The Political Economy of Indirect Control*
(PRELIMINARY AND INCOMPLETE)

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Abstract

This paper characterizes optimal policy in an environment in which a government uses indirect control to exert its authority. We develop a dynamic principal-agent model in which a principal (a government) delegates the prevention of a disturbance–such as riots, protests, terrorism, crime, or tax evasion–to an agent who has an advantage in accomplishing this task. Our setting is a standard dynamic principal-agent model with two additional features. First, the principal is allowed to exert direct control by intervening with an endogenously determined intensity of force. Second, the principal suffers from limited commitment. We focus on characterizing the optimal likelihood, intensity, and duration of intervention. Using recursive methods, we derive a fully analytical characterization of the optimal contract. The first main insight from our model is that repeated and costly interventions are a feature of optimal policy. This is because they serve as a punishment to induce the agent into desired behavior. The second main insight is a detailed analysis of a fundamental tradeoff between the intensity and duration of intervention which is driven by the principal's inability to commit. Finally, we derive sharp predictions regarding the impact of various factors on the optimal likelihood, intensity, and duration of intervention.

Keywords: Repeated Games, Asymmetric and Private Information, Political Economy

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1 Introduction

In exerting their authority, governments often delegate political responsibilities to local agents or warlords who have an advantage in fulfilling these responsibilities. These tasks include the provision of law and order, the prevention of riots and protests, the control of terrorism and insurgency, the collection of taxes, or the extraction of natural resources, for example. Not only was such a strategy the hallmark of British indirect rule during colonial times, but it applies today to many governments who use local agents for indirect control.[1] In many of these cases, dissatisfied governments occasionally abandon this policy of indirect control in favor of temporary direct control and costly intervention.[2] Such an interaction between a government and a local agent is inherently dynamic, which is important for the design of optimal policy. How should a government use rewards or interventions to allign the incentives of the local agent with its own?

There are three key political economy frictions to consider in answering this question. First, the government cannot commit to providing rewards or using interventions. Second, the local agent cannot commit to fulfilling his delegated task. Third, the local agent's actions, which often occur through informal channels, are imperfectly observed by the government. Given these frictions, the interaction between the government and the local agent can be described in a dynamic principal-agent framework in which the government is the principal. The design of optimal policy in such a framework must incorporate the joint interaction of double-sided lack of commitment and asymmetric information.

In this paper, we develop a such a model in which a principal delegates the prevention of a disturbance–such as riots, protests, terrorism, crime, or tax evasion–to an agent who has an advantage in accomplishing this task. Our setting is a standard dynamic principal-agent model with two additional features which are natural in our application.[3] First, the principal is allowed to exert direct control by intervening with an endogenously determined intensity of force. Second, the principal suffers from limited commitment. We focus on characterizing the optimal likelihood, intensity, and duration of intervention. Using the recursive methods of Abreu, Pearce, and Stacchetti (1990), we derive a fully analytical

[1] This is particularly the case in governments that have tenuous control over parts their territory, for instance, in Pakistan's Federally Administered Tribal Areas and in rural areas in many African countries. On this point, see Herbst (2000) and Reno (1998).

[2] Such a description can arguably apply for instance to the strategies of Pakistan in its tribal territories, Russia in Chechnya, Israel in the Palestinian Territories, or Indonesia in Banda Aceh. The United Kingdom also suspended local administration and deployed the army during the turmoil in Northern Ireland.

[3] The literature on dynamic principal-agent relationships is vast and cannot be summarized here. Some examples are Albuquerque and Hopenhayn (2002), Atkeson and Lucas (1992), Golosov, Kocherlakota, and Tsyvinski (2003), Phelan (1995), Thomas and Worrall (1990), and Sannikov (2009).

characterization of the optimal contract. The first main insight from our model is that repeated and costly interventions are a feature of optimal policy. This is because they serve as a punishment to induce the agent into desired behavior.[4] The second main insight from our model–which emerges from our explicit characterization–is a detailed analysis of a fundamental tradeoff between the intensity and duration of intervention which is driven by the principal's inability to commit. Finally, we derive sharp predictions regarding the impact of various factors on the optimal likelihood, intensity, and duration of intervention.

More specifically, we construct a repeated game between a principal and an agent where in every period, the principal decides whether or not to intervene to reduce the likelihood of a disturbance. Under intervention, he chooses the intensity of force, where higher intensity is costly to both the agent and the principal (i.e., it does not help to reduce the probability of a disturbance). The principal suffers from limited commitment. In the absence of incentives, he would therefore always choose the minimal level of force. If the principal does not intervene, he allows the agent to reduce disturbances on his own by exerting unobservable effort which can be high or low. Both players are strictly better off under high effort by the agent compared to intervention by the principal. Nonetheless, there are two limitations to the extent to which intervention can be avoided. First, the agent cannot commit to high effort once the threat of intervention has subsided. Second, the principal does not observe the agent's effort, and since disturbances might happen even under high effort, the agent can always unobservably deviate and pretend to have exerted high effort. We consider the efficient sequential equilibrium of this game in which reputation sustains equilibrium actions, and we fully characterize in closed form the long run dynamics of the optimal contract.

Our first result is that repeated and costly interventions are a feature of optimal policy. Specifically, the optimal contract after a sufficient number of disturbances features two phases of play: a cooperative phase and a punishment phase, where these two phases sustain each other. In the cooperative phase, the agent exerts high effort because he knows that failure to do so raises the probability of a disturbance which can trigger a transition to the punishment phase. In the punishment phase, the principal temporarily intervenes with a unique endogenous level of intensive force. The principal exerts costly force because he knows that failure to do so would trigger the agent to choose low effort in all future cooperative phases, making permanent intervention by the principal a necessity. Importantly, the optimal contract which maximizes the principal's welfare under

[4] The role of punishments in political delegation are also studied in Dal Bó and Di Tella (2003) and Dal Bó, Dal Bó and di Tella (2006), though their focus is a static model in which the principal can commit to punishment.

cooperation also minimizes the agent's welfare under punishment. More specifically, the optimal policy prolongs the duration of cooperation and hence minimizes the likelihood of punishment. The harsher the punishment, the more easily can the agent be forgiven for disturbances without altering his incentives to exert high effort, and hence, the longer cooperation.

Our second result is that there is a tradeoff between the duration and the intensity of intervention since satisfaction of the principal's incentives require the duration of intervention to decline with intensity. More specifically, during the punishment phase, the principal can only be induced to intervene with more intensive force if cooperation is expected to resume soon. This means that the agent's welfare under punishment is at first declining in intensity, though it is eventually rising in intensity because of the counteracting effect of a shorter duration of punishment. Consequently, the likelihood of intervention following a disturbance declines then rises with intensity since the agent's incentives strengthen and then weaken in intensity. Since punishment is costly to both players, the optimal contract selects the level of intensity and duration of intervention which minimize the likelihood of intervention and the process minimize the agent's welfare under punishment.

Our final result concerns the effect of three important factors on the optimal likelihood, intensity, and duration of intervention. First, we consider the effect of a decline in the cost of intensity to the principal, where this can occur for instance if there is less international rebuke for the use of punitive measures. Second, we consider the effect of a rise in the cost of disturbances to the principal. Finally, we consider the effect of a rise in the cost of effort to the agent, where this can occur for instance if it becomes more politically costly for the agent to antagonize rival factions contributing to disturbances or alternatively if he acquires a higher preference for the realization of disturbances.

We show that all three changes increase the optimal intensity and decrease the optimal duration of intervention. To see why intensity must rise, consider the first case. If the cost of intensity declines, then the principal's return to intensity rises since it is cheaper to provide incentives to the agent via intensive force. In the second case, if the cost of disturbances rise, the principal should raise the intensity of intervention since the return to delegating to the agent rises, and it is optimal to provide the agent with stronger incentives. In the third case, if the cost of effort for the agent rises, then it is harder for the principal to provide incentives to the agent with lower levels of intensity, and higher levels of intensity become optimal. In all three cases, because the principal needs more inducement to punish, these increases in the level of intensity necessitate a decline in the duration of intervention.

Though all three changes increase the optimal intensity and decrease the optimal duration of intervention, only the third also raises its likelihood. Specifically, if the cost of intensity to the principal declines or if the cost of disturbances to the principal rises, then higher intensity translates into a more severe punishment for the agent and therefore makes it easier for the principal to forgive the agent following the realization of disturbances without weakening incentives for the agent. Therefore, the likelihood of intervention declines. In contrast, if the cost of effort to the agent rises, then incentives are harder to provide for the agent, so that the likelihood of intervention must rise following the realization of a disturbance.

As an aside, note that our simple benchmark model ignores three additional issues. First, it ignores the possibility that permanent concessions by the principal can reduce the presence of disturbances in the future. Second, it ignores the possibility that the agent's identity can change over time because of political transitions. Third, it ignores the possibility that high intensity levels by the principal today can raise the cost of effort by the agent in the future, for example if the agent becomes more antagonistic. These issues are discussed in our extensions which show that our main conclusions are unchanged.

This paper makes three contributions. First, it contributes to the dynamic principal-agent literature described in footnote 3 by allowing for costly intervention by a principal who suffers from limited commitment. Given that we study a model with double-sided lack of commitment, our results are related to the literature on repeated games with imperfect monitoring and to the insights due to the seminal work of Green and Porter (1984). These authors present examples of sequential equilibria in which punishment in the form of temporary price wars sustain cooperation between oligopolistic firms. In contrast to this work, we use the recursive methods of Abreu, Pearce, and Stacchetti (1990) to explicitly characterize the efficient equilibrium under history-dependent strategies and this allows for a detailed analysis of a fundamental tradeoff between the intensity and duration of punishment. Second, our paper contributes to the literature on costly political conflict by providing a formal framework for investigating the transitional dynamics between conflict and cooperation.[5] In particular, our model bears a similar structure to Yared (2009), though in contrast to this work, we introduce variable intervention intensity which allows for payoffs below the repeated static Nash equilibrium. This implies that, in contrast to this work, phases of intervention cannot last forever and must necessarily precede phases of cooperation. Third, our paper contributes to the literature on punishments dating

[5] Some examples of work in this literature are Acemoglu and Robinson (2006), Anderlini, Gerardi, and Lagunoff (2009), Baliga and Sjostrom (2004), Chassang and Padró i Miquel (2009), Esteban and Ray (2008), Fearon (1995), Jackson and Morelli (2008), Powell (1999), and Schwarz and Sonin (2004).

back to the work of Becker (1968). In contrast to this work which considers static models, we consider a dynamic environment in which the government lacks the commitment to punish.[6] This allows for an analysis of the optimal time structure of punishments together with the tradeoff between the duration and intensity of punishments.

The paper is organized as follows. Section 2 describes the model. Section 3 defines the efficient sequential equilibrium. Section 4 characterizes the equilibrium and provides our main results. Section 5 considers extensions. Section 6 concludes. The Appendix contains all proofs and additional material not included in the text.

2 Model

We consider an environment in which a principal seeks to induce an agent into limiting disturbances.[7] In every period, the principal has two options. On the one hand, he can forcefully intervene to control disturbances and in doing so he chooses the intensity of force. On the other hand, the principal can withhold force and allow the agent to exert *unobservable* effort in controlling disturbances. In this situation, if a disturbance occurs, the principal cannot determine if it is due to the agent's negligence or due to bad luck. In addition to this informational asymmetry, both the principal and the agent suffer from limited commitment.

