Cooperation in *Chicken* and *Stag Hunt*

Sandeep Baliga and Tomas Sjöström

MEDS and Rutgers

November 2007

Introduction

- Stag Hunt, Chicken and Prisoners' Dilemma represent canonical strategic interactions.
- Stag Hunt: aggression feeds on itself and escalates conflict (actions are *strategic complements*)
- Chicken: toughness deters aggression (actions are *strategic substitutes)*
- Stag hunt and chicken have multiple Nash equilibria. Jervis, Schelling: mutual fear and uncertainty determine the outcome.
- Enough private information (about preferences or costs) selects a *unique* equilibrium. (Unlike global games, no need to assume highly correlated private information.)
- Are long-run relationships more peaceful? Is democracy good for peace? (Other issues: communication? mediation? signalling?)

The one-shot game

- Payoffs for player i (row-chooser)

$$\begin{array}{ccc} & H & D \\ H & h_i - c & h_i \\ D & -d & 0 \end{array} \tag{1}$$

- Stag-hunt: $c < d$. Chicken: $c > d$
- Player i's *hostility* h_i is his private information. Distribution of h_i conditional on $h_j = y$ is $F(\cdot|y)$. Define $\phi(y) \equiv F(y|y)$

Assumption 1. (i) $\frac{\partial F(x|y)}{\partial y} \leq 0$ (a more hostile player i is more pessimistic about j's hostility) (ii) $\phi'(y) > 0$ (a more hostile player i thinks it is more likely that he is more hostile than j).

- A unique equilibrium exists if there is enough uncertainty: if ϕ' is not too big.

Stag Hunt

- *Strategic complements:* H more appealing if the opponent is expected to choose H $(c < d)$
- Player i is a *dominant strategy hawk* if $h_i \geq 0$. Player i is a *dominant strategy dove* if $h_i - c \leq -d$.

Theorem

If $\phi'(h) < \frac{1}{d-c}$ then stag hunt has a unique BNE. This BNE is in symmetric cut-off strategies.

- Both dominant strategy hawks and dominant strategy doves exist if $\bar{h} > 0$ and $\underline{h} < c - d$. In this case, the unique equilibrium must be interior. The cut-off point $\tilde{h} \in (\underline{h}, \bar{h})$ solves

$$\tilde{h} + (d - c)\left(1 - \phi(\tilde{h})\right) = 0$$

Chicken

- *Strategic substitutes:* H less appealing if the opponent is expected to choose H ($c > d$)
- If there is enough uncertainty, and a technical condition holds, then a unique equilibrium exists in chicken as well.

Repeated Stag Hunt

- Intuition suggests that long-run stag hunt relationships are peaceful, if the players can "build trust".
- Suppose the stag-hunt game is played twice. Types h_1 and h_2, are independently drawn before the first round, and remain the same in both rounds. Between the two rounds, they learn what the opponent did in round 1.
- The distribution of types is uniform on $[\underline{h}, \bar{h}]$, where $\bar{h} - \underline{h} = 1$, $c < \bar{h} < d$ and $\underline{h} < c - d$.
- A strong incentive to choose D in round 1 in order to encourage the opponent to choose D in round 2. Since they reveal some information in round 1, there is less "fear" in round 2, so there can be less conflict in round 2 as well.

Theorem

The twice repeated stag hunt has a PBE where in each round the probability of conflict is strictly lower than in the unique equilibrium of the one-shot game.

- Is democracy good for peace in stag hunt games ?
- Between rounds, the median voter in each country can elect a new leader, drawn again from the uniform distribution on $[\underline{h}, \bar{h}]$.
- Democracy interfers with reputation-building.
- If leader i chooses H in round 1 then he reveals that he is hawkish. If he stays in power, leader j will be very likely to choose H in round 2 by strategic complements. To avoid this, the median voter of country i will prefer to elect a new leader.
- Two opposite effects. First, if leaders who choose H are replaced, the cost of a "bad reputation" can be avoided. This *increases* the incentive to choose H in round 1. Second, if the leader who chooses H in round 1 is replaced, he loses the "rents from office", which *reduces* the incentive to choose H in round 1. The second effect is small if rents are small. Thus, if rents from office are small, democracy is bad for peace.

Repeated Chicken

- Intuition suggests that long-run chicken relationships are hostile, because neither party wants to be perceived as weak.

"If we had bent our back, they would immediately have thrown a saddle on us, and then they would have sat themselves on top of us and begun to drive on us." Khrushchev

- Model as repeated stag hunt, but $c > d$. If c is much bigger than d, multiple equilibria tend to exist. So suppose $c - d < \min\{\bar{h}, -\underline{h}\}$.
- A strong incentive to choose H in round 1 in order to encourage the opponent to choose D in round 2. Indeed, can show that the unique cut-off equilibrium is such that if first-round outcome is (H, D), then second round outcome must be (H, D) as well.

Theorem

The twice repeated chicken game has a unique cut-off PBE, where in round 1 the probability of conflict is strictly greater than in the unique equilibrium of the one-shot game.

- Is democracy good for peace in chicken games ?
- If leader i chooses D in round 1 then he reveals that he is dovish. If he stays in power, leader j will be very likely to choose H in round 2 by strategic substitutes. To avoid this, the median voter of country i will prefer to elect a new leader.
- Two opposite effects. First, if leaders who choose D are replaced, the cost of a "reputation for weakness" can be avoided. This *increases* the incentive to choose D in round 1. Second, if the leader who chooses D in round 1 is replaced, he loses the "rents from office", which *reduces* the incentive to choose D in round 1. The second effect is small if rents are small. Thus, if rents from office are small, democracy is good for peace.