Revealed Political Power

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Abstract

How much bias in a political system can be inferred from a given set of policy data? This paper adopts a "revealed preference" approach to the question. We examine a dynamic policy environment in which individuals differ by income each period. Long run preference profiles are unobserved to an outside observer but are known to belong to a well behaved class in which citizens' preferences are ordered by income in each state. Policy data is summarized by a Markov policy rule. The observer makes inferences about the underlying distribution of political power as if political power were derived from a wealth-weighted voting system with weights that can vary across states. Positive weights on relative income in any period indicate an "elitist" bias in the political system whereas negative weights indicate a "populist" one.

We ask: what class of weighted systems can rationalize a given policy rule as a weighted-majority outcome each period? Our first result shows that without further knowledge, all forms of bias are possible: *any* Markov policy rule can be shown to be rationalized by *any* system of wealth-weighted voting. An additional single crossing restriction on preferences can, however, rule out certain weighting systems. We then show that when polling data augments policy data, the wealth-weights are shown to be bounded above and below. In some cases, polls can provide information about the change in political inequality across time.

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1 Introduction

The principle of *political equality* is widely accepted as a governing philosophy in most democracies. According to the principle, all individuals, regardless of income or background are to be endowed with the same political power or influence. On paper, electoral processes in most democracies satisfy some rough form of it, often taking the form of "one-man-one-vote" electoral systems. Examples include Winner-take-all Presidential elections (in the U.S. and Latin America) and Proportional Representation in Parliamentary elections (e.g., Western Europe).[1]

It is unlikely, however, that the de facto distribution of power in these countries is equal. There is anecdotal evidence, and some systematic evidence, that wealth matters in the political process. Recent work, for instance, by Benabou (2000) and Campante (2008) both document some form of pro-wealth bias in the U.S. Benabou presents patterns of the political participation, including rates of voter turnout, contribution, and influence activities among various groups in the income distribution. He finds that the propensity to participate in every reported form of political activities rises with income. Using the campaign contribution data in the 2000 US presidential election, Campante identifies that inequality increases the share of contributions coming from relatively wealthy individuals.

These studies suggest that, putting aside the formal, *de jure* features of a political system, the *de facto* allocation of power is such that richer individuals have a disproportionate influence in the policy process. The result is that policies enacted appear to favor wealthier rather than poorer individuals. Consequently, economic inequality apparently produces political inequality to some degree.

A few theories have been proposed recently to explain why this might be so. One prominent theory links bias to differential participation rates among the rich and poor. Examples include Benabou (2000) and Bourguignon and Verdier (2000). In these models, the poor vote less frequently, the effect being that wealthier voters have a disproportionate influence on policy. A second type theory concerns the effect of campaign contributions, for instance, Austen-Smith (1987), Grossman and Helpman (1996), Prat (2002), Coate (2004), Campante (2008), etc. In these models, the money either "buys" influence directly or it affects policy indirectly by changing the electoral odds toward candidates ideologically predisposed toward the rich. Because contributions skew toward the wealthy, policies are biased in their favor. Finally, a third type of theory centers on disenfranchising investments, e.g., Acemoglu and Robinson (2008), made by a wealthy elite in order to disinherit the poor from the political process.

Rather than put forth another explanation for bias, the present paper takes a step back by asking whether and how bias can be identified from policy data. In particular, what distributions of political power could possibly be consistent with the observed policies? Can

[1] Clearly, there are well known exceptions. In the U.S. representation in the Senate is equal across states, so that voters in small states have disproportionate political power in that governing body.

an egalitarian distributions of power based on "one-man-one-vote" be ruled out?

To address these questions we envision an "outside observer" who attempts to infer something about the underlying distribution of political power from the observable attributes of the economy. In this and other respects, we draw some obvious parallels to Revealed Preference Theory dating back to Afriat's (1967) influential work. Afriat examines whether some observed consumption data is consistent with utility maximization by a budget-constrained consumer.[2] The "consistency" in our case is whether a collection of policy data can result from equilibrium of a voting process. This imposes an extra degree of difficulty since we suppose that elements of both the political system and the underlying preferences of voters are latent. In proposing a construction well-suited for this environment, we make inroads to the testability of the voting theory, hence both expand and complement the existing testability theory.

We also assume that both the explicit policy data and implicit decision process are dynamic. This puts the present work closer to that of Boldrin and Montrucchio (1986) who examine whether, in a dynamic capital accumulation model, a given policy rule could have been rationalized by a single dynamically-consistent decision maker.

The present work can be loosely viewed as the political economy analogue of Boldrin and Montrucchio. We posit a simple dynamic policy environment consisting of a continuum of citizens or citizen-types, each type differentiated by income each period. The income distribution in period t is summarized by a state variable ω_t (e.g., public capital), and next period's state evolves from the current state and policy choice a_t. Each type's income grows as the state increases, but overall inequality may increase or decrease.

All these tangible attributes of the economy are observed by or known to the outside observer. The observer does *not* observe the preference profile. Instead, he knows only that long run preferences over policies each period belong to the class of single peaked preferences satisfying a single crossing condition. Each preference from this class is well ordered to that an individual's preferred policy is increasing in income.

Given this structure, the observer is a witness to the time series of states and policies. While the time horizon is infinite, we restrict attention to series in which every state occurs at least once over some finite time span. This restriction could easily be relaxed with the result that the outsider's inference about political bias will be generally weaker.

More significantly, we restrict attention to data consistent with a Markov rule of the form $\Psi(\omega_t) = a_t$. The Markov assumption clearly rules out many data series of interest. Nevertheless, it seems reasonable for large, anonymous polities in which reputation and other history-dependent enforcement mechanisms do not arise.

[2]See also Varian (1982) and Chiappori and Rochet (1987). See Brown and Matzkin (1996) for applications to general equilibrium theory, and Boldrin and Montrucchio (1986) for applications to dynamic decision theory.

We then ask whether there exist sequences of distributions of political power that rationalize the observed Markov data as majority-winning outcomes of a voting process. Specifically, consider a weighted voting rule where the weights are attached to one's income and can vary with the state, independent of the income-generating process. The weights can be positive, indicating a pro-wealth bias. In this case, a wealthy individual's vote is worth more than a poorer one. We refer to this case as an *elitist bias*. Though one might normally expect an elitist bias to prevail, we do not rule out the case where the weights can be negative, indicating a anti-wealth bias. We refer to the case of negative weights as a *populist bias*. Under a populist bias, the poorer individual's vote is worth more than a richer one.

