Political Institutions and Group Identity

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(PRELIMINARY AND INCOMPLETE DRAFT. PLEASE DON’T QUOTE!)
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1 Introduction

Empirically, the formation of political attitudes has been strongly linked to the concept of group identification. Classifications such as race, gender, religion and ethnicity have been shown to be sharp predictors of many aspects of political life including who turns out to vote, how people vote, and the issues that are important to people. Furthermore, many topics that are of interest to political scientists, such as leadership, majority-minority relationships, and coalition formation, are fundamentally group phenomena. However, the relationship between institutions and group identification has received little attention in the formal literature. This relationship is important because individual perceptions of institutional features such as procedural justice and distributive equity have been shown capable of heightening or easing levels of intergroup conflict in certain circumstances.

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The effect of political institutions on group identity and conflict is particularly salient in newly emerging democracies. The drafters of the United States constitution wrote passionately of the need to design institutions that would mitigate the problem of “factions.” Madison conceived of factions as:

... a number of citizens, whether amounting to a majority or minority of the whole, who are united and actuated by some common impulse of passion, or of interest, adverse to the rights of other citizens, or to the permanent and aggregate interests of the community. [2]

Similarly, in recent times experts have been called upon to design political systems to help ameliorate bitter ethnic rivalries and tensions. Playing in to the design of these systems are issues of representation, legitimacy, political moderation and feasibility. For example, United Nations electoral specialist Carina Perelli decided upon a single-district list system of proportional representation for the recent Iraqi legislative elections. Perelli’s rationale was driven in large part by concerns that the new system fully represent Iraq’s ethnic diversity while not aggravating ethnic tensions. Perelli argued that a single district would reduce regional factions and tribalism while allowing for the possibility of historically repressed and displaced communities of interest to accumulate their votes. Issues of intergroup conflict are not only of relevance in newly emerging democracies—these issues are also of concern in established democracies such as Germany, many of which have electoral laws banning extremist parties from fielding candidates.

In this paper I model the interplay between political institutions and the formation of group identities. I assume that each individual belongs to a fixed collection of social groups, and may choose to identify with one of these groups. A group could, for example, consist of individuals of a particular ethnicity, religion, race, gender or nationality. When an individual identifies with a group, he both wants to resemble other members of that group, and cares about the relative status of that group in relation to other groups in society.

In modeling an individual’s choice of group identification I utilize the concepts of social
identity theory (SIT) defined by Tajfel and Turner [4] and self-categorization theory (SCT) defined by Turner [6]. Social identity theory posits that individuals seek to enhance their own self-esteem, choose to identify with groups that positively affect their self-esteem, and evaluate their own groups by social comparisons to other groups. Self-categorization theory recognizes that these choices may change over time, and whether or not an individual chooses to identify with a group is a function of an individual’s present situation, or state of the world (Turner et al., [5]).

Political institutions affect an individual’s choice of group identity by dictating how individual and group preferences are translated into states of the world. “Political institutions” in this model are parameterized in part as a distortion between the voting weight each group is allotted and its size relative to the total population. This definition is in keeping with much of the literature on comparative electoral systems, which is concerned largely with the proportionality profiles of different electoral rules. Highly proportional systems minimize the distortion between the percentage of votes a group wins and the percentage of seats it is allocated in parliament. Measures of proportionality such as the Gallagher index are commonly used to evaluate different electoral systems, with the general finding that majoritarian and plurality systems are the least proportional, and the list systems of proportional representation the most proportional (although within the list systems there is considerable variation depending upon the electoral quota or series of divisors used). Similarly, an increase in district magnitude generally increases the proportionality of a system, and higher electoral thresholds decrease proportionality.

(**Literature review, discussion of results, etc. to come**)
2 The model

2.1 Individuals and their group identities

Consider a society that consists of a finite collection of social groups. Every individual belongs to some subset of this collection of groups, but an individual may or may not choose to personally identify with a group that he belongs to. Identification with a group implies that the individual cares about the relative status of that group in relation to other groups, and enjoys resembling other members of that group. Individuals make a cognitive choice over which social groups to identify with. These choices over group memberships affect the preferences of citizens, and thus affect any choice of social policy made by society.

Society $N$ consists of a continuum of citizens $i \in N$, each indexed by a real number on the $[0, 1]$ interval. Given a fixed political institution (described in the following section), society makes a decision over a policy space $X \subseteq \mathbb{R}^M$ with generic element $x \in X$. Each citizen $i$ has a strictly quasi-concave and additively separable material payoff function $\pi_i : X \rightarrow \mathbb{R}$, that describes his personal preferences over the policy space. Citizen $i$ has an ideal point, $p_i \in X$, that maximizes his material payoff function, with $p^k_i$ being the policy that maximizes $i$’s material payoff function along policy dimension $k$. When $X$ is a continuum, ideal points are distributed over $X$ according to a probability density function $f$, with cumulative density function $F$. When $X$ is finite, $f$ is a probability mass function.

