. To show that such an equilibrium indeed exists, suppose individual h ex ante contributes the entire amount, $\hat{g}_h = \hat{G}^*$. Doing so allows h to achieve a utility level strictly larger than with any other possible contribution, positive or zero. First, choosing $\hat{g}_h < \hat{G}^*$ would trigger larger private contributions (of $\hat{g}_h + \tilde{g}_h = \alpha_h$ in total) and at the same time, reduce h 's public goods consumption.³⁸ Second, for $\hat{g}_h > \hat{G}^*$, public goods consumption would be identical, but at the cost of larger private contributions. Hence, given that no other agent contributes, h cannot do better than choosing $\hat{g}_h = \hat{G}^*$.³⁹

³⁷This is true for contribution profiles $\hat{G} < \alpha_h$, on which we concentrate in the text. Contribution profiles $\hat{G} > \alpha_h$ imply $\Delta^i < 1$ for any individual i , including individual h . Hence, the only potential motive for an individual i to provide a \hat{g}_i causing $\hat{G} > \alpha_h$ would be to prompt the median voter into choosing $g = 0$ rather than g^+ . But this motive is not valid: individual i 's gross utility from policies $g = 0$ and g^+ is $\alpha_i \ln \hat{G}$ and $\alpha_i \ln N\alpha_m - (\alpha_m \hat{G}/N)$, respectively. Since $\hat{G}^* < \alpha_h$, the latter utility dominates for any contribution profile in the relevant range $\hat{G} \in [\alpha_h, N\alpha_m]$; gross utilities coincide for $\hat{G} = N\alpha_m$ where g^+ becomes zero. Accordingly, a contribution profile $\hat{G} > \alpha_h$ cannot be realized in equilibrium.

³⁸With $\hat{g}_h = \hat{G}^*$, agent h 's total contribution (including his tax contribution) towards the public good may be larger than α_h . However, his preferred stage-2 policy still exceeds g^+ , so that h is not forced into overconsumption of the public good.

³⁹Clearly, any other individual's best response to $\hat{g}_h = \hat{G}^*$ is not to privately contribute at all. Note there may exist additional other stage-1 contributions equilibria in which $\hat{g}_h < \hat{G}^*$, but again characterized by $\hat{G} = \hat{G}^*$. In any such equilibrium, only individuals of type $i > m$ potentially contribute: by the definition of \hat{G}^* , any individual $i < m$ will at stage 2 prefer a policy $g = 0$ over policy g^+ . Hence, private contributions that prompt policy g^+ cannot be beneficial for those types.

For the case of public goods and quasi-linear preferences, this completes the characterization of a ‘general system’ with endogenous timing of private contributions. Defining $\Delta = N\alpha_m - \alpha_h$, we can now state

Proposition 9. *Consider an endogenous timing of private contributions, quasi-linear preferences, and $\beta = 1$.*

- 1) *If $N\alpha_m \leq \alpha_h$, the equilibrium policy g_m^{gs} is zero and equilibrium outcomes coincide with those in the ex-post system.*
- 2) *If $N\alpha_m > \alpha_h$,*
 - a) *the unique equilibrium policy is $g_m^{gs} = g^+ > 0$, initial contributions satisfy $\hat{G} = \hat{G}^* (< \alpha_h)$, and there are no ex-post contributions. Moreover, a) public good consumption G^{gs} is never smaller and sometimes strictly larger than in regimes with only ex-ante or ex-post contributions; b) the equilibrium policy satisfies $g^{gs} < g^{ea}$, and in addition $g^{gs} \leq g^{ep}$, with strict inequality unless Δ is too large; c) private contributions exceed those in the ex-ante system, and are smaller (strictly so unless Δ is too large) than those in the ex-post system.*
 - b) *A majority of the population is strictly better off compared to both the ex-post system, and compared to the ex-ante system.*

Proof: see the Appendix.

For a broad range of parameter constellations, ex-post system and general system yield distinct outcomes in terms of equilibrium policies, contribution patterns, and consumption levels. Specifically, the feasibility of ex-ante contributions allows high preference individuals to escape – or at least alleviate – their exploitation by the majority. Remember that in the ex-post system, the median voter for strategic reasons chooses a low equilibrium policy, in order to encourage subsequent private contributions. Conversely, ex-ante contributions in the general system serve high-preference individuals as a commitment device not to make further contributions ex post. This shuts down the median voter’s interest in collective underprovision, and leads her to adopt a larger policy than in the ex-post regime.