More generally, an increase in the wealth-weight works in favor of the wealthy, while a decrease works in favor of the poor. The case where the weights are exactly zero corresponds to the standard system of "one-man-one vote" or equal representation. We refer to this as the *unbiased* system.[3]

In all cases, the system of wealth-weights corresponds to an implied distribution of political power. A policy rule Ψ is a *Weighted majority winner (WMW)* of the wealth-weighted voting rule if in each state, $\Psi(\omega)$ wins in a weighted majority vote against any alternative. The system of wealth-weighted voting is then said to *rationalize the policy rule* Ψ *in a given class of payoff profiles* if there exists a profile in the class under which Ψ is a weighted majority winner (WMW).

The main results address two types of questions. First, what type of wealth bias (if any) can rationalize a given policy rule? Second, what type of wealth bias (if any) *cannot* rationalize a given policy rule? Our first result addresses both questions. We show that *any* Markov policy rule Ψ can be rationalized by *any* wealth bias. The proof adapts a construction by Boldrin and Montrucchio (1986) to the present dynamic voting environment with heterogeneous voters. It demonstrates that without further structure on preferences or additional data, the policy data alone is not very discerning; the data is consistent with every type of income-weighted bias.

The observer's external information can then be augmented by allowing the observer access to polling data. Polls provide data on specific aggregate binary orderings between benchmark policies — typically those that are being considered in the political process. Two simple polls are analyzed in each state. The wealth-weighted majority winner $\Psi(\omega)$ is compared two alternatives, a policy located to its right, and one located to its left.

We characterize the both sufficient, and some necessary conditions on the wealth weights in each state. Regarding the latter, it turns out that fairly minimal amounts of polling data can nevertheless provide clear restrictions on the bias in each state. Upper and lower bounds are characterized in each state. An upper bound represents a maximal degree of positive wealth-bias — the largest possible bias in favor of wealthy individuals. The lower bound

[3]Though there may be other reasonable notions of unbiased, the present definition seems natural to us and is very much in line with that of the literature, including Benabou (2000) and Campante (2008).

represents the lowest possible bias. Together, these bounds define an admissible band of bias weights that can rationalize the policy data.

Other things equal, the larger is income inequality in a given state, the narrower is this band. This is not surprising when bias band implies an elitist system. In that case, the pro-wealth bias must be lower to have off-set the greater income inequality. However, the result holds even when the band implies a populist system. There, the bias may be larger (more favorable to the wealthy), despite the greater income inequality that already benefits richer individuals. The reason is that with a populist system, political inequality is a weighted mirror image of income inequality. Hence, an increase in relative income of the top 10% translates into an weighted decrease in their political power. Hence, the bias must increase to offset this fall in political power due to income change.

Finally, we later use information specific to the dynamics of the system. Period-by-period changes in poll data are used to examine whether political power to the wealthy increases or decreases over time.

The paper is organized as follows. Section 2 lays out the basic dynamic framework. There we describe an implied voting process with latent, exponential weights. Section 3 describes the first result showing that any weighted system can rationalize any policy rule. We also show, however, that a simple supermodularity requirement on preferences can imply some restriction on the bias. Section 4 examines the implications of polling data. When coupled with policy data, polls imply significant restrictions on the bias. Section 5 concludes with a discussion of the results and suggestions for future work.

2 The Model

Notationally, functions said to be "increasing" in their arguments are increasing the weak sense (includes equality). Otherwise, we refer to such functions as "strictly increasing".

2.1 The Physical Environment

An infinite horizon economy is populated by a continuum of $I = [0, 1]$ of *citizen-types*. A citizen-types is an index that orders individuals by income, with higher types accorded higher income. A citizen of type $i \in I$ holds income $y(i, \omega_t)$ in period t that depends on the value of an aggregate state variable ω_t. For concreteness, this state can be interpreted as an economy-wide public capital stock, such as public infrastructure. The set of possible states is denoted by Ω with Ω assumed to be a finite subset of $\mathbb{R}$. The function y is assumed to be continuous in i, strictly increasing in each of its arguments, and has strict monotone differences in i and

ω.[4] Finally, assume that $y(0, \omega) > 0$.

These assumptions imply the following. Higher citizen types are wealthier; everyone's income is increasing in the state; and by the Monotone differences property, Lorenz income inequality can be ranked unambiguously across any two states. Formally, the standard Lorenz curve, defined by

$$L(j, \omega) \;=\; \frac{\int_0^j y(i, \omega) di}{\int_0^1 y(i, \omega) di}, \tag{1}$$

describes the proportion of income held by the lowest $j\%$ in state ω. Monotone differences then implies either $L(j, \omega) \geq L(j, \hat{\omega})$ for all j, or $L(j, \omega) \leq L(j, \hat{\omega})$ for all j.

Each period the state (e.g., public infrastructure) can be augmented by a public investment denoted by a_t. We refer to a_t as the *policy choice*. Assume $a_t \in A$ with A a compact interval in $\mathbb{R}$. The state evolves according to a simple Markov transition technology whereby next period's state is determined by $\omega_{t+1} = Q(\omega_t, a_t)$. The transition function Q is strictly increasing in both arguments.

Putting these attributes together, the physical environment is summarized by following list (I, Ω, A, Q, y) and remains fixed for the rest of the analysis.

2.2 Policy Data

To the outside observer (as well the participants themselves), the state ω_t and policy a_t are observed in each period t. We limit the analysis to data that satisfy a Markov data property. Namely, the sequences $\{\omega_0, \omega_1, \ldots\}$ and $\{a_0, a_2, \ldots\}$ are such that every state occurs at least once in the state path, i.e., for all $\omega \in \Omega$, there exists t such that $\omega = \omega_t$, and for any two dates t and s, if $\omega_t = \omega_s$, then $a_t = a_s$. Data that satisfy this requirement consist of states and policies that policies vary only in the state variable, and all states (but not necessarily all policies) occur on the realized path.

The Markov data property comes at the price of some loss of generality. However, it has the benefit of admitting a tractable characterization of the data. Policy data that satisfy the Markov data property are fully characterized by an initial state ω_0 and a Markov policy function, $\Psi : \Omega \to A$. Given a state ω_t, the chosen policy is therefore given by

$$\Psi(\omega_t) \;=\; a_t.$$

Note also that if some states were never observed then it would be easier to show that the observations are consistent with any given type of political bias because an extra degree of freedom exists in the way that "off-path" behavior can be specified.

[4] y has monotone differences if for any pair of states, ω and $\hat{\omega}$, the difference $y(i, \omega) - y(i, \hat{\omega})$ is either strictly increasing or strictly decreasing in i.

2.3 Preferences

In revealed preference theory, the standard interpretation is that of an economy from the viewpoint of an outside observer. The outside observer knows something about the nonparametric structure of citizens' preference profiles, but would not know their precise, parametric form. That view is adopted here as well.