Society can be partitioned into an exogenous and finite collection of distinct groups, $G = \{g_1, \ldots, g_{|G|}\}$, with generic element $g \in G$. Any particular group $g$ is a strict subset of $N$ and is measurable with respect to the ideal points of its members. Let $G_i \subseteq G$ denote the collection of groups that individual $i$ is a member of. Then $i$ faces a choice over which $g_i \in G_i$ to identify with, as he may identify with only one particular group at a time.\footnote{It is assumed that individuals may only identify with a single group. While it is clear that many people strongly identify with multiple groups in the real world, I leave the topic of social identification with multiple groups to later research. To defend my choice of focusing only on pure strategies I note that in real-world interactions, the group memberships of individuals are often fixed and unchangeable.}

An
action $a_i \in A_i$ for individual $i$ is a choice of group identity. Individual $i$ can also choose $a_i = \emptyset$, which represents a choice of no group identification. Thus, $i$’s choice set $A_i$ can be defined as $A_i = G_i \cup \{\emptyset\}$. Let $a = \{a_i\}_{i \in N}$ be an action profile, and let $A$ be the set of all action profiles. Let $A = G \cup \{\emptyset\}$ be the set of all possible actions.

Individuals receive utility from both a policy outcome $x \in X$ and from their choice of group identity, $a_i$. Furthermore, individuals receive utility from an interaction between these two variables, because any given policy outcome may affect different groups differently, and individuals care about the relative status of their group in relation to other groups. Utility functions are assumed to be strictly quasi-concave and additively separable across dimensions, and are of the form:

$$u_i(\pi_i(x), R(a_i, x))$$

where $a_i \in A_i$ is the group that individual $i$ chooses to identify with and $R : A \times X \to \mathbb{R}$ is the relative status of identification with $a_i$ given policy $x$. As above, $\pi_i$ is $i$’s material payoff function. Utility is assumed to be increasing in both arguments.

Finally, note that individual utility functions are dependent upon actions. For each individual $i \in N$, $u_i$ is a function of both policy $x$ and identity choice $a_i$. For ease of notation it will be useful to think of utility as a function of $x$ and $a_i$. Let $v_i(x, a_i) = u_i(\pi_i(x), R(a_i, x))$ be termed individual $i$’s subjective utility function. Similarly, $v_i(x|a_i) = u_i(\pi_i(x), R(x|a_i))$ is individual $i$’s subjective utility function conditional upon a particular action $a_i \in A_i$ and $v_i(a_i|x) = u_i(\pi_i(x), R(a_i|x))$ is individual $i$’s subjective utility function conditional upon a particular policy $x \in X$.

Let $v(x, a) = \{v_i(x, a_i)\}_{i \in N}$ and $v(x|a) = \{v_i(x|a_i)\}_{i \in N}$ be profiles of subjective utilities and subjective utilities conditional upon action profile $a$ respectively, and let $V$ and $V_a$ be sets of all such profiles. Conditional subjective utilities are important because a given institution (as defined in the following section) only responds to an individual’s utility settings intergroup judgments frequently revolve around a single dimension of group differentiation. See [1], pp. 554-594.
function as specified by his choice of group identity. In other words, when individuals vote they have a well-defined set of preferences that affect their voting decision. These preferences are dependent upon the individual’s choice of group identity, and cannot be altered by the institution itself.

### 2.2 Political institutions

Political institutions in this model govern how the preferences of different societal groups are aggregated into policy. If an institution favors one group over another, then the policy preferences of members of the favored group are weighed more heavily in determining national policy. Intuitively, we can think of an institutionally favored group as being more capable of winning legislative seats (but once a legislature is chosen the preferences of each legislator count equally in determining policy). The voting weight given to a group by a given institution can be a function of many different factors, the most obvious of which is the electoral formula used within the institution. However, other characteristics of a political institution can also affect proportionality; examples include minimum electoral thresholds, district boundaries, ballot structure, voting and candidacy requirements, and access to the media. Thus, political institutions are not necessarily equivalent to electoral rules.

