IGNORANCE AND NAIVETE IN LARGE ELECTIONS

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ABSTRACT. We consider a two-alternative election with voluntary participation and nearly common interests in which voters may acquire information about which alternative is best. Voters may be *rational* or *naive* in the sense of being able, or not, to update their beliefs about the state of the world conditioning on the behavior of others. We show that, if all voters are rational and the distribution of the cost of information is not bounded way from zero, there is a sequence of equilibria such that along that sequence the probability of choosing the best alternative converges to one. Moreover, electoral participation converges to zero if and only if the probability that there is free information is zero. Per contra, if some voters are naive, equilibrium electoral participation remains bounded way from zero, and there is sequence of equilibria with the desirable information aggregation property only if there is some positive probability that voters receive free information.

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> On trove de plus, que si la probabilité de la voix de chaque Votant est plus grande que $\frac{1}{2}$, c'est-à-dire, s'il est plus probable qu'il jugera conformément à la vérité, plus le nombre des Votans augmentera, plus la probabilité de la vérité de la décision sera grande: la limite de cette probabilité sera la certitude [...]
>
> Une assemblée très-nombreuse ne peut pas être composée d'hommes très-éclaires; il est même vraisemblable que ceux qui la forment joindront sur bien des objets beaucoup d'ignorance à beaucoup de préjugés.
>
> Condorcet (1785)[1986, p. 29-30]

1. Introduction

The idea that elections serve to make good collective choices by aggregating the information dispersed among the voters was first given a statistical foundation by Condorcet and has been studied in the last fifteen years from a game-theoretic viewpoint, starting with the pioneer work of Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997) and Myerson (1998b). The conclusion of this literature has been, for the most, in the positive with respect to the ability of elections to lead to good choices when there are many voters under a variety of circumstances. Most of this literature has taken as given the information held by each voter. In many situations of interest, however, one may think that the decision of voters to acquire information may be affected by the circumstances of the election. It seems desirable, therefore, to analyze the aggregation of information in elections taking into account in the same game-theoretic treatment both the electoral behavior and the information acquisition behavior of voters.

With many voters, the ability of the election to aggregate information potentially held by voters may be weakened for two different reasons. Understanding that each voter has little probability of influencing the outcome, voters may decline acquiring any costly information. That is, it may be a rational decision for many or most voters to remain ignorant if becoming informed requires any time or effort. Even worse, realizing that there is little at stake in casting a single vote, voters may not behave in a manner fully consistent with rational behavior when facing difficult inference problems related to their behavior at the voting booth.[1]

In this paper, we propose a model of elections with incomplete information that recognizes both sources of difficulty for successful information

[1] Indeed, as the initial quotes make it clear, Condorcet was aware of the possibility of both ignorance and biased judgment clouding the opinion of voters as electoral bodies grow large.

aggregation. In the model, voters may have different access to free or costly information, thus making it possible the emergence of rational ignorance. In addition, voters may be rational or naive. Rational and naive voters are assumed to play best responses to their expectation about the behavior of other voters, but to form expectations differently. Unlike rational voters, naive voters are assumed to be unable to update their beliefs about the distribution of the state of the world conditional on the behavior of other voters, and in particular conditional on the event of being decisive. That is, naive voters are "fully cursed" in the sense of Eyster and Rabin (2005).[2] While naive voters, as we will see, behave sincerely at the voting booth in the sense of conditioning on their private information, naive voting cannot be reduced to the traditional notion of sincere voting to the extent that naive voters calculate correctly the (unconditional) distribution of voting profiles for other voters, and use this calculation to decide whether to acquire costly information.

Unlike our previous related work on rational ignorance in large elections (Martinelli 2006, 2007), we allow for abstention and for voters to have asymmetric heterogenous preferences. Both deviation from previous work have strong implication for equilibrium, as described below. The cost of information is assumed to be drawn independently for each voter from a distribution that is not bounded way from zero, with some voters possibly receiving free information. For technical reasons, we formalize the election as a Poisson game of population uncertainty, using the setup introduced by Myerson (1998ab, 2000). As in the Condorcet jury literature, we are interested in the ability of large electorates to reach correct decisions with large probability. We study this question considering the limit of sequences of equilibria corresponding to the game described above for the expected number of voters going to infinity, and finding conditions under which the probability of choosing alternative the alternative that voters would favor if the state of the world were known.

Table 1 below summarizes our findings with respect to information aggregation in the sense defined above in different cases. In all cases, the incentive to acquire information decline in equilibrium with the expected number of voters, so the fraction of voters who acquire costly information declines in equilibrium to zero. This, however, very different implications depending on whether there are or not naive voters. If all voters are rational, information aggregation is successful under a very general condition that put bounds on the preferences of voters in relation to the quality of the information possibly held by one voter so that they are willing to abstain rather than vote for their favorite alternative whenever they are uninformed.

[2]See Esponda (2007) and Jehiel and Koessler (2008) for similar ideas.

		Voters' behavior	
		rational	*rational and naive*
Information	*free and costly*	success; large turnout	success; large turnout
	costly	success; small turnout	failure; large turnout

TABLE 1. Information aggregation in large elections

If the probability of receiving free information is zero, successful information aggregation relies precisely on most voters abstaining as the size of the electorate increases. By way of contrast, in previous work were voting is assumed to be mandatory, successful aggregation of costly information relies on stringent conditions on the density of the cost of information.

If there are both rational and naive voters, the condition for rational voters to be willing to abstain becomes more stringent, as naive voters do not abstain when they are uninformed, so that the information contained on the event of the election being tied or nearly tied is reduced. In particular, the condition for rational voters to be willing to abstain cannot be satisfied if the probability of receiving free information is zero. Under these circumstances, the fraction of informed voters declines to zero, most voters, regardless of whether rational or naive, vote for the alternative they favor in the absence of information acquisition, and information aggregation fails.

If the probability of receiving free information is positive, then information aggregation is positive with naive and rational voters. While naive voters are less willing than rational voters to abstain when they are uninformed, they are more willing than rational voters to vote for the alternative they favor the least conditional on being informed about that alternative being likely to be the best for all voters. Thus, it is possible that all rational voters vote for their favorite alternatives, but information aggregation is successful because of naive voters using free information.