More formally, there are time periods $t = \{0, ..., \infty\}$ where in every period t, the principal (p) and the agent (a) repeat the following interaction. The principal publicly chooses $f_t = \{0, 1\}$, where $f_t = 1$ represents a decision to intervene. If $f_t = 1$, then the principal publicly decides the intensity of force $i_t \geq 0$. The payoff to the principal is $-\pi_p \chi - A i_t$ and the payoff to the agent is $w_a - g(i_t)$, where $A > 0$ and $g'(\cdot), -g''(\cdot) > 0$ with $g(0) = 0$ and $g'(0) = \infty$. The concavity of $g(\cdot)$ captures the fact that there are diminishing returns to the use of intensity by the principal, and it ensures an interior solution for the optimal level of intensity. The parameter A captures the cost of intensive force which can decline if there is less international rebuke for the use of force, for instance. Within the term $-\pi_p \chi - A i_t$ is embedded the cost of a stochastic disturbance, where π_p represents the probability of such a disturbance and χ represents its cost to the principal.[8] Analogously, within the term $w_a - Ag(i_t)$ is the cost of the damage suffered by the agent.

[6] Some examples of models of punishments are Acemoglu and Wolitsky (2009), Dal Bó and Di Tella (2003), Dal Bó, Dal Bó and di Tella (2006), and Polinski and Shavell (1979,1984). We discuss our relationship to the literature on punishments in greater detail in Section 4.2.

[7] In practice, the agent can be a leader, a political party, or an entire society.

[8] That we have chosen the minimum level of intensity to be zero is only a normalization and has no effect on our results.

Importantly, conditional on intervention by the principal, both the principal and the agent are strictly better off under minimal force. Intuitively, choosing $i_t > 0$ imposes more physical damage on the agent. Moreover, it is inefficient from the perspective of the principal since it is more costly to use and does not diminish the likelihood of a disturbance. Therefore, conditional on $f_t = 1$, the principal would always choose $i_t = 0$ in a one-shot version of this game.

If instead the principal decides to not intervene, then the agent *privately* chooses whether to exert high effort ($e_t = \eta$) or low effort ($e_t = 0 < \eta$) in preventing a disturbance. Nature then stochastically chooses the realization of a publicly observed disturbance $s_t = \{0, 1\}$, where $s_t = 0$ represents the prevention of a disturbance. If a disturbance is prevented, the principal receives 0, and if it occurs, the principal receives $-\chi$. Independently of the shock, the agent loses e_t from exerting effort. The stochastic realization of a disturbance occurs as follows. If $e_t = \eta$, then a disturbance occurs with probability $\pi_a(\eta) \in (0, 1)$ and if $e_t = 0$, then it occurs with probability $\pi_a(0) \in (\pi_a(\eta), 1]$. Therefore, high effort reduces the likelihood of a disturbance. The parameter η captures the cost of effort to the agent which can rise for instance if it becomes more politically costly for the agent to antagonize rival factions contributing to disturbances or alternatively if he acquires a higher preference for the realization of disturbances.[9] Notice that we have ruled out payments from the principal to the agent since our focus on is on the use of interventions. This is done only for expositional simplicity and without bearing on our main results.[10] The game is displayed in Figure 1.

Let $u_j(f_t, i_t, e_t, s_t)$ represent the payoff to j at t, where value of i_t is only relevant if $f_t = 1$ and the values of e_t and s_t are only relevant if $f_t = 0$. Each player j has a period zero welfare

$$E_0 \sum_{t=0}^{\infty} \beta^t u_j(f_t, i_t, e_t, s_t),\ \beta \in (0, 1).$$

We make the following assumptions.

Assumption 1 (inefficiency of intervention) $-\pi_a(\eta) > -\pi_p$ and $-\eta > w_a$.

Assumption 2 (desirability of intervention) $-\pi_p > -\pi_a(0)$.

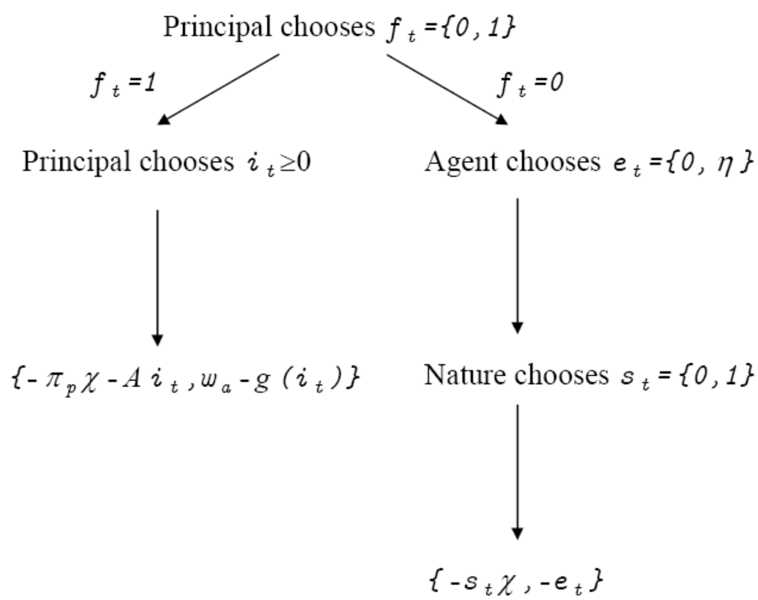
Assumption 1 states that, relative to payoffs under intervention, both the principal and

[9] Without affecting any of our results, one can modify the model so that e_t subsumes the fact that the agent receives utility from realization of a disturbance.

[10] Even in a model with payments, the presence of limited liability on the agent would imply that the use of intervention is optimal along the equilibrium path after a long enough sequences of disturbances. See footnote 19 for more details.

the agent are strictly better off if the agent exerts high effort in preventing a disturbance. Intuitively, the agent is better informed about the sources of disturbances and is better than the principal at preventing them. Moreover, from an ex-ante perspective, the agent prefers to exert high effort to prevent a disturbance versus enduring the damage from intervention by the principal.

Figure 1: Game



Assumption 2 states that the principal is strictly better off using intervention to prevent a disturbance versus letting the agent exert low effort in preventing such a disturbance. This assumption has an important implication. Specifically, in a one-shot version of this game, $f_t = 1$ and $i_t = 0$ is the unique static Nash equilibrium. This is because conditional on $f_t = 0$, the agent chooses $e_t = 0$. Thus, by Assumptions 2, the principal chooses $f_t = 1$ and $i_t = 0$. The agent cannot commit to controlling disturbances, and so the principal must intervene to do so himself.

Since the static Nash equilibrium is inefficient (by Assumption 1), one can imagine that in a dynamic framework, the principal may be able to enforce high effort from the agent by rewarding the prevention of disturbances today by refraining from intervention in the future. Nevertheless, there are two obstacles to this arrangement which are important to consider. First, the principal cannot commit to refraining from using intervention in the future, since he also suffers from limited commitment. Second, the principal does not observe the effort by the agent. Consequently, if a disturbance occurs, the principal cannot determine if this is accidental (i.e., $e_t = \eta$) or if this is intentional (i.e., $e_t = 0$).

Note that our simple benchmark model ignores three additional issues. First, it ignores the possibility that permanent concessions by the principal can reduce the presence of disturbances in the future. Second, it ignores the possibility that the agent's identity can change over time because of political transitions. Third, it ignores the possibility that high intensity levels by the principal today can raise the cost of effort by the agent in the future, for example if the agent becomes more antagonistic. These issues are discussed in Section 5 which shows that our main conclusions are unchanged.

3 Equilibrium Definition

In this section, we present our recursive method for the characterization of the efficient sequential equilibria between the two players. We provide a formal definition of these equilibria in the Appendix. The important feature of a sequential equilibrium is that each player dynamically chooses his best response given the strategy of his rival at every public history.[11]

Since we are concerned with optimal policy, we characterize the efficient sequential equilibria to this game, which are the set of equilibria which maximize the period 0 welfare of the principal subject to providing the agent with some minimal period 0 welfare U_0 (or vice versa). The most important feature of these equilibria due to the original insight achieved by Abreu (1988) is that they are sustained by the worst punishment. More specifically, all public deviations from equilibrium actions by a given player lead to his worst punishment off the equilibrium path, which we denote by $\underline{J}$ for the principal and $\underline{U}$ for the agent. Note that

$$\begin{aligned} \underline{J} &= -\frac{\pi_p \chi}{1-\beta} \text{ and} \\ \underline{U} &\leq \frac{w_a}{1-\beta} \end{aligned}$$

since the principal cannot receive a lower payoff than the repeated static Nash equilibrium (i.e., he can always choose it) and since the agent can be credibly punished by the principal at least as harshly as under the repeated static Nash equilibrium.

Note that in characterizing this equilibrium, we take into account that it may be efficient for players to choose correlated strategies so as to potentially randomize over the choice of intervention, intensity, and effort. Let $z_t = \{z_t^1, z_t^2\} \in Z \equiv [0,1]^2$ represent a

[11] Because the principal's strategy is public by definition, any deviation by the agent to a non-public strategy is irrelevant (see Fudenberg, Levine, and Maskin, 1994).

pair of i.i.d. publicly observed random variables independent of s_t, of all actions, and of each other, where these are drawn from a bivariate continuous c.d.f. $G(\cdot)$. Let z_t^1 be revealed prior to the choice of f_t so as to allow the principal to randomize over the use of intervention and let z_t^2 be revealed immediately following the choice of f_t so as to allow the principal to randomize over intensity or the agent to randomize over the effort.

As is the case in many incentive problems, an efficient sequential equilibrium can be represented in a recursive fashion, and this is a useful simplification for characterizing equilibrium dynamics.[12] Specifically, at any public history, the entire public history of the game is subsumed in the continuation value to each player, and associated with these two continuation values is a continuation sequence of actions and continuation values. More specifically, let U represent the continuation value of the agent at a given history. Associated with U is $J(U)$, which represents the highest continuation value achievable by the principal in a sequential equilibrium conditional on the agent achieving a continuation value of U. More formally, letting $\delta = \left\{f_z, i_z, e_z, U_z^F, U_z^H, U_z^L\right\}_{z\in Z}$, the recursive program which characterizes the efficient sequential equilibrium is

$$J(U) = \max_{\delta} \int \left[\begin{array}{c} f_z\left(-\pi_p\chi - Ai_z + \beta J\left(U_z^F\right)\right) + \\ (1-f_z)\left(-\pi_a(e_z)\chi + \beta\left((1-\pi_a(e_z))J\left(U_z^H\right) + \pi_a(e_z)J\left(U_z^L\right)\right)\right) \end{array} \right] dG_z \tag{1}$$

s.t.

$$U = \int \left[\begin{array}{c} f_z\left(w_a - g(i_z) + \beta U_z^F\right) + \\ (1-f_z)\left(-e_z + \beta\left((1-\pi_a(e_z))U_z^H + \pi_a(e_z)U_z^L\right)\right) \end{array} \right] dG_z, \tag{2}$$

$$J\left(U_z^F\right), J\left(U_z^H\right), J\left(U_z^L\right) \geq \underline{J}\ \forall z \tag{3}$$

$$U_z^F, U_z^H, U_z^L \geq \underline{U}\ \forall z \tag{4}$$

$$-\pi_p\chi - Ai_z + \beta J\left(U_z^F\right) \geq \underline{J}\ \forall z \tag{5}$$

$$\beta\left(U_z^H - U_z^L\right)(\pi_a(0) - \pi_a(e_z)) \geq e_z\ \forall z \tag{6}$$

$$f_z \in [0,1],\ i_z \geq 0, \text{ and } e_z = \{0, \eta\}\ \forall z. \tag{7}$$

(1) represents the continuation value to the principal written in a recursive fashion at a given history. f_z, i_z, and e_z represent the use of intervention, the choice of intensity, and the choice of effort, respectively, conditional on today's random public signal $z = \{z^1, z^2\}$. U_z^F represents the continuation value promised to the agent for tomorrow conditional on intervention being used today at z. If intervention is not used, then the continuation

[12] This is consequence of the insights from the work of Abreu, Pearce, and Stacchetti (1990).

value promised to the agent for tomorrow conditional on z is U_z^H if $s = 0$ (there is no disturbance) and U_z^L if $s = 1$ (there is a disturbance). Note that f_z depends only on z^1 since it is chosen prior to the realization of z^2, but all other variables depend on z^1 as well as z^2.