Formally, a political institution consists of two parts: a weighting function $\rho$ and an aggregation rule $s$. Let $|X| = M$. Then function $\rho$ is denoted $\rho = \{(\rho^k_1, \ldots, \rho^k_{|G|})\}_{k=1}^M$, where for all $g \in G$ and $k \leq M$, $\rho^k_g > 0$ and $\sum_{g \in G} \rho^k_g = 1$. Thus each $\rho^k$ is a probability distribution over $G$. Let $\mathcal{P}$ be the set of all weighting functions, with $\rho \in \mathcal{P}$. The term $\rho^j_k$ represents the voting weight group $g_j$ is given in the political institution’s determination of national policy along dimension $k$. For example, if $\rho^k_g = \frac{\int_{i \in g} \frac{dF(p_i)}{\int_{i \in h} dF(p_i)}}{\sum_{h \in G} \int_{i \in h} dF(p_i)}$ for all groups $g$ and along all dimensions $k$, then the system is completely proportional, in that the interests

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2The weighing function could be incorporated into the aggregation rule. I prefer to define them separately in part because of future work I plan to do on this topic concerning distributive versus procedural equity.
of each group are weighted exactly in proportion to the group’s relative size: the legislative body is a perfect microcosm of society.  

Aggregation rule $s$ translates the weighting function $\rho$ and the preferences of individuals (contingent upon a prior choice of group identity) into a specific policy outcome. Thus $s : P \times V_a \rightarrow X$. Let $S$ be the set of all aggregation rules. This paper will focus primarily on the following three aggregation rules: dimension-by-dimension majority rule, convex combinations of voters’ utility-maximizing policies, and a social planner. The outcome induced by a given institution is denoted $s(\rho, v(\cdot|a)) \in X$. Note that this outcome takes action profile $a$ to be fixed.

**Dimension-by-dimension majority rule: $s_{\text{dim}}$**

Institution $(s_{\text{dim}}, \rho)$ produces outcome $s_{\text{dim}}(\rho, v(\cdot|a))$ equal to the utility-maximizing policy of the dimension-by-dimension weighted median voter. To characterize $s_{\text{dim}}(\rho, v(\cdot|a))$ we need definitions of a *decisive coalition* and an *induced median voter* in this setting.

A *decisive coalition* on dimension $k$, or $D(s_{\text{dim}}, \rho)$, in this setting is a collection of measurable sets of individuals whose voting weights sum to a number greater than $\frac{1}{2}$ on a given policy dimension. Let $D(s_{\text{dim}}, \rho)$ denote the set of all decisive coalitions induced by political institution $(s_{\text{dim}}, \rho)$. Formally, $D \in D(s_{\text{dim}}, \rho) \iff \left[ \sum_{g \in G} \rho_g^k \int_{i \in D} 1_{i \in D} dF(p_i^k) \right] \geq \frac{1}{2}$, where $1_{i \in D}$ is an indicator function that equals 1 if $i \in D$ and 0 otherwise and $k$ is a policy dimension.

Since it is assumed that utility functions are additively separable and single-peaked, there is a unique median voter induced by political institution $(s_{\text{dim}}, \rho)$ and action profile $a$ along any given dimension $k$. Define the *median voter induced by institution $(s_{\text{dim}}, \rho)$ along dimension $k$*, or $m^k(s_{\text{dim}}, \rho, v(\cdot|a))$, to be individual $m^k \in N$ such that there exists a

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3Note however that this notion of “complete proportionality” does not imply that every individual’s vote counts equally; individuals belonging to several groups will have a greater “voting weight” than those belonging to fewer, although technically any particular voter is given zero weight under any political institution.
\( D, D' \in D(s_{\text{dim}}, \rho) \) where:

\[
D = \{ i \in N : \arg\max_{\alpha \in \mathbb{R}} v_i((x^k, \alpha)|a) \in (-\infty, \arg\max_{\alpha \in \mathbb{R}} v_{m_k}((x^k, \alpha)|a)] \},
\]

\[
D' = \{ i \in N : \arg\max_{\alpha \in \mathbb{R}} v_i((x^k, \alpha)|a) \in [\arg\max_{\alpha \in \mathbb{R}} v_{m_k}((x^k, \alpha)|a), \infty) \},
\]

and \((x^k, \alpha) = (x^1, ..., x^{k-1}, \alpha, x^{k+1}, ..., x^m)\).