Among the game-theoretic work on the Condorcet jury theorem, most relevant for this paper is the work of Feddersen and Pesendorfer (1996) analyzing "swing voter's curse," that is, by analogy with the winner's curse in auctions with common values, the fact that rational voters may find it optimal to abstain rather than to vote sincerely when, conditional on being pivotal, they would regret changing the result of the election. Guarnaschelli, McKelvey and Palfrey (2000) and Battaglini, Morton and Palfrey (2010) study experimentally jury situations and in particular the swing voter's curse. Eyster and Rabin (2005) introduce the notion of cursed equilibrium trying to capture precisely the experimental evidence in the context of auctions that bidders fall prey of the winner's curse; they also discuss the possibility that the experimental evidence of Guarnaschelli et al. (2000) is consistent with cursed behavior. Besides the mentioned work on information acquisition in large elections, Mukhopadhaya (2005), Persico (2004) and Gerardi and Yariv (2008) have worked on endogenous information in committees. Preliminary experimental evidence from Elbittar et al. (2011) seems to be consistent with the idea that rational and naive voters coexist in the context of information acquisition in juries and provides some of the motivation for this work.

2. The Model

We analyze an election with two alternatives, A and B. The number of voters is a random variable drawn from a Poisson distribution with mean n; that is, the probability that the number of voters is z is given by

$$P(z|n) \equiv e^{-n}n^{z}/z!$$

for all nonnegative integers.

There are two possible states of the world, ω_A and ω_B. Both states of the world are equally likely. Let

$$V(d,\omega'_d) = \begin{cases} 1 & \text{if } d = d' \\ 0 & \text{if } d \neq d' \end{cases}$$

for $d,d' \in \{A,B\}$, where the first argument is the winning alternative and the second argument is the state of the world. V represents the *common value* component of voters' payoffs.

Each voter has a bias ε in favor of A that is independently drawn from $\{-\underline{\varepsilon},\overline{\varepsilon}\}$, where $-\underline{\varepsilon} < 0 < \overline{\varepsilon}$. We denote $\underline{p} = \Pr(\varepsilon = -\underline{\varepsilon})$ and $\overline{p} = \Pr(\varepsilon = \overline{\varepsilon})$. Let

$$W(d,\varepsilon) = \begin{cases} \varepsilon & \text{if } d = A \\ 0 & \text{if } d = B \end{cases}$$

for $d \in \{A,B\}$ and $\varepsilon \in \{\varepsilon_k\}$. W represents the *partisan* component of voters' payoffs.

Voters do not know the realization of the state of the world. Each voter, however, may acquire some costly information. The cost of information is independently and identically distributed across voters according to a distribution function F. F is strictly increasing and continuously differentiable over the interval $(0,\overline{c})$ for some $\overline{c} \in \Re_+$, with $F(0) \geq 0$ and $F(\overline{c}) = 1$. Note that, if $F(0) > 0$, then voters receive free information with positive probability. After learning their idiosyncratic cost of information, each voter must decide whether to acquire information or not. Each voter who acquires information receives a signal $s \in \{s_A, s_B\}$. The probability or receiving signal s_d in state ω_d is equal to $1/2+q$ for $d \in \{A,B\}$, where $q \in (0,1/2)$.

The election takes place after voters receive their signals. A voter may vote for A, vote for B, or abstain. The alternative with most votes is chosen, with ties broken by a fair coin toss.

Given the winning alternative d, the state of the world ω, a voter cost of information c, and a voter bias ε, the utility of the voter is

$$V(d,\omega)+W(d,\varepsilon)-c$$

if the voter acquires information, and

$$V(d,\omega)+W(d,\varepsilon)$$

otherwise.

Since prior beliefs give equal probability to each state of the world, the posterior probability of state ω_d conditional on having observed a signal s_d is equal to $1/2+q$. Thus, a voter with bias ε acting like a single decision-maker would choose A if $\varepsilon > 0$ and would choose B if $\varepsilon < 0$ if the voter has not acquired information about the state of the world, and would choose whichever alternative is favored by the voter's private signal if the voter has acquired information and $|\varepsilon| < 2q$. Of course, a voter is not a single decision-maker. Whether or not casting a vote influences the winning alternative, and hence the utility of the voter, depends on other voters' decisions. Therefore, in order to choose optimally whether to acquire information and which alternative to support, the voter must form some beliefs about other voters' actions, and how these actions depend on the state of the world.

We assume that each voter is either *rational* or *naive*. Both rational and naive voters play best responses to their beliefs about other voters' actions, but form these beliefs differently. Rational voters understand that the the probability distribution of other voters' decisions is influenced by the state of the world, and calculate correctly this probability distribution for the two states of the world. Naive voters mistakenly believe that other voters' actions are independent of the state of the world, and calculate the probability distribution of other voters' decisions in each state of the world to be equal to the unconditional distribution. Each voter's behavioral type β is independently drawn from $\{\rho,\nu\}$ (for rational and naive, respectively), were $\Pr(\beta=\rho)=r$.

A voter's type is a vector $t=(\beta,\varepsilon,c,s)$ specifying the voter's behavioral type, partisan bias, cost of information and private signal (which the voter only gets to observe if the voter acquires information). An action is a pair $a=(i,v)$, $i\in\{1,0\}$, $v\in\{A,B,0\}$, indicating wether the voter acquires or not information and whether the voter votes for alternative A, for alternative B, or abstains.

A *strategy function* is a measurable mapping σ assigning to each voter type a probability distribution over the set of actions, where $\sigma(a|t)$ is interpreted as the probability that a voter will choose action a given type t, with the constraint

$$\sigma((0,v)|(\beta,\varepsilon,c,s_A)) = \sigma((0,v)|(\beta,\varepsilon,c,s_B))$$

for all v, β, ε and c. That is, an uninformed voter's behavior is independent of the voter's signal (and hence, independent of the state of the world).

Let $\tau(v|\sigma,\omega)$ be the probability that a voter chooses $v \in \{A,B,0\}$ given strategy σ conditional on the state of the world ω. From the independent action property of Poisson games, the number of votes for A, votes for B and abstentions in state ω are independent Poisson random variables with mean $n\tau(A|\sigma,\omega)$, $n\tau(B|\sigma,\omega)$, and $n\tau(0|\sigma,\omega)$. Let

$$\tau(\sigma,\omega) = (\tau(v|\sigma,\omega))_{v\in\{A,B,0\}}.$$

Then the probability of the voting profile $x = (x(A),x(B),x(0))$, describing how many voters vote for A, vote for B, or abstain, is equal to

$$P(x|n\tau(\sigma,\omega)) = \prod_{v\in\{A,B,0\}} P(x(v)|n\tau(v|\sigma,\omega)).$$

Moreover, from the perspective of any given voter, the probability that the voting profile of the other voters is given by $x = (x(A),x(B),x(0))$ is also $P(x|\lambda(\sigma,\omega))$, where this result follows from the environmental equivalence property of Poisson games (Myerson 1998a).