Equation (2) represents the promise keeping constraint which ensures that the agent is achieving a continuation value of U. Constraints (7) ensure that the allocation is feasible. Constraints (3) – (6) represent the incentive compatibility constraints of this game. Without these constraints, the solution to the problem starting from an initial U_0 is simple: Players refrain from intervention forever. Constraints (3) – (6) capture the inefficiencies introduced by the presence of limited commitment and imperfect information which ultimately lead to the need for intervention. Constraint (3) captures the fact that at any history, the principal cannot commit to refraining from intervention forever with zero effort intensity which provides a continuation welfare of $\underline{J}$. Constraint (4) captures the fact that at any history, the agent cannot commit to high effort, as he can choose low effort forever and ensure himself a continuation value of at least $\underline{U}$. Importantly, constraint (5) captures the fact that at any history, the principal cannot commit to using intensive force since this is costly. Constraint (5) ensures that the principal prefers to use intensive force and be rewarded for it in the future compared to his best deviation which involves using intervention with zero intensive force forever. Constraints (3) – (5) capture the constraint of limited commitment. Under perfect information, they imply that if players are sufficiently patient, the permanent absence of intervention can be sustained by the off-equilibrium threat of intervention. Constraint (6) captures the additional constraint of imperfect information: The principal does not observe the effort e_z. If the principal requests $e_z = \eta$, the agent can always privately choose $e_z = 0$ without detection. Constraint (6) ensures that the agent's punishment from this deviation is weakly exceeded by the equilibrium path reward for choosing high effort.[13]

4 Analysis

We focus our analysis on the likelihood, the intensity, and the duration of intervention which are formally defined below.

Definition 1 *(i) The likelihood of intervention at t is* $\Pr\{f_{t+1} = 1 | f_t = 0$ *and* $s_t = 1\}$, *(ii) the intensity of intervention at t is* $E\{i_t | f_t = 1\}$, *and (iii) the duration of intervention*

[13] Note that we have ignored the constraint that the agent does not deviate to high effort if $e_z = 0$ since such a constraint never binds in equilibrium.

at t is $\Pr\{f_{t+1} = 1 | f_t = 1\}$.

This definition states that the likelihood of intervention is the probability that the principal intervenes following a disturbance; the intensity of intervention is the expected intensity of the force used by the principal; and the duration of the intervention is the probability that intervention continues into the next period.

We also focus on *long run* equilibrium dynamics. We do so because these dynamics can be explicitly characterized in closed form, and because we can show that phases of intervention occur only in the long run.[14] More specifically, we first show in Section 4.1 that the optimal contract in the long run is characterized by two phases of play: a cooperative phase and a punishment phase, where these two phases sustain each other. Second, we describe in Section 4.2 an important tradeoff in the optimal contract between the duration and the intensity of intervention. Finally, in Section 4.3 we consider comparative statics.

To facilitate exposition, we assume that players are sufficiently patient for the remainder of our discussion.

Assumption 3 (High Patience) $\beta > \widehat{\beta}$.

The exact value of $\widehat{\beta}$ is described in the Appendix.[15]

4.1 Characterization

Let

$$\delta^*(U) = \left\{f_z^*(U), i_z^*(U), e_z^*(U), U_z^{F*}(U), U_z^{H*}(U), U_z^{L*}(U)\right\}_{z \in Z}$$

represent an argument which solves $(1) - (7)$. Since $\delta^*(U)$ may not be unique, we focus on the unique solution which satisfies the *Bang-Bang* property as described by Abreu, Pearce, and Stacchetti (1990).[16] In our context, the *Bang-Bang* property is satisfied if the equilibrium continuation value pairs at t following the realization of z_t^1 are extreme points in the set of sequential equilibrium continuation values. Define

$$\overline{U} = -\frac{\pi_a(0)\,\eta}{(1-\beta)(\pi_a(0) - \pi_a(\eta))}. \tag{8}$$

[14] See Yared (2009) for a similar model which more explicitly describes short run transitional dynamics.

[15] This assumption guarantees that the likelihood of punishment is bounded away from 1 and that the duration of punishment is bounded away from 0, which guarantees that the long run equilibrium can be explicitly characterized. The value of $\widehat{\beta}$ is below 1 as long as η is sufficiently bounded away from w_a so that permanent reversion to the static Nash equilibrium is a sufficient enough threat to induce high effort.

[16] Efficient equilibria which do not satisfy the *Bang-Bang* property emerge here in part because information is coarse. See Yared (2009) for a discussion. Also see Sannikov and Skrzypacz (2009) for an additional discussion of the characteristics of equilibria under different information structures.

Let $\lim_{t\to\infty} \Pr\left\{U_t \leq \overline{U}\right\}$ represent the long run probability that the agent receives a continuation value (following the realization of z_t^1) which is weakly below $\overline{U}$ in the solution to the program.

Proposition 1 ***(characterization)***

1. *$\lim_{t\to\infty} \Pr\left\{U_t \leq \overline{U}\right\} = 1\ \forall U_0$, and*

2. *If $U \leq \overline{U}$, then $Ef_z^*(U) = \left(\overline{U} - U\right) / \left(\overline{U} - \underline{U}\right)$ and $\forall z$*

$$\begin{aligned} i_z^*(U) &= i^*, \\ e_z^*(U) &= \eta, \\ U_z^{F*}(U) &= \left(\underline{U} - w_a + g(i^*)\right)/\beta, \\ U_z^{H*}(U) &= \overline{U}, \text{ and} \\ U_z^{L*}(U) &= \overline{U} - \eta/\left(\beta\left(\pi_a(0) - \pi_a(\eta)\right)\right) \end{aligned}$$

for i^* and $\underline{U}$ which satisfy

$$\begin{aligned} 1 &= \frac{g'(i^*)}{A}\frac{(\pi_p - \pi_a(\eta))\chi + Ai^*}{-\eta - w_a + g(i^*)}, \text{ and} \\ \underline{U} &= -\frac{(\pi_p - \pi_a(\eta))\chi\eta + Ai^*(w_a - g(i^*))}{(1-\beta)((\pi_p - \pi_a(\eta))\chi + Ai^*)}. \end{aligned} \tag{9}$$

This proposition states that in the long run, continuation values are weakly below $\overline{U}$ and it explicitly characterizes the solution for $U \leq \overline{U}$. More specifically, in the long run, the principal exerts a unique level of intensity i^*, the agent exerts high effort, and continuation values for tomorrow are conditioned on whether or not intervention is used and whether or not a disturbance occurs in the absence of intervention. The continuation value U is therefore provided to the agent by randomizing over a *cooperative phase* and a *punishment phase*. In the cooperative phase, intervention is not used and the agent and principal receive $\overline{U}$ and $J\left(\overline{U}\right)$, respectively, following the realization of z_t^1. In the punishment phase, intervention is used and the agent and principal receive $\underline{U}$ and $J(\underline{U}) = \underline{J}$, respectively, following the realization of z_t^1.

More specifically, in the cooperative phase at t, the principal does not intervene ($f_t = 0$) and the agent chooses high effort ($e_t = \eta$). If there is no disturbance at t ($s_t = 0$), then the cooperative phase at $t+1$ occurs with probability 1. If there is a disturbance at t ($s_t = 1$), then the cooperative phase at $t+1$ occurs with probability $1 - l^*$, and the punishment phase at $t+1$ occurs with probability l^*. In contrast, in the punishment phase at t, principal chooses intervention ($f_t = 1$) and a unique level of intensity $i_t = i^*$. The punishment phase at $t+1$ occurs with probability d^* and the cooperative phase at $t+1$ occurs with probability $1 - d^*$. Note that given Definition 1, it is clear from this characterization that the optimal likelihood, intensity, and duration of intervention correspond to l^*, i^*, and d^*, respectively, and these can be characterized explicitly in our framework.[17]

To understand the first part of Proposition 1 consider Figure 2 which depicts $J(U)$ as a function of U. The y-axis represents $J(U)$ and the x-axis represents U, with $\overline{U}$ situated on the x-axis. $\overline{U}$ is important for two reasons. First, it can be shown that if $U \geq \overline{U}$ that $f_z^*(U) = 0 \; \forall z$, so that intervention is used with zero probability since it is too costly for the principal and for the agent. Now suppose there was zero probability of continuation values traveling below $\overline{U}$. Then there would be zero probability of intervention along the equilibrium path, and the agent would optimally choose low effort forever, but this would violate the incentive compatibility constraint of the principal by Assumption 2. Therefore, intervention must occur along the equilibrium path to induce high effort and continuation values must eventually decline below $\overline{U}$.[18] Second, $\overline{U}$ is important since continuation values in the future cannot increase above $\overline{U}$ once they have declined below $\overline{U}$. The intuition is that below $\overline{U}$ the principal would like to extract as much from the agent as possible in the absence of intervention, and he does this by requesting high effort, which hurts the agent and benefits the principal.[19] If instead continuation values were to increase above $\overline{U}$, then the agent would be able to exert low effort going forward. However, this would imply that the principal must also punish the agent more often in order to satisfy (2). Nonetheless, this is inefficient from the principal's perspective since both low effort by the agent as well as a higher likelihood of punishment are costly to the principal.[20]

The intuition behind the second part of Proposition 1 is that in equilibrium, phases of cooperation and phases of punishment sustain each other. In the cooperative phase, the agent exerts high effort because he knows that failure to do so raises the probability of a disturbance which can trigger a transition to the punishment phase. In the punishment

[17] More specifically, $U_z^{F*}(\underline{U}) = d^*\underline{U} + (1-d^*)\overline{U}$ and $U_z^{L*}(\overline{U}) = l^*\underline{U} + (1-l^*)\overline{U}$.

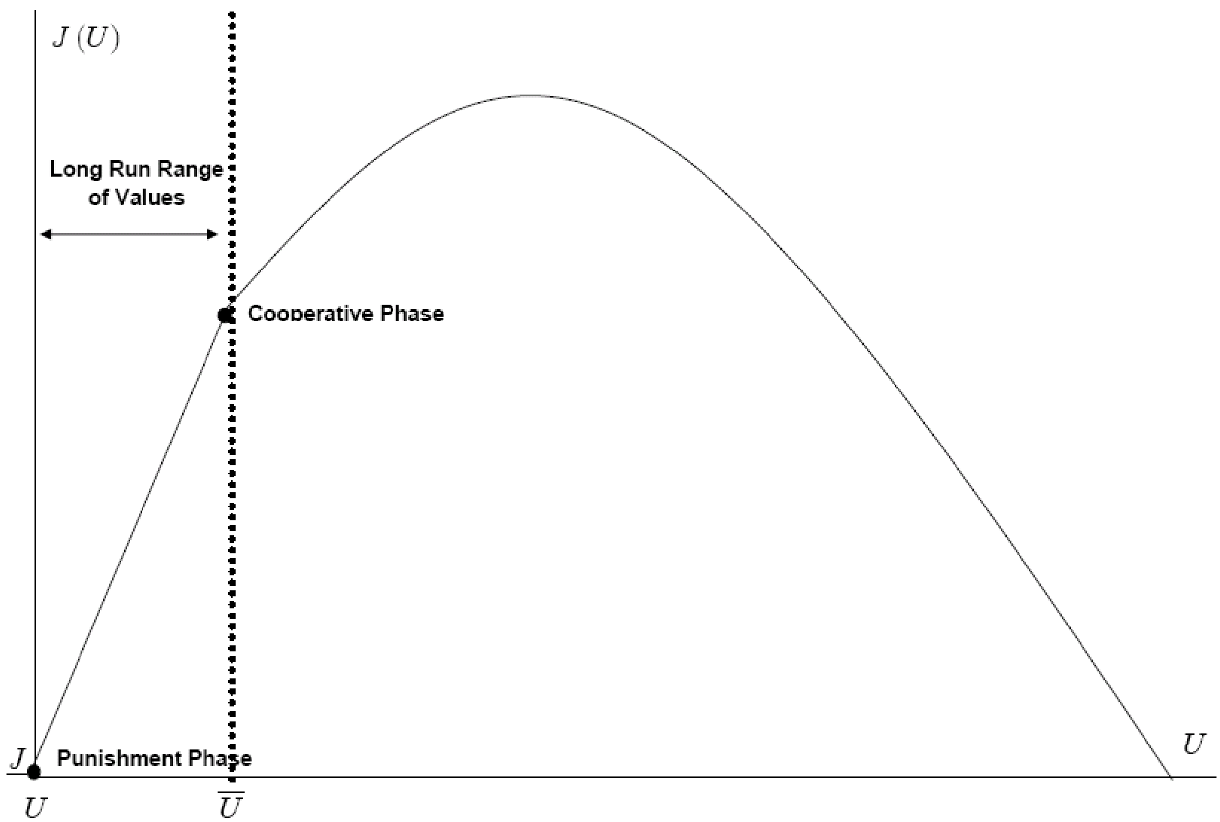
[18] Note that an efficient equilibrium necessarily begins on the downward sloping portion of $J(U)$. See the Appendix for a deeper discussion of the shape and endpoints of $J(U)$.

