

A Theory of Conflict as a Coordination Failure in Anarchic Environments*

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Abstract

This paper presents a theory of conflict in which violence occurs as an outcome of a difficult balancing act between the fear of being attacked and the opportunity cost of breaking peace. We link the propensity of conflict to current and future economic conditions and discuss the effects of growth, inequality and military technology on the ability of groups to escape the Security Dilemma.

VERY PRELIMINARY – COMMENTS WELCOME

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1 Introduction

In weak states the government does not have a monopoly on the coercive use of violence. As a consequence, groups are permanently tempted to engage in predatory behavior. This creates an unstable environment in which groups have to be entrepreneurial in providing protection. When there is a large offensive advantage, this situation can easily create coordination problems: an attack may happen just to avoid being attacked. In other words, there can be violence in situations in which a credible commitment not to attack by any of the parts would be enough to ensure peaceful coexistence. This mechanism, designated as the Security Dilemma, is at the heart of numerous episodes of ethnic rioting, pillaging, or ethnic wars. However, as Fearon and Laitin (1996) point out, ethnic strife is, by no means, prevalent, even in these unstable settings. How do groups manage to avoid continuous violence? What is the relationship between the prospect of violence and economic performance during peaceful interludes? In what circumstances will an offensive advantage precipitate the occurrence of violence?

We are interested in exploring the outbreak of violence as a failure of groups to coordinate in a potentially peaceful situation. The decision to go to war or not will be influenced by two basic forces.

On the one hand, the fear of being attacked is conducive to violence. This argument is already present in Hobbes' *Leviathan*: "if one plant, sow, build or possess a convenient seat, others may probably be expected to come prepared with forces united to dispossess and deprive him not only of the fruit of his labor, but also of his life or liberty. [...] And from this diffidence of one another, there is no way for any man to secure himself so reasonable as anticipation, that is, by force or wiles, to master the persons of all men he can so long till he see no other power great enough to endanger him."

On the other hand, the proceeds of peace are a powerful incentive try to coordinate and avoid conflict. Also in the *Leviathan*, Hobbes writes that "The passions that incline men to peace are fear of death, desire of such things as are necessary to commodious living, and a hope by their industry to obtain them." The higher is the opportunity cost of going to war, the lower should be the propensity to engage in violence. This hypothesis is confirmed by Miguel et al (2004) which provides conclusive evidence that bad economic outcomes are conducive to civil wars.

The multiplicity of equilibria inherent to coordination games imposes that some selection criterion be chosen before one can study comparative statics. It is typical in such cases to select either the worse or the best equilibrium. Here we support the use of risk-dominance

as a selection criterion for two reasons. First and foremost, it captures the idea that at the time of taking decisions, players balance their desire for good outcomes and their fear caused by strategic uncertainty. Neither the best nor the worse equilibrium capture this trade-off and we think it is at the heart of the security dilemma. Second, risk-dominant selection can be founded in non-cooperative game theory following the global games approach of Carlsson and van Damme (1993). This approach consists of introducing a state of the world θ , which affects payoffs and on which players get slightly noisy signals. As an added benefit, this approach allows to define a continuously moving propensity of conflict, which makes comparative statics particularly simple to formulate.

The risk-dominant equilibrium takes a very simple form: groups coexist peacefully if and only if θ is greater than a threshold θ^{RD} . Below this threshold the fear of being attacked builds up to such an extent that it overcomes the desire for peace, and groups fight although peace might have been sustainable under common knowledge. This risk-dominance threshold has interesting comparative statics that appeal to most intuitions on the nature of conflict and that are difficult to generate with other equilibrium selection criteria. First, rich countries are less likely to experience conflict. Second, inequality across groups within a country increases the probability of conflict. Third, the model exhibits deterrence in the sense that arming the contenders reduces the probability of conflict (this is not to say that it is good for welfare). Finally, increasing the offensive advantage increases the likelihood of conflict.

We also show that the global games framework can be extended to discuss dynamic considerations, which, as Fearon and Laitin (1996) emphasize, can be essential to explain why constant conflict isn't prevalent. We model conflict as a exit game, in which war causes the game to end. This simplification of the standard repeated game framework allows us to study questions related to economic growth and investment.

The first implication to be drawn from this analysis is that there is a static and a dynamic link between the economy and the occurrence of violence: the risk-dominance threshold is decreasing both in the current stock of capital and in the future expected returns from the economy¹. As these future returns increase, the value of peace increases, helping groups to coordinate. For instance, the expectation of fast growth (in the absence of conflict) is conducive to stability. More generally, any future shock that increases the future proceeds of peace reduces the current probability of war. Obviously, the opposite occurs with future shocks that are expected to reduce economic returns. Note that none of these predictions

¹In the model, an economy with a larger stock of capital, provides higher economic returns for the same economic shock.

could be made if the game retained multiple equilibria.

Second, future economic inequality also affects the probability of war. The more unbalanced the proceeds of peace are, the more difficult it is to refrain the poor side from engaging into pillage and thus the higher is the probability of violence. In addition, future inequality has a compounding effect because the increased probability of future violence reduces the option value of peace thus increasing the incentives to deviate in the current period. An important implication of this analysis is that economic growth is not enough to prevent outbreaks of violence. The proceeds of economic development have to be reasonably shared among the different groups to prevent inequality from fueling conflict.

Third, the model can generate a war trap in which poor countries cannot escape their plight because the high probability of conflict depresses investment, thus keeping the country poor and the probability of conflict high. By themselves, even if there is a string of good economic shocks, these countries cannot escape their poverty-violence vicious cycle. On the other hand, if a country reaches a certain level of income per capita, a virtuous cycle can set off in which investment and future growth reduce the current probability of conflict and hence countries can grow out of violence.

This paper provides a framework to address policy questions such as the form and optimal duration of a peace-keeping intervention operation. The analysis shows that when there is economic growth during peace periods the probability of reversion to violence is decreasing in the duration of the peace. Hence, keeping a force that prevents violence for some initial period, may have dramatic long term effects on the stabilization of a country: the economy can be allowed to grow up to a point in which the incentives to keep peace make coordination easy.

This paper is related to various streams of literature. Starting with Herz (1950) and structured in Jervis (1976, 1978) the concept of the “security dilemma” is used as the staple theory of realist security studies scholars to analyze the causes of escalation and ultimately of war. At the heart of this concept is the acknowledgement of a state of anarchy in international relations which makes commitment difficult. In such circumstances, whenever a country sees a neighbour making military preparations, it can deduce that it intends to attack (as opposed to prepare for defense) and respond by escalating its own preparations. This presumably leads to a spiral that generates actual conflict. This argument has been analyzed formally by Kydd (1997) in a model in which the driving force is the potential existence of some types of countries that are predatory². Baliga et al. (2004) also analyze these dynamics

²A state is considered predatory when it intends to violently upset the status quo even if it has no fear

giving a formal account of an unraveling argument present in Schelling (1960). In this case, any (small) positive probability of the opponent being predatory is enough for escalation to occur via a contagion effect produced by iterated elimination of dominated strategies. In addition, they show the existence of cheap talk equilibria that mitigate the coordination problem. These approaches rely on the existence of aggressive types to generate a security dilemma. Ours hinges on observational uncertainty about the state of the world. In a sense, type based approaches study how the possibility of the opponent being Hitler affects conflict, while our approach emphasizes the effects that the economic environment has on the contenders' ability to coordinate into peaceful coexistence or conflict. We view the two approaches as highly complementary.

We remain agnostic as to whether our model should be applied to conflict within or between countries. Posen (1993) pioneered the application of the “security dilemma” concept to a situation of ethnic confrontation. In his analysis, the collapse of such states as the Soviet Union and Yugoslavia left the different ethnic entities living within their borders in a situation tantamount to the traditional realist anarchy. We think that this idea is applicable to any circumstance in which there is no strong overarching authority, such as in sub-saharan Africa where states are weak, or in international relations where there is no strong extra-governmental peace enforcer. In fact, the basic mechanism by which agents manage to cooperate when the opportunity cost of conflict is high enough, has already been recognized in the literature on ethnic conflict by Fearon and Laitin (1996), and in the literature on international organization by Glaeser (1994).

Section 2 presents the static version of the game and discusses war as a coordination problem. Section 3 analyzes the dynamic model with exogenous growth dynamics. Section 4 analyzes the nature of the War Trap that can arise when we endogenize investment decisions. Section 5 draws policy implications and section 6 concludes. All proofs are contained in the appendix unless noted otherwise.

of being attacked.

2 Coordination and the Security Dilemma

2.1 The Model

Consider the following game in which two groups A and B decide to remain peaceful P or go to war W . The payoff matrix of player $i \in \{A, B\}$ is:

$$\begin{array}{cc} & P & W \\ P & \Pi_i + \theta & \theta - S \\ W & M & -W \end{array}$$

When a group decides to play peace P , it devotes its effort to some productive activity. In the absence of an attack, this productive activity yields a return that depends on the amount of productive capital (irrigation, for instance) of the group – denoted $\Pi_i \geq 0$ – and on the state of the world θ (rain, for example). The state θ is common³ to both groups. It has a density f_θ and support in \mathbb{R} . We assume an additive technology for simplicity. If a group is attacked and does not defend, its productive capital is destroyed. In addition, it suffers a loss of utility S that accounts for the violence it has to suffer. When a group attacks, we assume it completely abandons any productive activity. If it faces no resistance (if the opponent plays P) it obtains a payoff M from plunder. Finally, when both players attack, no production is made and both groups incur a cost W .

To simplify notations and without loss of generality, we assume $\Pi_A \geq \Pi_B$. We also make a sustained assumption on payoff to avoid the existence of asymmetric equilibria in which a group accepts passively the attack of the opponent. More precisely, we assume that,

$$(1) \quad S - W > M - \Pi_B$$

This ensures that the best response to an attack when the opponent finds it profitable to do so, is also to attack. Note that if we assume that $M < \Pi_B$, plundering is a destructive transfer. In addition to this destruction, violence entails two further efficiency losses: first, the attacking group(s) does not produce at all. Second, if one group plays peace, its utility is reduced by an extra S .

This payoff matrix incorporates the two elements of decentralized violence that we want to emphasize: the fear of being attacked as a reason to go to war and the economic opportunity

³In fact we only need the state of the world to be common at the stage where groups make decisions. Idiosyncratic noise could be added to the realized returns.

cost as a reason to keep the peace. For intermediary states of the world, this payoff matrix involves non-trivial coordination problems . More precisely, when θ is common knowledge, this game falls into one of three regimes:

1. When $\theta < M - \Pi_B$, the game features a single Nash equilibrium at (W, W) .⁴
2. When $M - \Pi_B \leq \theta \leq S - W$ there are multiple equilibria at (P, P) and (W, W) .
3. When $\theta > S - W$ a unique Nash equilibrium exists at (P, P)

The central region is the area of interest in which war can arise because of a coordination problem. The "fear" of being attacked enters through the $S - W$ term which measures the extent to which a surprise attack is harmful independently of income levels. Note that, not surprisingly, the region of multiplicity expands and encroaches into the region of peace as $S - W$ increases.