We say that a strategy σ' is a *best response* to the strategy σ if

(i) for almost every triple ρ,ε,c (that is, for each possible rational voter) the strategy σ' maximizes the expected utility of the voter given the distributions $P(x|n\tau(\sigma,\omega_A))$ and $P(x|n\tau(\sigma,\omega_B))$ of voting profiles of other voters in state ω_A and ω_B, respectively, and

(ii) for almost every triple ν,ε,c (that is, for each possible naive voter) the strategy σ' maximizes the expected utility of the voter given the distribution $\frac{1}{2}P(x|n\tau(\sigma,\omega_A))+\frac{1}{2}P(x|n\tau(v|\sigma,\omega_B)))$ of voting profiles of other voters in both states of the world.

An *equilibrium* is a strategy that is a best response to itself. Note that, in equilibrium, both rational and naive voters play best responses, but naive voters ignore the dependence of the distribution of other voters'profiles on the state of the world.

If $r = 1$, that is, if all voters are rational, then the equilibrium definition is similar to Myerson's (1998) definition of equilibrium for extended Poisson games. If $r = 0$, that is, if all voters are naive, then the equilibrium definition is similar to Eyster and Rabin's (2005) definition of fully cursed equilibrium, adapted to a context of private information acquisition and population uncertainty. If $0 < r < 1$, then the equilibrium definition is analogous to the generalized cursed equilibrium of Eyster and Rabin (2005).

3. BELIEFS AND BEST RESPONSES

Given any strategy γ, the number of votes for A minus the number of votes for B in state ω is the difference between two independent Poisson random variables with mean $n\tau(A|\sigma,\omega)$ and $n\tau(B|\sigma,\omega)$, respectively. This random variable is sometimes referred to as a Skellam random variable (after Skellam 1946) with parameters equal to the means of the two independent Poisson variables.[3]

In particular, the probability that the difference between the number of votes for A and the number of votes for B is equal to some integer z is $S(z|n\tau(A|\sigma,\omega), n\tau(B|\sigma,\omega))$, where

$$S(z|m_1,m_2) = e^{-(m_1+m_2)}\left(\frac{m_1}{m_2}\right)^{z/2} I_{|z|}(2\sqrt{m_1m_2})$$

is the *Skellam distribution function* with parameters m_1 and m_2, and

$$I_z(y) = \sum_{k=0}^{\infty} \frac{1}{k!(k+z+1)!}\left(\frac{y}{2}\right)^{2k+z}$$

is a *modified Bessel function of the first kind* (Bronshtein et al 2003). Note that $I_z(y)$ is increasing in y, and $I_0(0) = 1$ and $I_z(0) = 0$ for $z > 0$.

We will make use of the following asymptotic approximation

$$I_z(y) = \frac{e^y}{\sqrt{2\pi y}}(1+O(1/y)),$$

where $g(y) = O(1/y)$ stands for $\lim_{y\to\infty} yg(y) = C$ for some constant $C \neq 0$ (Bronshtein et al 2003).

We will also make use of the following known result. Consider a sequence $(\lambda_{1n}, \lambda_{2n})$ of positive numbers for increasing values of n, and let z_n be a Skellam random variable with parameters $n\lambda_{1n}$ and $n\lambda_{2n}$ for each n. If $n\lambda_{1n}$ and $n\lambda_{2n}$ go to infinity and $\lambda_{1n}/\lambda_{2n}$ converges to a number larger than one as n increases, then the probability that z_n is strictly larger than zero converges to one.[4]

[3]Recent use of the distribution of the difference between Poisson random variables includes Myerson (1998, 2000, 2002) and Krishna and Morgan (2010) in the context of voting, and Karlis and Ntzoufras (2003, 2006) in the context of sport competitions and epidemiology.

[4]This follows from the fact that the Skellam distribution is asymptotically normal. This can be shown treating the Skellam random variable z_n as a summation of n iid random variables as in Fisz (1955a,1955b) and using the Berry-Esseen theorem; see Feller (1971) for a classical formulation of the Berry-Esseen theorem.

Let

$$D(z|\rho,\sigma,\omega) = S(z|n\tau(A|\sigma,\omega), n\tau(B|\sigma,\omega))$$

and

$$D(z|\nu,\sigma,\omega) = \tfrac{1}{2}\textstyle\sum_{\omega'} S(z|n\tau(A|\sigma,\omega'), n\tau(B|\sigma,\omega'))$$

represent, respectively, the beliefs of a rational and a naive voter about the distribution of the net difference of votes in favor of A of other voters in state ω if other voters are playing according to the strategy σ. That is, rational voters assess correctly the distribution of the net difference in favor of A while naive voters believe incorrectly this distribution is independent of the state of the world, but assess correctly the unconditional distribution.

The difference in expected utility between voting for alternative A after receiving signal s_A and abstaining after receiving signal B versus abstaining regardless of the signal for a voter of behavioral type β and bias ε is

$$\begin{aligned} G(s_A|\beta,\varepsilon,\sigma) \equiv \tfrac{1+\varepsilon}{4}(\tfrac{1}{2}+q)(D(0|\beta,\sigma,\omega_A) + D(-1|\beta,\sigma,\omega_A)) \\ - \tfrac{1-\varepsilon}{4}(\tfrac{1}{2}-q)(D(0|\beta,\sigma,\omega_B) + D(-1|\beta,\sigma,\omega_B)). \end{aligned}$$

Similarly, the difference in expected utility between abstaining after signal A and voting for alternative B after receiving signal s_B versus abstaining regardless of the signal for a voter of behavioral type β and bias ε is

$$\begin{aligned} G(s_B|\beta,\varepsilon,\sigma) \equiv -\tfrac{1+\varepsilon}{4}(\tfrac{1}{2}-q)(D(0|\beta,\sigma,\omega_A) + D(1|\beta,\sigma,\omega_A)) \\ + \tfrac{1-\varepsilon}{4}(\tfrac{1}{2}+q)(D(0|\beta,\sigma,\omega_B) + D(1|\beta,\sigma,\omega_B)). \end{aligned}$$