In words, the median voter on dimension \( k \) induced by a political institution \((s_{\text{dim}}, \rho)\) and action profile \( a \) is the individual whose ideal policy (contingent upon his choice of group identity) lies at the median of the distribution of voter ideal policies on dimension \( k \), weighted by the voting weights specified by \( \rho^k \). Institution \((s_{\text{dim}}, \rho)\) thus produces outcome

\[
s_{\text{dim}}(\rho, v(\cdot|a)) = (\arg\max_{\alpha \in \mathbb{R}} v_{m_k}((x^k, \alpha)|a_m))^M_{k=1}.
\]

**Convex combination of voter ideal points: \( s_{\text{conv}} \)**

Institution \((s_{\text{conv}}, \rho)\) produces an outcome equal to a convex combination of the voters’ utility-maximizing policies. Let \( \overline{v}_{g}^k(a) \) be the average utility-maximizing policy of the members of group \( g \) along dimension \( k \) when action profile \( a \) has been taken. Thus,

\[
\overline{v}_{g}^k(a) = \frac{\int_{i \in g} \arg\max_{\alpha \in \mathbb{R}} v_i((x^k, \alpha)|a) dF(p_i)}{\int_{i \in g} dF(p_i)}.
\]

Institution \((s_{\text{conv}}, \rho)\) produces outcome

\[
s_{\text{conv}}(\rho, v(\cdot|a)) = \left( \sum_{g \in G} \rho^k_g \cdot \overline{v}_{g}^k(a) \right)^M_{k=1}.
\]

**Social planner: \( s_{\text{plan}} \)**

Institution \((s_{\text{plan}}, \rho)\) produces an outcome equal to a fixed policy \( x \in X \). Thus, \( s_{\text{plan}}(\rho, v(\cdot|a)) = x \) for some \( x \in X \).
Note that a given political institution is not affected by actions taken by individuals. Thus, an institution biases policy in favor of particular groups regardless of whether individuals choose to identify with those groups or not. An example is the institutional norm in some countries of staggering male and female candidates on party lists (a practice called “zipping”), in order to ensure that a certain number of female candidates win seats. This institution can be regarded as biasing policy outcomes toward the interests of women, regardless of whether women choose to identify with “women” as a group, and regardless of what the preferences of women actually are. In other words, individual preferences and actions can affect the outcomes induced by political institutions, but not the institutions themselves.

### 2.3 Equilibrium

The equilibrium concept is a variant of social identity equilibrium, defined by Shayo [3], and based on social identity theory, defined by Tajfel and Turner (1979). At equilibrium, people choose to associate with the group that maximizes their utility out of the set of groups that they possibly could associate with. When people associate with such groups, and make decisions based upon their group association, they still want to remain members of their group. Equilibrium is a collection of group memberships (one per person) and a social policy \( x^* \) that satisfy this property. The following definition formalizes this idea.

At a given \((s, \rho) \in S \times \mathcal{P}\) equilibrium is a social policy \( x^* \) and vector of identities \( \{a_i^*\}_{i \in N} \) such that the following two conditions hold:

1. For all \( i \in N, a_i^* \in \arg\max_{a \in A_i} v_i(x^*, a) \)
2. \( x^* = s(\rho, v(\cdot|a^*)) \).

The first condition says that in equilibrium, no individual wants to deviate from his choice
of group identity, $a^*_i$, given equilibrium policy $x^*$. The second condition implies that policy $x^*$ is a policy induced by institution $(s, \rho)$ under action profile $a^*$.

The following proposition presents a sufficient condition for the existence of an equilibrium under aggregation rule $s_{dim}$ (majority rule) when the policy space is one-dimensional. Note that the proposition relies on the fact that at a given policy $x \in X$, individual best responses $a^*_i$ depend only upon $x$ and not the identity choices of other individuals. Throughout the proof let $A^*_i(x)$ be the set of utility-maximizing choices of group identity for player $i$, given policy $x$. Thus, $A^*_i(x) = \arg\max_{a \in A_i} v_i(a|x) \subseteq A_i$, and has generic element $a^*_i(x)$. Let $A^*(x) = \times_{i \in N} A^*_i(x)$, with generic element $a^*(x)$.

**Proposition 1** Let $m(s_{dim}, \rho, v(\cdot|\emptyset))$ denote the median voter induced by institution $(s_{dim}, \rho)$ if each individual had utility function $u_i(x) = \pi_i(x)$ (i.e. if each individual chose action $a_i = \emptyset$). If for all $x \in X$ and all $a^*(x) \in A^*(x)$ it is the case that $m(s_{dim}, \rho, v(\cdot|a^*)) = m(s_{dim}, \rho, v(\cdot|\emptyset)) = m$, then there exists a social identity equilibrium.

**Proof:** For any $a_m \in A_m$, median voter $m$ has a unique ideal policy $x^*(a_m)$ where

$$x^*(a_m) = \arg\max_{x \in X} v_m(x|a_m).$$

Let

$$\tilde{a}_m \in \arg\max_{a_m \in A_m} v_m(x(a_m), a_m).$$

Thus, $\tilde{a}_m$ is the choice of group identity that maximizes $m$’s utility when $m$ gets to select his ideal policy.