Equilibrium multiplicity makes comparative statics difficult to interpret. In particular, in the absence of a theory of equilibrium switching, it is not clear how to analyze the effect of a change in a parameter on the equilibrium played. For all we know, changing this parameter could trigger an equilibrium switch to any of other the multiple equilibria that are available. In order to draw comparative statics, the standard approach is to select either the best or the worst out come. In this setting, these equilibria will be entirely characterized by cooperation thresholds. The worst equilibrium corresponds to a threshold $\theta_* = S - W$ while the best equilibrium corresponds to a threshold $\theta^* = M - \Pi_B$. This best threshold θ^* is increasing in M and decreasing in Π_B . Thus it is linked to the opportunity cost of war for the poorest country. However it isn't influenced by either S or W or the rich group's income. The conditions of war have no impact on equilibrium selection and hence there is no Security Dilemma. Inversely, the worst equilibrium θ_* is increasing in S and decreasing in W but does not depend on M at all or on the income of either the rich or the poor group. There is a security dilemma but the opportunity cost of war plays no role.

To get more realistic comparative statics, we choose to study the risk-dominant equilibrium. To justify risk-dominant selection, we follow Carlsson and van Damme (1993) which show that if players have precise but still imperfect information about the state of the world, the set of rationalizable strategies is a singleton that converges to the risk-dominant equilibrium as information gets perfect. In our context, this means that when groups do not have

⁴The game is actually a Prisoners' dilemma only if $\theta < M - \Pi_A$.

common knowledge of the state of the world, a single equilibrium is selected. In developing economies, economic returns are very closely correlated due to the reliance on weather. However it can be quite difficult to predict with precision and to agree on the state of the world that will occur. Hence, we feel that the application of global games in this context is particularly appealing: when a group looks at the sky and predicts whether rain will be forthcoming or not it cannot be sure of how many clouds the neighboring group is noticing. By breaking common knowledge in this way, multiplicity of equilibria is broken in favor of the risk-dominant equilibrium.⁵ In a world where the best response to violence is violence, it is appropriate that the groups play it safe.

To examine this argument formally, assume that players observe a signal $s = \theta + \sigma \varepsilon_i$ where θ is the state of the world and ε_i , $i = A, B$ is idiosyncratic observational noise that is independently distributed. Denote by Γ_σ the resulting game with incomplete information.

Lemma 1. *As σ goes to 0, the set of rationalizable strategies of game Γ_σ converges to a singleton (x_A, x_B) . Moreover, strategies x_A and x_B are the risk dominant strategies: players cooperate if and only if θ is greater than the risk dominant threshold θ^{RD} .*

The risk dominant equilibrium corresponds to the one for which the product of unilateral deviations is largest.⁶ Hence, in the context of our game, the risk dominant threshold, is defined by the equation

$$(2) \quad (\Pi_A + \theta - M)(\Pi_B + \theta - M) = (-W - \theta + S)^2$$

The left hand side (LHS) of this equation is the product of the unilateral deviation gain from the peaceful equilibrium for both players, while the right hand side (RHS) is the same expression with deviations from the warring equilibrium. If there are multiple equilibria, θ^{RD}

⁵See Harsanyi and Selten (1988). The intuition behind risk dominance in 2x2 games is as follows. Assume each player does not know what the other player thinks she will do and they have uniform second order beliefs. Each player i , j can take one of two actions, α , β and there are two symmetric equilibria. Call \bar{s}_j the probability agent i playing action α_i that makes agent j indifferent between her actions. Higher s_j makes playing α_j optimal. Hence, the higher \bar{s}_j , the larger the range of beliefs that rationalize playing β_j . Equilibrium β risk-dominates equilibrium α if $\bar{s}_j + \bar{s}_i \geq 1$, that is, if the sum of ranges of second order beliefs across players that rationalize playing the β strategies is larger than the sum of the ranges for the α equilibrium.

⁶See Carlsson and Van Damme (1993)

must be between $M - \Pi_B$ and $S - W$. We obtain the following expression for the threshold

$$\begin{aligned}
 (3) \quad \theta^{RD} &= \frac{(S - W)^2 - (\Pi_A - M)(\Pi_B - M)}{(\Pi_A + \Pi_B) + 2(S - W - M)} \\
 &= \frac{(S - W - M)^2 - \Pi_A \Pi_B}{(\Pi_A + \Pi_B) + 2(S - W - M)} + M
 \end{aligned}$$

Note that the LHS of equation (2) is increasing in θ while the RHS is decreasing. Whenever $\theta > \theta^{RD}$, the LHS will be larger than the RHS and hence the peaceful equilibrium will risk-dominate and groups will remain peaceful. Whenever $\theta < \theta^{RD}$ they will go to war.

Note that θ^{RD} is always in the range of states of the world for which conflict occurs as a coordination failure because groups cannot commit not to attack because of the absence of common knowledge of the state of the world. In other words, a state of the world realized within $(M - \Pi_B, \theta^{RD})$ induces conflict even though none of the groups actually would attack if they were could be assured that their opponents would stay peaceful. In this range of realizations, Security Dilemma concerns trigger conflict.

It is useful to note at this stage that from differentiating equation (2) we obtain,

$$(4) \quad -1 < \frac{\partial \theta^{RD}}{\partial \Pi_i} < 0$$

2.2 Comparative Statics

One benefit of the global games approach is that it provides us with a continuously moving threshold for peace which we can use to formulate comparative statics in a simple way.

The effect of wealth and military technology.

Lemma 2. *The risk-dominant threshold θ^{RD} is increasing in $S - W$, increasing in M and decreasing in Π_i .*

Thus, the higher the temptation to attack M and the greater the fear of being attacked, $S - W$ the more likely is war to happen.

Note that the fear of being attacked creates a Security Dilemma in this model. In particular, whenever $\theta \in [M - \Pi_B, \theta^{RD}]$, groups are fighting even though each one of them would not attack if they could be assured that the other would stay peaceful. As Lemma 2 shows, this region of coordination failure is increasing in $S - W$, that is, in the cost

that a failure to coordinate would impose on the player that plays peace. We think that this coordination failure formally captures the intuition of Hobbes and the Realist school of international relations but enriches it by showing why failure is not ever-present: when the opportunity cost of war is too high, groups can escape the trap of fear.

Inequality. In addition, the risk-dominance threshold features a characteristic that is absent in the alternative thresholds: it is sensitive to inequality. To see this, normalize the sum $\Pi_A + \Pi_B = \Pi$ and define λ such that $\Pi_A = \lambda\Pi$ and $\Pi_B = (1 - \lambda)\Pi$. The threshold θ^{RD} is expressed as,

$$(5) \quad \theta^{RD} = \frac{(S - W - M)^2 - \lambda(1 - \lambda)\Pi}{\Pi + 2(S - W - M)} + M$$

This expression is minimized at $\lambda = 1/2$, which implies that increasing inequality increases the probability of conflict. The propensity of the poor group to attack increases because their opportunity cost of war is smaller. Even though the opportunity cost of fighting has increased for the rich, the rich do not reduce their propensity to attack because the best response to violence is always violence. Hence, the net effect of an increase in inequality is an increase in the incidence of violence. Figure 1 plots the probability of conflict on the level of inequality.

Note, however that we are keeping total income constant in this comparative statics. We know from Lemma 2 that θ^{RD} is decreasing in Π_i for $i \in \{A, B\}$, and hence increasing income for any of the groups can only be conducive to increased peace. In particular, increasing the income of the rich reduces violence. It is instructive to examine the intuition for this result. If the rich group becomes richer, there is *common knowledge* that the rich have become more reluctant to use violence. This reduces the fear of the poor of being caught off guard and hence it allows both groups to coordinate into a larger range of peace.

Note also that this does not imply that any increase in aggregate income increases the chances to coordinate into peace. An increase in total income that comes at a cost for some groups can increase the probability of violence as expression (5) shows. Hence, when redistribution mechanisms are not in place, some policies that provide lopsided gains may be harmful by directly increasing social instability and violence.

For instance, if we believe in a basic Heckscher-Olin model of trade, opening to trade may be beneficial in developing countries to the extent that it both increases wealth and reduces inequality. However this result also highlights the fact that the stabilization benefits of trade

openness will only be realized if its proceeds are relatively equally distributed.

The Effect of Deterrence. Lemma 2 shows that increasing W reduces θ^{RD} and thus the probability of violence. Hence, this model exhibits deterrence in the sense that increasing the costs of war always diminishes the probability of violence. However, since this means that wars are more costly in terms of utility, the net effect on the ex-ante utility of players is ambiguous. Intuitively, we would expect rich countries to benefit more from an increase in W and the associated reduction in the probability of war for two reasons: first, their underlying propensity of conflict is lower, and hence the increased costs of war are realized with lower probability.⁷ Second, the proceeds of peace are higher in rich countries and they have greater value for peace. However, these simple comparative statics are obscured by the fact that the reduction in the probability of war caused by an increase in W will not be the same for different countries. Lemmas 3 and 4 discuss deterrence while keeping the countries propensity for conflict constant.

Lemma 3. *For any real number r and any W , we consider the family of groups $\mathcal{F}_{r,W}$ characterized by $\Pi_A = \Pi_B = \Pi$ and $S + M = \Pi + r$. Then all groups in $\mathcal{F}_{r,W}$ have the same propensity to go to war : $\theta^{RD} = (r - W)/2$.*

This lemma allows us to consider changes in wealth that do not imply a change in the underlying probability of war. The conditions for this lemma hold when, for instance, the amount plundered M varies one to one with wealth Π and destruction S is independent of wealth. Keeping the likelihood of war constant, we can unambiguously compare the value of deterrence for rich and poor countries.

Lemma 4. *Consider a family $\mathcal{F}_{r,W}$. For any group in that family denote V the limit value of playing game Γ_σ as σ goes to 0. Then, we have*

$$(6) \quad \frac{\partial}{\partial \Pi} \left(\frac{\partial V}{\partial W} \right)_{S+M=\Pi+r} > 0$$

That is, keeping the propensity to go to war constant, the value of deterrence increases with the wealth of the groups.

⁷The intuition is simple: it is better not to give atomic devices to countries that face civil war with high probability because the marginal reduction in this probability will not compensate the huge costs of a realized nuclear war. On the contrary, for countries that almost never face violence, the costs of increasing their destructive capacity are seldom realized.

Moreover, there always exists Π large enough so that $\frac{\partial V}{\partial W} > 0$, and parameters W, r and Π small enough, $\frac{\partial V}{\partial W} < 0$.

This lemma shows that for identical levels of conflict, richer countries benefit from deterrence more than poor ones because of the higher benefits they enjoy from peace. In fact, while deterrence will always be welfare improving for rich enough countries, arms proliferation will have adverse consequences in underdeveloped countries. The mirror implication is that while it is probably welfare improving to send aid to poor countries that are experiencing conflict, we should be wary of sending aid to relatively well off countries that choose to give way to violence.