The difference in expected utility between voting for alternative A and abstaining, regardless if the signal in both cases, is

$$\begin{aligned} G(A|\beta,\varepsilon,\sigma) \equiv \tfrac{1+\varepsilon}{4}\left(D(0|\beta,\sigma,\omega_A) + D(-1|\beta,\sigma,\omega_A)\right) \\ - \tfrac{1-\varepsilon}{4}\left(D(0|\beta,\sigma,\omega_B) + D(-1|\beta,\sigma,\omega_B)\right). \end{aligned}$$

Finally, the difference in expected utility between voting for alternative B and abstaining, regardless if the signal in both cases, is

$$\begin{aligned} G(B|\beta,\varepsilon,\sigma) \equiv -\tfrac{1+\varepsilon}{4}\left(D(0|\beta,\sigma,\omega_A) + D(1|\beta,\sigma,\omega_A)\right) \\ + \tfrac{1-\varepsilon}{4}\left(D(0|\beta,\sigma,\omega_B) + D(1|\beta,\sigma,\omega_B)\right). \end{aligned}$$

We claim that, if a best-responding voter acquires information at some positive cost, then the voter either votes for A if the signal is s_A and abstains if the signal is s_B, or votes for B if the signal is s_B and abstains if the signal is s_A, or votes according to the signal received regardless of the signal. To see this, note that after receiving signal d a voter must be strictly better off voting for d or abstaining rather than voting for the other alternative. Otherwise, the voter is strictly better off voting for the other alternative after receiving the other signal, which decreases the expected utility of voting for alternative d or abstaining for any strategy of other players. But then the voter is better off voting for the other alternative and not acquiring costly information. Moreover, the voter cannot prefer abstaining or be indifferent between abstaining and voting for alternative d if the signal is s_d for both alternatives, since then the voter would be better off abstaining and not acquiring costly information.

Let

$$c(\beta,\varepsilon,\sigma) \equiv \max\{G(s_A|\beta,\varepsilon,\sigma), G(s_B|\beta,\varepsilon,\sigma), G(s_A|\beta,\varepsilon,\sigma)+G(s_B|\beta,\varepsilon,\sigma)\} \\ -\max\{0, G(A|\beta,\varepsilon,\sigma), G(B|\beta,\varepsilon,\sigma)\}.$$

From the preceding argument it follows that this is the utility gain of acquiring information, net of the cost of information acquisition.

Given a strategy σ, we can characterize completely best responses using the using the definition of $G(\cdot|\beta,\varepsilon,\sigma)$ as follows.

Lemma 1. *A strategy σ' is a best response to σ if for almost every triple β,ε,c, under the strategy σ',*

(i) *if $c < c(\beta,\varepsilon,\sigma)$ then the voter acquires information, and after signal s_d the voter votes for d if $G(s_d|\beta,\varepsilon,\sigma) > 0$, abstains if $G(s_d|\beta,\varepsilon,\sigma) < 0$, and either votes for d or abstains if $G(s_d|\beta,\varepsilon,\sigma) = 0$*

(ii) *if $c > c(\beta,\varepsilon,\sigma)$, then the voter does not acquire information, and votes for d only if $G(d|\beta,\varepsilon,\sigma) = \max\{0, G(A|\beta,\varepsilon,\sigma), G(B|\beta,\varepsilon,\sigma)\}$ and abstains only if $0 = \max\{0, G(A|\beta,\varepsilon,\sigma), G(B|\beta,\varepsilon,\sigma)\}$.*

Since $D(z|\nu,\sigma,\omega_A) = D(z|\nu,\sigma,\omega_B)$ for all z, we have that, for any given σ,

$$\operatorname{sgn}(G(A|\nu,\varepsilon,\sigma)) = -\operatorname{sgn}(G(B|\nu,\varepsilon,\sigma)) = \operatorname{sgn}(\varepsilon).$$

Thus, (best-responding) *uninformed naive voters always vote for the alternative favored by their bias regardless of the strategy followed by other voters*. In particular, uninformed naive voters do not abstain. This posses particular difficulties for successful information aggregation as will be argued below. In contrast, uninformed rational voters may decide to abstain if the strategy followed by other voters is such that the voter's expected utility worsens by casting an uninformed vote, in fact delegating the collective decision to other voters as in the swing voter's curse model of Feddersen and Pesendorfer (1996).

Best-responding naive voters can be said to be "sincere," that is behave as if each of them were the sole decision maker, with regard to the decision of which alternative to support. Best-responding naive voters, however, will not be "sincere" with respect to the decision of acquiring information as long as other voters vote with positive probability.

4. RATIONAL VOTERS

In this section we consider as a benchmark the case in which all voters are rational. We use the following condition in the statements of theorems below:

$$\max\{\underline{\varepsilon}, \overline{\varepsilon}\} < \frac{2q}{1+2\sqrt{\frac{1}{4}-q^2}}. \tag{A1}$$

We say that *condition* (A1) *fails strictly* if the opposite strict inequality holds.

Note that condition (A1) implies $|\varepsilon| < 2q$; i.e., biases in favor of either alternative are small enough compared to the quality of a signal so that a single signal would sway any voter if the voter were a single decision-maker. As we will show, condition (A1) is sufficient for (i) an uninformed rational voter to be willing to abstain rather than vote for the voter's favorite alternative, and (ii) for an informed rational voter to vote according to the signal received rather than abstaining, if all other uninformed voters abstain in a large election. Moreover, if condition (A1) fails strictly, neither (i) nor (ii) are possible.

To provide an intuition for condition (A1), suppose $\overline{\varepsilon} > \underline{\varepsilon}$ and consider the problem of an uninformed rational voter who is biased in favor of A and is considering voting for A rather than abstaining. There are two possibilities for a vote for A to be decisive: either A and B are tied, or A is behind by one vote. In either case, a vote for A would change the result from A winning the election to B winning the election with probability 1/2. If the voter knew that A and B are tied, the voter would think both states are equally likely, and the voter would gain

$$(1/2)[(1+\overline{\varepsilon})-0]+(1/2)[\overline{\varepsilon}-1]=\overline{\varepsilon}$$

in expected terms by changing the result in favor of A. If the voter knew that A is behind by one vote, the voter would think that the probability of state ω_A is $1/2-q$ and the probability of state ω_B is $1/2+q$, and the voter would lose

$$(1/2-q)[(1+\overline{\varepsilon})-0]+(1/2+q)[\overline{\varepsilon}-1]=2q-\overline{\varepsilon}$$

in expected terms by changing the result in favor of A. The voter will be willing to abstain, then, if

$$\frac{2q-\overline{\varepsilon}}{\overline{\varepsilon}} > \frac{\Pr(A \text{ and } B \text{ are tied})}{\Pr(A \text{ is behind by one vote})},$$

where the probabilities in the right-hand side are unconditional. In the proof of the theorem we show that the ratio in the right-hand side converges to $2\sqrt{1/4-q^2}$ if all voters who participate are informed, yielding condition

(A1) as necessary for rational abstention. The quality of information play a double role of the quality of information q in facilitating rational abstention when preferences among voters are not perfectly aligned. Increasing the value of q both makes abstention more attractive when a voter's favorite alternative is behind by one vote, since the information conveyed by one vote is increasing in q, and makes the event of one's favorite alternative being behind by one vote more likely in relation to the event of both alternatives being tied.