It follows that $x(\tilde{a}_m)$ and $a^*(x(\tilde{a}_m))$ constitute an equilibrium. By the construction of $a^*(x(\tilde{a}_m))$ each individual is best-responding to policy choice $x(\tilde{a}_m)$. Because we have assumed that $m = m(s_{dim}, \rho, v(\cdot|a^*(x(\tilde{a}_m))))$ it follows that $x(\tilde{a}_m)$ is in the core of $X$. □

The proposition implies that when the median voter is invariant to any collection of best
responses of members of society, then there always exists an equilibrium. Although this condition is strong, it holds in general when society consists of two disjoint and convex groups, and can hold in more restrictive settings with three or more groups. Also note that the proposition does not imply that the constructed equilibrium is unique. In the examples that follow there often exist multiple equilibria.

3 An application to “exclusionary groups” in one dimension

In this section we will examine a particular specification of this model in which the policy space is one-dimensional and group membership is exclusionary; each individual faces a choice between identifying with one particular social subgroup, or with no group. Groups in this setting could (for example) consist of individuals of a particular ethnicity or religion, or individuals of a certain age or gender. The main point is that groups be disjoint from each other. The following assumptions are used throughout the remainder of this section:

Assumptions:

1. The policy space is one-dimensional: \( X \subseteq \mathbb{R} \).

2. Groups are disjoint: \( \forall i \in N, G_i = \{g_i\} \) and \( A_i = \{g_i, \emptyset\} \).

3. Groups are connected: \( \{p_i\}_{i \in g} \) is an interval on \( \mathbb{R} \) for all \( g \in G \).

4. Utility functions are additively separable: \( \forall i \in N, u_i(\pi_i(x), R(a_i, x)) = \pi_i(x) + \gamma R(a_i, x) \) with \( \gamma \in \mathbb{R}_+ \).

5. Material payoff functions are quadratic in \( x \): \( \pi_i(x) = -(p_i - x)^2 \), where \( p_i \) is \( i \)'s ideal point.\(^4\)

\(^4\)Note that an ideal point may not be the utility-maximizing policy for an individual; such a policy will
Let \( \mu_g(x) \) be the expected material payoff of a member of group \( g \) when policy \( x \) is chosen:

\[
\mu_g(x) = \frac{\int_{i \in g} \pi_i(x) dF(p_i)}{\int_{i \in g} dF(p_i)}.
\]

6. Relative status is the sum of differences in \( \mu_g \) across groups:

\[
R(g, x) = \sum_{h \neq g} \mu_g - \mu_h.
\]

7. The status associated with choosing no identification is a constant:

\[
R(\emptyset, x) = \nu \text{ with } \nu \in \mathbb{R}_+.
\]

These assumptions have several useful implications. The first is that, at a given \( x \in X \), individuals will choose to identify with group \( j \) if \( R(g, x) > \nu \), and will choose no group identity otherwise.\(^5\) Thus, when two individuals \( i \) and \( i' \) are members of the same group (i.e. \( g_i = g_{i'} \)), then they will make the same decision. A second useful implication is that the relative status of group \( g \), \( R(g, x) \), is linear in \( x \), and in particular

\[
R(g, x) = \sum_{h \neq g} \left[ \frac{\int_{i \in g} -(p_{i}^{2}-2xp_{i})dF(p_i)}{\int_{i \in g} dF(p_i)} - \frac{\int_{i \in h} -(p_{i}^{2}-2xp_{i})dF(p_i)}{\int_{i \in h} dF(p_i)} \right]. \tag{1}
\]

This is a consequence of the quadratic specification of \( \pi_i \), and implies that identification with a particular subgroup simply shifts an individual’s utility-maximizing policy to either the right or left of his ideal point by a fixed amount. Let \( \overline{p}_g \) denote the expected ideal point (or material payoff-maximizing policy) of a member of group \( g \), so that

\[
\overline{p}_g = \frac{\int_{i \in g} p_i dF(p_i)}{\int_{i \in g} dF(p_i)}.
\]

Similarly, let

\[
\overline{p}_g^2 = \frac{\int_{i \in g} p_{i}^{2}dF(p_i)}{\int_{i \in g} dF(p_i)}.
\]

depend upon the individual’s choice of group identity.

\(^5\)This assumes that when individuals are indifferent, they choose not to identify with a group.
If an individual $i$ identifies with group $g$ his utility is maximized at policy $x = p_i + \gamma \sum_{h \neq g}(\bar{p}_g - \bar{p}_h)$. If an individual chooses not to identify with a group then his utility is maximized at his ideal point, $p_i$.