[19] In a model which allows for non-negative payments from the principal to the agent, the second part of Proposition 1 holds exactly, though the first part may not necessarily do so since a long enough absence of disturbances can lead to the permanent absence of intervention. An intuition for this is provided in the extension in Section 5.1.

[20] Technically, if $U_z^{H*}(U) > \overline{U}$, then (6) would not bind which is inefficient by the concavity of $J(\cdot)$. Note that the first part of Proposition 1 holds for all solutions, not just those which satisfy the *Bang-Bang* property.

phase, the principal temporarily intervenes with a unique level of intensive force. The principal exerts costly force since he knows that failure to do so would trigger the agent to choose low effort in all future cooperative phases, making permanent intervention by the principal a necessity.[21]

Figure 2: $J(U)$



Importantly, the values of $\underline{U}$ and $J\left(\overline{U}\right)$ are intimately linked. To see why, consider the system of equations which characterizes the long run equilibrium:

$$\overline{U} = -\eta + \beta\left(\left(1-\pi_a(\eta)\,l^*\right)\overline{U} + \pi_a(\eta)\,l^*\underline{U}\right) \quad (10)$$

$$\underline{U} = w_a - g(i^*) + \beta\left(d^*\underline{U} + (1-d^*)\,\overline{U}\right) \quad (11)$$

$$J\left(\overline{U}\right) = -\pi_a(\eta)\,\chi + \beta\left(\left(1-\pi_a(\eta)\,l^*\right)J\left(\overline{U}\right) + \pi_a(\eta)\,l^*\underline{J}\right) \quad (12)$$

$$\underline{J} = -\pi_p\chi - Ai^* + \beta\left((1-d^*)\,J\left(\overline{U}\right) + d^*\underline{J}\right). \quad (13)$$

[21]Note that if $g'(0) < \infty$, then one could construct environments in which $i^* = 0$ so that the principal does not need inducement to intervene and intervention lasts forever as in Yared (2009).

Equations (10) and (13) are derived from equations (6) and (5), the incentive compatibility constraints on the agent and principal, respectively. Importantly, the value of $\overline{U}$ does not depend on the value of i^* since $\overline{U}$ is self-generating in equilibrium, and $\underline{J}$ is independent of i^* since it corresponds to the repeated static Nash payoff to the principal.[22] Therefore, $(10)-(13)$ is a system of four equations and five unknowns–$\underline{U}$, $J\left(\overline{U}\right)$, l^*, i^*, and d^*–where the value of i^* is selected to maximize $J\left(\overline{U}\right)$.[23]

Since $\overline{U}$ and $\underline{J}$ are exogenously determined, equations (10) and (12), imply that the lower is $\underline{U}$, then the lower is the implied value of l^*, and the higher is the implied value of $J\left(\overline{U}\right)$. Intuitively, the harsher the punishment, the more easily can the agent be forgiven for disturbances without altering his incentives to exert high effort, and hence, the longer cooperation and the higher the principal's welfare under cooperation. Analogously, equations (11) and (13) imply that, conditional on i^*, the higher is $J\left(\overline{U}\right)$, then the higher is the implied value of d^*, and the lower is the implied value of $\underline{U}$. Intuitively, the higher the principal's welfare under cooperation, the more easily can the principal be induced to punish for longer, and the lower the punishment to the agent. Consequently, the optimal choice of i^* which maximizes the principal's value of cooperation also simultaneously minimizes the agent's value of punishment since the cooperative and punishment phases sustain each other.

4.2 Tradeoff between Intensity and Duration of Intervention

In this section, we consider the choice of intensity in the optimal contract together with its implications for the optimal likelihood and duration of intervention. In doing so, we highlight an important tradeoff between the intensity and duration of intervention.

To this end, it is useful to construct an equilibrium with the exact same structure as the long run characterization of the efficient sequential equilibrium described in Proposition 1 but under an alternative level of intensity i to replace i^*. More specifically, let $l(i)$ and $d(i)$ correspond to the implied values of l^* and d^*, respectively, in $(10)-(13)$ for a given $i^* = i$. $l(i)$ corresponds to the likelihood of intervention under intensity i and $d(i)$ corresponds to the duration of intervention under intensity i.

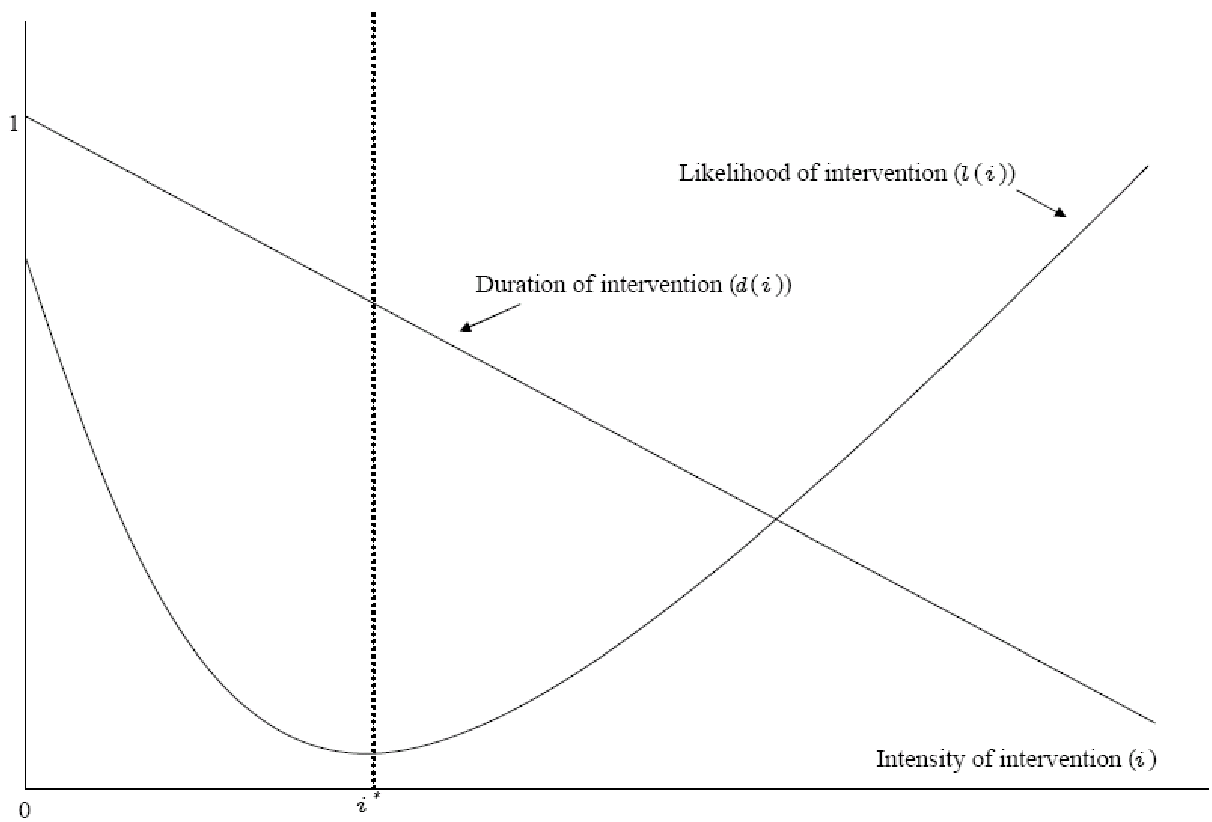
Proposition 2 ***(optimal intervention)*** *The optimal levels of l^*, i^*, and d^* satisfy $l^* = l(i^*)$ and $d^* = d(i^*)$ for i^* defined in (9) where $l(\cdot)$ and $d(\cdot)$ are continuously differentiable functions with $l'(i) < (>)\, 0$ if $i < (>)\, i^*$ and $d'(i) < 0$.*

[22] That is, $\overline{U}$ is derived by combining (2) with (6) (which binds) given that $e_z^*\left(\overline{U}\right) = \eta$ and $U_z^{H*}\left(\overline{U}\right) = \overline{U}$.

[23] Note that the uniqueness of i^* defined in (9) is guaranteed by the global concavity of $g(\cdot)$.

Proposition 2 states that, in the set of equilibria with the same structure as the efficient equilibrium, an increase in intensity reduces the likelihood of intervention for $i < i^*$ and it increases the likelihood of intervention for $i > i^*$. Moreover, an increase in intensity reduces the duration of intervention. This proposition implies that there is a tradeoff between the intensity and duration of intervention, and that the optimal level of intensity i^* corresponds to the point which minimizes the likelihood of intervention. This proposition is displayed graphically in Figure 3, where intensity i is on the x-axis and the implied likelihood and duration of intervention–$l(i)$ and $d(i)$, respectively–are on the y-axis.

Figure 3: Likelihood, Intensity, and Duration of Intervention



The principal's incentives to intervene are the driving force behind Proposition 2. More specifically, if the intensity of the intervention rises, then the principal can only be induced to exert this intensive intervention if the resumption of cooperation following intervention is more likely. Therefore, $d'(i) < 0$, so that the duration of intervention is declining in intensity. An implication of this is that the agent's continuation value under punishment declines then rises with intensity. Initially, there are high returns to intensity so that the agent suffers by more under punishment as intensity rises. Eventually, however, returns to intensity decline and these are outweighed by the reduction in the duration of intervention which serve to reduce the suffering of the agent under punishment.

Since the agent's value under punishment decreases then increases with intensity, the likelihood of intervention $l(i)$ decreases then increases with intensity. This is because as the punishment for the agent becomes worse, the likelihood of punishment declines without adversely affecting his incentives, and this is optimal from the perspective of the principal since it maximizes the duration of cooperation. In contrast, as the punishment for the agent becomes less severe, the likelihood of punishment rises so as to induce high effort by the agent, and this diminishes the welfare of the principal by reducing the duration of cooperation. As such, the levels of intensity i^* and duration $d(i^*)$ which maximize the principal's welfare under cooperation also minimize the agent's welfare under punishment, and they also minimize the likelihood of punishment.

Our results are related to static political economy models of punishment which study variety of situations, such as extortion and slavery.[24] They are also related to the law and economics literature which considers the tradeoff between the likelihood of punishment (i.e., the probability of capturing criminals) and the harshness of punishment (i.e., the length of incarceration).[25] Our analysis shares some similarities to the literature on static punishments. First, as in our environment, this literature establishes that choosing the harshest punishment is suboptimal because costly punishments must be exercised *in equilibrium*. Second, the law and economics literature highlights a complementarity between the likelihood and the harshness of punishment which is also present in our framework. More specifically, in our model an increase l^* and a reduction in $\underline{U}$ are complementary tools for the reduction of the punishment continuation value $U^L(\overline{U})$. Nonetheless, in contrast to our dynamic model, static models by definition cannot distinguish between the intensity and the duration of punishment, and hence they cannot provide any answers to the motivating questions of our analysis. In this regard, the tradeoff in our model between the intensity and duration of punishment and its relationship to the absence of commitment on the side of the principal is novel to the literature on punishment.[26]

[24] See Dal Bó and Di Tella (2003) and Dal Bó, Dal Bó and di Tella (2006) for an application to political capture and Chwe (1990) and Acemoglu and Wolitzky (2009) for labor contracts with limited liability.