Now suppose $\overline{\varepsilon} > \underline{\varepsilon}$ and consider the problem of a rational voter who is biased in favor of A, who has received signal s_B and is considering voting for B rather than abstaining. If the voter knew that A and B are tied, the voter would think that the probability of state ω_A is $1/2 - q$ and the probability of state ω_B is $1/2 + q$, and the voter would gain $2q - \overline{\varepsilon}$ in expected terms by changing the result in favor of B. If the voter knew that B is behind by one vote, the voter would think both states are equally likely, and the voter would lose $\overline{\varepsilon}$ in expected terms by changing the result in favor of B. Thus, for a rational voter, the decision of abstaining when uninformed is entirely analogous to the decision of voting for the voter's least favorite alternative when the voter has received a signal favoring that alternative. As discussed in the next section, these two situations are very different for a naive voter.

Theorem 1. *If $r = 1$ and (A1) holds, there is some $\underline{n}$ such that, for every $n > \underline{n}$, there is an equilibrium such that voters acquire information if their cost is below some cutoff c_n, and vote for A if they observe s_A and for B if they observe s_B, and do not acquire information and abstain if their cost is above c_n. As n goes to infinity, along any sequence of such equilibria the probability that a voter acquires information converges to $F(0)$, the probability of abstention converges to $1 - F(0)$, and the probability of choosing alternative A in state ω_A and alternative B in state ω_B converges to 1.*

This is a special case of Theorem 3 below, but we include a separate proof as it makes the arguments more transparent and describes in detail the equilibrium strategy.

Proof. Suppose $r = 1$ and (A1) holds. Let $\sigma(\hat{c})$ denote the strategy of acquiring information if the cost is below $\hat{c}$ independently of ε, and voting for A if the signal is s_A and for B if the signal is s_B, and do not acquire information and abstain if their cost is above $\hat{c}$, for any $\hat{c} \geq 0$. We have

$$G(s_A|\rho,\varepsilon,\sigma(\hat{c})) = e^{-nF(\hat{c})}\left[\left(\frac{q}{2}+\frac{\varepsilon}{4}\right)I_0\left(2nF(\hat{c})\sqrt{\tfrac{1}{4}-q^2}\right)\right.$$
$$\left.+\frac{\varepsilon}{2}\sqrt{\tfrac{1}{4}-q^2}\,I_1\left(2nF(\hat{c})\sqrt{\tfrac{1}{4}-q^2}\right)\right]$$

and

$$G(s_B|\rho,\varepsilon,\sigma(\hat{c})) = e^{-nF(\hat{c})}\left[\left(\frac{q}{2}-\frac{\varepsilon}{4}\right)I_0\left(2nF(\hat{c})\sqrt{\tfrac{1}{4}-q^2}\right)\right.$$
$$\left. -\frac{\varepsilon}{2}\sqrt{\tfrac{1}{4}-q^2}I_1\left(2nF(\hat{c})\sqrt{\tfrac{1}{4}-q^2}\right)\right].$$

Thus,

$$G(s_A|\rho,\varepsilon,\sigma(\hat{c}))+G(s_B|\rho,\varepsilon,\sigma(\hat{c})) = qe^{-nF(\hat{c})}I_0\left(2nF(\hat{c})\sqrt{\tfrac{1}{4}-q^2}\right) > 0$$

for all $\hat{c} \geq 0$. Moreover, using the asymptotic approximation for $I_0(\cdot)$, there is some $\tilde{n}$ such that

$$G(s_A|\rho,\varepsilon,\sigma(\overline{c}))+G(s_B|\rho,\varepsilon,\sigma(\overline{c})) = qe^{-nF(\overline{c})}I_0\left(2nF(\overline{c})\sqrt{\tfrac{1}{4}-q^2}\right) < \overline{c}$$

for all $n > \tilde{n}$. Thus, for large enough n there is some $c_n \in (0,\overline{c})$ such that

$$e^{-nF(c_n)}qI_0\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right) = c_n. \tag{1}$$

Now let n go to infinity. We claim that c_n converges to zero. For suppose there is a subsequence such that along that subsequence c_n converges to any number in $(0,\overline{c}]$. Then, using the asymptotic approximation for $I_0(\cdot)$, along that subsequence the expression in the left-hand side of equation 1 converges to 0, but the expression in the right-hand side converges to a positive number. Similarly, we claim that $nF(c_n)$ increases without bound. For suppose there is a subsequence such that along that subsequence $nF(c_n)$ converges to either zero or a positive number. Then, since $I_0(y) \geq 1$ for $y \geq 0$, along that subsequence the expression in the left-hand side of equation 1 converges to a positive number, but the expression in the right-hand side converges to 0. (Note that $nF(c_n)$ increases without bound regardless of whether $F(0) = 0$ or $F(0) > 0$. In the former case, $F(c_n)$ decreases at a rate slower that $1/n$.)