### 3.1 Two groups

In this section we will analyze the case of a society consisting of two distinct groups, $j^1 = L$ and $j^2 = R$. Assume that $i \in L$ when $p_i < \bar{p}$, and $i \in R$ otherwise. Thus, the proportion of the population in group $L$ is $F(\bar{p})$ and the proportion in $R$ is $1 - F(\bar{p})$. Let $\rho_L$ and $\rho_R = 1 - \rho_L$ be the vote shares each party receives from electoral system $(s, \rho)$.

As discussed in the previous section, all members of the same group will choose the same action. Let $a^* = (a_L^*, a_R^*)$, where $a_g^*$ denotes the equilibrium action taken by all members of group $g$. Then three types of equilibria are possible: $a^* = (L, \emptyset)$, $a^* = (\emptyset, R)$, and $a^* = (\emptyset, \emptyset)$.

From Equation 1 we get that the relative statuses of the two groups at policy $x$ are

$$R(R, x) = -p_R^2 + p_L^2 + 2x(\bar{p}_R - \bar{p}_L)$$

and

$$R(L, x) = -p_L^2 + p_R^2 + 2x(\bar{p}_L - \bar{p}_R).$$

Note that the variance in the distribution of ideal points of members of group $g$, written $\sigma_g^2$, equals $\bar{p}_g^2 - (\bar{p}_g)^2$. We can rewrite $R(R, x)$ (and similarly $R(L, x)$) as:

$$R(R, x) = -(\sigma_R^2 + (\bar{p}_R)^2) + (\sigma_L^2 + (\bar{p}_L)^2) + 2x(\bar{p}_R - \bar{p}_L).$$

We now get that if an equilibrium exists it can be characterized by the following conditions:
\[ a^* = \begin{cases} 
(L, \emptyset) & \text{if } s(\rho, v(\cdot|a^*)) < \frac{\nu + \sigma^2_L - \sigma^2_R}{2(\overline{p}_L - \overline{p}_R)} + \frac{1}{2}(\overline{p}_L + \overline{p}_R) \\
(\emptyset, R) & \text{if } s(\rho, v(\cdot|a^*)) > \frac{\nu + \sigma^2_R - \sigma^2_L}{2(\overline{p}_R - \overline{p}_L)} + \frac{1}{2}(\overline{p}_R + \overline{p}_L) \\
(\emptyset, \emptyset) & \text{if } \nu + \sigma^2_L - \sigma^2_R \\ & \leq s(\rho, v(\cdot|a^*)) \leq \frac{\nu + \sigma^2_R - \sigma^2_L}{2(\overline{p}_R - \overline{p}_L)} + \frac{1}{2}(\overline{p}_R + \overline{p}_L). \end{cases} \]

The following proposition shows that a group is more likely to choose an ethnic identity as it becomes more homogeneous. Similarly, a group is more likely to choose an ethnic identity as the other societal group becomes more heterogeneous. This result is in keeping with the social psychology literature, and in particular social identity theory, which posits that individuals value ethnic identification more highly the more similar they are to other members of their ethnic group [4]. The perceived difference between individual \( i \) and his social group \( a_i \) is termed cognitive distance, and here it is characterized as \( \sigma^2_i \).

**Proposition 2** Members of a group are more likely to pursue an ethnic identity as their own cognitive distance decreases, and as their rival group’s cognitive distance increases.

**Proof:** Without loss of generality I will focus only on group \( L \). \( L \) chooses an ethnic identity when the policy outcome \( s(\rho, v(\cdot|a)) \) is less than a cutoff point which I will call \( c_L = \frac{\nu + \sigma^2_L - \sigma^2_R}{2(\overline{p}_L - \overline{p}_R)} + \frac{1}{2}(\overline{p}_L + \overline{p}_R) \). Clearly,

\[ \frac{\partial c_L}{\partial \sigma^2_L} = \frac{1}{2(\overline{p}_L - \overline{p}_R)} < 0 \]

and

\[ \frac{\partial c_L}{\partial \sigma^2_R} = \frac{-1}{2(\overline{p}_L - \overline{p}_R)} > 0. \]

Thus, this cutoff is decreasing in \( \sigma^2_L \) and increasing in \( \sigma^2_R \). A lower cutoff implies that there are fewer policies generated by any given institution that will induce members of \( L \) to pursue an ethnic identity. Thus, members of \( L \) are more likely to pursue an ethnic identity as \( \sigma^2_L \) decreases and as \( \sigma^2_R \) increases. \( \square \)
An interesting consequence of this proposition is that under certain circumstances groups have an incentive to pretend that they are more cohesive than they actually are. For example, under aggregation rule $s_{\text{conv}}$ (a convex combination of voter utility-maximizing policies), when a group decides to pursue an ethnic identity, the utility-maximizing policies of its members are shifted further away from the opposing group. This would induce the opposing group to pretend to be more cohesive than it actually is, in order to convince its rival to pursue no choice of group identity and to consequently drive policy towards the ideal points of the opposing group’s members. Similarly, under aggregation rule $s_{\text{dim}}$ (majority rule) the group that the induced median voter is not a member of, such as a minority party for example, has an incentive to pretend that it is more cohesive than it actually is because when the median voter selects a nationalistic identity, the equilibrium policy is closer to the opposing group.