[25] See, for instance, the seminal articles by Becker (1968) and Polinsky and Shavell (1979,1984).

[26] Because applying punishments is costly to the principal, static models need to assume that the principal can commit to some punishment intensity as a function of observable outcomes.

4.3 Comparative Statics

In this section, we consider the effect of three factors on the optimal likelihood, intensity, and duration of intervention. First, we consider the effect of a decline in the cost of intensity to the principal (A), where this can occur for instance if there is less international rebuke for the use of punitive measures. Second, we consider the effect of a rise in the cost of disturbances to the principal (χ). Finally, we consider the effect of a rise in the cost of effort to the agent (η), where this can occur for instance if it becomes more politically costly for the agent to antagonize rival factions contributing to disturbances or alternatively if he acquires a higher preference for the realization of disturbances. We make the following assumption to facilitate our discussion.

Assumption 4 $g(i) = i^{\theta}$ for $0 < \theta < 1$.

As we discuss further below, the only purpose of this assumption is to make the effect on duration of a change in A or χ unambiguous. The comparative statics are summarized in the below proposition.[27]

Proposition 3 ***(comparative statics)***

1. *If A decreases (increases), then l^* decreases (increases), i^* increases (decreases), and d^* decreases (increases),*

2. *If χ increases (decreases), then l^* decreases (increases), i^* increases (decreases), and d^* decreases (increases), and*

3. *If η increases (decreases), then l^* increases (decreases), i^* increases (decreases), and d^* decreases (increases).*

This proposition states that all three changes increase the optimal intensity and decrease the optimal duration of intervention. However, only the third change also raises its likelihood whereas the first two changes decrease its likelihood.

To see why intensity must rise, consider the first case. If the cost of intensity declines, then the principal's return to intensity rises since it is cheaper to provide incentives to the agent via intensive force. In the second case, if the cost of disturbances rise, the principal should raise the intensity of intervention since the return to delegating to the agent rises, and it is optimal to provide the agent with stronger incentives. In the third case, if the

[27] Performing comparative statics with respect to the probability of a disturbance is not straightforward given that this would affect the values of π_p, $\pi_a(\eta)$, and $\pi_a(0)$ jointly.

cost of effort for the agent rises, then it is harder for the principal to provide incentives to the agent with lower levels of intensity, and higher levels of intensity become optimal. In all three cases, because the principal needs more inducement to use more intensive punishments, these increases in the level of intensity necessitate a decline in the duration of intervention.

Though all three changes increase the optimal intensity and decrease the optimal duration of intervention, only the third also raises its likelihood. Specifically, if the cost of intensity to the principal declines or if the cost of disturbances to the principal rises, then higher intensity translates into a more severe punishment for the agent and therefore makes it easier for the principal to forgive the agent following the realization of disturbances without weakening incentives for the agent. Therefore, the likelihood of intervention declines. In contrast, if the cost of effort to the agent rises, then incentives are harder to provide for the agent, so that likelihood of intervention must rise following the realization of a disturbance.

Note that the comparative statics with respect to the likelihood and the duration of intervention rely on the fact that the principal responds optimally to changes in the environment by increasing the level of intensity. To see why, consider the effect of each of these factors absent any change in the level of intensity, where the ensuing hypothetical suboptimal equilibrium can be constructed as in Section 4.2. Consider the effect of a decrease in the cost of intensity to the principal or an increase in the cost of disturbances to the principal absent any change in i. In this circumstance, the implied likelihood of intervention declines and implied duration of intervention *rises.* This is because it becomes easier to provide incentives to the principal to use intensive force (i.e., either the cost of force is lower or the marginal benefit of resuming cooperation rises). Since incentives to the principal are easier to provide, the duration of intervention rises, which means that punishment becomes more severe for the agent, and the likelihood of intervention declines.[28] These comparative statics do not take into account that the optimal level of intensity rises and it reinforces the decline in the likelihood of intervention. Moreover, under Assumption 4, the implied rise in intensity is sufficiently large that it generates an overall *reduction* in the duration of intervention so as to provide incentives to the principal to punish. Therefore, the reduction in duration due to the rise in intensity more than fully offsets the rise duration absent such a change in intensity. Note that this final comparative static relies on Assumption 4, and one can construct environments in which a decline in A or a rise in χ would instead increase the duration of intervention since the

[28] Formally, this is equivalent to stating that $d(i)$ is decreasing in A and increasing in χ.

implied rise in intensity would be too small.[29]

Analogously, one can consider the effect of a rise in the cost of effort to the agent, absent any change in i. In this circumstance, the implied likelihood of intervention rises and the implied duration of punishment declines. This is because it becomes more difficult to provide incentives to the agent to exert high effort, so that the likelihood of intervention rises, reducing the value of cooperation for the principal. Because the principal puts lower value on cooperation, the duration of intervention must decline so as to provide the principal with enough inducement to exert the same level of intensity. In this circumstance, the optimal level of intensity rises and therefore *mitigates* the rise in the likelihood of intervention, and this reinforces the decline in the duration of intervention.[30]

5 Extensions

Our simple benchmark model ignores three additional issues. First, it ignores the possibility that permanent concessions by the principal can reduce the presence of disturbances in the future. Second, it ignores the possibility that the agent's identity can change over time because of political transitions. Third, it ignores the possibility that high intensity levels by the principal today can raise the cost of effort by the agent in the future, for example if the agent becomes more antagonistic. These issues are discussed in the below three extensions which show that our main conclusions are unchanged.[31]

5.1 Permanent Concessions

Consider an extension of our benchmark model where if the principal does not intervene at t ($f_t = 0$), he chooses a concession $c_t = \{0, 1\}$. If $c_t = 0$, then no concession is made and the rest of the period proceeds as in our benchmark model. In contrast, if $c_t = 1$, a permanent concession is made which ends the game an provides a continuation value J^C to the principal and U^C to the agent starting from t. Such a concession can come in the form of land or political representation, for instance, and we assume that it satisfies the agent and ends all disturbances. Specifically, suppose that $U^C > 0$, so that it provides the agent with more utility than low effort forever.

[29] This would be true for instance if $g(\cdot)$ and i^* are such that $-i^* g''(i^*) / g'(i^*) > 1$.

[30] The rise in the likelihood of intervention occurs independently of Assumption 4 since the principal must be strictly worse off if η rises.

[31] Due to space restrictions, we describe these results informally, but more details are available upon request.

Clearly, if $J^C < \underline{J}$, then the principal cannot possibly be induced to make a concession since he prefers to use intervention with zero intensity of force forever. Therefore, the equilibrium would be exactly as the one we have characterized. Moreover, if $J^C > -\pi_a(\eta)\chi/(1-\beta)$, then the efficient equilibrium involves no intervention since the principal makes the concession in period 0 and the game ends. We therefore consider the more interesting case in which $J^C \in (\underline{J}, -\pi_a(\eta)\chi/(1-\beta))$.

In this situation, the provision of concessions serves as a reward for the successful avoidance of disturbances and the use of intervention continues to serve as a punishment for disturbances.[32] Clearly, if a sufficient number of disturbances are avoided, then intervention never takes place and the long run equilibrium features a concession by the principal together with the end of all conflict so as to reward the agent for good behavior.[33] In contrast, if a sufficient number of disturbances occur, then continuation values decline below $\overline{U}$ defined in (8) and punishment necessarily occurs. Moreover, by analogous reasoning as in Proposition 1, continuation values cannot rise above $\overline{U}$ once they have declined below it. This is because the principal prefers to have the agent exert high effort in preventing disturbances versus making a concession going forward. Therefore, continuation values must be trapped below $\overline{U}$ if intervention is ever used along the equilibrium path, and no concession will ever be made going forward in this situation.

The equilibrium of the extended model thus admits two potential long run outcomes, one with a permanent concession and the other which is analogous in structure to the one which we consider. Thus, as in our benchmark environment, the second equilibrium features phases of cooperation and punishment which sustain each other, it features a tradeoff between the intensity and duration of punishment, and it features the same comparative statics. Nevertheless, the equilibrium is not quantitatively identical to the one in the benchmark model precisely because the min-max for the principal is now J^C as opposed to $\underline{J}$. In other words, the principal cannot experience a continuation value below that which he can guarantee himself by making a concession to the agent. This implies that the punishment to the agent $\underline{U}$ must be higher in the extended model. Thus, the likelihood of punishment is higher and its duration shorter because it is harder to provide incentives to the principal and to the agent.[34]

As an aside, note that if the principal lacks commitment to concessions and if a concession costs the principal $J^C(1-\beta)$ in every period, then nothing changes as long as

[32] This is because rewarding the agent by allowing low effort is inefficient for the principal as well as the agent.

[33] This is also the case if the initial condition U_0 is chosen to be sufficiently high.

[34] We have implicitly assumed that an analogous condition to Assumption 3 holds so that the implied duration of punishment is bounded away from zero.

$J^C > \underline{J}$, since concessions can be enforced. If instead $J^C < \underline{J}$, then temporary concessions may be featured along the equilibrium path, but the long run characterization of the equilibrium is exactly as in our benchmark model.

5.2 Political Transitions

Our model additionally ignores the role of political transitions since it assumes that the two players interact with each other forever. This issue is particularly relevant for the case of the agent since the dynamics of the equilibrium are generated by the need for the principal to punish the agent for the realization of past disturbances. Clearly, there is no need for the principal to punish an agent who cannot possibly be blamed for past disturbances.

To explore this issue further, imagine if in every period there is a probability $1-q$ that the incumbent agent is replaced by another identical agent, where replacement yields a continuation value of zero to the incumbent. Moreover, to simplify discussion, consider the efficient sequential equilibrium which maximizes the principal's period 0 welfare, where the optimal contract now clearly specifies the identity of the agent whom the principal faces.

It is easy to show that in such a setting, the second part of Proposition 1 will hold for the long run interaction between the principal and a given agent, where β in Proposition 1 and in (8) is replaced by βq which corresponds to the relevant discount factor for the agent. In other words, our characterization of the cooperative and punishment phases holds for the interaction between the principal and an agent after several disturbances have occurred during the agent's tenure. This equilibrium features phases of cooperation and punishment which sustain each other. Moreover, one can show that for q sufficiently close to 1, it features the same tradeoff between the intensity and duration of intervention, and it features same exact comparative statics. Nonetheless, the model is not quantitatively equivalent to our benchmark environment since the principal's and the agent's discount factors differ from one another. Moreover, one can show that as q declines, it becomes more difficult for the principal to provide incentives to the agent so that the likelihood of intervention rises and the duration of intervention declines.

An important new feature of the extended model which is not present in the benchmark model in that a political transition causes the continuation value to the agent to rise above $\overline{U}$. This is because it is inefficient for the principal to punish an agent who is not responsible for the exertion of effort in the past by providing him with low welfare. Note further that it is straightforward to combine this extension with that of Section 5.1

which allows the principal to make a permanent concession. In such a setting, the long run will always feature a permanent concession by the principal. This is because even if one agent is punished and may never receive the concession himself, there is always a positive probability going forward that the agent which replaces him will be successful at preventing disturbances and will therefore be rewarded with a permanent concession.