We have

$$G(s_A|\rho,\varepsilon,\sigma(c_n)) = e^{-nF(c_n)}\left[\left(\frac{q}{2}+\frac{\varepsilon}{4}\right)I_0\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right)\right.$$
$$\left. +\frac{\varepsilon}{2}\sqrt{\tfrac{1}{4}-q^2}I_1\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right)\right]$$

and

$$G(A|\rho,\varepsilon,\sigma(c_n)) = e^{-nF(c_n)} \times \left[\frac{\varepsilon}{2} I_0\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right)\right.$$
$$\left. + \left(\frac{1+\varepsilon}{4}\left(\frac{\frac{1}{2}-q}{\frac{1}{2}+q}\right)^{1/2} - \frac{1-\varepsilon}{4}\left(\frac{\frac{1}{2}+q}{\frac{1}{2}-q}\right)^{1/2}\right) I_1\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right)\right].$$

Using the fact that $nF(c_n)$ increases without bound and the asymptotic approximation for $I_z(\cdot)$, we have that

$$I_0\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right) \Big/ I_1\left(2nF(c_n)\sqrt{\tfrac{1}{4}-q^2}\right)$$

converges to one as n goes to infinity. But then under assumption (A1) the terms in brackets in the expression for $G(s_A|\rho,\varepsilon,\sigma(c_n))$ becomes positive for n large enough, and the term in brackets in the expression for $G(A|\rho,\varepsilon,\sigma(c_n))$ becomes negative for n large enough. Therefore, for n large enough,

$$G(s_A|\rho,\varepsilon,\sigma(c_n)) > 0 \quad \text{and} \quad G(A|\rho,\varepsilon,\sigma(c_n)) < 0.$$

A similar argument shows that, for n large enough,

$$G(s_B|\rho,\varepsilon,\sigma(c_n)) > 0 \quad \text{and} \quad G(B|\rho,\varepsilon,\sigma(c_n)) < 0.$$

Thus, there is some $\underline{n}$ such that for $n > \underline{n}$, $\sigma(c_n)$ is an equilibrium.

Now, pick $n > \underline{n}$. Given the equilibrium $\sigma(c_n)$, the probability that a voter acquires information and votes according to the signal received is $F(c_n)$, and the probability that a voter abstains is $1-F(c_n)$. Since c_n converges to zero, the probability that a voter is informed converges to $F(0)$, and the probability that a voter abstains converges to $1-F(0)$. Moreover, the probability that A wins the election in state ω_A is equal to the probability that B wins the election in state ω_B and is equal to the probability that a Skellam random variable with parameters $nF(c_n)(1/2+q)$ and $nF(c_n)(1/2-q)$ is positive. Since

$$\frac{nF(c_n)(1/2+q)}{nF(c_n)(1/2-q)} = \frac{1/2+q}{1/2-q} > 1,$$

the probability that this random variable is positive converges to one as n goes to infinity. □

Theorem 2. *If $r = 1$, (A1) fails strictly and $\overline{p} \neq \underline{p}$, then as n goes to infinity, along any sequence of equilibria the probability of choosing the alternative favored by most voters' partisan biases converges to one regardless of the state.*

This is just a special case of theorem 4 below.

5. RATIONAL AND NAIVE VOTERS

In this section we consider the case in which some voters are naive. From lemma 1, we know that naive voters, whenever ignorant, will vote for the alternative they are biased in favor. The fact that uninformed voters participate in the election reduces the information that rational voters can deduce conditional on being pivotal, and makes voting for the alternative they are biased in favor more attractive. We use the following condition, expressing this additional constraint for rational abstention, in the statement of theorems below:

$$\text{(A2)} \qquad \max\{\underline{\varepsilon},\overline{\varepsilon}\} < \frac{2H(F,r,\underline{p},\overline{p})q}{1+2\sqrt{\frac{1}{4}-H(F,r,\underline{p},\overline{p})^2q^2}},$$

where $H(F,r,\underline{p},\overline{p})$ stands for

$$\frac{F(0)}{2(1-r)(\max\{\underline{p},\overline{p}\})(1-F(0))+F(0)}.$$

We say that *condition* (A2) *fails strictly* if the opposite strict inequality holds. Note that $H(F,r,\underline{p},\overline{p}) \leq 1$, with strict inequality unless $r=1$ or $F(0)=1$. Thus, (A3) is equivalent to (A1) if $r=1$ or $F(0)=1$, and is more restrictive otherwise. In particular, (A2) fails strictly if $r<1$ and $F(0)=0$.

To provide an intuition for condition (A2), note that if uninformed rational voters are (close to fully) offsetting the biased voting of naive voters, the probability that an uninformed voter participates in the election must be nearly equal to $2(1-r)\max\{\underline{p},\overline{p}\}$. Since the probability that a voter is uninformed converges to $1-F(0)$, H represents, in the limit, the probability that a voter who participates in the election is in fact informed.

In particular, if a voter knew that A is behind by one vote, the voter would think that the probabilities of state ω_A and ω_B are, respectively,

$$(1-H)(1/2)+H(1/2-q) = 1/2-Hq \quad \text{and} \quad 1/2+Hq.$$

Thus, condition (A2) is similar to condition (A1), taking into account that the informativeness of a single vote is now Hq rather than q.

Condition (A2) guarantees that, if rational voters were to fully offset the biased voting of naive voters, uninformed rational voters would prefer to abstain, and informed voters would prefer to vote according to the signal received. In the equilibrium we describe below, uninformed rational voters do not fully offset the biased voting of naive voters in order to keep some uninformed voters participating in the election in the first place. This makes equilibrium behavior more involved than in the previous section.

As described below, if condition (A2) holds, for successful information aggregation to occur in equilibrium it is sufficient that there are enough rational voters to offset uninformed naive voters. The following condition, which we use below, expresses this constraint:

$$r \geq \frac{|\overline{p}-\underline{p}|}{1+|\overline{p}-\underline{p}|}. \tag{A3}$$

Theorem 3. *If (A2) and (A3) hold, there is some $\overline{n}$ such that, for every $n > \overline{n}$, there is an equilibrium such that type $(\beta, \varepsilon, c, s)$ voters acquire information and vote according to the signal received if and only if $c \leq c_n(\beta, \varepsilon)$ for some vector*

$$(c_n(\beta,\varepsilon)))\ \textit{such that}\ 0 < c(\beta,\varepsilon) < \overline{c}\ \textit{for each pair}\ \beta,\varepsilon,$$

naive voters vote for their favorite alternative if they are uninformed, and rational voters abstain with positive probability if they are uninformed. As n goes to infinity, along any sequence of such equilibria the probability that a voter acquires information converges to $F(0)$, and the probability of choosing alternative A in state ω_A and alternative B in state ω_B converges to 1.

Proof. The case $r = 1$ is covered in Theorem 2, so for the remainder of the proof suppose (A2) hold and $r < 1$. Note that (A2) and $r < 1$ imply $F(0) > 0$.