Consider Figure 1 under aggregation rule $s_{\text{dim}}$ and assuming that the median voter is a member of group $R$. Figure 1 shows a situation where Group $R$ is distributed uniformly and Group $L$ is distributed symmetrically about the point $\bar{\rho} = \frac{1}{2}$. The cutoff point between the two groups is $\bar{\rho} = \frac{1}{2}$. In this example the equilibrium policy when the median voter chooses $a^*_m = \emptyset$ is $1 - \frac{1}{4\rho_R}$. When the median voter chooses an ethnic identity (i.e. $a^*_m = R$), then the equilibrium policy $s_{\text{dim}}(\rho, v(\cdot|a^*))$ equals $1 - \frac{1}{4\rho_R} + \frac{\gamma}{2}$.

### 3.2 Two-group examples

The following examples illustrate how different distributions of individuals within society and different political institutions can affect the constructed social identity equilibrium.

**Example 1** Comparing aggregation rules $s_{\text{dim}}$ (majority rule) and $s_{\text{conv}}$ (convex combination).

In this example we will assume a fixed weighting function $\rho = (\rho_L, \rho_R)$ and look at how equilibria differ under the different aggregation rules $s_{\text{dim}}$ and $s_{\text{conv}}$. A setting similar to
that depicted in Figure 1 is assumed: group $R$ is distributed uniformly over $[\frac{1}{2}, 1]$ and group $L$ is distributed symmetrically over $[0, \frac{1}{2}]$ with variance $\sigma^2_L$. It is assumed that $\rho_R \geq \frac{1}{2}$, so that under $s_{dim}$ the induced median voter is always a member of group $R$. It is also assumed that $\gamma$, the weight placed on group status in individuals’ utility functions, equals $\frac{1}{2}$ and that the relative status of no identification, $\nu$, equals $\frac{1}{4}$.

Figures 2 and 3 depict the equilibrium actions induced by different values of $\rho_L \in [0, \frac{1}{2}]$ (the voting weight allotted to group $L$), and $\sigma^2_L$, the variance of group $L$. When $a^* = (\emptyset, R)$, then the only possible equilibrium is one in which members of group $R$ assume an ethnic identity and members of group $L$ choose not to identify with their group. When $a^* = (\emptyset, R)$ or $a^* = (\emptyset, \emptyset)$ then two equilibria are possible: one where $R$ assumes an ethnic identity and $L$ does not, and one where neither group assumes an ethnic identity. When $a^* = (\emptyset, \emptyset)$ then the only possible equilibrium is one where neither group assumes an ethnic identity.

It is clear from Figures 2 and 3 that both higher variance across group $L$ and a lower voting weight for group $L$ makes members of $R$ more likely to pursue an ethnic identity. Furthermore, majority rule is always more capable of producing ethnic equilibria than is
the convex combination aggregation rule. The two rules can be conceptualized as representing different norms of bargaining in legislatures. By holding the weighting function \( \rho \) fixed across the two figures, the “composition” of the legislature is held constant. Whether the legislature generates policy through majority rule or through more proportional means (such as bargaining that might occur in a coalition government) is captured by the two different aggregation rules.

![Figure 2: Equilibrium action profiles when \( s = s_{\text{conv}} \) (convex combination)](image)

**Example 2 Increasing the size of Group \( R \) under \( s_{\text{dim}} \).**

This example focuses only on majority rule, and looks at how an increase in the size of one group affects the constructed equilibrium. We will assume that individual ideal points are distributed according to the beta distribution \( f(p_i | \mu, 1) \), with \( \mu > 0 \) (the PDF is decreasing for \( \mu \in (0, 1) \) and increasing for \( \mu > 1 \)). When \( \mu = 1 \) this distribution reduces to the uniform(0, 1). The probability density function of citizen ideal points is

\[
f(p_i | \mu, 1) = \mu p_i^{\mu - 1}.
\]