The overall outcome in the general system features smaller private contributions, a larger policy, and larger total consumption levels of the public good. Another interesting question is the welfare ranking of regimes. To compare general and ex-post system, notice that the median voter could always adopt the equilibrium policy of the ex-post system, and (at least) achieve the same utility. By choosing a larger policy, she and all higher-preference individuals must be better off, making the general system majority-preferred. Interestingly, this system also majority-dominates the ex-ante regime: since the equilibrium policies are best responses to the ex-ante contributions in either case, public goods consumption levels must be the same. Since the general system displays larger ex-ante contributions by high-preference agents, a majority of lower-preference agents strictly prefers an institutional setting in which ex-ante and ex-post contributions are feasible.

These findings have interesting policy implications. When citizens can choose the institutional structure by majority vote, the general system with endogenous timing emerges as the Condorcet winner: it is majority preferred over the pure public system and the ex-ante system (by all lower-preference agents), and over the ex-post system (by all higher-preference agents). In an evolutionary sense, this institutional system can be expected to prevail in the long run.

Overall, our results suggest that equilibrium outcomes in the general system originate from a complex strategic interaction of the median voter, and of high-preference individual(s). For the highest-preference individual, allowing additional ex-ante contributions reduces the overall hold-up power of the majority, and makes him better off. Likewise, the general system benefits the median voter because her equilibrium policy does not need to incorporate a downwards bias for incentive reasons.

8 Private Provision as Status Quo Regime

The previous Sections compared public provision and dual public-private systems. In many situations of empirical importance, though, the status quo institution is one in which individuals privately provide those services. For example, many countries

display market based health care systems, some municipalities do not finance public transportation, and there are many fields of decentralized policymaking within federations.

As is well known, only a subset of highest-preference individuals delivers the public good in a private-contributions regime. In equilibrium, individual contributions of any agent i are described by the system of first-order conditions $\Delta^i(c_i, G_i) \leq 1$, and hold with equality for any contributor. Recalling our previous findings, these properties suggest a basic tradeoff between each dual systems, and pure private provision. In a private system, low-preference individuals can free ride on the private supply of public goods, which they obviously find beneficial. On the other hand, public provision alleviates or even mitigates the underprovision of public services that characterizes a private system, which benefits everybody in society. Notwithstanding these tradeoffs, our comparison of regimes yields some clear cut results.

Proposition 10. *Compare a private market system and a dual public-private regime. For any $\beta > 0$,*

- 1) *adopting the dual ex-post regime and a fortiori, the general system, is preferred by a majority of the population (strictly so if $g_m^{ep} > 0$), including any $i \geq m$. Conversely, a subset of lower-preference individuals may oppose.*
- 2) *adopting an ex-ante system may be opposed by a majority of individuals, including all low-preference individuals $j \leq m$.*

Proof: To establish part 1), notice that for a policy $g = 0$, the outcome is identical to the outcome in the private system. If the pivotal median voter m implements $g^{ep} > 0$, she must be better off by revealed preference. In addition, every individual $i > m$ prefers g^{ep} over any smaller policy (Lemma 5), which validates the result. Finally, notice that for $\beta \rightarrow 0$ where $g^{ep} \rightarrow g^{pp}$, each individual $j < m$ (and thus, a strong minority of citizens) opposes the regime change from private to ex-post system.

To prove part 2), reconsider the quasi-linear example analyzed in Section 6. If G is a pure public good, only the highest-preference h individual will contribute to its

provision in a decentralized setting, while all other individuals free ride. In the ex-ante system, the highest-preference agent h may not privately contribute, but rely on the public provision of G instead. Specifically, he will do so if $\alpha_m N$ and α_h are sufficiently similar. Under m 's optimal policy response, both regimes then feature a similar level of public goods consumption; however, the funding is provided entirely by h in the private regime, and uniformly by all society members in the ex-ante regime. Clearly, this makes everybody (other than h) worse off in the ex-ante regime. \square

When private provision represents the status quo, adopting a dual system does not find unanimous consent, regardless of the degree of externalities. Perhaps counterintuitively at first glance, this is because the low-preference (or poor) individuals often prefer a private system over one which entails public provision. The reasons are twofold. The median voter's policy decision forces each individual with smaller preferences to pay more towards public goods consumption than privately desired. In addition, and as said above, the pure private system allows individuals to free ride on the contributions of high-preference individuals, in the extreme without providing any private contribution at all.⁴⁰ Both arguments shape the preferences of low-preference individuals in the same direction: their loss on private endowment in a dual system can be so severe that even a larger public goods consumption provides no sufficient compensation.