Consider the case $\overline{p} > \underline{p}$ and $r \geq 1 - \underline{p}/\overline{p}$. For any given vector

$$(c(\beta,\varepsilon)))\ \text{ such that }\ 0 \leq c(\beta,\varepsilon) \leq \overline{c}\ \text{ for each pair }\ \beta,\varepsilon,$$

and any real number $w \in [0,1]$, let $\sigma((c(\beta,\varepsilon)),w)$ denote the strategy such that voters acquire information and vote according to the signal received if $c \leq c(\beta,\varepsilon)$, naive voters vote for their favorite alternative if they are uninformed, rational voters with bias $\overline{\varepsilon}$ abstain if they are uninformed, and rational voters with bias $\underline{\varepsilon}$ abstain with probability w if they are uninformed and vote for B with probability $1-w$ if they are uninformed.

Let

$$\Lambda : [0,\overline{c}]^4 \times [0,1] \to [0,\overline{c}]^4 \times [0,1]$$

with

$$\Lambda((c(\beta,\varepsilon)),w) = ((\hat{c}(\beta,\varepsilon)),\hat{w})$$

be a mapping such that

$$\begin{aligned}
\hat{c}(\rho,\overline{\varepsilon}) &= \min\{\overline{c},\max\{0,G(s_A|\rho,\overline{\varepsilon},\sigma((c),w))+G(s_B|\rho,\overline{\varepsilon},\sigma((c),w))\}\},\\
\hat{c}(\rho,-\underline{\varepsilon}) &= \min\{\overline{c},\max\{0,G(s_A|\rho,-\underline{\varepsilon},\sigma((c),w))\\
&\qquad\qquad\qquad +G(s_B|\rho,-\underline{\varepsilon},\sigma((c),w))\}\},\\
\hat{c}(\nu,\overline{\varepsilon}) &= \min\{\overline{c},\max\{0,G(A|\nu,\overline{\varepsilon},\sigma((c),w))\}\},\\
\hat{c}(\nu,-\underline{\varepsilon}) &= \min\{\overline{c},\max\{0,G(B|\nu,-\underline{\varepsilon},\sigma((c),w))\}\},
\end{aligned}$$

and $\hat{w}$ solves the equation

$$G(B|\nu,-\underline{\varepsilon},\sigma((\hat{c}),\hat{w}))=0 \tag{2}$$

if a solution in the interval $[0,1]$ exists, and $\hat{w}=1$ otherwise.

We claim that there is some $\underline{n}$, depending on the parameters $F(0)$, $\underline{\varepsilon}$ and q, such that for any $n\geq\underline{n}$ a solution to equation 2 in the interval $[0,1]$ exists for any vector $(c(\beta,\varepsilon))\in[0,\overline{c}]^4$. To see this, note that equation 2 is equivalent to

$$\frac{D(0|\rho,\sigma((\hat{c}),\hat{w}),\omega_A)+D(1|\rho,\sigma((\hat{c}),\hat{w}),\omega_A)}{D(0|\rho,\sigma((\hat{c}),\hat{w}),\omega_B)+D(1|\rho,\sigma((\hat{c}),\hat{w}),\omega_B)}=\frac{1+\underline{\varepsilon}}{1-\underline{\varepsilon}}.$$

Let $\tau(v|w,\omega)$ represent the probability that a voter votes for alternative $v\in\{A,B\}$ in state ω given a strategy $\sigma((\hat{c}),w)$ for any $w\in[0,1]$. Let

$$L(w)=$$

$$\frac{I_0(2n\sqrt{\tau(A|w,\omega_A)\tau(B|w,\omega_A)})+\left(\frac{\tau(A|w,\omega_A)}{\tau(B|w,\omega_A)}\right)^{\frac{1}{2}}I_1(2n\sqrt{\tau(A|w,\omega_A)\tau(B|w,\omega_A)})}{I_0(2n\sqrt{\tau(A|w,\omega_B)\tau(B|w,\omega_B)})+\left(\frac{\tau(A|w,\omega_B)}{\tau(B|w,\omega_B)}\right)^{\frac{1}{2}}I_1(2n\sqrt{\tau(A|w,\omega_B)\tau(B|w,\omega_B)})}$$
$$\times\exp\left(-n((\tau(A|w,\omega_A)+\tau(B|w,\omega_A))-(\tau(A|w,\omega_B)+\tau(B|w,\omega_B)))\right).$$

Then equation 2 has a solution if and only if there is some $w\in[0,1]$ such that $L(w)=(1+\underline{\varepsilon})/(1-\underline{\varepsilon})$.

It is straightforward to check that any vector $(c(\beta,\varepsilon))$, each cutoff $\hat{c}(\beta,\varepsilon)$ is arbitrarily close to zero for large n, where the bound on the approximation depends on n, q and $F(0)$. Thus, letting n go to infinity, $\tau(A|w,\omega_A)$, $\tau(B|w,\omega_A)$, $\tau(A|w,\omega_B)$ and $\tau(B|w,\omega_B)$ converge respectively to

$$\begin{aligned}\overline{\tau}(A|w,\omega_A) &= (1-r)(1-F(0)\overline{p}+F(0)(1/2+q)\\ \overline{\tau}(B|w,\omega_A) &= (1-r)(1-F(0))\underline{p}+r(1-F(0))\underline{p}w+F(0)(1/2-q),\\ \overline{\tau}(A|w,\omega_B) &= (1-r)(1-F(0))\overline{p}+F(0)(1/2-q),\end{aligned}$$

and

$$\overline{\tau}(B|w,\omega_B) = (1-r)(1-F(0))\underline{p}+r(1-F(0))\underline{p}w+F(0)(1/2+q),$$

and $L(w)$ is arbitrarily close to

$$\exp\Big(-n\Big((\sqrt{\overline{\tau}(A,w|\omega_A)}-\sqrt{\overline{\tau}(B|w,\omega_A)})^2$$
$$-(\sqrt{\overline{\tau}(A|w,\omega_B)}-\sqrt{\overline{\tau}(B|w,\omega_B)})^2\Big)\Big)$$
$$\times\left[\frac{\sqrt[4]{\overline{\tau}(A|w,\omega_B)\overline{\tau}(B|w,\omega_B)}\left(1+\sqrt{\frac{\overline{\tau}(A|w,\omega_A)}{\overline{\tau}(B|w,\omega_A)}}\right)}{\sqrt[4]{\overline{\tau}(A|w,\omega_A)\overline{\tau}(B|w,\omega_A)}\left(1+\sqrt{\frac{\overline{\tau}(A|w,\omega_B)}{\overline{\tau}(B|w,\omega_B)}}\right)}\right],$$

where the bound on the approximation depends on n, q and $F(0)$.