The Effect of Technological Differences in War. The fact that θ^{RD} is decreasing in both players' wealth does not support the intuition according to which the richer is my neighbor, the more I want to loot him. In the basic model, by keeping M constant, we do not allow the value of plunder to increase in my neighbors' capital stock and therefore we are shutting down this mechanism. Note that Lemma 2 already implies that increasing M increases the probability of conflict. We can in fact say something more with respect to increasing *inequality* in M . In particular, we can restate the payoff matrix to allow for inequality in other parameters:

$$\begin{array}{cc}
 & P & W \\
 P & \Pi_i + \theta & \theta - S_i \\
 W & M_i & -W_i
 \end{array}$$

Lemma 5. *Take an economy with the payoff matrix above. Denote $S_A + S_B = S$, $M_A + M_B = M$ and $W_A + W_B = W$.*

1. *If $S_A = S_B$, $M_A = M_B$ and $\Pi_A = \Pi_B$, inequality in W reduces θ^{RD}*
2. *If $W_A = W_B$, $M_A = M_B$ and $\Pi_A = \Pi_B$, inequality in S reduces θ^{RD}*
3. *If $S_A = S_B$, $W_A = W_B$ and $\Pi_A = \Pi_B$, inequality in M increases θ^{RD}*

Why are these effects so different? The intuition is that W and S affect the direct costs of violence, while M and Π determine the opportunity cost of violence. In particular, imagine that $W_A > W_B$. Then, it is common knowledge that group A is very reluctant to attack. Knowing this, group B has no need to mount a pre-emptive strike because it knows that

group A 's propensity to attack is very low. This helps groups to coordinate into peace. If $S_A > S_B$, it is common knowledge that group B does not suffer much from an undefended attack and hence their propensity to defensively attack is low and the same reasoning goes through. Extremely unequal military prowess can help explain extended periods of stability, such as the Ptolemaic dynasty in classic Egypt or the Yuan and Qing dynasties in China, in which an foreign elite group specialized in warfare monopolized power over a mass of autochthonous peasants for centuries.

Why does inequality in M increase violence? For exactly the same reason inequality in income did: if $M_A > M_B$, it is common knowledge that A 's opportunity cost of violence has gone down. This increases B 's propensity to mount a pre-emptive strike and hence the area of coordination failure increases.

Joint departure from equality can have many effects. Note, however, that we can always write:

$$(7) \quad \theta^{RD} = \frac{(S_A - W_A)(S_B - W_B) - (\Pi_A - M_A)(\Pi_B - M_B)}{\Pi - M + S - W}$$

This expression makes two things clear: first, inequality in S only interacts with inequality in W . The same is true for M and Π . Second, joint departures that keep $S_i - W_i$ or $\Pi_i - M_i$ constant, will not affect the probability of civil war.

The intuition that wealth may increase the temptation of plunder presented at the beginning of this subsection can be modelled by assuming an increasing functional form $M_i(\Pi_j)$. In this case, inequality in income would be doubly conducting to violence: the poor group not only would get less returns from peace, it would actually obtain higher returns from looting and hence its opportunity cost of violence would be doubly reduced.

Other situations suggest that increasing inequality in income may allow the rich group access to better military technology. This could be captured in the model by assuming $W_i(\Pi_j)$ increasing. In this case, the effect of inequality on the occurrence of violence would be ambiguous. The direct effect of income is bad as it makes the poor group more prone to attack. However, war would be now more costly for them. The net result would thus depend on the particular functional form assumed.

Our model of the combination of Security Dilemma concerns with opportunity costs of violence is very flexible and with the addition of specific functional relationships, can accommodate a closer examination of the effects of inequality. In addition, we turn now to show that it can be adapted to introduce dynamic considerations.

3 War as an exit game

The one-shot game severely underestimates the capacity of groups to avoid violence when offensive advantages are paramount. For instance, in Macedonia, Slavs and Albanians could be potentially locked in a prisoners' dilemma situation in terms of current payoffs and first strike advantages. However, the possibility of joining the European Union in the future provides a strong incentive to coordinate into peaceful coexistence. In general, the value of coordinating into peace today is higher than the current payoffs groups realize if only because they avoid a change of regime into one of future widespread violence. The weight of the future was already emphasized by Fearon and Laitin (1996) to explain why ethnic groups cooperate more often than an anarchic world would make us expect. How can the one-shot coordination game be modified in order to capture the importance of the future?

Because it is difficult to extend the global games framework to the standard repeated game setting, we consider instead the dynamic exit game structure proposed by Chassang (2005) which considerably simplifies the strategic considerations and allows to both extend the global games framework and introduce time varying capital stocks. Note also that since the exit game only differs from a repeated game in that payoffs upon deviation are exogenously given, any strategic consideration of an equilibrium of the fully repeated game before the punishment phase can be accommodated. In particular, we work with payoffs upon exit that replicate the trigger strategies equilibrium of the standard repeated game.

In the context of conflict, the assumption that whenever a group attacks, the game ends does not seem inappropriate because the median duration of a civil war is 6 years (see Fearon and Laitin (2003)). In addition the assumption of exit is in fact equivalent to the assumption that incentives that occur conditionally on there being a war are unimportant. Given the amount of turmoil caused by civil conflict, this does not seem a restrictive assumption.

3.1 Global games information structures in exit games

The game we consider has two players $i \in \{A, B\}$ with action space $\{P, W\}$ and infinite horizon $t \in \{1, \dots, \infty\}$. At time t , players get flow payoffs,

$$\begin{array}{cc}
 & P & W \\
 P & \Pi(k_{i,t}) + \theta_t & \theta_t - S \\
 W & M & -W
 \end{array}$$

Where $k_{i,t}$ denotes the productive capital of group i at time t . Π is weakly concave and

increasing in k . Capital follows an exogenously given, deterministic, recurrence equation. Denoting $\mathbf{k} = (k_i, k_{-i})$, we have,

$$(8) \quad \mathbf{k}_{t+1} = L(\mathbf{k}_t)$$

Where L is a continuous and increasing mapping. Hence, in the absence of conflict, this model exhibits (exogenous) economic growth.⁸ We assume that L is such that any long run capital stock must belong to a compact range R .

Whenever a group chooses to go to war, the game stops and players get a continuation payoff equal to zero. The sequence of states of the world $\{\theta_t\}$ is i.i.d. with distribution f_θ and support in \mathbb{R} . Players get a signal on the state of the world $s_{i,t} = \theta_t + \sigma\varepsilon_{i,t}$. We denote by Γ_σ this dynamic game.

Definition 1 (histories). *Because of the exit game structure, at any decision node, the histories of players take the form $h_{i,t} = \{s_{i,1}, k_{i,1}, \dots, s_{i,t}, k_{i,t}\}$. We denote \mathcal{H} the set of possible histories.*

A strategy x is a mapping from \mathcal{H} to $\{P, W\}$. At any history $h_{i,t}$, we denote $V_i(h_{i,t})$ the expected value of playing the game for player i . We must have $V_i > -W$ since any player can guarantee this payoff by going to war.

As in the static game, we want to ensure that the best response to an attack is to attack. For this reason, we assume that $S > (1 + \beta)W + M$. This is a sufficient condition to insure that no group accepts passively to be attacked.

A few definitions are required to present the selection results.

Definition 2. *We denote $\bar{\Pi} = \max_{k \in R} \Pi(k)$ and \bar{V} an upper bound to the value of playing, for instance the value of playing the best equilibrium under common-knowledge.*

Definition 3 (order on strategies). *We define an order, \prec , on strategies by,*

$$x \prec x' \iff \{\forall h \in \mathcal{H}, x(h) = P \Rightarrow x'(h) = P\}$$

Definition 4 (threshold form). *A strategy x has a threshold form if and only if:*

1. *There exists a mapping \tilde{x} such that for all $h_{i,t} \in \mathcal{H}$, $x(h_{i,t}) = \tilde{x}(s_{i,t}, \mathbf{k}_t)$*

⁸The structure can easily be adapted to stochastic evolution of capital stocks. For simplicity, we concentrate on the case of deterministic growth.

2. For all \mathbf{k}_t , there exists $\theta^T(\mathbf{k}_t) \in \mathbb{R}$ such that, $x(h_{i,t}) = P\mathbf{1}_{s_{i,t} > \theta^T(\mathbf{k}_t)} + W\mathbf{1}_{s_{i,t} \leq \theta^T(\mathbf{k}_t)}$

Lemma 6. *The game Γ_σ satisfies the assumptions of Theorem 7 of Chassang (2005). This implies that for σ small enough, the set of rationalizable strategies of Γ_σ is bounded by a greatest and a smallest Nash equilibria with respect to order \prec . These equilibria, denoted $(x_A^{H,\sigma}, x_B^{H,\sigma})$ and $(x_A^{L,\sigma}, x_B^{L,\sigma})$ take a threshold form. These equilibria are associated with value functions upon continuation $\vec{V}^{\overrightarrow{H,\sigma}}(\cdot) = (V_A^{H,\sigma}(\cdot), V_B^{H,\sigma}(\cdot))$ and $\vec{V}^{\overrightarrow{L,\sigma}}(\cdot) = (V_A^{L,\sigma}(\cdot), V_B^{L,\sigma}(\cdot))$. As σ goes to zero, $\vec{V}^{\overrightarrow{H,\sigma}}$ and $\vec{V}^{\overrightarrow{L,\sigma}}$ converge to the highest and lowest fixed points of an increasing and continuous mapping Φ defined by,*

$$(9) \quad \Phi(V_i, V_{-i})(\mathbf{k}_{t-1}) = \begin{pmatrix} \mathbf{E} \left[-W\mathbf{1}_{\theta < \theta_t^{RD}} + (\Pi_i(\mathbf{k}_t) + \theta + \beta V_i(\mathbf{k}_t)\mathbf{1}_{\theta > \theta_t^{RD}}) \right] \\ \mathbf{E} \left[-W\mathbf{1}_{\theta < \theta_t^{RD}} + (\Pi_{-i}(\mathbf{k}_t) + \theta + \beta V_{-i}(\mathbf{k}_t)\mathbf{1}_{\theta > \theta_t^{RD}}) \right] \end{pmatrix}$$

Where θ_t^{RD} is defined by

$$(10) \quad \theta_t^{RD} = \frac{(S - W - M)^2 - (\Pi_i(\mathbf{k}_t) + \beta V_i(\mathbf{k}_t))(\Pi_{-i}(\mathbf{k}_t) + \beta V_{-i}(\mathbf{k}_t))}{(\Pi_i(\mathbf{k}_t) + \beta V_i(\mathbf{k}_t)) + (\Pi_j(\mathbf{k}_t) + \beta V_j(\mathbf{k}_t)) + 2(S - W - M)} + M$$

An immediate corollary of this result is that equilibrium is unique at the limit whenever Φ has a unique fixed point. We now provide a sufficient condition under which this will be the case.