Specifically, preferences are assumed to be represented by a function, $U(i, \omega_t, a_t; \Psi)$ denoting the long run payoff to a citizen-type i of policy a_t in state ω_t under a policy rule Ψ there determines future policies. The precise form of function U is not known to the outside observer. However, U is known to belong to a set $\mathcal{U}$ of payoff functions satisfying:

(A1) U is continuous in the index i, and strictly concave in its a_tth argument when the policy space is extended to the entire real line.

(A2) U is satisfies the single crossing property in (i, a): for all $a > \hat{a}$,

$$U(i, \omega_t, a; \Psi) - U(i, \omega_t, \hat{a}; \Psi) > 0 \text{ implies } U(j, \omega_t, a; \Psi) - U(j, \omega_t, \hat{a}; \Psi) > 0 \;\; \forall j > i.$$

(A3) U is time separable and expressed as a geometrically discounted sum of flow payoffs of the form $u(\omega_t, y_t, a_t)$ so that i's long run payoff is given by

$$U(i, \omega_t, a_t; \Psi) \equiv u(\omega_t, y(i, \omega_t), a_t) + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u(\omega_\tau, y(i, \omega_\tau), \Psi(\omega_\tau)) \tag{2}$$

where all future states follow the transition law $\omega_{\tau+1} = Q(\omega_\tau, \Psi(\omega_\tau))$.

Note that the single crossing property implies that in every state, wealthier citizens always prefer larger policies than poorer citizens.[5] Note also that flow payoffs do not depend directly on one's type; all types have the same underlying preferences, and consequently heterogeneity comes exclusively from differences in income.

Though these assumptions are fairly standard in political economy, they may appear restrictive to a revealed preference theorist. It is useful to remember, however, that equilibrium existence is not an issue since Ψ is *already given* to the outside observer. More importantly, as a general rule the larger the set of feasible preference orderings, the easier it is to find one that "works" in the sense that a voting system can produce Ψ under such preferences. The narrower the class of preferences the the more difficult it is for a particular voting system to have generated the data. Hence, narrower classes of preferences give stronger possibility results (and weaker impossibility results), all else equal.

[5] Because policies have no specific interpretation, notions of "large" and "small" arbitrary. Hence, one could just as easily have assumed decreasing differences between i and a.

Clearly, we do require, and later verify, that the class of U satisfying (A1)-(A3) is nonempty. The theory would be vacuous if this were not the case. Let $\mathcal{U}$ denotes the collection of all payoff functions U satisfying (A1)-(A3). Henceforth, we refer to $\mathcal{U}$ as the class of *feasible preferences.*

3 Revealed Political Power

In this section the payoff function U is fixed and notationally suppressed where convenient. We attempt to identify the de facto distribution of political power from a class of parameterized distributions that all resemble, and have similar properties to, a standard Lorenz curve. Each "political Lorenz curve" in this class describes a proportion of political power held by the poorest $j\%$ of the population in each state. The interpretation is that of an implied, weighted vote. Power is measured by whether and how much of a weight would one have to give to income or wealth so that the observed policy is consistent with voting.

We proceed in two steps. First, we fix given state and define political bias as a weighting parameter that measures the departure from equal representation (political equality). Next, we allow this parameter to vary across states. In a polity with significant inertia, the weights on votes (though not necessarily the de facto distribution of power) will change very little across time.

3.1 Elitist versus Populist Bias

Time subscripts may be dropped for now, with ω representing a generic state. Define the share of political power allocated to citizen-type i in state ω by

$$\lambda(i, \omega, \alpha) = \frac{y(i, \omega)^{\alpha(\omega)}}{\int_0^1 y(j, \omega)^{\alpha(\omega)} dj} \tag{3}$$

Equation (3) describes a weighting system that determines the effective political power of each citizen-type. Because higher i-types have higher incomes, political power is increasing in income if $\alpha(\omega) > 0$, decreasing in income if $\alpha(\omega) < 0$, and invariant to income if $\alpha(\omega) = 0$. We allow that $\alpha(\omega)$ can take values in the entire real line.

This specification has a very simple interpretation. Consider policies are determined by some unspecified pairwise voting process. Each time a vote is taken, $\lambda(i, \omega, \alpha)$ is i's endowment of *vote share* in state ω. The exponent $\alpha(\omega)$ may then be thought of as the geometric weight attached to one's relative wealth, and $1 - \alpha(\omega)$ is the weight attached to equal vote share or *equal representation* in voting. To see this more transparently, write (3) as

$$\lambda(i, \omega, \alpha) \;=\; \frac{y(i, \omega)^{\alpha(\omega)} 1^{1-\alpha(\omega)}}{\int_0^1 y(j, \omega)^{\alpha(\omega)} 1^{1-\alpha(\omega)} dj}$$

When $\alpha(\omega) = 0$, the polity may be said to be *unbiased* in the sense that each person's vote, hence their political weight in the distribution, is the same. We will refer to the $\alpha(\omega) > 0$ case as an *elitist bias* since power accrues to the wealthy elite. For instance when $\alpha(\omega) = 1$ then an individual who possesses twice as much wealth as another has twice as many votes, hence twice as much political power.

The case of $\alpha(\omega) < 0$ is referred to as a *populist bias* since political power is redistributed away from wealth. In either case, the absolute value $|\alpha(\omega)|$ measures the intensity of the bias. The cases where $|\alpha(\omega)| > 1$ are particularly stark since this indicates a distribution of power that disproportionately rewards the fringes of the distribution. Extreme inequality occurs in the limit as $|\alpha(\omega)| \to \infty$.

As with income inequality, political inequality can be similarly by measured by a "Lorenz curve,"

$$L^P(j, \omega, \alpha) \;=\; \int_0^j \lambda(i, \omega, \alpha) di \tag{4}$$

which gives the proportion of *political power* held by the lowest $j\%$ of types in state ω.

Figure 1 displays the graph of L^P when $0 < \alpha(\omega) < 1$. Political inequality therefore lies somewhere between income inequality and full equality, depending on the wealth weight, $\alpha(\omega)$. From the assumptions on $y(\cdot)$, it is not difficult to show that, changes in the state imply unambiguous changes in political inequality just as with income inequality. Graphically this means that the Political Lorenz curves corresponding to two different states ω and ω' do not cross.

Figure 2 illustrates the case of a populist bias. When $\alpha(\omega) < 0$ the Political Lorenz curve is a "mirror image" of one with bias $-\alpha(\omega) > 0$. Most theories we are aware of predict an elitist bias if any. Nevertheless, it does not seem sensible to rule out the $\alpha(\omega) < 0$ case, a priori.

Notice also that political inequality, as measured by L^P, can change over time. It can change for two distinct reasons. First, changes in the state directly affect income inequality, and political inequality is a function of income inequality. Second, even if income inequality does not change, political inequality (can change because the wealth weight $\alpha(\omega)$ itself varies with changes in the state.