Remarkably, we nevertheless find that the ex-post system - and by extension of our previous results the general system with endogenous timing - unambiguously dominates the private market system for a majority of agents. Not only this, the preferences of these agents are strict (whenever $g^{ep} > 0$) for any arbitrary degree of externalities.⁴¹

⁴⁰The first point has been made in Cremer and Plafrey (2000), who analyze a setting without externalities. Note that the arguments against public provision qualitatively extend to a scenario with income taxation: despite making smaller tax payments, poor individuals may still view their tax contributions as excessive.

⁴¹We have shown above that for $\beta = 0$, a majority of agents is indifferent. Slightly different results have been established in the literature for this case. In Cremer and Palfrey (2000), voters within districts are heterogenous and by assumption, the federal policy decision is made by the federal median individual. Then, a majority of voters in regions with median preferences below those of the federal median agent suffer from a transition to the ex-post system, because they would prefer a smaller provision level. Conversely, all other individuals and thus a majority of the overall population strictly welcome the transition. Epple and Romano (1996) also show that the majority of voters

The intuition is again found in a revealed preference argument. If the median voter chooses a policy $g = 0$, she simply replicates the outcome of the private-provision setting. Conversely, if she chooses $g > 0$, and since all individuals $i > m$ prefer even larger policies, all those individuals must be better off.

In contrast, the ex-ante system may be rejected by a majority. Because they move first, high preference individuals now have the opportunity to reduce their contributions for strategic reasons. In the limit, they may not contribute at all, effectively turning the ex ante system into one with pure public provision. Not surprisingly, this can lead a large majority to favor the private system over the ex-ante setting, in stark contrast to the case where public provision represents the status quo regime.

Overall, our findings show that irrespective of the size of externalities, a transition from a private system to a dual system is always majority preferred, as long as the system admits private ex-post contributions. This outcome is especially encouraging because at the same time, the move to a purely public system will often be rejected by a majority of the population. Adopting a dual institution thus relaxes political feasibility constraints, and it can also serve as a response to distributional concerns in society.

9 Conclusions

This paper offers an analysis of political decisionmaking, in a framework where individuals can privately enhance their consumption of an impure public good that is through democratic choice. It explores different scenarios in which individuals can make their private provisions in a non-cooperative fashion, before and/or after the political outcome is decided by majority vote. Notwithstanding the relative complexity of the model a variety of strong and interesting results can be established. First and foremost, we find that while the feasibility of private contributions in presence of externalities may cause a break down of preference single peakedness, the outcome of the median voter

strictly prefers the dual system; all these voters benefit from the redistributive aspects of income taxation in their setting.

theorem still applies: in equilibrium, the median voter's preferred provision level beats any other policy alternative in pairwise comparison. Relatedly, irrespective of the assumed timing, private contributions are never rank-reversing in the sense that some individual other than the 'natural' median (the median individual in a pure public setting) might become the political decision maker in equilibrium. We then use these insights to explore the characteristics of equilibria, and to provide a comparison of alternative regimes. When public goods are provided exclusively through the political process in the status quo, moving to a system with additional ex-ante contributions is beneficial for everybody in society. In contrast, unanimity is lost when moving to a system which allows only ex-post contributions; this latter regime is still preferred by majority but will in general be opposed by some high-preference individuals. Interestingly, allowing for private ex-post *and* ex-ante contributions finds the support of the majority of individuals. A majority of higher preference individuals prefer endogenous timing over the ex-post regime; conversely, a different majority including all low-preference individuals prefers it over the ex-ante regime. These findings are especially encouraging insofar that in many empirically relevant situations, individuals will likely have full discretion on the timing of their individual contributions. The paper also quantified and compared equilibrium policies and the equilibrium level of private contributions for each institutional setting.