Lemma 7 (sufficient condition for uniqueness). *Whenever the distribution of states of the world is such that,*

$$(11) \quad \sup |f_\theta| < \frac{1 - \beta}{4\beta(\bar{\Pi} + \beta\bar{V} + S)}$$

Then, the mapping Φ is contracting with a rate $\lambda < 1$. This implies that as σ goes to 0, the set of rationalizable strategies of game Γ_σ converges to a unique equilibrium associated with continuation value functions, (V_i, V_{-i}) .

Assumption 1. *For the rest of the paper we consider a range of parameter values such that Φ is a contraction mapping.*

The structure of the unique equilibrium is similar to the one in the static version of the game. The equilibrium is characterized by a sequence of thresholds $\{\theta_t^{RD}\}_{t \in \mathbb{N}}$. The threshold θ^{RD} depends on the contemporary stocks of capital and, through $V_i(\mathbf{k}_t)$ and $V_{-i}(\mathbf{k}_t)$ on the future path of the economy and the future probability of conflict. As before, at any given

point in time, groups go to war if and only if the realized state of the world is below the threshold for that period. Hence, this model still exhibits a correlation between poor states of the world and the occurrence of conflict.

3.2 Comparative statics with stationary capital

Many interesting comparative statics can be obtained in the case where capital is constant:

$$L(\mathbf{k}) = \mathbf{k}$$

The effect of wealth and military technology.

Lemma 8. V_i is increasing in Π_i , Π_{-i} and decreasing in S and M . θ^{RD} is decreasing in Π_i and increasing in S and M .

Most of the comparative statics of the one-shot game are maintained in the stationary game. Note, however, that the effect of a change in any of the parameters on the current probability of conflict is amplified because future probabilities of conflict and payoffs enter the expression for θ^{RD} via V_A and V_B .

Precisely because of this feedback, the comparative static with respect to W does not survive the introduction of dynamics. In the static model, the value function was not necessarily monotonous in W , however the threshold for peace was strictly decreasing in W . Now, since V_i affects the current probability of conflict, the effect of military technology on the probability of war also becomes ambiguous.

Next, we examine whether inequality affects propensity for violence in the exit game.

Inequality and conflict.

Lemma 9. Assume without loss of generality that $k_i < k_{-i}$, then

$$(12) \quad \frac{\partial \theta^{RD}}{\partial k_i} < \frac{\partial \theta^{RD}}{\partial k_{-i}}$$

Moreover we have

$$(13) \quad \frac{\partial V_i}{\partial k_i} - \frac{\partial V_i}{\partial k_{-i}} > \frac{\partial V_{-i}}{\partial k_{-i}} - \frac{\partial V_{-i}}{\partial k_i} \quad \text{and} \quad \frac{\partial V_i}{\partial k_i} - \frac{\partial V_i}{\partial k_{-i}} > 0$$

This lemma shows that θ^{RD} is sensitive to inequality in the sense that an increase in one unit of capital is more beneficial if it increases the income of the poor rather than the rich. Moreover, the second part of the lemma shows that this is also true if we take a utilitarian view of welfare, as we can rewrite

$$\frac{\partial V_i}{\partial k_i} + \frac{\partial V_{-i}}{\partial k_i} \geq \frac{\partial V_{-i}}{\partial k_{-i}} + \frac{\partial V_i}{\partial k_{-i}}$$

which implies that it is always positive to reduce inequality. It is important to emphasize that this result does not depend on the concavity of Π . Even if production is linear in capital, it is strictly better to transfer aid to the poor. This is the case because in the static game the threshold is increasing in inequality. This increased probability of conflict is compounded in the dynamic exit game because it enters V_i and V_{-i} .

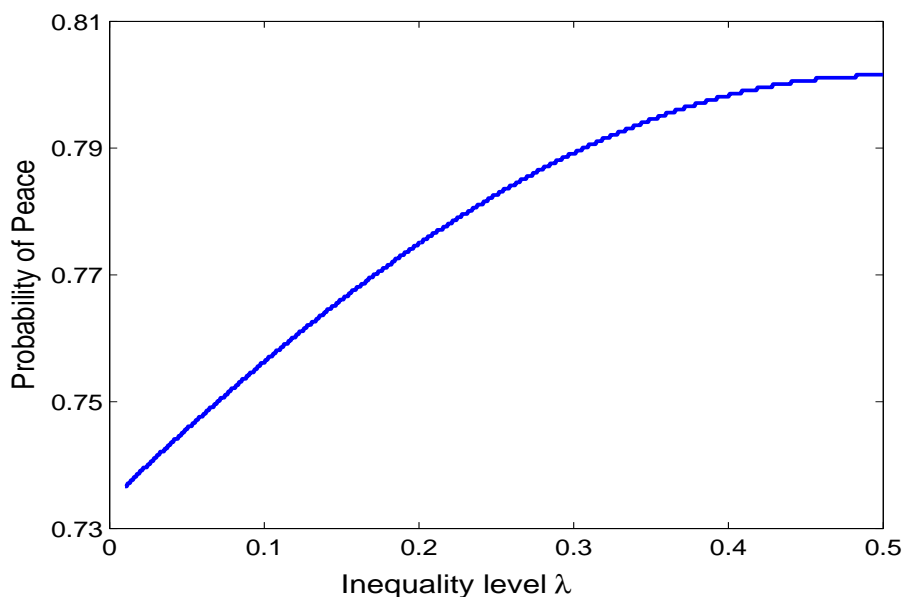


Figure 1: Probability of Peace as a function of inequality parameter λ . Uniform distribution over $[-8, 12]$, $k = 5$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

Patience and Conflict. Now we turn to examine the effect of patience on the probability of resorting to violence. Because of the exit structure of the game, the way we model payoffs upon exit, M, W, S becomes important. There are two approaches:

Approach 1: M , W and S are fixed independent of β

Approach 2: M , W and S depend on β . More precisely, we consider functional forms,

$$W = \frac{1}{1-\beta}w ; \quad M = m - \frac{\beta}{1-\beta}w ; \quad S = s + \frac{\beta}{1-\beta}w$$

The first approach takes exit at face value, which may be appropriate in matters of war. The second approaches uses the discounted value of the flow payoffs from being in a war. The essential difference between these two cases is that in the second one, equilibrium behavior is invariant with respect to parallel shifts in payoffs while this is not true in the first case. As a consequence, in the second case the comparative statics are not ambiguous. This is shown in the following lemmas.

Lemma 10. *When M, W and S are fixed, the comparative statics in β depend on the sign of $V_i(\beta = 0)$. If $V_A(\beta = 0) < 0$ and $V_B(\beta = 0) < 0$, then V_i is decreasing in β and θ^{RD} is increasing in β . Inversely, if $V_A(\beta = 0) > 0$ and $V_B(\beta = 0) > 0$, then V_i is increasing in β and θ^{RD} is decreasing in β .*

Lemma 10 has a simple interpretation. If the game has a negative continuation value for both players, then it is clear that the value of peace is diminishing in patience. As a consequence, the more patient groups become, the more prone to ending the game via violence. This effect is not there in the second formulation as the costs of fighting are also increasing in patience.

Consider now the second specification:

$$W = \frac{1}{1-\beta}w ; \quad M = m - \frac{\beta}{1-\beta}w ; \quad S = s + \frac{\beta}{1-\beta}w$$

Lemma 11. *With this specification of W , M , and S , the equilibrium behavior of the players is invariant with respect to shifts in the flow payoffs of the form: $\tilde{w} = w - h$; $\tilde{s} = s - h$; $\tilde{m} = m + h$; $\tilde{\Pi}_i = \Pi_i + h$.*

Lemma 12. *With this specification of S, W and M , θ^{RD} is decreasing in β .*

The interpretation of this result is clear: forward looking groups take into account that there is an option value to play peace which is greater the more patient they are. This helps groups coordinate into peaceful coexistence as it is common knowledge that the opportunity cost of violence is higher for everybody.

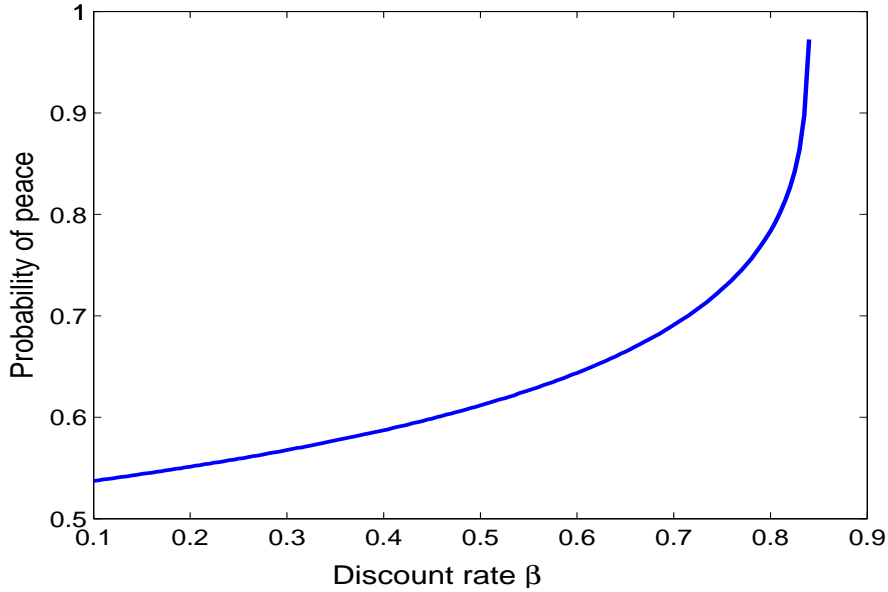


Figure 2: Probability of peace as a function of the discount rate β . Uniform distribution over $[-8, 12]$, $k = 1$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

Because of its properties, from now on we always work under this specification of payoffs upon exit. The next result establishes that this effect of patience is also increasing in wealth. In other words, patience and wealth reinforce each other in helping groups to coordinate into peace.

Lemma 13 (complementarity of patience and wealth). *Consider symmetric groups with wealth k and discount rate β . Whenever $\frac{\partial f_\theta}{\partial \theta}(\theta^{RD}) \leq 0$ then*

$$(14) \quad \frac{\partial^2 \theta^{RD}}{\partial k_i \partial \beta} < 0$$

and

$$(15) \quad \frac{\partial^2 V_i}{\partial k_i \partial \beta} > 0 \quad \text{and} \quad \frac{\partial^2 V_i}{\partial k_{-i} \partial \beta} > 0$$

Note that Lemma 13 implies that as long as $f'_\theta(\theta^{RD})$ is not too high, then $\frac{\partial^2 P(\theta^{RD} < \theta)}{\partial k_i \partial \beta} > 0$. The intuition for this result is that forward looking agents experiment a time-multiplier effect. An increase in a unit of capital decreases the probability of conflict in all future periods. The value of these future changes is increasing in β , and thus the marginal effect

of capital on peace increases in the patience of citizens. The reason why we must in fact restrict ourselves to cases where $\frac{\partial f_{\theta}}{\partial \theta}(\theta^{RD}) \leq 0$ is that the amount by which the probability of war diminishes when the capital stock is increased also depends on where in the distribution of states of the world the current threshold for peace is. If by increasing β one shifts θ^{RD} to a zone where there is no mass, then Lemma 13 may not hold.