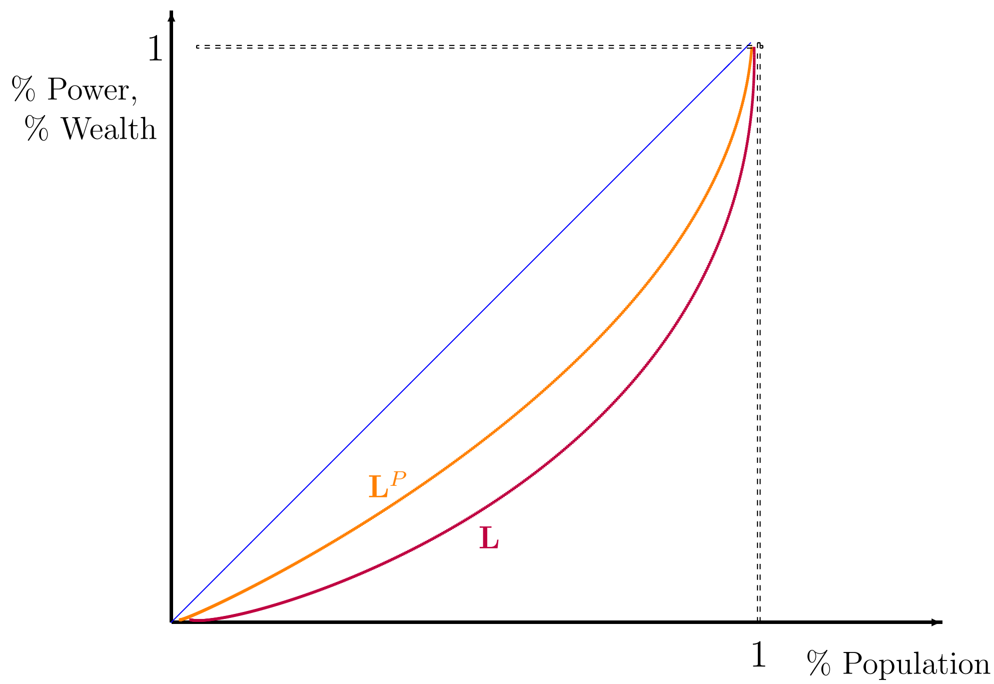

Figure 1: Political Lorenz Curve with an Elitist Bias

3.2 Rationalizing a Policy Rule

So far, political inequality has been implicitly associated with a weighted vote share. To make this idea operational, one must also describe how these vote shares are used. The special case of $\alpha(\omega) = 0$ illustrates this well. Each individual is allocated one vote in a simple majority vote between any pair of policies. A Majority Winning (sometimes referred to as Condorcet Winning) policy exists if that policy survives against all others in a simple majority vote. This idea can be extended to any weighted voting rule with $\alpha(\omega) \neq 0$.

We first introduce notation for voters who prefer one policy vis-a-vis another. For any policy rule Ψ, any state ω, and any arbitrary pair of policies a and $\hat{a}$, let

$$B(\omega, a, \hat{a};\ U, \Psi) \ \equiv\ \{i:\ U(i, \omega, a; \Psi) > U(i, \omega, \hat{a}; \Psi)\} \tag{5}$$

This notation will be used throughout the paper. Our equilibrium restriction is now stated in the following definition.

Definition 1 Given a policy rule Ψ, a policy a is an α-*Weighted Majority Winner (WMW)* in state ω under payoff function U if, for all policies $\hat{a}$,

$$\int_{B(\omega,a,\hat{a};\ U,\Psi)} \lambda(i, \omega, \alpha)\ di \ \ \geq\ \ 1/2$$

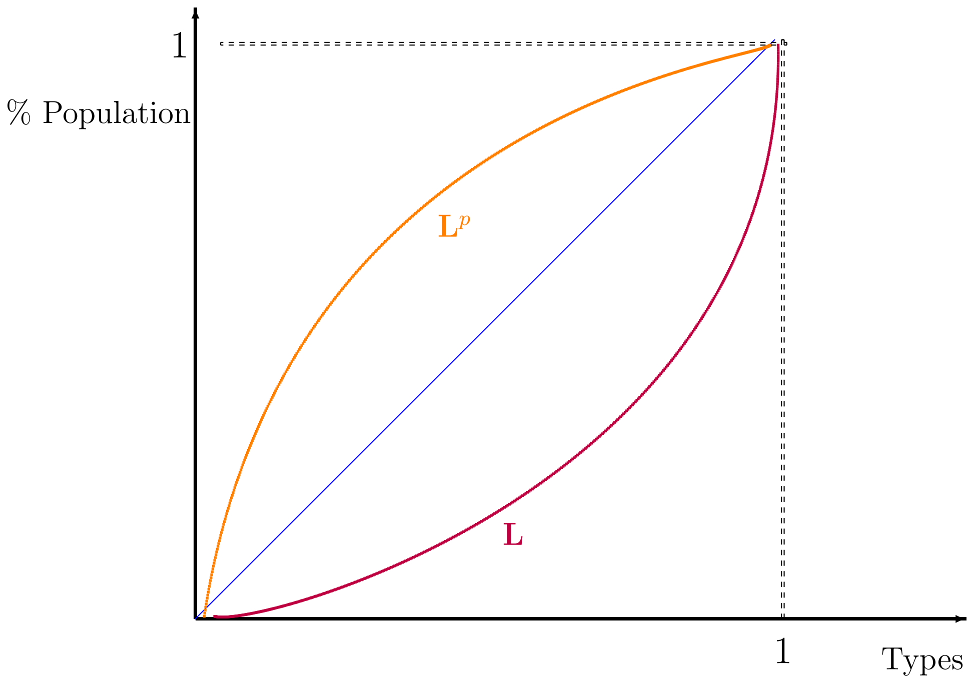

Figure 2: Political Lorenz Curve with Populist Bias

In other words, an α-weighted majority winner, or α-WMW, is a policy that survives against all others in a majority vote when each type i is allocated $\lambda(i, \omega, \alpha)$ votes and long run payoffs are given by U. In all the subsequent analysis, it will always be assumed that observed policy $\Psi(\omega)$ is a WMW under some weighting function α.

Definition 2 A weighting function α *rationalizes the policy rule* Ψ *in the feasible preference class* $\mathcal{U}$ if there exists a payoff function $U \in \mathcal{U}$ such that for all ω, $\Psi(\omega)$ is an α-weighted majority winner under U.

If the payoff function U were known precisely to the outside observer, then the political distribution of power that rationalizes Ψ is indeed be pinned down by Ψ. But because U is not known, it natural to ask whether a policy rule Ψ might be rationalized by the weighting function α for at least some U in a class preferences. This question is most germane when the class of preferences is defined by some natural or compelling properties. The point then is to understand when/whether policy data can be rationalized a preference profile with the requisite properties.