Finally, we showed that if a market based private-provision regime is taken as the status quo, the institutional comparisons change significantly. While moving to the ex-post system (or a fortiori, the general system) is still preferred by a majority, a strong supermajority may now oppose any transition to the ex-ante system. These results suggest that overwhelming support for a regime change may be less likely when a private system represents the status quo. However, when only a majority is required for a regime change, a dual public-private system will be adopted regardless of the status institution: according to our preliminary analysis, endogenous contributions then emerge as the stable equilibrium institution. Further research on these issues is clearly desirable to enhance our understanding of the evolution of institutions in setting with both dual private and public provision.

Appendix

Proof of Proposition 3

To establish the monotonicity properties as stated in the Proposition, we consider the private contributions equilibrium in stage 1. The utility-maximizing stage-1 contribution of an individual i is

$$\hat{g}_i^* = \arg \max_{\hat{g}_i} U^i(y_i - \hat{g}_i - g^{*M}(\hat{g}_{-i}^*, \hat{g}_i), g^{*M}(\hat{g}_{-i}^*, \hat{g}_i)[1 + \beta(N-1)] + \hat{g}_i + \beta \sum_{j \neq i} \hat{g}_j^*). \quad (11)$$

The set of contributors C must always include the highest-preference individuals. To see this, notice that the first-order derivative for individual i ,

$$\frac{dU^i(\cdot)}{d\hat{g}_i} = -U_c^i(\cdot)[1 + \frac{dg(\hat{g})}{d\hat{g}_i}] + U_G^i[1 + \frac{dg(\hat{g})}{d\hat{g}_i}(1 + \beta(N-1))],$$

is strictly increasing in index i : first, $dg/d\hat{g}_i$ is uniform across individuals, and second, the marginal rate of substitution $\Delta^i(\cdot) = U_G^i/U_c^i$ is increasing in i . Invoking normality, it is then impossible to have $dU^l(\hat{g}_{-l}, \hat{g}_l = 0)/d\hat{g}_l \leq 0$ and at the same time $U^k(\hat{g}_{-k}, \hat{g}_k = 0)/d\hat{g}_k > 0$ for any two individuals k, l with rank order $l > k$.⁴²

For any contributor i , the first-order condition to (11) holds with equality and can be rewritten as

$$\frac{U_G^i(c_i, G_i)}{U_c^i(c_i, G_i)} = \frac{1 + \frac{dg(\hat{g})}{d\hat{g}_i}}{1 + \frac{dg(\hat{g})}{d\hat{g}_i}[1 + \beta(N-1)]}. \quad (12)$$

This system of first-order conditions defines the equilibrium contributions for any $i \in C$. To analyze these conditions, note first that for any $\beta > 0$, the marginal rate of substitution is not unity because any increase in private contributions triggers a smaller equilibrium public policy, $dg(\hat{g})/d\hat{g}_i < 0$. This feature is intuitive. With normal preferences, larger private contributions of some $i > m$ cause the median voter m to raise her private consumption, with the consequence of a decrease in the public

⁴²Hence, the set of contributors is non-empty iff the first-order derivative for individual n ,

$$-U_c^n(\cdot)[1 + \frac{dg(\hat{g})}{d\hat{g}_n}] + U_G^n[1 + \frac{dg(\hat{g})}{d\hat{g}_n}(1 + \beta(N-1))],$$

is positive if evaluated at a contribution vector $\hat{g} = 0$.

policy g . In anticipation of this negative response, each contributor provides a smaller contribution than he would do otherwise. Specifically, contributions are kept below the level which equates (taken the subsequent public provision into account) marginal rate of substitution and marginal costs of provision. Finally, inspection shows that because of the symmetry property $dg/\hat{d}g_i = dg/\hat{d}g_j$ for any two contributors i, j , equilibrium contributions must be strictly increasing in the index of natural preferences.⁴³ Hence, public-goods consumption is increasing in the natural preference index, and strictly so for the set of contributors and unless $\beta = 1$. \square

Proof of Proposition 6

For any private contributor i , the second-stage first-order condition $\Delta^i(g_m^{ep}, \tilde{g}) = 1$ holds, which equalizes his marginal utilities from consuming public and private goods. In contrast, $\Delta_j(g_m^{ep}, \tilde{g}_{-j}, \tilde{g}_j = 0) < 1$ for any non-contributor j . By standard arguments, the set of contributors in stage 2 is decreasing in g and at certain threshold levels g , the lowest-preference contributing individual becomes a non-contributor.