5.3 Endogenous Effort Cost

Our model additionally ignores the fact that the use of intensity by the principal can potentially embitter the agent and hence make it more difficult for the agent to exert effort in preventing disturbances. This would occur also if the agent loses political credibility with the population he seeks to influence. To explore this issue further, imagine if the cost of high effort η depends on time so that it is denoted by η_t and it can either be low ($\eta_t = \eta^L$) or high ($\eta_t = \eta^H$). Suppose $\eta_0 = \eta^L$ and imagine the following process for η_t:

$$\eta_t = \begin{cases} \eta^H & \text{if } f_k = 1 \text{ and } i_k > \widetilde{i} \text{ for any } k < t \\ \eta^L & \text{otherwise} \end{cases}.$$

This means that if the principal ever exceeds a certain level of intensity, then the cost of high effort for the agent permanently rises. Moreover, suppose $\widetilde{i}$ is below the optimal level of intensity in an environment in which $\eta_t = \eta^L$ for all t. This means that if the principal uses the same level of intensity as in our benchmark environment, the cost of effort for the agent permanently rises.

Imagine if the level of η^H is sufficiently low that one can construct an equilibrium with the same structure as in our benchmark setting in which the agent can be induced to exert this level of effort. We can show that in this case the principal always lets the cost of effort rise in the extended model. The intuition for this is that the rise in the cost of effort to the agent serves as an additional form of long run punishment for the agent and therefore provides even better incentives to the agent to exert high effort along the equilibrium path.

More specifically, in the efficient equilibrium of the extended model, the principal chooses the likelihood, intensity, and duration of intervention associated with the level of effort equal to η^H in our benchmark model. Given Proposition 3, this means that the likelihood of intervention is higher, the intensity of intervention is higher, and the duration of intervention is lower compared to the original equilibrium in which the cost of effort does not rise and remains at η^L. Therefore, the level of intensity rises to reinforce the rise in the cost of effort to the agent.

To understand this, note that the first instance of a punishment phase provides the principal with a continuation value of $\underline{J}$ independently of whether the cost of effort to the agent rises or remains the same going forward. Therefore, from an ex-ante perspective, the optimal policy for the principal is to minimize the welfare under a punishment phase for the agent so as to provide the best incentives for the agent to exert effort *along the equilibrium path*. In providing these ex-ante incentives, the principal therefore has two options. One option is to choose $i_t = \widetilde{i}$ so as to prevent the cost of effort from rising. The second option is to choose $i^* > \widetilde{i}$ and to let the cost of effort rise, where i^* represents the level of intensity which minimizes the agent's welfare from punishment conditional on the cost of effort equal to η^H going forward. It is clear that the principal should choose the second option since, starting from the punishment phase, the agent expects higher levels of intensive force and a higher cost of effort going forward under i^* versus $\widetilde{i}$.

Therefore, the long run equilibrium in this extended model features a cooperative and punishment phase which sustain each other as in our benchmark environment, though these are associated with a higher cost of effort to the agent. Moreover, the tradeoff between the intensity and duration of intervention remain and none of our comparative statics change.

As an aside, note that these conclusions change if instead η^H is so high that one cannot construct any equilibrium which sustains high effort by the agent. In this situation, levels of intensity above $\widetilde{i}$ cannot be credibly used by the principal since the agent will never exert high effort in the future. Consequently, the optimal punishment for the principal features a cooperative phase and a punishment phase as in our benchmark environment, though the principal sets the level of intensity at $\widetilde{i}$ in order to prevent the cost of effort to the agent from rising. Given Proposition 2, this means that there is a higher likelihood of intervention, a lower intensity of intervention, and a longer duration of intervention in comparison to our benchmark environment. Moreover, note that our comparative statics in Proposition 3 must be modified to take into account the fact that the level of intensity does not change with small changes in the environment. Consequently, not only is it the case that the level of intensity does not change, but the duration of intervention actually rises if A declines or if χ rises. This is because, holding the level of intensity constant, these changes enhance the incentives of the principal to punish and hence increase the duration of intervention, and this effect cannot be undone by a rise in intensity as in our benchmark environment.

6 Conclusion

We have analyzed a dynamic principal-agent model in which repeated interventions are a feature of optimal policy. This has allowed us to analyze factors behind the optimal likelihood, intensity, and duration of interventions. Our explicit closed form solution for the long run dynamics of the efficient sequential equilibrium highlights a novel tradeoff between the intensity and duration of interventions. It also allows us to consider the separate effects of a fall in the cost of intensity to the principal, a rise in the cost of disturbances to the principal, and a rise in the cost of effort to the agent.

Our model abstracts from a number of potentially important issues. First, in answering our motivating questions, we have abstracted away from the *static* components of intervention and the means by which a principal *directly* affects the level of disturbances (i.e., we let π_p be exogenous). Future work should also focus on the static features of optimal intervention and consider how they interact with the dynamic features which we describe. Second, we have ignored the possibility that the agent may have a permanent source of private information, a situation which would occur for example if the agent's cost of effort were unobservable to the principal. We have ignored this issue not for realism but for convenience since it maintains the common knowledge of preferences over continuation contracts and simplifies the recursive structure of the efficient sequential equilibria. Future work should consider the effect of relaxing this assumption.

7 Appendix

7.1 Equilibrium Definition

We consider equilibria in which each player conditions his strategy on past public information. Let $h_t^0 = \{z^{1t}, f^{t-1}, z^{2t-1}, i^{t-1}, s^{t-1}\}$, the history of public information at t after the realization of z_t^1.[35] Let $h_t^1 = \{h_t^0, f^{t-1}, z^{2t}\}$, the history of public information at t after the realization of z_t^2. Define a strategy $\sigma = \{\sigma_p, \sigma_p\}$ where $\sigma_p = \{f_t(h_t^0), i_t(h_t^1)\}_{t=0}^{\infty}$ and $\sigma_a = \{e_t(h_t^1)\}_{t=0}^{\infty}$ for σ_p and σ_a which are feasible if $f_t(h_t^0) \in \{0,1\}$ $\forall h_t^0$, $i_t(h_t^1) \geq 0$ $\forall h_t^1$, and $e_t(h_t^1) = \{0, \eta\}$ $\forall h_t^1$.

Given σ, define the equilibrium expected continuation values for player j at h_t^0 and h_t^1, respectively, as $U_j\left(\sigma|_{h_t^0}\right)$ and $U_j\left(\sigma|_{h_t^1}\right)$ where $\sigma|_{h_t^0}$ and $\sigma|_{h_t^1}$ correspond to continuation strategies following h_t^0 and h_t^1, respectively. Let $\Sigma_j|_{h_t^0}$ and $\Sigma_j|_{h_t^1}$ denote the entire set of feasible continuation strategies for j after h_t^0 and h_t^1, respectively.

Definition 2 *σ is a sequential equilibrium if it is feasible and if for $j = p, a$*

$$\begin{aligned}
U_j\left(\sigma|_{h_t^0}\right) &\geq U_j\left(\sigma'_j|_{h_t^0}, \sigma_{-j}|_{h_t^0}\right) \ \forall \sigma'_j|_{h_t^0} \in \Sigma_j|_{h_t^0} \ \forall h_t^0 \text{ and} \\
U_j\left(\sigma|_{h_t^1}\right) &\geq U_j\left(\sigma'_j|_{h_t^1}, \sigma_{-j}|_{h_t^1}\right) \ \forall \sigma'_j|_{h_t^1} \in \Sigma_j|_{h_t^1} \ \forall h_t^1.
\end{aligned}$$

In order to build a sequential equilibrium allocation which is generated by a particular strategy, let $q_t^0 = \{z^{1t}, z^{2t-1}, s^{t-1}\}$ and $q_t^1 = \{z^{1t}, z^{2t}, s^{t-1}\}$, the *exogenous* equilibrium history of public signals and states after the realizations of z_t^1 and z_t^2, respectively. Define an equilibrium allocation as a function of the exogenous history:

$$\alpha = \left\{f_t\left(q_t^0\right), i_t\left(q_t^1\right), e_t\left(q_t^1\right)\right\}_{t=0}^{\infty}.$$

Let $\mathcal{F}$ denote the set of feasible allocations α with continuation allocations from t onward which are measurable with respect to public information generated up to t. Let $U_j\left(\alpha|_{q_t^0}\right)$ and $U_j\left(\alpha|_{q_t^1}\right)$ correspond to the equilibrium continuation value to player j following the realization of q_t^0 and q_t^1, respectively. The following lemma provides necessary and sufficient conditions for α to be generated by sequential equilibrium strategies.

[35] Without loss of generality, we let $i_t = 0$ if $f_t = 0$ and $e_t = 0$ if $f_t = 1$.

Lemma 1 *$\alpha \in \mathcal{F}$ is a sequential equilibrium allocation if and only if*

$$U_j\left(\alpha|_{q_t^0}\right) \geq \underline{U}_j \text{ for } j = p, a \ \forall q_t^0, \tag{14}$$

$$U_p\left(\alpha|_{q_t^1}\right) \geq -\pi_p \chi + \beta \underline{U}_p \ \forall q_t^1 \text{ s.t. } f_t\left(q_t^0\right) = 1, \text{ and} \tag{15}$$

$$U_a\left(\alpha|_{q_t^1}\right) \geq \max \left\{ \begin{array}{c} -\eta + \beta \left(\begin{array}{c} (1-\pi_a(\eta)) \left\{U_j\left(\alpha|_{q_{t+1}^0}\right) | q_t^1, s_t = 0\right\} \\ +\pi_a(\eta) E\left\{U_j\left(\alpha|_{q_{t+1}^0}\right) | q_t^1, s_t = 1\right\} \end{array} \right), \\ \beta \left(\begin{array}{c} (1-\pi_a(0)) \left\{U_j\left(\alpha|_{q_{t+1}^0}\right) | q_t^1, s_t = 0\right\} \\ +\pi_a(0) E\left\{U_j\left(\alpha|_{q_{t+1}^0}\right) | q_t^1, s_t = 1\right\} \end{array} \right) \end{array} \right\} \forall q_t^1 \text{ s.t. } f_t\left(q_t^0\right) = 0 \tag{16}$$

for $\underline{U}_p = -\pi_p \chi / (1-\beta)$ and some $\underline{U}_a \leq w_a / (1-\beta)$.

Proof. Step 1. *The necessity of (14) for $j = p$ follows from the fact that the principal can choose $f_k'\left(q_k^0\right) = 1 \ \forall k \geq t$ and $\forall q_k^0$ and $i_k'\left(q_k^1\right) = 0 \ \forall k \geq t$ and $\forall q_k^1$, and this delivers continuation value $\underline{U}_p$. The necessity of (14) for $j = a$ follows from the fact that the agent can choose $e_k'\left(q_k^1\right) = 0 \ \forall k \geq t$ and $\forall q_k^1$, and this delivers a continuation value of at least $\underline{U}_a$.* ***Step 2.*** *The necessity of (15) follows from the fact that conditional on $f_t\left(q_t^0\right) = 1$, the principal can choose $f_k'\left(q_k^0\right) = 1 \ \forall k > t$ and $\forall q_k^0$ and $i_k'\left(q_k^1\right) = 0 \ \forall k \geq t$ and $\forall q_k^1$, and this delivers continuation value $-\pi_p \chi + \beta \underline{U}_p$. The necessity of (16) follows from the fact that conditional on $f_t'\left(q_t^0\right) = 0$, the agent can unobservably choose $e_t'\left(q_t^1\right) \neq e_t\left(q_t^1\right)$ and follow the equilibrium strategy $\forall k > t$ and $\forall q_k^1$.* ***Step 3.*** *For sufficiency, consider a feasible allocation which satisfies (14) – (16) and construct the following off-equilibrium strategy. Any observable deviation by the principal results in a reversion to the repeated static Nash equilibrium. We only consider single period deviations since $\beta < 1$ and since continuation values are bounded. If $f_t'\left(q_t^0\right) = 1$, then a deviation by the principal to $f_t'\left(q_t^0\right) = 0$ is weakly dominated by (14) and Assumption 2. Moreover, a deviation by the principal to $i'\left(q_t^1\right) \neq i\left(q_t^1\right)$ is weakly dominated by (15). If $f_t\left(q_t^0\right) = 0$, then a deviation by the principal to $f_t'\left(q_t^0\right) = 1$ is weakly dominated by (14). If $f_t\left(q_t^0\right) = 0$, then a deviation by the agent to $e_t'\left(q_t^1\right) \neq e_t\left(q_t^1\right)$ is weakly dominated by (16).* ■

7.2 Implications of Assumption 3

The value of $\widehat{\beta}$ satisfies

$$\widehat{\beta} = \max \left\{ \frac{\eta}{\eta (1 - \pi_a (0)) - w_a (\pi_a (0) - \pi_a (\eta))}, \frac{1}{A} \frac{g' (i^*) i^*}{-\frac{\pi_a (0)}{\pi_a (0) - \pi_a (\eta)} \eta - w_a + g (i^*)} \right\}$$

for i^* which satisfies (9). Given the functions $l(i)$ and $d(i)$ defined in Section 4.2, the first part of this assumption implies that $l(0) < 1$ so that an equilibrium in which high effort is sustained by the threat of the repeated static Nash equilibrium exists. Since $l(i)$ is declining in i for $i < i^*$ by Proposition 2, this assumption guarantees that $l(i^*) < 1$. The second part of this assumption implies that $d(i^*) > 0$. These features guarantee that the set of values $U \in \left[\underline{U}, \overline{U}\right]$ are self-generating so that the long run equilibrium can be explicitly characterized.