Theorem 1 *For any policy rule* Ψ*, every bias function* α *rationalizes* Ψ *in the feasible pref-*

erence class $\mathcal{U}$.

The proof is given below. According to Theorem 1, without specific information about preference orderings, the policy data does not tell us anything about political bias, whether it exists or whether its magnitude is large. Since, among all other α, the unbiased weight $\alpha(\omega) = 0 \ \forall \omega$ can also rationalize Ψ we cannot rule out the possibility of a completely unbiased polity.

Proof. First, notice that as direct consequence of the increasing difference property, for any policy rule Ψ, and any two policies a and $\hat{a}$ such that $a < \hat{a}$. Then the set $\{i : U(i, \omega, a; \Psi) \geq U(i, \omega, \hat{a}; \Psi)\}$ is either empty or of the form $[0, j]$ for some $0 \leq j \leq 1$. That is, the set of individuals who prefer the smaller policy always constitutes the poorer interval of citizen-types.

Define μ as the function for which $\mu(\omega)$ implicitly solves

$$L^P(\mu(\omega), \omega, \alpha) \ = \ \frac{1}{2}$$

For instance, when $\alpha(\omega) > 0$ see Figure 3.

Next, let U be a long run payoff function defined by

$$U(i, \omega, a) = -\ \frac{1}{2}\mu(\omega)[a^2 - (\Psi(\omega))^2] \ + \ i\Psi(\omega)[a - \Psi(\omega)]$$

We now verify that $U \in \mathcal{U}$.

First, observe that U is clearly concave in a and continuous in i. The first order condition for citizen $i = \mu(\omega)$ gives $\Psi(\omega) = a$ as the solution to max U. Moreover, U satisfies single crossing in (i, a), and so the preferences are ordered by citizen-types. Higher types (wealthier individuals) prefer higher policies, as required. It is easier to show then that these payoffs admit a weighted median voter — in this case, $i = \mu(\omega)$.

Given the assumptions on y, there is an inverse function $i = h(\omega, y)$ such that h is increasing in y and ω and has increasing differences in y and ω. To verify (A3), we now find the flow payoff u as the difference:

$$\begin{aligned} u(\omega, y, a) \ &= \ U(h(\omega, y), \omega, a) \ - \ \delta U(h(\omega, y), Q(\omega, a), \Psi(Q(\omega, a))\ ) \\ &= \ \ -\ \tfrac{1}{2}\mu(\omega)[a^2 - (\Psi(\omega))^2] \ + \ h(\omega, y)\Psi(\omega)[a - \Psi(\omega)] \end{aligned}$$

Hence $U \in \mathcal{U}$ and we conclude the proof. ∎

At this stage there are two possible ways one could rule out certain bias weights. First, one could add direct information about specific binary rankings. Such information could come,

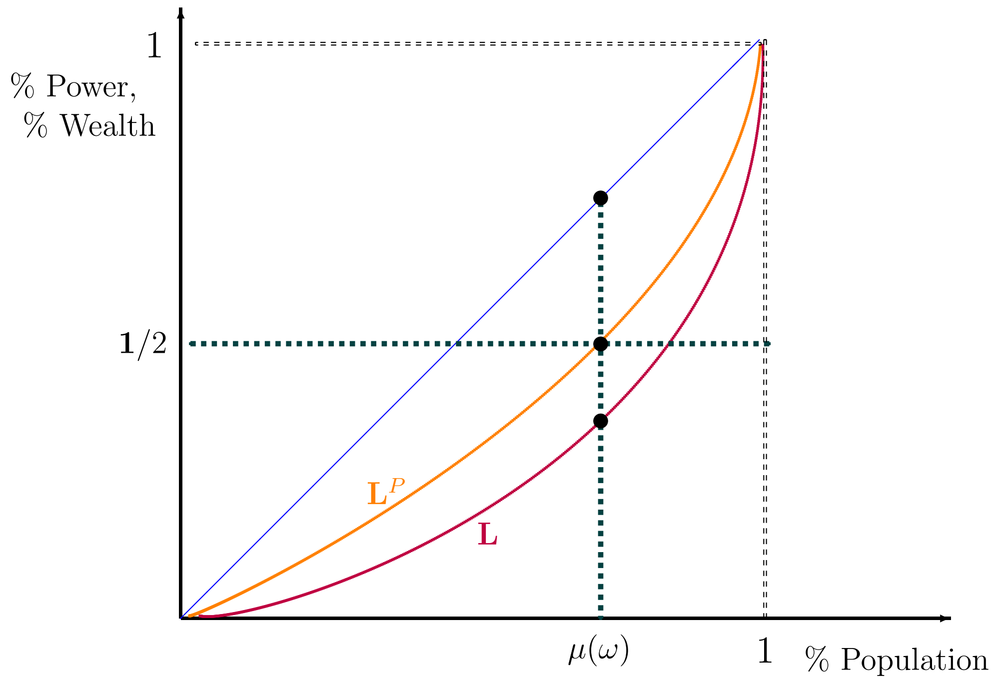

Figure 3: Identifying a Pivotal Voter under an Elitist Bias

for instance, from polls. We consider this option in the next Section. Before proceeding with that option, however, it is worth exploring a second option: that the class of preferences considered here is "too large." Is there a sensible subset $\mathcal{U}^* \subset \mathcal{U}$ such that there are α that *cannot* rationalize the policy data Ψ in $\mathcal{U}^*$?

One possible, though by no means only, requirement is that an individual's bliss rule is monotone, increasing in the value of the state. Formally, consider

(A4) U satisfies single crossing in the pair (ω, a) for each i.

Assumption (A4) implies that every type's most preferred policy is (weakly) increasing in the state. This monotonicity restriction is fairly natural when the policy is a complementary input in the production process. A standard example is investment in human capital. In growing economies citizens prefer greater investment.

Let $\mathcal{U}^* \subset \mathcal{U}$ be the set of payoff functions that satisfy (A1)-(A4). If $U \in \mathcal{U}^*$ and $\Psi(i, \omega_t)$ is policy that maximizes $U(i, \omega_t, a_t; \Psi)$, then it straightforward to show that $\Psi(i, \omega_t)$ is increasing in both i and ω_t. The following result shows that Theorem 1 does not hold in the more restricted class $\mathcal{U}^*$.