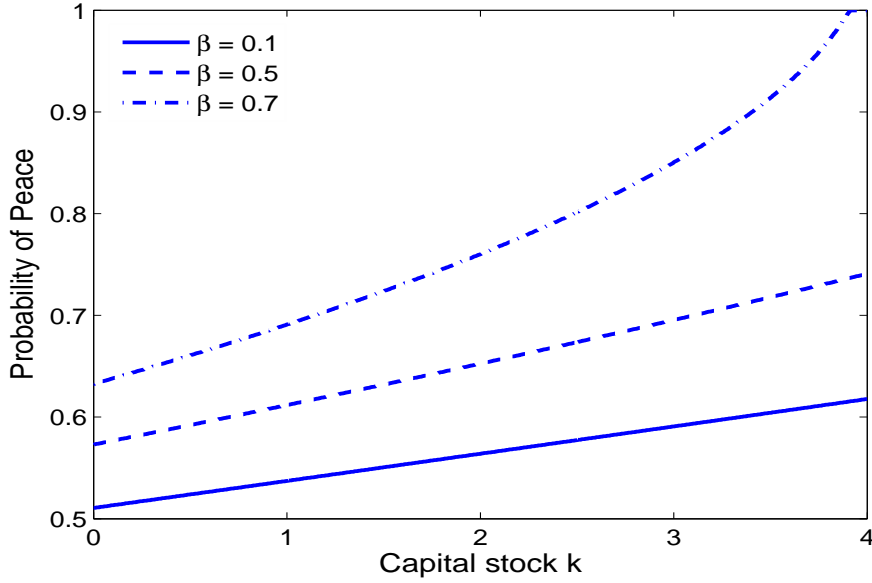


Figure 3: Probability of Peace as a function of capital stock k and discount rate β . Uniform distribution over $[-8, 12]$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

Figures 3 and 4 show the effect of capital and patience on the current probability of peace and the value of the game. Indeed, the marginal impact of capital increases with β . Note that at high β , V is highly increasing in k . This is the time-multiplier effect. Note also that when groups are poor $k = 0$, V is small and hence does not help much in coordinating into peace. In this case, making agents forward looking does not make much of a difference. On the contrary, when groups are rich, V makes a great contribution to the opportunity cost of violence. As a consequence, the expected duration of peace is strongly increasing in k because the static effect (the current economic proceeds of peace are higher) and the dynamic effect (the option value of peace increases dramatically) reinforce each other.

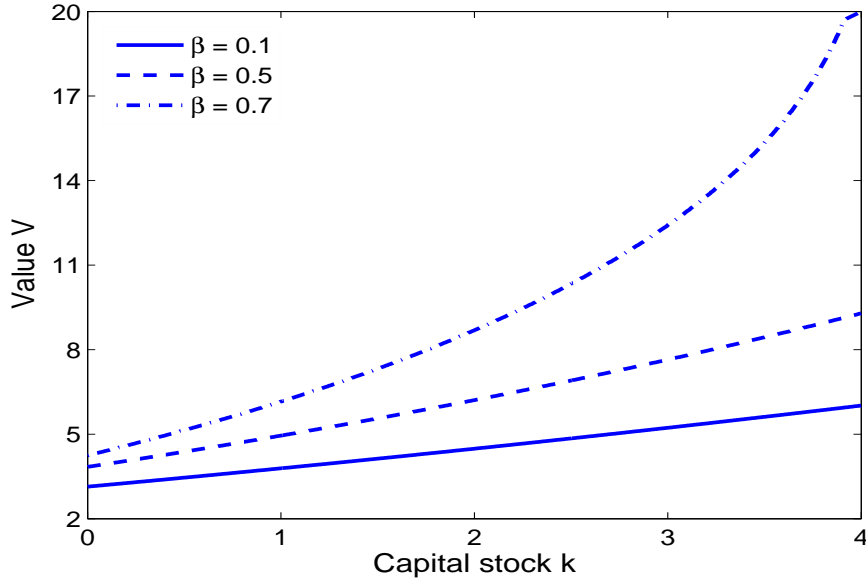


Figure 4: Value of playing as a function of capital stock k and discount rate β . Uniform distribution over $[-8, 12]$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

3.3 Comparative statics with non-stationary capital stocks

Can the promise of entry in the European Union stem inter-communal violence in places like Macedonia or Turkey? How do expectations of future growth affect conflict? What about inequality in expected growth patterns? The exit game framework allows us to ask a variety of questions involving the effect of future growth on current propensity for violence.

First, we address the effect of future exogenous economic growth on the current probability of conflict.

Lemma 14 (growth and conflict). *Index the growth process by a variable $z \in \mathbb{R}$: $\mathbf{k}_{t+1} = L_z(\mathbf{k}_t)$, such that, L_z is increasing in z . Then for any initial capital stocks \mathbf{k} , we have that*

$$(16) \quad \frac{\partial \theta_z^{RD}(\mathbf{k})}{\partial z} \leq 0 \quad \text{and} \quad \frac{\partial V_z(\mathbf{k})}{\partial z} \geq 0$$

Lemma 14 makes clear that increasing the slope of the growth process reduces the current propensity of violence. Hence, taking into account expectations of growth may explain variations in conflict propensity that do not correspond to observable variations in the current state of the economy. This may help explain the largely peaceful assimilation of Spanish immigrants into Catalonia and the Basque Country in the 1960s. Even though a "sons of

the soil” dynamic could have started, this massive immigration coincided with a period of robust economic growth.

Lemma 15 (complementarity of patience and growth). *Consider symmetric groups, with a common capital stock following the recurrence equation $k_{t+1} = L_z(k_t)$. Denote $\theta^R D_t$ the threshold of peace at time t . Then whenever for all $t \in \mathbb{N}$, $f'_\theta(\theta_t^{RD}) \leq 0$, we have,*

$$(17) \quad \forall t \in \mathbb{N}, \quad \frac{\partial^2 \theta_t^{RD}}{\partial z \partial \beta} < 0$$

Lemma 15 simply indicates that the time-multiplier effect is also at work when we examine the role of growth. This is not surprising, as the mechanism is basically the same as in the stationary model: the increased future economic returns and reduced probabilities of conflict compound into a current reduced probability of violence via their effect on V .

Now we turn to the impact of unequal sharing of the proceeds of growth within a country.

Lemma 16 (inequality in growth). *Assume that each group’s capital stock follows its own growth process, that is,*

$$(18) \quad \forall i \in \{1, 2\}, \quad k_{t+1}^i = L_{z_i}(k_t^i)$$

With L increasing and weakly concave in both k and z .

Then, whenever $k_i \leq k_{-i}$ and $z_i \leq z_{-i}$, we have,

$$(19) \quad \frac{\partial \theta^{RD}(\mathbf{k})}{\partial z_i} \leq \frac{\partial \theta^{RD}(\mathbf{k})}{\partial z_{-i}}$$

Moreover we have

$$(20) \quad \frac{\partial V_i}{\partial z_i} - \frac{\partial V_i}{\partial z_{-i}} > \frac{\partial V_{-i}}{\partial z_{-i}} - \frac{\partial V_{-i}}{\partial z_i} \quad \text{and} \quad \frac{\partial V_i}{\partial z_i} - \frac{\partial V_i}{\partial z_{-i}} > 0$$

Lemma 16 shows that disparity in growth rates increases the propensity of conflict. For the same aggregate growth rate, a country that has all its groups enjoying the average rate of growth will be more peaceful than a country in which a group monopolizes economic growth. This result sheds light on the violent dynamics in many countries in which a group controls access to education or civil service. In such a country, it is common knowledge that the excluded groups have little to gain from remaining peaceful and hence it is easy for a failure of coordination to arise.

In general, this sections makes clear that any expected future shock in income affects the current propensity of violence, keeping the current level of income constant. This being the case, and assuming that joining the European Union provides widespread growth, we can conjecture that the promise of future joining may be a force at place in stemming communal violence in Eastern Europe and in the Balkans. On the contrary, an expected drop in the economic situation could fuel conflict well before the actual shock takes place. To put it starkly, future conflict breeds current conflict.

4 The War Trap

The time preferences of citizens are clearly affected by the potential of future conflict. When citizens expect the incidence of violence to be high, they will put little value in future income, if it is conditional on peace. This impact of violence on effective discount rates has adverse implications for the accumulation of savings in countries plagued by conflict. Our opportunity cost approach to conflict adds a feedback mechanism by which countries that don't manage to save also face a greater likelihood of conflict.

We first address the question of effective time preference in a setup where the two groups have constant capital stocks. The following lemma computes the discount rate at which groups value an extra unit of capital at different points in time.

Lemma 17. *Consider two groups with constant capital stocks : $\forall t, k_{i,t} = k_i$. Denote $V_{i,1}$ the value of group i at period 1, before the state of the world is revealed. The effective discount rate is,*

$$(21) \quad \rho \equiv \frac{\frac{\partial V_{i,1}}{\partial k_{i,2}}}{\frac{\partial V_{i,1}}{\partial k_{i,1}}} = \beta P(\theta \geq \theta^{RD}) - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} f_{\theta}(\theta^{RD}) [\Pi_i(k_i) + \beta V_i + \theta^{RD} + W]$$

Condition (11) ensures that $\rho < 1$. However, the benchmark discount rate in the absence of conflict is β and it is in fact possible to have $\rho > \beta$. The reason for this unintuitive possibility is that on the one hand, the probability of peace is smaller than one, which reduces the expected returns from investment. On the other hand, increasing tomorrow's payoff increases the likelihood of peace today and tomorrow. How those two effects balance out is ambiguous. However, because rich countries are less likely to go to war, the natural intuition is that effective discount rates will be increasing in wealth. The following lemma gives conditions under which this intuition holds.

Lemma 18. *Consider groups with identical and constant capital stocks k and such that $f'_\theta(\theta^{RD}) \leq 0$. Then the intertemporal substitution rate of capital is increasing in wealth, that is, $\frac{\partial \rho}{\partial k} > 0$.*

Lemma 18 is of interest for two reasons. First, it shows how a war trap may build up even in the presence of a central planner. In impoverished countries, the likelihood of conflict is very high. This reduces optimal investment rates, which in turn guarantees that the country will not rise out of poverty thus further fueling conflict. This paints a situation of economic stagnation preluding to violent conflict. In that case even if a good string of states of the world allows peace to survive for a few periods, it may fail to generate economic growth for the hazard rate of violence to diminish.