7.3 Proofs of Additional Lemmas

In this section we prove several important lemmas which are required for proving our propositions. Let Γ represent the set of sequential equilibrium continuation values and let $U^{\max}$ the highest continuation value to the agent in this set.

Lemma 2 *(i) Γ is convex and compact, (ii) $J(\underline{U}) = J(U^{\max}) = \underline{J}$, and (iii) $J(U)$ is weakly concave.*

Proof. ***Step 1.*** *The weak concavity of the program and the convexity of the constraint set in (1) – (7) guarantees that Γ is convex.* ***Step 2.*** *If we set an arbitrarily high upper bound for i in (1) – (7), then the compactness of the constraint set together with the fact that $\beta < 1$ guarantees that that Γ is closed and bounded by the Dominated Convergence Theorem.* ***Step 3.*** *By (3), $J(\underline{U}) \geq \underline{J}$ and $J(U^{\max}) \geq \underline{J}$.* ***Step 4.*** *By Assumptions 1 and 2 and equations (4) and (6), it must be that $f_z^*(\underline{U}) = 1$ $\forall z$ since otherwise an increase in f_z for some z must satisfy (3) – (7) and strictly reduces the welfare of the agent. If $J(\underline{U}) > \underline{J}$, then an increase i_z must satisfy (3) – (7) and strictly reduces the welfare of the agent. Therefore $J(\underline{U}) = \underline{J}$.* ***Step 5.*** *By Assumption 1 and equations (4) and (6), $f_z^*(U^{\max}) = 0$ $\forall z$ since otherwise a decrease in f_z for some z must satisfy (3) – (7) and strictly increase the welfare of the agent. If $J(U^{\max}) > \underline{J}$, then a decrease in e_z or an increase in U_z^H strictly increases the welfare of the principal while satisfying (3) – (7), and if this were not feasible then $U^{\max} = 0$, which violates (3) since it implies $J(U^{\max}) = -\pi_a(0)\chi$. Therefore, $J(U^{\max}) = \underline{J}$.* ***Step 6.*** *The weak concavity of $J(\cdot)$ follows directly from the first and second parts of the lemma.* ■

Lemma 3 $\exists i^*$ *s.t. the solution to* $(1)-(7)$ *cannot admit* $i_z^*(U) \neq i^*$ *for any* z *given* $f_z^*(U) = 1$.

Proof. Step 1. Define $i^* = Ei_z^*(\underline{U})$. By Step 4 of the proof of Lemma 2, $f_z^*(\underline{U}) = 1\ \forall z$. It must be that $i_z^*(\underline{U}) = i^*\ \forall z$ since otherwise a perturbation which sets $i_z^*(\underline{U}) = Ei_z^*(\underline{U})\ \forall z$ continues to satisfy $(3)-(7)$ and strictly reduces the welfare of the agent by the concavity of $g(i)$ and $J(U)$. **Step 2**. Let $\widehat{J}\left(U|\widehat{i}\right)$ correspond to the maximizer of $(1)-(7)$ subject to the additional constraints that $f_z = 1$ and $i_z = \widehat{i}\ \forall z$ for some $\widehat{i}$. Note that for any two value U' and U'' where $\left(w_a - g\left(\widehat{i}\right)\right)/(1-\beta) \leq U' < U''$, it must be that

$$\frac{\widehat{J}\left(U''|\widehat{i}\right) - \widehat{J}\left(U'|\widehat{i}\right)}{U''-U'} = \frac{J\left(\dfrac{U''-w_a+g\left(\widehat{i}\right)}{\beta}\right) - J\left(\dfrac{U'-w_a+g\left(\widehat{i}\right)}{\beta}\right)}{\dfrac{U''-U'}{\beta}} \tag{17}$$

$$\leq \frac{J(U'')-J(U')}{U''-U'}, \tag{18}$$

where we have appealed to the concavity of $J(\cdot)$. **Step 3.** Imagine if $\exists\widehat{i} \neq i^*$ s.t. $\widehat{J}\left(U|\widehat{i}\right) = J(U)$ for some U. Let $\widehat{U}\left(\widehat{i}\right) \geq \left(w_a - g\left(\widehat{i}\right)\right)/(1-\beta)$ denote the value which solves $\widehat{J}\left(\widehat{U}\left(\widehat{i}\right)|\widehat{i}\right) = \underline{J}$ for such $\widehat{i}$, which must exist by the concavity of $\widehat{J}(\cdot)$ since $\widehat{J}\left(U|\widehat{i}\right) \geq \underline{J}$ for some U. By step 1 and Assumption 3, $J\left(\widehat{U}\left(\widehat{i}\right)\right) > \widehat{J}\left(\widehat{U}\left(\widehat{i}\right)|\widehat{i}\right)$, so that by (18) $\widehat{J}\left(U|\widehat{i}\right) < J(U)\ \forall U \geq \widehat{U}\left(\widehat{i}\right)$. Therefore, $\widehat{J}\left(U|\widehat{i}\right) < J(U)\ \forall U$ and $\forall\widehat{i} \neq i^*$. **Step 4.** By step 3, $i_z^*(U) = i^*$ if $f_z^*(U) = 1\ \forall z$. ■

Lemma 4 $\exists\widetilde{U} \in (\underline{U}, U^{\max})$ *and some* $m > 0$ *s.t.*

$$f_z^*(U) = 0\ \forall z \text{ and } \forall U \geq \widetilde{U} \text{ and}$$
$$J(U)\begin{cases} = \underline{J} + m(U-\underline{U}) & \text{if } U \leq \widetilde{U}. \\ < \underline{J} + m(U-\underline{U}) & \text{if } U > \widetilde{U} \end{cases}$$

Proof. Step 1. *Consider two continuation values* $U' < U''$ *s.t.* $Ef_z^*(U') > 0$ *and* $Ef_z^*(U'') > 0$. *It follows given Lemma 3 that*

$$J(U) = J(U') + m(U-U')\ \ \forall U \in [U', U''] \tag{19}$$
$$\text{where } m = \frac{J(U'')-J(U')}{U''-U'}.$$

To see why, let $U^{W*}(U)$ *correspond to the expected continuation value to the agent conditional on* $f_z = 1$ *and let* $U^{P*}(U)$ *correspond to the expected continuation value to the agent conditional on* $f_z = 0$. *Optimality and the concavity of* $J(\cdot)$ *thus require*

$$J(U) = J\left(U^{W*}(U)\right) Ef_z^*(U) + J\left(U^{P*}(U)\right)(1 - Ef_z^*(U)). \tag{20}$$

By (20) *and the concavity of* $J(\cdot)$, *it follows that* $U^{W*}(U)$ *and* $U^{P*}(U)$ *are on the same line segment in* $J(\cdot)$ *for a given* U. *By the concavity of* $J(\cdot)$, *one can choose* $\forall z$, $U_z^{F*}\left(U^{W*}(U)\right) = \frac{U^{W*}(U) - w_a + g(i^*)}{\beta} \geq U^{W*}(U)$ *which is weak if* $i^* > 0$, *so that*

$$\frac{J\left(U^{W*}(U'')\right) - J\left(U^{W*}(U')\right)}{U^{W*}(U'') - U^{W*}(U')} = \frac{\left(J\left(\frac{U^{W*}(U'') - w_a + g(i^*)}{\beta}\right) - J\left(\frac{U^{W*}(U') - w_a + g(i^*)}{\beta}\right)\right)}{\frac{U^{W*}(U'') - U^{W*}(U')}{\beta}}. \tag{21}$$

By the concavity of $J(\cdot)$, *this implies* $U^{W*}(U'')$ *and* $U^{W*}(U')$ *are on the same line segment. Therefore,* (19) *applies.* ***Step 2.*** *Since* $Ef_z^*(\underline{U}) = 1$ *by step 3 of the proof of Lemma 2, it follows from step 1 that* (19) *applies for* $U' = \underline{U}$ *and some* $U'' = \widetilde{U} \geq \underline{U}$. *It follows that* $f_z^*(U) = 0$ $\forall z$ *and* $\forall U \geq \widetilde{U}$ *if* $\widetilde{U} > \underline{U}$ *and* $f_z^*(U) = 0$ $\forall z$ *and* $\forall U > \widetilde{U}$ *if* $\widetilde{U} = \underline{U}$. ***Step 3.*** *If* $\widetilde{U} = \underline{U}$, *then* $Ef_z^*(\underline{U}) = 0$ $\forall U > \underline{U}$, *but this is not possible since* (2) *and* (6) *imply that* $EU_z^{L*}(U) < U$ *and cannot be arbitrarily close to* U. *Therefore* $m > 0$. ***Step 4.*** *It cannot be that* $\widetilde{U} = U^{\max}$ *since this violates part 2 of Lemma 2.* ■

Lemma 5 $\widetilde{U} = \overline{U}$.

Proof. Step 1. $e_z^*(U) = \eta$ *if* $f_z^*(U) = 0$ *and* $U \in \left[\underline{U}, \widetilde{U}\right]$. *Suppose this is not the case and consider a solution for which* $e_z^*(U) = 0$ *and* $f_z^*(U) = 0$. *Because the constraint set is convex, one can perturb this solution without changing welfare so that* (6) *binds and* $U_z^{L*}(U) = U_z^{H*}(U)$. *However, this implies that* $U_z^{L*}(U) < -e_z^*(U) + \beta U_z^{L*}(U)$. *Because optimality given the concavity of* $J(\cdot)$ *requires* $-e_z^*(U) + \beta U_z^{L*}(U) \in \left[\underline{U}, \widetilde{U}\right]$, *this means given Lemma 4 that* $-\pi_a(0)\chi + \beta J\left(U_z^{L*}(U)\right) < \underline{J}$, *which violates* (3). ***Step 2.*** *Suppose* $\overline{U} < \widetilde{U}$. *By Assumption 3, there exists a solution to* (1) $-$ (7) *s.t.* $f_z^*(U) = 0$ *and* $e_z^*(U) = \eta$ $\forall z$ *and* $\forall U \in \left[\overline{U}, \widetilde{U}\right]$. *Moreover, given the concavity of the program and convexity of the constraint set in* (1) $-$ (7) *such a solution can feature* $U_z^{H*}(U) = U^{H*}(U)$ *and* $U_z^{L*}(U) = U^{L*}(U)$ $\forall z$. *This implies that*

$$m = \frac{J\left(\widetilde{U}\right) - J\left(\overline{U}\right)}{\widetilde{U} - \overline{U}} = (1 - \pi_a(\eta)) \frac{J\left(U^H\left(\widetilde{U}\right)\right) - J\left(U^H\left(\overline{U}\right)\right)}{U^H\left(\widetilde{U}\right) - U^H\left(\overline{U}\right)} + \pi_a(\eta)\, m,$$

but since $U^H\left(\widetilde{U}\right) > \widetilde{U}$, *this violates Lemma 4.* ***Step 3.*** *Suppose* $\overline{U} > \widetilde{U}$ *so that by Lemma 4,* $J\left(\widetilde{U}+\epsilon\right) < \underline{J} + m\left(U - \underline{U}\right)$ *for* $\epsilon > 0$ *arbitrarily small. Consider a perturbation which sets* $e_z^*\left(\widetilde{U}+\epsilon\right) = \eta$ *and lets* (6) *bind so that* $U_z^{L*}\left(\widetilde{U}+\epsilon\right) < U_z^{H*}\left(\widetilde{U}+\epsilon\right) < \widetilde{U}\ \forall z$. *This perturbation yields a payoff to the principal equal to* $\underline{J} + m\left(\widetilde{U}+\epsilon - \underline{U}\right)$, *violating the definition of* $\widetilde{U}$ *in Lemma 4.* ■