Theorem 2 *Let ω^1 and ω^2 be any two states with $\omega^2 > \omega^1$. Let α be any weighting system such that $i^2 \geq i^1$ and $L^p(i^1, \omega^1; \alpha) = 1/2 = L^p(i^2, \omega^2; \alpha)$. Then any policy rule Ψ such that $\Psi(\omega^2) < \Psi(\omega^1)$ cannot be rationalized by α in the preference class $\mathcal{U}^*$.*

An important special case of a weighting system satisfying the conditions of the Theorem is the unbiased benchmark, $\alpha(\omega) = 0$ for all ω. In that case, $i^2 = i^1 = 1/2$, the median voter who does not vary across states. The Theorem then implies that the unbiased weighting system cannot rationalize a Psi that is anywhere decreasing.

Proof. We proceed by contradiction. Let α, Ψ, ω^1, ω^2, and i^1 and i^2 be as hypothesized. Suppose α rationalizes Ψ in $\mathcal{U}^*$. Fix some $U \in \mathcal{U}^*$ and let

$$\tilde{\Psi}(i, \omega) \in \arg\max U(i, \omega, a;\ \Psi)$$

By (A4), $\tilde{\Psi}(i, \omega)$ is increasing in ω and in i. Notice that Since $\Psi(\omega^2) < \Psi(\omega^1)$ and $\Psi(\omega^\ell) = \tilde{\Psi}(i^\ell, \omega^\ell)$ for each of $\ell = 1, 2$. Consequently, we must have $i^2 < i^1$, contradicting the fact that $i^2 \geq i^1$ as hypothesized in the Theorem statement. Hence, α cannot rationalize Ψ. ∎

4 Revealed Political Power and the Power of Polls

Often, external information about policy preferences exist in the form of polls. This section examines how simple aggregate data from polls might reveal information about political bias.

Consider the following scenario. Each period t, a poll is taken in which citizens are asked to compare the actual policy choice $\Psi(\omega_t)$ to some small collection of fixed alternatives in the feasible policy set A. Typically, these alternatives are some much discussed policy alternatives, always on the table but not necessarily adopted (e.g., the "public option" in health care in the U.S).

We examine the case of two anonymous binary polls that ask individuals to rank $\Psi(\omega)$ against each of two alternatives, $\bar{a}$ and $\underline{a}$ such that $\underline{a} < \Psi(\omega) < \bar{a}$. Policy alternative $\underline{a}$ can be thought of as the "left wing" alternative to the chosen policy, $\bar{a}$ the "right-wing" alternative. For tractability, these polls are assumed to be accurate in the sense that the sampling error is ignored.

Since preferences are fully summarized by the state, the poll data may be summarized by a pair (p, q) of Markov functions that give fractions $p(\omega)$ and $q(\omega)$ of the population that weakly prefer the weighted-majority winner (WMW) $\Psi(\omega)$ to the alternatives $\bar{a}$ and $\underline{a}$, respectively, in each state ω.

Since underlying payoff function U that generates the poll data is itself unobservable, the poll data must be consistent with both U and the observable policy data Ψ. The following definition makes use of the notation defined in (5).

Definition 3 A weighting function α *jointly rationalizes a policy rule* Ψ *and poll data* (p, q) *in the feasible preference class* $\mathcal{U}$ if there exists a long run payoff function $U \in \mathcal{U}$ such that

(i) the policy rule $\Psi(\omega)$ is an α-weighted majority winner under payoff function U, and

(ii) U satisfies

$$\begin{aligned} p(\omega) &= |B(\omega, \Psi(\omega), \bar{a};\ U, \Psi)|, \quad \textit{and} \\ q(\omega) &= |B(\omega, \Psi(\omega), \underline{a};\ U, \Psi)| \end{aligned} \tag{6}$$

Part (i) is the equilibrium requirement as before. Part (ii) is a consistency requirement. The long run payoff function U must reflect preferences that can generate the observed poll data (p, q) given Ψ.

In the next subsection, we examine necessary conditions for political weights rationalizing policy rules and poll data.

4.1 Necessary Conditions for Rationalizing Policies

It is not difficult to verify from the single crossing property (A2) that, given poll data (p, q) and a state ω, the poorest $p(\omega)$% prefer $\Psi(\omega)$ to alternative $\bar{a}$, and the the richest $q(\omega)$% prefer $\Psi(\omega)$ to alternative $\underline{a}$. Formally,

$$\begin{aligned} B(\omega, \Psi(\omega), \bar{a};\ U, \Psi) &= [0, p(\omega)], \quad \textit{and} \\ B(\omega, \Psi(\omega), \underline{a};\ U, \Psi) &= [1 - q(\omega), 1] \end{aligned}$$

whenever these sets are nonempty. In fact, the assumption that Ψ must be a weighted-majority winner (WMW) implies certain restrictions on $p(\omega)$ and $q(\omega)$.

First, observe that

$$1 - q(\omega) < p(\omega) \tag{7}$$

If this were not the case then a nonempty interval $[p(\omega), 1 - q(\omega)]$ exists, consisting of types that weakly prefer both the smallest policy, $\underline{a}$, and the largest, $\bar{a}$, to $\Psi(\omega)$. This, in turn, is a contradiction of the strict concavity assumption (A1) on U.

Second, notice that for any $0 < p(\omega) < 1$, the bias weight α cannot be too large since otherwise the richest $(1 - p(\omega))$ fraction could have used its political influence to veto $\Psi(\omega)$, in which case $\Psi(\omega)$ could not have been a WMW. This puts an upper bound on on $\alpha(\omega)$. To find the upper bound, consider the "worst case" against $\Psi(\omega)$ in which the fraction $(p(\omega), 1]$ who prefer the right-wing alternative $\bar{a}$ is largest. For that to happen, $p(\omega)$ must be pivotal. The $\alpha(\omega)$ that makes $i = p(\omega)$ pivotal is given by the equation,

$$L^P(p(\omega), \omega, \alpha) = 1/2. \tag{8}$$

Since L^P is strictly decreasing in the weight $\alpha(\omega)$, the equation (8) implicitly defines a function $M : [0, 1] \times \Omega \to \mathbb{R}$ such that $\alpha(\omega) \leq M(p(\omega), \omega)$.

Third and finally, notice that for any $0 < q(\omega) < 1$, the bias cannot be too small. Otherwise, the poor could have vetoed the WMW, $\Psi(\omega)$, by opting for the left wing alternative $\underline{a}$. This then implies a lower bound on $\alpha(\omega)$. Using the same logic as before, the lower bound can be found by assuming that $1 - q(\omega)$ is pivotal. Hence, the lower bound can be calculated from the equation

$$1 - L^P(1 - q(\omega), \alpha(\omega)) \;=\; 1/2. \tag{9}$$

In fact, equation (9) defines the very same implicit function M as above. In this case, $\alpha(\omega) \geq M(1 - q(\omega), \omega)$.

The following result distills these three points.