The proof proceeds in several steps. We consider the range of first-period policies g_m for which m contributes in the second stage, $\tilde{g}_m(g_m) > 0$. We show that within this range $g \in [\underline{g}_m, \bar{g}_m]$, an increase in g raises the total contributions $G^T \equiv Ng_m + \sum_i \tilde{g}_i$ of all $i \in N$ individuals. As an implication of this result, the total contributions $G_m^{-m}(g)$ of individuals other than m are shown to increase in g as well. Since $U^m(g_m)$ is increasing if and only if $dG_m^{-m}/dg_m > 0$ (see Lemmas 1 and 2 above), the median voter m cannot be a stage-2 contributor in equilibrium. In other words, m 's globally optimal policy g_m^{ep} is (weakly) larger than \bar{g}_m , and therefore larger than any upper-boundary threshold \bar{g}_j for individuals indexed $j < m$. This proves the result.

Step 1: $dG^T(g_m)/dg_m > 0$ whenever the number of contributors is $\hat{N} < N$, and if $\beta < 1$.

The proof is by contradiction. Suppose a raise in g_m from g'_m to g''_m would lower total contributions $G^T(g_m)$ towards the public good. Then, there must be at least

⁴³Suppose not, and the lower-preference individual j contributes more in equilibrium. Then, the first-order conditions () and () cannot simultaneously hold for individuals i and j .

one individual k (necessarily, a contributor under policy g'_m) who reduces his total contribution g_k^T under policy g''_m . We show that this behavior would be inconsistent with normality. To see this, suppose first that k 's public goods consumption G_k is smaller under policy g''_m . But since his private goods consumption must be larger under policy g''_m than under policy g'_m , this would contradict normality. Second, suppose other individuals compensate for the reduction in k 's total contribution in a way which allows k to (weakly) raise his public goods consumption G_k under policy g''_m . To see that this is impossible, notice that for any unit reduction in g_k^T , the total provision of all other individuals, G_k^{-k} , must increase by at least $1/\beta > 1$ to avoid a reduction in G_k . But then, total public goods provision would be characterized by $G^T(g''_m) > G^T(g'_m)$, a contradiction.

Step 2: $dG^T(g_m)/dg_m > 0$ implies $dG_m^{-m}(g_m)/dg_m > 0$.

Suppose not, and suppose instead $G_m^{-m}(g''_m) - G_m^{-m}(g'_m) = -\epsilon$ for $g''_m > g'_m$, where ϵ is a positive number. We show that m 's stage-2 contributions are characterized by $\tilde{g}_m(g''_m) - \tilde{g}_m(g'_m) < \beta\epsilon$. To see why, note that otherwise, m would raise her total public-goods consumption in state g''_m relative to state g'_m . Her private consumption would fall at the same time, which is inconsistent with the assumed normality of preferences. Since $dG^T(g_m)/dg_m > 0$, the difference in total public goods supply between both states would satisfy $G(g''_m) - G(g'_m) \leq -(1 - \beta)\epsilon$, which is negative for any $\beta < 1$ and a contradiction to the result in Step 1.

Step 3: $dG_m^{-m}(g_m)/dg_m > 0$ implies $dU^m(g_m)/dg_m > 0$ for any $g_m < \bar{g}_m$.

For any $g_m < \bar{g}_m$, m contributes a positive amount in stage 2. In the considered range $g_m \in [\underline{g}_m, \bar{g}_m]$, m can always raise g_m in such a way that her total contribution $g_m^T = g_m + \tilde{g}_m$ constant. Since $G_m^{-m}(g_m)$ increases in g_m , this strategy would leave m 's private consumption constant while raising her public goods consumption. Accordingly, m always raises her utility by raising g_m to a level $g_m \geq \bar{g}_m$ where she does not voluntarily contribute in continuation equilibrium. Finally, since $\arg \max_{g_m} U^m(g_m) \geq \bar{g}_m$, neither m nor any individual $j < m$ contributes $\tilde{g}_j > 0$ in stage 2: this is because $\Delta^m(g_m^{ep}, \tilde{g}_m = 0, \tilde{G}_m^{-m}(g_m^{ep})) \leq 1$ implies $\Delta^m(g_m^{ep}, \tilde{g}_j = 0, \tilde{G}_j^{-j}(g_m^{ep})) < 1$ by normality of preferences. \square