An additional effect appears when investment decisions are decentralized. Lemma 18 implies that when conflict is a possibility, there will be positive externalities at the investment stage. As is well known, in the presence of such externalities investment will be inefficiently low, worsening the war trap. Furthermore, those inefficiencies are likely to be greater in poor countries than in rich countries since those are unlikely to experience violent conflict in the first place.

We provide a simple examination of these ideas in section 4.1. A full fledged study of endogenous investment is beyond the scope of this paper.

4.1 A Simple Example with Endogenous Investment

We consider a country with symmetric groups and enrich the former model by having capital follow a simple recurrence equation: $k_{t+1} = (1 - \delta)k_t + d$, where d is a simple investment decision $d \in \{0, I\}$, associated with costs $C(0) = 0$ and $C(I) = C$. We also assume that the distribution of states of the world is uniform.

A unique investor. Let us first consider the case in which investment is made by a unique investor that takes into account her impact on conflict and has the possibility to commit to future investments. Assume that she obtains a benefit of the form Ak from holding a capital stock k . Her value function W satisfies the Bellman equation,

$$(22) \quad W(k_t) = \max_{d(\cdot) \in \{0, I\}^{\mathbb{R}}} Ak_t - C(d) + \beta \text{Proba}(\theta > \theta_{d(\cdot)}^{RD}(k_t)) W(k_{t+1})$$

We underline the fact that the investor takes into account that θ^{RD} depends on her policy

function $d(\cdot)$. From Lemma 18 we know that her value for additional investment increases with her capital stock. This implies that her optimal investment policy will take a threshold form. More precisely, there exists a threshold k^* such that her optimal investment rule is,

$$d(k) = \begin{cases} 0 & \text{if } k < k^* \\ I & \text{if } k \geq k^* \end{cases}$$

Whenever $k^* > 0$ there will be multiple steady states. One of them can be characterized as a war trap. More precisely, if $k_0 < k^*$, then conditionally on peace $\lim_{t \rightarrow +\infty} k_t = 0$. The country does not experience economic growth and hence the probability of conflict remains high in every period.

Note that in this setting, the hypothesis that the investor can commit to future investments is not binding. Whenever she finds it optimal to invest today, she will find it optimal to invest thereafter.

Multiple investors. We now assume that investment decisions are made by a continuum of investors. This implies that an investor will take the probability of war as given when making her investment decisions. Given a threshold function $\theta^{RD}(k)$, an investor's value function W satisfies the Bellman equation,

$$(23) \quad W(k_t) = \max_{d \in \{0, I\}} Ak_t - C(d) + \beta \text{Proba}(\theta > \theta^{RD}(k_t))W(k_{t+1})$$

It is possible for this game to have multiple equilibria. The exit game framework guarantees however that they have a relatively simple structure. An equilibrium of the current game is characterized by a policy function $d(\cdot)$ from investors and a conflict threshold function θ^{RD} from the two groups. Definition 2 introduced a partial order denoted \prec on peace and war decisions. We now introduce a natural order on investment decisions:

$$d \triangleleft d' \iff \forall k, d(k) \leq d'(k)$$

It is straightforward to show that the game between groups and investors exhibits monotonous best-responses with respect to \prec and \triangleleft . Thus the set of rationalizable strategies is bound by two extreme equilibria (d_H, θ_H^{RD}) and (d_L, θ_L^{RD}) , such that for all k ,

$$d_H(k) \geq d_L(k) \quad \text{and} \quad \theta_H^{RD}(k) < \theta_L^{RD}(k)$$

Moreover, these extreme equilibria can be obtained by iterating the best response mapping starting from the highest and the lowest possible pairs of strategies. It follows from this iteration process that the extreme equilibria are such that:

1. There exist $k^H < k^L$ such that

$$d_H(k) = I \iff k \geq k^H \quad \text{and} \quad d_L(k) = I \iff k \geq k^L$$

2. $\theta_H^{RD}(\cdot)$ and $\theta_L^{RD}(\cdot)$ are both increasing in k .

Finally, note that even in the high equilibrium, investment levels are lower than the socially efficient investment level, that is $k^H > k^*$, since decentralized investors do not take into account the positive externality they have on each other by making peace more likely.

Again, as long as $k^H > 0$, the model with multiple investors will exhibit war traps. Countries that happen to begin with capital levels below k^H experience no growth due to the high probability of conflict that poverty entails. But obviously, even if peace remains through a good string of states of the world, the probability of conflict is not reduced and the country cannot grow out of the trap to a better steady state in which economic growth generates a steady reduction of the threat of conflict. On the contrary, for countries that have a initial level of capital above k^H , the hazard rate is diminishing in the length of the period of peace, which reinforces investment and growth, thus accelerating the process.

5 Consequences for Intervention Strategies

From our analysis of conflict as coordination failure we can draw a number of policy prescriptions. There are two types of interventions that we observe in reality that we are interested in discussing : the first is economic aid in its various forms, the econd is peace keeping interventions in which soldiers from a third party are deployed between groups in potential conflict. With respect to economic aid, we make four points.

First we should be cautious about providing war relief to countries that are in open conflict to the extent that it also increases the likelihood of war. War relief has an unambiguously positive effect in really poor countries but may actually make relatively wealthy countries worse off.

Second, reducing inequality across groups within a country reduces the incidence of violence. This is a direct consequence of Lemma 9. The donor community should always direct

its transfers conditional on peace to the poorest group as the returns for the country as a whole are unambiguously higher thanks to the coordination effect.

Third, it is important to note that the previous point does not apply across countries. We envisage the donor community as having to decide the allocation of funds across a number of countries with symmetric groups that are locked in potential conflict. If this is the situation, then the transfers should typically not be directed to the poorest country in the sample. The reason is that the effect of increasing capital on the probability of peace is highly non-linear, especially if citizens are patient. This convexity is apparent, for instance, in figures 5 and 6. From these figures it is clear that an extra unit of capital given to a country that has $k = 2$ obtains better returns than given to a country with $k = 0$. Moreover, the existence of increasing returns over a range of capital implies that the optimal allocation does not entail spreading aid across countries even if they have the same level of income. Concentrating on a case at a time yields higher global reduction in the incidence of coordination failures.

Finally, if countries differ in the degree of patience that their citizens exhibit, aid should be directed to the most patient countries. This is a corollary from Lemma 13 that can be appreciated in Figures 3 and 4. The intuition behind this result was discussed above.

The model also sheds light on the role of peace keeping interventions. Peace-keeping operations are sent to mediate between contenders that have already reached a cease fire or truce of some sort. The focus of our framework on the expected duration of peace makes the exit structure especially adequate for this analysis.

First, note that in a completely stationary world a temporary intervention seems pointless: the probability of conflict is constant and hence whatever the consideration that prompted the intervention, it should also keep it in place. In other words, either a permanent intervention is optimal or there should be no intervention at all. However, the exit game with economic growth provides a rationale for a temporal intervention: as we have seen, the probability of conflict conditional on the time length of peace is rapidly diminishing because of the accumulation of capital in times of peace. Hence, forcing the contenders not to fight for a small number of periods may have huge welfare returns. Eventually the probability of peace is close enough to 1 and the returns to each additional period of intervention decline, which provides a rationale for finite time interventions.

Second, since peace keeping interventions only make sense in a context where peace permits some amount of economic growth, peace keeping interventions should be accompanied by measures encouraging investment. Those are in fact complementary instruments. Peace intervention make investment subsidies more effective and investment subsidies increase the

long term impact of peace keeping interventions.

Finally, endogenizing investment highlights that it is essential that peace corps be able to commit to stay for a minimum amount of time otherwise, peace interventions will have no impact on investment. With endogenous investment and i.i.d. states of the world, a lucky string of good states will have no effect on future peace whereas the same number of peaceful periods guaranteed *ex-ante* by a peace keeping intervention may trigger an investment boom that lifts the country out of the war trap.

6 Conclusion

In this paper we have presented a theory of intercommunal conflict that takes seriously the coordination problem central to the Security Dilemma. To emphasize the fear of being caught off guard as a driving force in this story, we focus on the risk-dominant equilibrium, which is a natural equilibrium concept when strategic risk considerations weigh heavily in the decision making process.

We show that the risk-dominant equilibrium displays behavior consistent with the agents trying to balance out the opportunity cost of violence with the fear of being attacked. In particular, the likelihood of conflict increases when the country is poor, the proceeds from looting are high and the offensive advantage large. The equilibrium also exhibits deterrence in the sense that reducing the payoffs in the situation of open conflict helps sustain peace. In addition, we show that inequality in income across groups is also conducive to violence while inequality in the offensive advantage is good for coordination into peaceful coexistence.

Besides this static model, we analyze a dynamic extension in which the game continues until a group defects and resorts to violence. This model allows us to examine the weight that the future has into current coordination. The value of peace increases because it contains the option value of continuing the game. This value is increasing in future economic growth and in patience. Hence, any expected positive future shock has effects in the current ability of groups to coordinate into peace. In reverse, expecting a bad economic situation for the future may trigger conflict today.

This dynamic version of the model allows us to extend the analysis to endogenous investment, unveiling the existence of a war trap. This is a situation in which poor countries do not invest because they expect conflict with high probability, which reinforces violence precisely because of the absence of economic growth. On the contrary, middle income countries can grow out of conflict by investing.

Finally, from the analysis, we draw policy prescriptions on two dimensions. First, the model provides as a corollary a characterization of the countries in which foreign economic aid would be most helpful. These are not the poorest countries, because the time-multiplier effect of the future generally induces increasing returns to capital. Second, the model provides a rationale for the use of temporary peace-keeping operations when there times of peace are accompanied by sufficient economic growth. It also suggests that investment enhancing measures would be strategic complements to peace-keeping measures.