Theorem 3 *Let Ψ be any policy rule for which $\underline{a} < \Psi(\omega) < \bar{a}$ for all ω and let (p, q) be the corresponding poll data relating $\Psi(\omega)$ to $\bar{a}$ and $\underline{a}$, resp. Then a weighting function α jointly rationalizes policy rule Ψ and poll data (p, q) in $\mathcal{U}$ only if for all states ω, $1 - q(\omega) < p(\omega)$, and*

$$M(1 - q(\omega), \omega) \;\leq\; \alpha(\omega) \;\leq\; M(p(\omega), \omega), \;\; \forall\, \omega$$

It is easily verified that M is strictly increasing in j, and so the condition $1 - q(\omega) < p(\omega)$ also implies that the interval of biases that can rationalize Ψ is nonempty. We refer to this interval $[M(1 - q(\omega), \omega), M(p(\omega), \omega)]$ as the *bias band.*

Figure 4 expresses a graph of M and the bias band. The bounds of the band are are displayed on the vertical axis. In the graph, the range of bias band includes 0, the unbiased weight. It also includes a subinterval of elitist biases, as well as a subinterval of populist ones. Given properties of M, some simple comparative statics facts can easily be discerned.

Corollary. *Fix ω, Ψ and poll data (p, q). Then*

(i) $M(1 - q(\omega), \omega) > 0$ whenever $q(\omega) < 1/2$ in which case the polity is known to express an elitist bias.

(ii) $M(p(\omega), \omega) < 0$ *whenever* $p(\omega) < 1/2$ *in which case the polity is known to express a populist bias.*

(iii) *The larger is income inequality then the smaller is* $|M(j, \omega)|$ *for any fixed* j*. In particular, if* 0 *(the unbiased weight) belongs to the band, then larger income inequality reduces the size of the band around 0.*

Other things equal, greater income inequality has an equalizing effect politically. When 0 is an admissible weight, then the band shrinks around it. If the band is entirely above 0 (elitism) then it moves closer to 0. This is not surprising since in that case, the pro-wealth bias must be lower to have off-set the greater income inequality. Somewhat more surprising is the fact that when the band is entirely below 0 (populism), greater inequality moves the band closer to 0 as well. In other words the band becomes less populist implying that wealthier individuals receive increased political weight from the bias in addition to increased weight from income alone. Why? Because with a populist system, political inequality is a weighted mirror image of income inequality. Hence, holding the bias weight constant, an increase in relative income of the top 10% translates into an weighted decrease in this group's political power. The bias weight must therefore increase to offset this fall in political power due to income change.

In addition to state-by-state bounds implied by polling data, the polls may impose some dynamic restrictions as well. Observer that the Political Lorenz curve can change over time for two reasons. First, changes in the state directly affect income inequality, and power is wealth-weighted. Hence, political inequality changes with income inequality. Second, Political Lorenz curve changes as the bias $\alpha(\omega_t)$ changes.

The inferential problem is complicated in either case by the fact that the distribution of policy preferences changes as the state varies over time. To illustrate this suppose the popularity of the policy choice has increased relative to $\bar{a}$, i.e., $p(\omega_{t+1}) > p(\omega_t)$. Popularity increased as the state changed from ω_t to ω_{t+1}. On the one hand, it could be due to the fact that the taste distribution on the policy set A has shifted away from $\bar{a}$. On the other hand, it could be due to a change in the distribution of political power. For instance, suppose that bias is fixed such that $\alpha(\omega_{t+1}) = \alpha(\omega_t) \neq 0$. Suppose income inequality decreased, i.e., $L(j, \omega_{t+1}) < L(j, \omega_t)\ \forall j$. Clearly, political inequality must have decreased as well. Holding tastes constant, the fact that $p(\omega_{t+1}) > p(\omega_t)$ is therefore explained by the fact that the new chosen policy in $t + 1$ is more widely supported than the old policy at date t.

Generally, the two explanations: a change in tastes versus a change in political inequality are hard to decouple. Sometimes, spatial information can be used sort them out. One simple extreme case occurs when $1 - q(\omega_{t+1}) > p(\omega_t)$. That is, the individuals who prefer $\underline{a}$ to $\Psi(\omega_{t+1})$ in state ω_{t+1} is more numerous than those who prefer $\Psi(\omega_t)$ to $\bar{a}$ in state ω_t. Though it sounds unintuitive, this condition implies that the distribution of ideal points has, roughly speaking, shifted to the left. It is straightforward to show