Let $\overline{w} \equiv (1-r)(\overline{p}-\underline{p})/(r\underline{p})$. If $w = \overline{w}$, the argument of the exponential function in the expression above is zero, and, under assumption (A3), the term in brackets is larger than or equal to $(1+\underline{\varepsilon})/(1-\underline{\varepsilon})$, so for large enough n, $L(\overline{w}) \geq (1+\underline{\varepsilon})/(1-\underline{\varepsilon})$. Let instead w take any value smaller than $\overline{w}$ and larger than or equal to

$$\max\left\{0, \frac{(1-r)(\overline{p}-\underline{p})}{r\underline{p}} - \frac{2q}{r\underline{p}}\frac{F(0)}{1-F(0)}\right\},$$

so that $\overline{\tau}(A|w,\omega_A) > \overline{\tau}(B|w,\omega_B) \geq \overline{\tau}(A|w,\omega_B) > \overline{\tau}(B|w,\omega_A)$. Then the argument in the exponential function in the expression above goes to $-\infty$ as n goes to infinity, so for large enough n, $L(w) < 1$. Thus, for large enough n, for any strategy $\sigma((c(\beta,\varepsilon)),w)$ there is some $\hat{w}$ that solves equation 2. Moreover, $\hat{w}$ is arbitrarily close to $\overline{w}$ from below.

Since Λ is a mapping from a compact set into itself and is continuous for large n, it has a fixed point $((c_n(\beta,\varepsilon)),w_n)$. Moreover, for large n, $c_n(\beta,\varepsilon)$ is arbitrarily close to 0 from above for any pair β,ε, and w_n is arbitrarily close to $\overline{w}$ from below.

We claim that for large enough n, $\sigma((c_n(\beta,\varepsilon)),w_n)$ is an equilibrium strategy. To see this, it is straightforward to check using condition (A2) that

$$G(s_A|\beta,\varepsilon,\sigma((c_n(\beta,\varepsilon)),w_n)) \geq 0, \quad G(A|\beta,\varepsilon,\sigma((c_n(\beta,\varepsilon)),w_n)) \leq 0,$$

$$G(s_B|\beta,\varepsilon,\sigma((c_n(\beta,\varepsilon)),w_n)) \geq 0, \quad G(B|\beta,\varepsilon,\sigma((c_n(\beta,\varepsilon)),w_n)) \leq 0.$$

Along the sequence of equilibria $\sigma((c_n(\beta,\varepsilon)),w_n)$, the probability of voting for alternative A in state ω_A and the probability of voting for alternative B in state ω_A converge respectively to $\overline{\tau}(A|\overline{w},\omega_A)$ and $\overline{\tau}(B|\overline{w},\omega_A)$. Since

$$\frac{\overline{\tau}(A|\overline{w},\omega_A)}{\overline{\tau}(B|\overline{w},\omega_A)} > 1,$$

the probability that A wins the election in state ω_A converges to one as n goes to infinity. A similar argument shows that the probability that B wins the election in state ω_B converges to one as n goes to infinity.

The proof for the case $\overline{p} > \underline{p}$ and $1-\underline{p}/\overline{p} > r \geq (1/2)(1-\underline{p}/\overline{p})$ is similar to the one above, but the equilibrium involves rational voters biased in favor of B voting for B with probability one when uninformed, and rational voters biased in favor of A randomizing between abstention and voting for B when uninformed. The proof for the case $\overline{p} = \underline{p}$ is also similar to the one above, but the equilibrium involves rational voters abstaining with probability one. Finally, the case $\overline{p} < \underline{p}$ is analogous to the one above. □

Recall that uninformed naive voters consider both states equally likely regardless of whether A and B are tied or one of the alternatives is ahead of the other by one vote, so that they rather vote for their favorite alternative than abstaining. Informed naive voters, per contra, consider that state w_d has probability $1/2+q$ if they receive signal d regardless of whether A and B are tied or one of the alternatives is ahead of the other by one vote, so that they are willing to vote for their least favorite alternative if $2q \geq \max\{\underline{\varepsilon}, \overline{\varepsilon}\}$. Note that this condition is weaker than (A2). That is, *while naive voters are not willing to abstain when ignorant, they are more willing to vote with their signals than rational voters.* If condition (A2) fails, this means that successful information aggregation in the limit is possible only there are enough naive voters.

Theorem 4. *If condition (A2) fails strictly, there is a sequence of equilibria for large n such that along that sequence the probability of choosing alternative A in state ω_A and alternative B in state ω_B converges to 1 if and only if*

$$2q \geq \max\{\underline{\varepsilon}, \overline{\varepsilon}\} \text{ and } 2q > |\overline{p}-\underline{p}|\left(\frac{1}{(1-r)F(0)}-1\right).$$

Proof. (Sketch.) For sufficiency of the condition stated in the theorem, note that under the condition informed naive voters vote according to their signals, and there are enough naive voters so that they compensate any bias from the remainder of voters if all other voters vote for their favorite alternatives. The remainder of the argument is showing that if (A2) fails strictly there is in fact an equilibrium in which rational voters do not acquire information and vote for their favorite alternatives, and naive voters acquire information with cutoffs going to 0. Necessity of the condition $2q > \max\{\underline{\varepsilon}, \overline{\varepsilon}\}$ is easily established. Necessity of the condition $2q > |\overline{p}-\underline{p}|(1/(1-r)F(0)-1)$ requires showing that if (A2) fails strictly, in any equilibrium rational voters do not acquire information and vote for their favorite alternatives. □

6. Conclusions

We have shown that, when discovering their private information is costly for all voters, successful information aggregation occurs in equilibrium only if all voters are rational, and their preferences are close enough to being aligned for them to be willing to abstain when uninformed, which most of them do. In a way, the model resembles collective choice by means of committees that self-select on the basis of the opportunity cost of expertise and are small in relation to the potential population, perhaps like some faculty meetings or voluntary consumer reviews in the internet. If discovering private information is costly for all voters and some voters are naive, the incentive for abstention is destroyed since uninformed naive voters rather vote for their favorite alternative than abstain, so that most voters who participate in the election are uninformed.

If discovering their private information is costless for voters with some probability, successful information aggregation may occur even if some voters are naive. One possibility is that rational voters offset any net bias given by naive voters participating in the election, and the condition for rational abstention is satisfied. An alternative set of circumstances may rely on informed naive voters voting according to the information they have. The model brings to mind collective choice by means of mass participation in elections, with good results in terms of information aggregation being possible only if enough free information is available to voters to begin with.

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