Proof of Proposition 7

For a given policy level g_m , private contributions are larger in the ex-post regime. To see this, notice that for each contributor i , the second stage optimality condition reads $\Delta^i(c_i, G_i) = 1$, as compared to the optimality condition $\Delta^i(c_i, G_i) > 1$ that prevails in the ex-ante setting for commitment reasons. If in addition the equilibrium policies are characterized by $g_m^{ep} \leq g^{ea}$ (this will be established below), the respective Nash equilibria unambiguously satisfy $\tilde{g}_i > \hat{g}_i$ for each contributor i .

In what follows, we show that the equilibrium policy level in the ex-post regime is indeed smaller. To see this, note that in the alternative ex-ante setting, m 's optimal policy choice satisfies the first-order condition $\Delta^m(c_m, G_m) = 1/z$. In the ex-post scenario, the corresponding condition reads

$$\Delta^m(\cdot) = \frac{U_G(\cdot)}{U_c(\cdot)} = \frac{1}{z + \beta \sum_j \frac{d\tilde{g}_j}{dg_m}}.$$

By our previous arguments (See Step 2 in the Proof of Proposition 5), this implies $\Delta^m(c_m, G_m) > 1/z$ whenever the set of contributors is nonempty. By way of contradiction, suppose now that $g_m^{ep} \geq g^{ea}$, i.e., the equilibrium policy in the ex-post setting is (weakly) larger. Since the median voter m does not privately contribute in either regime, her private consumption in the ex-post regime is then (weakly) smaller. But in order to satisfy the respective first-order conditions for equilibrium policy choice in each regime, m 's public goods consumption G_m in the ex-post regime would have to be smaller as well. As a consequence, her utility in this regime would be smaller, which is impossible: by our above results, implementing a policy g_m^{ea} would raise m 's public-goods consumption relative to the ex-ante regime (for any g_m , private contributions are larger in the ex-post regime) while leaving her private consumption the same, a contradiction. Hence, $g_m^{ep} < g_m^{ea}$.

By revealed preference, our previous arguments imply that any non-contributor is better off in the ex-post regime: the median voter could have chosen the same policy in each regime, rendering private consumption for non-contributors in both regimes identical. At the same time, public consumption in the ex-post setting had been larger

because of the higher level of private contributions (see above).⁴⁴

Conversely, contributors i are worse off in the ex-post system: since the equilibrium policy is lower, achieving the same consumption G_i requires each of these individuals to raise their total contributions in the ex-post regime. A contributor in the ex-post regime may do even worse compared to the pure public system: the equilibrium policy in the ex-post system is smaller because first, private contributions reduce m 's preferred policy. Second, the equilibrium policy in the ex-post setting is even smaller than the best response to private contributions for commitment reasons. Hence, non-contributors contribute less towards a contributor's public-goods consumption than in the pure public system, trading off the utility-enhancing effect of the feasibility of private contributions. \square

Proof of Proposition 9

Most of these results have been established before. In what follows, assume $N\alpha_m > \alpha_h$. In an institutional system with ex-ante contributions, no agent then makes a private contribution, and policy $g_m^{ea} = g_m^{pp} = N\alpha_m (> \alpha_h > g^+)$ is adopted. Moreover, since wealth effects are absent and the equilibrium policy in the ex-ante and the general regime constitutes a best response to private ex-ante contributions (recall $\tilde{g} = 0$), public goods consumption levels must satisfy $G^{gs} = G^{ea}$. In the ex-post system, equilibrium outcomes depend on the size of $\Delta = N\alpha_m - \alpha_h (> 0)$. If Δ is not too large, the median voter implements a policy $g_m^{ep} = 0$, and h provides an ex-post contribution $\tilde{g}_h = \alpha_h$. In this case, relative consumption levels are such that $G^{gs} > G^{ep}$ because G^{ep} satisfies $\Delta = 1/N\alpha_m$ whereas G^{ep} satisfies $\Delta = 1/\alpha_h$. Conversely, if Δ is large, policies will instead be $g_m^{ep} = g_m^{ea} = N\alpha_m = g^+(\hat{G} = 0) = g^{gs}$, and no individual privately contributes in either regime. Hence, $G^{gs} = G^{ep}$ in this case.