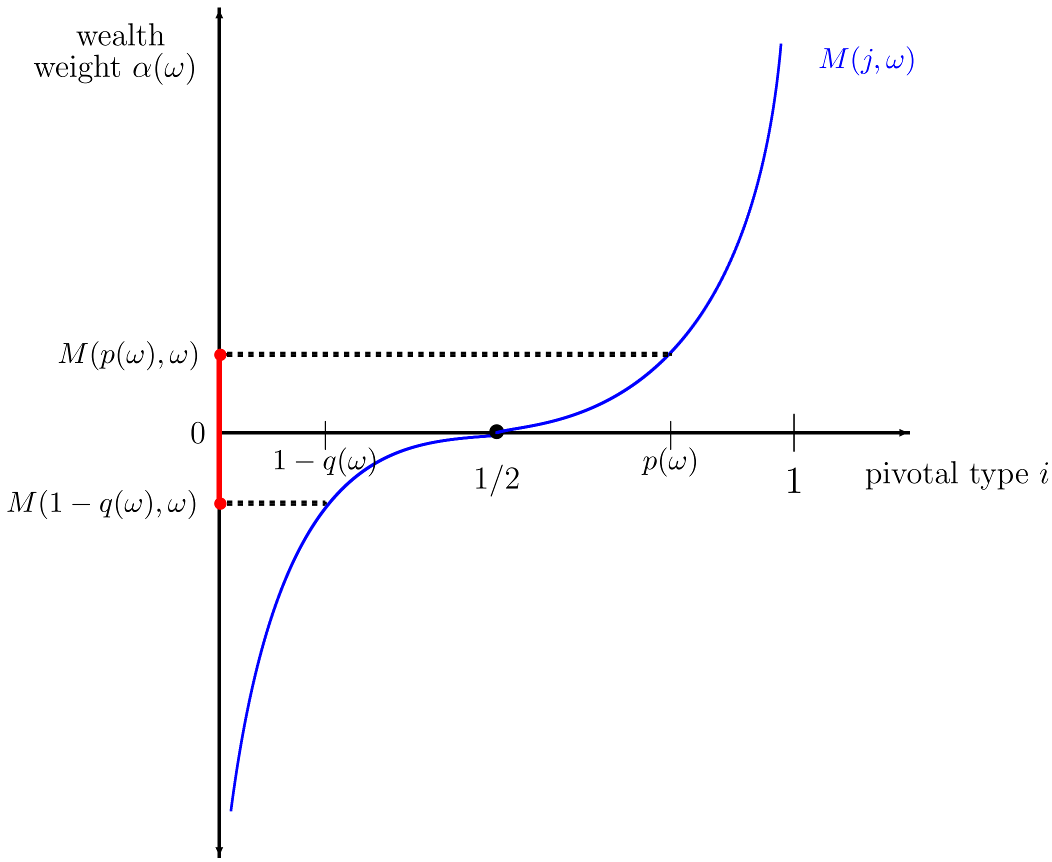

Figure 4: Bias Band and Bounding Function

Theorem 4 *If $1 - q(\omega_{t+1}) > p(\omega_t)$ then $L^p(j, \omega_{t+1}) < L^p(j, \omega_t)$, i.e., political power of the wealthy increased in $t+1$ relative to t. If, in addition, income inequality decreased in $t+1$, then $\alpha(\omega_{t+1}) > \alpha(\omega_t$ so that the bias became more elitist/less populist than before.*

Similarly, If $1 - q(\omega_t) > p(\omega_{t+1})$, then $L^p(j, \omega_{t+1}) > L^p(j, \omega_t)$, i.e., political power of the wealthy decreased in $t+1$ relative to t. If, in addition, income inequality increased in $t+1$, then $\alpha(\omega_{t+1}) < \alpha(\omega_{t+1}$ so that the bias became less elitist/more populist than before.

Proof. Take the first assumption $1 - q(\omega_{t+1}) > p(\omega_t)$. It is straightforward to verify that $p(\omega_t) > \mu(\omega_t)$ and $\mu(\omega_{t+1}) > 1 - q(\omega_{t+1})$. Consequently, $\mu(\omega_{t+1}) > \mu(\omega_t)$, and so political power of the wealthy has increased. If in this time income inequality decreased, then the increased power of the wealthy comes from an increase in wealth-bias. The second case follows by similar reasoning. ∎

5 Sufficient Conditions for Rationalizing Policy

We now show that the necessary conditions established in Theorem 3 are, in fact, sufficient.

Theorem 5 *Let Ψ be any policy rule for which $\underline{a} < \Psi(\omega) < \bar{a}$ for all ω and let (p, q) be the corresponding poll data relating $\Psi(\omega)$ to $\bar{a}$ and $\underline{a}$, resp. Then a weighting function α jointly rationalizes Ψ and (p, q) in the preference class $\mathcal{U}$ if for all ω,*

$$M(1 - q(\omega), \omega) \ \leq \ \alpha(\omega) \ \leq \ M(p(\omega), \omega) \tag{10}$$

Proof. Given the structure of M, (10) implies

$$1 - q(\omega) \ \leq \ \mu(\omega) \ \leq \ p(\omega)$$

with strict inequality somewhere in the string and where $\mu(\omega)$ is defined, as before, $L^P(\mu(\omega), \omega; \alpha) = 1/2$. We now construct a long run payoff function $U \in \mathcal{U}$ as follows. Let

$$U(i, \omega, a; \Psi) = -\frac{1}{2}\left(a^2 - \Psi^2(\omega)\right) + f(i; \omega)(a - \Psi(\omega)), \tag{11}$$

where $f(i; \omega)$ is a bivariate function in (i, ω). Notice that this U easily satisfies (A1). It may also be shown to satisfy (A3) using the same argument as in Proof of Theorem 1. Hence it remains to show that $f(i; \omega)$ can be constructed in a way that satisfies Assumption (A2) and the equilibrium requirement that Ψ is a α-WMW. These conditions imply that U must satisfy the following four restrictions:

First, U needs to satisfy the equilibrium requirement. From the First Order conditions

$$f(i; \omega) = a \tag{12}$$

$$\Longrightarrow \quad f(\mu(\omega); \omega) = \Psi(\omega). \tag{13}$$

Second, U must satisfy the polling equation for $\overline{a}$. If the support rate for $\Psi(\omega)$ against $\overline{a}$ is $p(\omega)$. Under increasing difference property, it implies that the pivotal decision maker is $p(\omega)$. Then we have

$$U(p(\omega), \omega, \overline{a}) = U(p(\omega), \omega, \Psi(\omega)) = 0 \tag{14}$$

$$\Longrightarrow \quad -\frac{1}{2}\left(\overline{a}^2 - \Psi^2(\omega)\right) + f(p(\omega); \omega)(\overline{a} - \Psi(\omega)) = 0 \tag{15}$$

$$\Longrightarrow \quad f(p(\omega); \omega) = \frac{1}{2}(\overline{a} + \Psi(\omega)). \tag{16}$$

Third, U must satisfy the polling equation for $\underline{a}$. Let the polling support rate for $\Psi(\omega)$ against $\underline{a}$ equal to $q(\omega)$. Under increasing difference property, it implies that pivotal decision maker is $(1 - q(\omega))$. We have

$$U(1 - q(\omega), \omega, \underline{a}) = U(1 - q(\omega), \omega, \Psi(\omega)) = 0 \tag{17}$$

$$\implies -\frac{1}{2}\left(\underline{a}^2 - \Psi^2(\omega)\right) + f(1 - q(\omega); \omega)(\underline{a} - \Psi(\omega)) = 0 \tag{18}$$

$$\implies f(1 - q(\omega); \omega) = \frac{1}{2}(\underline{a} + \Psi(\omega)). \tag{19}$$

Fourth, $U(i, \omega, a)$ needs to satisfy increasing difference property (A2), i.e., $f(i; \omega)$ is increasing in i for each ω.

To summarize, the four restrictions lead to the following equation system

$$\begin{pmatrix} f(1 - q(\omega); \omega) \\ f(\mu(\omega); \omega) \\ f(p(\omega); \omega) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\underline{a} + \Psi(\omega)) \\ \Psi(\omega) \\ \frac{1}{2}(\overline{a} + \Psi(\omega)) \end{pmatrix}, \tag{20}$$

such that $f(i; \omega)$ is increasing. In other words, it is a standard interpolation problem at three data points $\left(1 - q(\omega), \frac{1}{2}(\underline{a} + \Psi(\omega))\right)$, $(\mu(\omega), \Psi(\omega))$ and $\left(p(\omega), \frac{1}{2}(\overline{a} + \Psi(\omega))\right)$, with the class of interpolants as increasing functions. Notice that $\frac{1}{2}(\underline{a} + \Psi(\omega)) < \Psi(\omega) < \frac{1}{2}(\overline{a} + \Psi(\omega))$.

There are numerous ways to construct such an f. For instance a piecewise linear spline easily works. We omit the remaining details and conclude the proof. ∎

6 Summary

[TO BE COMPLETED]

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