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7 Appendix

Proof of Lemma 1. See Carlson and van Damme (1993). ■

Proof of Lemma 2. Recall equation (2). At θ^{RD} both equilibria exist which means that each of the factors on both sides are positive. Hence the left hand side is increasing in θ and Π_i and decreasing in M . The right hand side is decreasing in θ , Π_i and k and increasing in $S - W$. The comparative statics stated in the lemma are a direct consequence of these facts. ■

Proof of Lemma 3. Replace in expression (2): $\theta^{RD} = (S + M - W - \Pi)/2 = (r - W)/2$. ■

Proof of Lemma 4. We have

$$V = -WF(\theta^{RD}) + \int_{\theta^{RD}}^{+\infty} (\Pi_i + \theta) f_\theta d\theta$$

Hence,

$$\frac{\partial V}{\partial W} = -\frac{\partial \theta^{RD}}{\partial W} f_\theta(\theta^{RD}) [W + \Pi_i + \theta^{RD}] - F(\theta^{RD})$$

Since we know that $\theta^{RD} = (r - W)/2$, we obtain,

$$(24) \quad \frac{\partial V}{\partial W} = \frac{1}{4} f_\theta \left(\frac{r - W}{2} \right) [2\Pi + W + r] - F \left(\frac{r - W}{2} \right)$$

Thus,

$$\frac{\partial}{\partial \Pi} \left(\frac{\partial V}{\partial W} \right)_{S+M=\Pi+r} = \frac{1}{2} f_\theta \left(\frac{r - W}{2} \right)$$

Finally since $f_\theta(\frac{r-W}{2}) > 0$ and $F(\frac{r-W}{2}) > 0$, it's clear from expression (24) that for Π large enough, $\frac{\partial V}{\partial W} > 0$ and that for r , Π and W small enough, $\frac{\partial V}{\partial W} \sim -F(0) < 0$. ■

Proof of Lemma 6. See Chassang (2005). ■

Proof of Lemma 7. Consider a function $\mathbf{g} = (g_i, g_{-i})$, mapping \mathbb{R}^2 onto \mathbb{R}^2 . We define the norm of \mathbf{g} by $\|\mathbf{g}\| = \|g_i\|_\infty + \|g_{-i}\|_\infty$. Let us show that condition (11) implies that Φ is a

contraction mapping with rate $\lambda < 1$. Consider \mathbf{V} and \mathbf{V}' two possible pairs of continuation value functions and denote \mathbf{W} and \mathbf{W}' their respective images by Φ . From equation (9), we have,

$$\|\mathbf{W} - \mathbf{W}'\| \leq 4\|f_\theta\|_\infty \left| \frac{\partial \theta^{RD}}{\partial V} \right| \|\mathbf{V} - \mathbf{V}'\| (\bar{\Pi} + \theta^{RD} + \beta \bar{V} + W) + \beta P(\theta > \theta^{RD}) \|\mathbf{V} - \mathbf{V}'\|$$

Equation (4) implies $\left| \frac{\partial \theta^{RD}}{\partial V} \right| < \beta$, since in addition $\theta^{RD} < S - W$, we get that, $\|\mathbf{W} - \mathbf{W}'\| \leq \lambda \|\mathbf{V} - \mathbf{V}'\|$, with

$$\lambda = 4\beta \|f_\theta\|_\infty (\bar{\Pi} + S + \beta \bar{V}) + \beta < 1$$

This implies that Φ is a contraction mapping and that equilibrium is unique. \blacksquare

Proof of Lemma 8. Given continuation values V_i, V_{-i} , the equation defining the risk dominant threshold is

$$(25) \quad (\Pi_A + \beta V_A + \theta^{RD} - M)(\Pi_B + \beta V_B + \theta^{RD} - M) = (S - W - \theta^{RD})^2$$

At θ^{RD} , $\Pi_i + \beta V_i + \theta^{RD} - M > 0$ and $S - W - \theta^{RD} > 0$, thus the left hand side is increasing in θ , Π_i , βV_i and decreasing in M , while the right hand side is decreasing in θ , and increasing in $S - W$. Therefore, θ^{RD} is increasing in M and S and decreasing in Π_i, V_i and W . We thus get that the first part of Lemma 8 implies the second. We can consider V_i as a function of Π_i, Π_{-i}, M, W, S and the mapping Φ as some mapping from bounded value functions to bounded value functions. From Assumption 1, Φ is a contraction mapping over the range of parameters we are concerned with. To any vector of functions \vec{V} increasing in Π_i , decreasing in S and M , Φ associates a vector of functions that's also increasing in Π_i , decreasing in S and M . By iteratively applying Φ , this gives us the first part of the lemma. \blacksquare

Proof of Lemma 9. V_i and V_{-i} can be seen as functions mapping (k_i, k_{-i}) to real numbers. Those functions can be computed by iterating the contraction mapping Φ starting from some initial vector of functions \vec{V}^0 . This iterative process produces sequences of value functions $\{\vec{V}^k\}_{k \in \mathbb{N}}$ and thresholds $\{\theta^{RD,k}\}_{k \in \mathbb{N}}$. We prove Lemma 9 by showing that when the initial value functions \vec{V}^0 *weakly* satisfy inequality (13), then all elements of $\{\theta^{RD,k}\}_{k \in \mathbb{N}}$ *strictly* satisfy (12) and all elements of $\{\vec{V}^k\}_{k \in \mathbb{N}}$ *strictly* satisfy (13).

We first begin by showing that if \vec{V}^k weakly satisfies (13) then $\theta^{RD,k}$ strictly satisfies (12).

Recall the equation defining θ^{RD} :

$$(\theta + \Pi(k_i) + \beta V_i - M)(\theta + \Pi(k_{-i}) + \beta V_{-i} - M) = (S - \theta - W)^2$$

Differentiate this equation with respect to k_i . We obtain that,

$$\begin{aligned} [\Pi(k_i) + \beta V_i + \Pi(k_{-i}) + \beta V_{-i} + 2(S - M - W)] \frac{\partial \theta}{\partial k_i} &= -\frac{\partial(\Pi + \beta V_i)}{\partial k_i} (\theta + \Pi_{-i} + \beta V_{-i} - M) \\ &\quad - \beta \frac{\partial V_{-i}}{\partial k_i} (\theta + \Pi_i + \beta V_i - M) \end{aligned}$$

Recall that by assumption $[\Pi(k_i) + \beta V_i + \Pi(k_{-i}) + \beta V_{-i}]$ is positive. It's also symmetric in i and $-i$. Therefore to prove that $\theta^{RD,k}$ satisfies inequation (12), we only need to prove that,

$$\begin{aligned} \Delta \equiv & -\frac{\partial(\Pi(k_i) + \beta V_i)}{\partial k_i} (\theta + \Pi(k_{-i}) + \beta V_{-i} - M) - \beta \frac{\partial V_{-i}}{\partial k_i} (\theta + \Pi(k_i) + \beta V_i - M) \\ & + \frac{\partial(\Pi(k_{-i}) + \beta V_{-i})}{\partial k_{-i}} (\theta + \Pi(k_i) + \beta V_i - M) + \beta \frac{\partial V_i}{\partial k_{-i}} (\theta + \Pi(k_{-i}) + \beta V_{-i} - M) < 0 \end{aligned}$$

We can write,

$$\begin{aligned} \Delta = & -[\theta + \Pi(k_{-i}) + \beta V_{-i} - M] \left(\Pi'(k_i) + \beta \frac{\partial V_i}{\partial k_i} - \beta \frac{\partial V_{-i}}{\partial k_{-i}} \right) \\ & + [\theta + \Pi(k_i) + \beta V_i - M] \left(\Pi'(k_{-i}) + \beta \frac{\partial V_{-i}}{\partial k_{-i}} - \beta \frac{\partial V_i}{\partial k_i} \right) \end{aligned}$$

From the fact that $[\theta + \Pi(k_{-i}) + \beta V_{-i} - M] > [\theta + \Pi(k_i) + \beta V_i - M] > 0$ and the fact that \vec{V}^k satisfies (13), simple algebra shows that indeed $\Delta < 0$.

Let us now show that if $\theta^{RD,k}$ and \vec{V}^k respectively satisfy inequalities (12) and (13) then \vec{V}^{k+1} satisfies (13). By definition, $\vec{V}^{k+1} = \Phi(\vec{V}^k)$, thus,

$$\begin{aligned} \frac{\partial V_i^{k+1}}{\partial k_i} - \frac{\partial V_i^{k+1}}{\partial k_{-i}} &= \left[\Pi'(k_i) + \beta \left(\frac{\partial V_i^k}{\partial k_i} - \frac{\partial V_i^k}{\partial k_{-i}} \right) \right] P(\theta^{RD,k} \leq \theta) \\ &\quad + f_\theta(\theta^{RD,k})(-W - \Pi_i - \theta^{RD,k} - \beta V_i^k) \left(\frac{\partial \theta^{RD,k}}{\partial k_i} - \frac{\partial \theta^{RD,k}}{\partial k_{-i}} \right) \end{aligned}$$

By assumption, $\frac{\partial V_i^k}{\partial k_i} - \frac{\partial V_i^k}{\partial k_{-i}} \geq \frac{\partial V_{-i}^k}{\partial k_{-i}} - \frac{\partial V_{-i}^k}{\partial k_i}$ and $\frac{\partial \theta^{RD,k}}{\partial k_i} - \frac{\partial \theta^{RD,k}}{\partial k_{-i}} < 0$. This implies that indeed,

$$\frac{\partial V_i^{k+1}}{\partial k_i} - \frac{\partial V_i^{k+1}}{\partial k_{-i}} > \frac{\partial V_{-i}^{k+1}}{\partial k_{-i}} - \frac{\partial V_{-i}^{k+1}}{\partial k_i}$$

Applying Φ iteratively, the sequences $\{\mathbf{V}^k\}_{k \in \mathbb{N}}$ and $\{\theta^{RD,k}\}_{k \in \mathbb{N}}$ converge to the equilibrium \mathbf{V} and θ^{RD} . Inequalities (12) and (13) hold weakly at the limit. In addition, the proof shows that if (12) and (13) hold weakly at a fix point, they must hold strictly by iteration of Φ . ■

Proof of Lemma 10. Consider the case where $V_A(\beta = 0) < 0$ and $V_B(\beta = 0) < 0$. The equilibrium \vec{V} is a function of β that can be obtained by iterating the contraction mapping Φ . Assume that \vec{V}^0 is negative and decreasing in β . Then $\theta^{RD,0}$ is increasing in β which implies that \vec{V}^1 is decreasing in β . Begin the iteration at $\vec{V}^0 = (V_A(\beta = 0), V_B(\beta = 0))$. Then for all iterated \vec{V}^k , we have $\vec{V}^k(\beta = 0) = (V_A(\beta = 0), V_B(\beta = 0))$. Therefore \vec{V}^k decreasing in β also implies that \vec{V}^k is negative. We can thus conclude by iteration that all \vec{V}^k are negative and decreasing in β . The proof for the case with $V_A(\beta = 0) > 0$ and $V_B(\beta = 0) > 0$ identical. ■

Proof of Lemma 11. Denote $\tilde{V}_i = V_i + \frac{1}{1-\beta}h$. Let us show that \tilde{V}_i is a fixed point of $\tilde{\Phi}$. First note from expression (10) that with this \tilde{V}_i , we must have $\tilde{\theta}^{RD} = \theta^{RD}$. Finally, all payoffs in the expression of $\tilde{\Phi}$ are shifted by a term $\frac{1}{1-\beta}h$. Thus \tilde{V}_i is the unique equilibrium continuation value of the new game with shifted payoffs. This implies that the equilibrium thresholds are indeed the same. ■

Proof of Lemma 12. Consider the case where $\beta = 0$. Using the notations of lemma 11, pick $h = w$, then $V_i + \frac{1}{1-\beta}h > 0$. Therefore, using Lemma 11, we can equivalently study players behavior in a game where $V_A(\beta = 0) > 0$ and $V_B(\beta = 0) > 0$ and $w = 0$.