Finally, compare the general system with the ex-ante system and the ex-post system, respectively, with respect to individual utilities. Since $G^{gs} = G^{ea}$ and $g_m^{gs} = g^+ < g_m^{ea}$,

⁴⁴Since $g_m^{ep} < g_m^{ea}$ and since for commitment reasons, g_m^{ep} is smaller than the best response to the vector of subsequent private contributions, a comparison of public good consumption in both regimes is generally ambiguous. However, note that if $\beta = 1$ and preferences are quasilinear, public goods consumption in the ex-post regime will be larger.

a majority of citizens including all agents $i \leq m$ prefers the general system over the ex-ante system. To compare general system and ex post system, consider first a situation in which Δ is sufficiently small that $g_m^{ep} = 0$ (and $\tilde{g}_h = \alpha_h$). In the general system, m could instead adopt a policy $g = 0$, and induce the same economic outcome as in the post system. If she chooses g^+ , she raises her well being by revealed preferences. Utilities are also raised (at least) for any agent with rank $i > m$, including the ex-ante contributor(s).⁴⁵ Next, suppose Δ is sufficiently large that m implements $g_m^{ep} > 0$ and collective contribution fund public goods supply in the ex-post system. Then, total threshold contributions in the general system satisfy $\hat{G}^* = 0$, and economic outcomes in both regimes coincide. \square

⁴⁵To see this, assume an equilibrium contributor i had chosen not to contribute ex ante. He would then still prefer a policy larger than g^+ at stage 2, notwithstanding the fact that policy $g = 0$ gives him the same utility as in the ex-post system. By revealed preferences, contributing $\hat{g}_i > 0$ ex ante must further improve i 's equilibrium utility.

References

- Alesina, A., Angeloni, I. and F. Etro** (2005), "International Unions", *American Economic Review*, 95(3), 602-615.
- Bergstrom, T.C., Blume, L., and H.R. Varian** (1986) , "On the Private Provision of Public Goods," *Journal of Public Economics*, 29(1), 25-49.
- Cornes, R., Hartley, R., and T. Sandler** (1999) , "Equilibrium Existence and Uniqueness in Public Good Models: An Elementary Proof via Contraction," *Journal of Public Economic Theory*, 1(4), 499-509.
- Crémer, J. and T.R. Palfrey** (2000) , "Federal Mandates by Popular Demand," *Journal of Political Economy*, 108, 905-927.
- Crémer, J. and T.R. Palfrey** (2006) , "An Equilibrium Voting Model of Standards with Externalities," *Journal of Public Economics*, 90(10-11), 2091-2106.
- Epple, D. and R. Romano** (1996) , "Public Provision of Private Goods," *Journal of Political Economy*, 104(1), 57-84.
- Epple, D. and R. Romano** (2003) , "Collective Choice and Voluntary Provision of Public Goods", *International Economic Review*, 44(2), 545 - 572.
- Fernandez, R. and R. Rogerson** (1995) , "On the Political-Economy of Education Subsidies," *Review of Economic Studies*, 62(2), 249-262.
- Fernandez, R. and R. Rogerson** (2003) , "Equity and Efficiency: An Analysis of Education Finance Systems," *Journal of Political Economy*, 111, 858-897.
- Fraser, C.D.** (1992) , "The Uniqueness of nash Equilibrium in the Private Provision of Public Goods: An Alternative Proof," *Journal of Public Economics*, 49, 389-390.
- Glomm, G. and B. Ravikumar** (1998), "Opting Out of Publicly Provided Services A Majority Voting Result," *Social Choice and Welfare*, 15, 187-199.

Hafer, C. and D. Landa (2005), "Public Goods in Federal Systems," NYU, *mimeo*.

Rubinchik-Pessach, A. (2005), "Can Decentralization Be Beneficial?," *Journal of Public Economics*, 89(7), 1231-1250.

Stiglitz, J.E. (1974), "The Demand For Education in Public and Private School Systems," *Journal of Public Economics*, 3, 349-385.