7.4 Proof of Proposition 1

Step 1. We begin by characterizing the solution for $U \in \left[\underline{U}, \overline{U}\right]$ to prove the second part of the proposition and having done this we prove the first part of the proposition. By steps 4 and 5 of the proof of Lemma 2 and by Lemma 4, the solution which satisfies the *Bang-Bang* property is characterized by a probability $Ef_z^*(U) = \left(\overline{U} - U\right) / \left(\overline{U} - \underline{U}\right)$, where

$$\begin{aligned} \underline{U} &= E\left\{w_a - g\left(i_z^*(U)\right) + \beta U_z^{F*}(U) \,|\, f_z^*(U) = 1\right\} \\ \overline{U} &= E\left\{-e_z^*(U) + \beta\left(\left(1 - \pi_a\left(e_z^*(U)\right)\right) U_z^{H*}(U) + \pi_a\left(e_z^*(U)\, U_z^{L*}(U)\right)\right) |\, f_z^*(U) = 0\right\}, \end{aligned}$$

and the analogous expected continuation values for the principal are $J\left(\overline{U}\right)$ and $J\left(\underline{U}\right) = \underline{J}$, respectively. Therefore, one only needs to characterize $i_z^*\left(\underline{U}\right)$, $e_z^*\left(\overline{U}\right)$, $U_z^{F*}\left(\underline{U}\right)$, $U_z^{H*}\left(\overline{U}\right)$, and $U_z^{L*}\left(\overline{U}\right)$ to achieve an full description of equilibrium actions. **Step 2.** By Lemma 3 $i_z^*\left(\underline{U}\right) = i^*\ \forall z$. By step 2 of the proof of Lemma 5, $e_z^*\left(\overline{U}\right) = \eta\ \forall z$. **Step 3.** By Lemmas 4 and 5 $U_z^{H*}\left(\overline{U}\right) = \overline{U}$ and $U_z^{L*}(U) = \overline{U} - \eta / \left(\beta\left(\pi_a(0) - \pi_a(\eta)\right)\right) < \overline{U}$ since otherwise (6) does not bind and a perturbation which reduces U_z^H and raises U_z^L strictly raises welfare. **Step 4.** The fact that $U_z^{F*}\left(\underline{U}\right) = \left(\underline{U} - w_a + g\left(i^*\right)\right)/\beta\ \forall z$ is implied by (2) and the fact that (5) binds since otherwise the principal is receiving a continuation value above $\underline{J}$. **Step 5.** We are left to characterize i^* and $\underline{U}$. Note that the equilibrium can be represented by a system of 4 equations: (10) – (12) and

$$\underline{J} \leq -\pi_p \chi - Ai^* + \beta\left(\left(1 - d^*\right) J\left(\overline{U}\right) + d^* \underline{J}\right). \tag{22}$$

(22) is an equality if $d^* \geq 0$ which occurs if $\left(\underline{U} - w_a + g\left(i^*\right)\right)/\beta \leq \overline{U}$, where we have taken Lemma 4 into account. (10) – (12) and (22) represent a system of 4 equations and 5 unknowns: $J\left(\overline{U}\right)$, $\underline{U}$, l^*, i^*, and d^*, where the fifth unknown is pinned down by the fact that these variables are chosen to maximize $J\left(\overline{U}\right)$. Note that given steps 1-4, $d^* < 1$ and

$l^* \in (0,1)$ so that by algebraic substitution, it is the case that

$$J\left(\overline{U}\right)(1-\beta) \leq \frac{-\frac{\pi_a(0)}{\pi_a(0)-\pi_a(\eta)}\eta - w_a + g(i^*)}{-\eta - w_a + g(i^*)}\left((\pi_p - \pi_a(\eta))\chi + Ai^*\right) - (\pi_p \chi + Ai^*), \tag{23}$$

which is an equality if and only if (22) is an equality. **Step 6.** Note that i^* which satisfies (9) maximizes the right hand side of (23). Moreover, by Assumption 3, it is the case in the optimum that (22) binds since the implied value of d^* exceeds 0 so that $U_z^{F*}(\underline{U}) = (\underline{U} - w_a + g(i^*))/\beta \leq \overline{U}$. Substitution into (11) yields $\underline{U}$ which completes the proof of the second part. **Step 7.** Lemmas 4 and 5 imply that if $\Pr\left\{U_t \geq \overline{U}\ \forall t\right\} > 0$, then $\Pr\{f_t = 0\ \forall t\} > 0$. However, (2) and (6) imply that $\Pr\{U_{t+1} < U_t - \epsilon | f_t = 0\} > 0\ \forall t$ for some $\epsilon > 0$, which means that $\Pr\left\{U_t \geq \overline{U}\ \forall t\right\} = 0$. **Step 8.** $\Pr\left\{U_{t+1} \leq \overline{U} | U_t \leq \overline{U}\right\} = 1$ by steps 3 and 6, so that by step 7, $\lim_{t\to\infty} \Pr\left\{U_t \leq \overline{U}\right\} = 1\ \forall U_0$. **Q.E.D.**

7.5 Proof of Proposition 2

Step 1. An equilibrium with the given structure satisfies $(10)-(13)$ and entails functions $l(i)$ and $d(i)$ defined by:

$$\begin{aligned} 1 - \pi_a(\eta)\, l(i) &= \frac{\gamma_a + \beta\gamma_p - 1}{\beta\left(\gamma_a + \gamma_p - 1\right)} \\ d(i) &= \frac{\gamma_p + \beta\gamma_a - 1}{\beta\left(\gamma_a + \gamma_p - 1\right)} \end{aligned}$$

for

$$\gamma_a = \frac{-\frac{\pi_a(0)}{\pi_a(0)-\pi_a(\eta)}\eta - w_a + g(i)}{-\eta - w_a + g(i)} \tag{24}$$

$$\gamma_p = \frac{(\pi_p - \pi_a(\eta))\chi}{(\pi_p - \pi_a(\eta))\chi + Ai} \tag{25}$$

where Assumption 3 and the fact that $\beta < 1$ implies $\gamma_a \in [0,1]$, $\gamma_p \in [0,1]$, and $\gamma_a + \gamma_p - 1 > 0$ for all $i \leq \bar{i}$, where $\bar{i} > i^*$. **Step 2.** By some algebra, $l'(i)$ has the same sign as $-\gamma_p \partial\gamma_a/\partial i - (1-\gamma_a)\partial\gamma_p/\partial i$ which equals

$$-\gamma_p(1-\gamma_a)\left(\frac{g'(i)}{-\eta - w_a + g(i)} - \frac{A}{(\pi_p - \pi_a(\eta))\chi + Ai}\right). \tag{26}$$

Since $g(\cdot)$ is concave, it follows that (26) is negative if $i < i^*$ and positive if $i > i^*$. **Step 3.** By some algebra, $d'(i)$ has the same sign as $\left(1-\gamma_p\right)\partial\gamma_a/\partial i + \gamma_a\partial\gamma_p/\partial i$ which equals

$$-\frac{A}{[-\eta-(\omega_a-g(i))]\left[(\pi_a(\eta)-\pi_p)\chi-Ai\right]}\times \\ \left[ig'(i)(1-\gamma_a)-\left[\frac{-\eta\pi_a(0)}{\pi_a(0)-\pi_a(\eta)}-(\omega_a-g(i))\right]\gamma_p\right] \tag{27}$$

The element outside the square brackets is always positive. Consider $i < i^*$, where the concavity of $g(\cdot)$ guarantees that

$$-g'(i)\left[(\pi_a(\eta)-\pi_p)\chi-Ai\right]-A(-\eta-(\omega_a-g(i)))>0. \tag{28}$$

By some algebra, one can show that given (28), the element inside the square brackets in (27) is decreasing in i for $i < i^*$. Since $d(0) = 1$, it follows that $d'(0) \leq 0$, so this fact implies that $d'(i) < 0$ for $i < i^*$. Consider $i \geq i^*$. By rearranging terms, $\left(1-\gamma_p\right)\partial\gamma_a/\partial i + \gamma_a\partial\gamma_p/\partial i$ can also be expressed as

$$\left(1-\gamma_p-\gamma_a\right)\frac{g'(i)}{-\eta-(\omega_a-g(i))}+\gamma_a\gamma_p\left[\frac{g'(i)}{-\eta-(\omega_a-g(i))}+\frac{A}{(\pi_a(\eta)-\pi_p)\chi-Ai}\right],$$

which is negative for $i \geq i^*$ since the left hand side of (28) is weakly negative in this case. **Q.E.D.**

7.6 Proof of Proposition 3

Step 1. Implicit differentiation of (9) taking into account the concavity of $g(\cdot)$ yields the comparative statics with respect to i^*. **Step 2.** Given γ_a and γ_p defined in the proof of Proposition 2, it is the case that if a particular parameter $x = \{A, \chi, \eta\}$ changes, the effect on l^* has the same sign as

$$-\gamma_p\frac{\partial\gamma_a}{\partial x}-(1-\gamma_a)\frac{\partial\gamma_p}{\partial x}, \tag{29}$$

where we have used the fact that $-\gamma_p\partial\gamma_a/\partial i-(1-\gamma_a)\partial\gamma_p/\partial i = 0$ at i^*. The effect on d^* has the same sign as

$$\left(\left(1-\gamma_p\right)\frac{\partial\gamma_a}{\partial x}+\gamma_a\frac{\partial\gamma_p}{\partial x}\right)+\left(\left(1-\gamma_p\right)\frac{\partial\gamma_a}{\partial i}+\gamma_a\frac{\partial\gamma_p}{\partial i}\right)\frac{\partial i}{\partial x}. \tag{30}$$

Step 3. Given (29), the comparative statics with respect to l^* are implied by the fact that $\partial\gamma_a/\partial A = \partial\gamma_a/\partial\chi = 0$, $\partial\gamma_a/\partial\eta < 0$, $\partial\gamma_p/\partial A < 0$, $\partial\gamma_p/\partial\chi > 0$, and $\partial\gamma_p/\partial\eta = 0$. **Step 4.** Given (30), the effect of an increase in η on d^* is implied by the fact that $\partial\gamma_a/\partial\eta < 0$, $\partial\gamma_p/\partial\eta = 0$, $\partial i/\partial\eta > 0$, and $\left(1-\gamma_p\right)\partial\gamma_a/\partial i + \gamma_a\partial\gamma_p/\partial i < 0$ from Proposition 2. Letting $x = A$, substitution into (30) taking Assumption 4 into account together with $\partial i/\partial A = \gamma_p g'(i)/Ag''(i)$ yields:

$$\left(\frac{\gamma_a\gamma_p - \left(1-\gamma_p\right)\left(1-\gamma_a\right)}{\gamma_p}\right)\left(\frac{\partial\gamma_p}{\partial i}\right)\left(-\gamma_p\frac{i}{A}\frac{1}{(1-\theta)}\right) + \gamma_a\frac{\partial\gamma_p}{\partial i}\frac{i}{A}$$

which has the same sign as $\theta\gamma_a + \gamma_p - 1$, which is unambiguously positive given the definition of i^*. Analogous arguments imply the comparative static with respect to χ. **Q.E.D.**

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