As before, we use a proof by induction. Assume that \vec{V}^k is positive and increasing in β . Note that $S - W$ is constant in β and that M is decreasing in β . Therefore, we know from expression (10) that the risk-dominant threshold is decreasing in β . From the expression of Φ , this implies that \vec{V}^{k+1} is increasing in β . Now consider the particular sequence of continuation values started from $\vec{V}^0 = (V_A(\beta = 0), V_B(\beta = 0))$. Then for all k , $\vec{V}^k(\beta = 0) = (V_A(\beta = 0), V_B(\beta = 0))$. For this particular sequence, being increasing in β also implies that all values \vec{V}^k are strictly positive which finishes the induction step. For all k , \vec{V}^k is strictly positive and increasing in β and $\theta^{RD,k}$ is decreasing in β .

These properties hold at the limit. ■

Proof of Lemma 13. From Lemma 11, without loss of generality, we consider a game for which $W = 0$. As usual, we prove the result by iteratively applying the contraction mapping

Φ . In particular we show that if \vec{V}^0 is symmetric and $\theta^{RD,0}$ and \vec{V}^0 satisfy conditions (14) and (15) then, $\theta^{RD,1}$ and \vec{V}^1 also satisfy conditions (14) and (15). Let us show that if \vec{V}^1 satisfies condition (15) then $\theta^{RD,1}$ satisfies inequality (14). Indeed, we have that,

$$\begin{aligned} [\Pi(k_i) + \beta V_i + \Pi(k_{-i}) + \beta V_{-i} + 2(S - M - W)] \frac{\partial \theta^{RD,1}}{\partial k_i} &= -\frac{\partial(\Pi + \beta V_i^1)}{\partial k_i} (\theta^{RD,1} + \Pi_{-i} + \beta V_{-i}^1 - M) \\ &\quad - \beta \frac{\partial V_{-i}^1}{\partial k_i} (\theta^{RD,1} + \Pi_i + \beta V_i^1 - M) \end{aligned} \quad (26)$$

Which yields, differentiating with respect to β ,

$$\begin{aligned} \frac{\partial^2 \theta^{RD,1}}{\partial k_i \partial \beta} [\Pi(k_i) + \beta V_i^1 + \Pi(k_{-i}) + \beta V_{-i}^1 + 2(S - M - W)] &+ \frac{\partial \theta^{RD,1}}{\partial k_i} \left[V_i^1 + V_{-i}^1 + \beta \frac{\partial V_i^1}{\partial \beta} + \beta \frac{\partial V_{-i}^1}{\partial \beta} \right] \\ &= -\frac{\partial \Pi_i + \beta V_i^1}{\partial k_i} \left[\frac{\partial \theta^{RD,1}}{\partial \beta} + V_{-i}^1 + \beta \frac{\partial V_{-i}^1}{\partial \beta} \right] - \beta \frac{\partial V_{-i}^1}{\partial k_i} \left[\frac{\partial \theta^{RD,1}}{\partial \beta} + V_i^1 + \beta \frac{\partial V_i^1}{\partial \beta} \right] \\ &- \left[\beta \frac{\partial^2 V_i^1}{\partial k_i \partial \beta} + \frac{\partial V_i^1}{\partial k_i} \right] (\theta^{RD,1} + \Pi(k_{-i}) + \beta V_{-i} - M) - \left[\beta \frac{\partial^2 V_{-i}^1}{\partial k_i \partial \beta} + \frac{\partial V_{-i}^1}{\partial k_i} \right] (\theta^{RD,1} + \Pi(k_i) + \beta V_i - M) \end{aligned}$$

Since the two ethnic groups are symmetric,

$$V_i^1 = V_{-i}^1 \quad \text{and} \quad \frac{\partial V_i^1}{\partial \beta} = \frac{\partial V_{-i}^1}{\partial \beta}$$

Moreover it is clear from equation (15) that,

$$-\left[\beta \frac{\partial^2 V_i^1}{\partial k_i \partial \beta} + \frac{\partial V_i^1}{\partial k_i} \right] (\theta^{RD,1} + \Pi(k_{-i}) + \beta V_{-i} - M) - \left[\beta \frac{\partial^2 V_{-i}^1}{\partial k_i \partial \beta} + \frac{\partial V_{-i}^1}{\partial k_i} \right] (\theta^{RD,1} + \Pi(k_i) + \beta V_i - M) < 0$$

Thus, to prove that equation (14) holds, it is sufficient to prove that,

$$(27) \quad -\frac{\partial \theta^{RD,1}}{\partial k_i} \left[2V_i^1 + 2\beta \frac{\partial V_i^1}{\partial \beta} \right] - \left[\frac{\partial \Pi_i + \beta V_i^1}{\partial k_i} + \beta \frac{\partial V_{-i}^1}{\partial \beta} \right] \left[\frac{\partial \theta^{RD,1}}{\partial \beta} + V_i^1 + \beta \frac{\partial V_i^1}{\partial \beta} \right] \leq 0$$

Since we are in the symmetric case, $\theta^{RD,1} = (S - W + M + \Pi_i + \beta V_i)/2$. Thus,

$$\frac{\partial \theta^{RD,1}}{\partial \beta} = -\frac{1}{2} \left[V_i + \beta \frac{\partial V_i}{\partial \beta} \right]$$

Which yields that condition (27) is equivalent to $-\frac{\partial \theta^{RD,1}}{\partial k_i} - \frac{1}{4} \left[\frac{\partial \Pi_i + \beta V_i^1}{\partial k_i} + \beta \frac{\partial V_{-i}^1}{\partial \beta} \right] \leq 0$. This

is indeed the case: combining equation (26) and the expression of $\theta^{RD,1}$, we get that in fact,

$$-\frac{\partial\theta^{RD,1}}{\partial k_i} - \frac{1}{4} \left[\frac{\partial\Pi_i + \beta V_i^1}{\partial k_i} + \beta \frac{\partial V_{-i}^1}{\partial\beta} \right] = 0$$

This concludes the first step of the proof. We now show that when $\theta^{RD,0}$ and \vec{V}^0 satisfy inequalities (14) and (15), then \vec{V}^1 satisfies (15). We have

$$\begin{aligned} \frac{\partial^2 V_i^1}{\partial k_i \partial \beta} &= P(\theta > \theta^{RD,0}) \left(\frac{\partial V_i^0}{\partial k_i} + \beta \frac{\partial^2 V_i^0}{\partial k_i \partial \beta} \right) - f_\theta(\theta^{RD,0}) \frac{\partial \theta^{RD,0}}{\partial \beta} \beta \frac{\partial V_i^0}{\partial k_i} \\ &\quad - f'_\theta(\theta^{RD,0}) \frac{\partial \theta^{RD,0}}{\partial \beta} \frac{\partial \theta^{RD,0}}{\partial k_i} (\Pi_i + \beta V_i + \theta^{RD,0}) \\ &\quad - f_\theta(\theta^{RD,0}) \frac{\partial^2 \theta^{RD,0}}{\partial k_i \partial \beta} (\Pi_i + \beta V_i + \theta^{RD,0}) \\ &\quad - f(\theta^{RD,0}) \frac{\partial \theta^{RD,0}}{\partial k_i} \left(\frac{\partial \theta^{RD,0}}{\partial \beta} + V_i + \frac{\partial V_i}{\partial \beta} \right) \end{aligned}$$

Since we have already shown that $\frac{\partial \theta^{RD,0}}{\partial \beta} + V_i + \frac{\partial V_i}{\partial \beta} \geq 0$, simple manipulations of the previous expression show that $\frac{\partial^2 V_i^1}{\partial k_i \partial \beta} > 0$. Similar reasoning shows that $\frac{\partial^2 V_{-i}^1}{\partial k_{-i} \partial \beta} > 0$. This concludes the induction. ■

Proof of Lemma 14. The proof by iteration given in Lemma 8 can be adapted in a straightforward manner. ■

Proof of Lemma 16. The proof of Lemma 9 goes through, replacing ks by zs . ■

Proof of Lemma 15. The proof of Lemma 13 can be adapted in a straightforward manner. ■

Proof of Lemma 17. We have that,

$$V_{i,t} = \mathbf{E}[-W \mathbf{1}_{\theta \leq \theta^{RD}} + (\Pi_{i,t} + \beta V_{i,t+1} + \theta) \mathbf{1}_{\theta \geq \theta^{RD}}]$$

Thus,

$$\frac{\partial V_{i,1}}{\partial \Pi_{i,2}} = \beta P(\theta \geq \theta^{RD}) \frac{\partial V_{i,2}}{\partial \Pi_{i,2}} - \beta \frac{\partial V_{i,2}}{\partial \Pi_{i,2}} \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} (\Pi_{i,1} + \beta V_{i,2} + \theta^{RD} + W) f_\theta(\theta^{RD})$$

Noting that $\frac{\partial V_{i,2}}{\partial \Pi_{i,2}} = \frac{\partial V_{i,1}}{\partial \Pi_{i,1}}$, we obtain that,

$$\frac{\frac{\partial V_{i,1}}{\partial k_{i,2}}}{\frac{\partial V_{i,1}}{\partial k_{i,1}}} = \frac{\frac{\partial V_{i,1}}{\partial \Pi_{i,2}}}{\frac{\partial V_{i,1}}{\partial \Pi_{i,1}}} = \beta P(\theta \geq \theta^{RD}) - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} f_{\theta}(\theta^{RD}) [\Pi_i(k_i) + \beta V_i + \theta^{RD} + W]$$

■

Proof of Lemma 18. From the previous lemma we have that,

$$\rho = \beta P(\theta \geq \theta^{RD}) - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} f_{\theta}(\theta^{RD}) [\Pi_i(k_i) + \beta V_i + \theta^{RD} + W]$$

Thus,

$$\begin{aligned} \frac{\partial \rho}{\partial k} &= -\beta f_{\theta}(\theta^{RD}) \frac{\partial \theta^{RD}}{\partial k} - \beta \frac{\partial}{\partial k} \left(\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} \right) f_{\theta}(\theta^{RD}) [\Pi(k) + \beta V + \theta^{RD} + W] \\ &\quad - \beta f'_{\theta}(\theta^{RD}) \frac{\partial \theta^{RD}}{\partial k} \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} [\Pi(k) + \beta V + \theta^{RD} + W] - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} \left[\frac{\partial \Pi}{\partial k} + \beta \frac{\partial V}{\partial k} + \frac{\partial \theta^{RD}}{\partial k} \right] \end{aligned}$$

Since capital stocks are symmetric, we have that $\theta^{RD} = (M + S - \Pi(k) - \beta V - W)/2$. This implies that

$$\frac{\partial}{\partial k} \left(\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} \right) = 0$$

and

$$\frac{\partial \Pi}{\partial k} + \beta \frac{\partial V}{\partial k} + \frac{\partial \theta^{RD}}{\partial k} > 0$$

Thus, we have indeed, $\frac{\partial \rho}{\partial k} > 0$. ■