---

6This result is most likely generalizable, and will hopefully be proved in the next draft of this paper.
Given this distribution over individual ideal points, we also get the following: the CDF of individual ideal points \( F(p_i|\mu, 1) = p_i^\mu \), the median voter’s ideal point is \( p_m = (1 - \frac{1 - \bar{p}_R}{2\rho_R})^{\frac{1}{\mu}} \), the average ideal point of a member of group \( R \) is \( \bar{p}_R = \frac{\mu(1-p_{\mu+1})}{(1-\bar{p})/(1+\mu)} \), the average ideal point of a member of group \( L \) is \( \bar{p}_L = \frac{\mu\bar{p}}{1+\mu} \), and \( p_{\mu+2}^R = \frac{\mu(1-p_{\mu+2})}{(1-\bar{p})(\mu+2)} \) and \( p_{\mu+2}^L = \frac{\mu\bar{p}}{\mu+2} \).

Figure 4 depicts the beta distribution for values of \( \mu \) between 0.5 and 9. Note that for a fixed \( \bar{p} \) (or cutoff point between the two groups), the size of group \( R \) is increasing in parameter \( \mu \).

Figure 5 shows the effect of parameters \( \mu \) and \( \rho_R \) on the equilibrium actions for the case where \( \gamma = \frac{1}{4}, \nu = \frac{1}{6}, \) and \( \bar{p} = \frac{1}{2} \). It shows that an increase in \( \rho_R \) increases the likelihood of an ethnic equilibrium, and that increasing \( \mu \), the percentage of people in group \( R \), increases the likelihood of an ethnic equilibrium when \( \mu \) is approximately greater than three. The first result is straightforward: an increase in \( \rho_R \) moves the induced median voter farther right, while having no effect on the equilibrium cutoff strategies characterized in Equation2.
At first glance it would appear that an increase in $\mu$ makes individuals in group $R$ worse-off, and would thus decrease the likelihood of an ethnic equilibrium. This is because increasing $\mu$ while holding weight $\rho_R$ constant decreases the institutional advantage given to group $R$, and thus decreases the vote share of each member of $R$. Furthermore, the cutoff for an $a^* = (\emptyset, R)$ equilibrium (again, as specified in Equation 2) shifts right as $\mu$
increases, making the likelihood of an ethnic equilibrium less likely. However, increasing $\mu$ also shifts the ideal point of the induced median voter farther right, and this shift is greater than the shift in the cutoff for large values of $\mu$. What really appears to be driving this result is the asymmetry of the beta distribution. Figure 6 depicts the difference in average ideal points of members of Group $R$ and Group $L$, and closely predicts the non-monotonic effect of $\mu$ on the equilibrium correspondence in Figure 5. The smaller the difference in average members of each group, the higher the likelihood of a nationalistic equilibrium.

![Figure 6: Difference in voter ideal points across the two groups](image)

This example illustrates a consequence of the functional form of the relative status term $R(g, x)$ in individuals’ utility functions. Under majority rule, institutions affect individual preferences differently than group identification does. While institutions affect the location of the median voter, relative group status is evaluated on the basis of mean voters. This is most apparent in the extreme case where $\mu$ is driven to infinity. As $\mu$ approaches infinity, the percentage of the population in Group $R$ approaches 100%, $\bar{p}_R$ approaches one, and $\bar{p}_L$ approaches $\frac{1}{2}$. However, for $\rho_R = \frac{1}{2}$ and $\bar{p} = \frac{1}{2}$, the median voter induced by institution $(s_{dim}, \rho)$ is located at $\frac{1}{2}$. Other possibilities are that $R(g, x)$ could compare groups on the basis of their median, mode, maximum or minimum material payoffs.
4  Issue Salience in Two Dimensions

In this section the same assumptions as in Section 3 are made with two changes: the policy space is now two-dimensional \((X \subseteq \mathbb{R}^2)\) and when \(i \in g\) material payoffs are of the form 

\[
\pi_i(x) = -\omega^1_g(p_i - x^1)^2 - \omega^2_g(p_i - x^2)^2.
\]

Thus, members of group \(g\) assign weight \(\omega^1_g\) to the first policy dimension and \(\omega^2_g\) to the second (regardless of whether members of \(g\) choose to identify with \(g\)). To extend the definition of \(\mu_g(x)\) given in the previous section, \(\mu^k_g(x)\) is now the average material payoff of a member of \(g\) on policy dimension \(k\) at policy \(x\). Relative status is now the dimension-by-dimension sum of the differences in \(\mu^k_g\) across groups: 

\[
R(g, x) = \sum_{k=1}^{2} \sum_{h \neq g} \mu^k_g(x) - \mu^k_h(x).
\]

Rewritten,

\[
R(g, x) = \sum_{k=1,2} \left[ \sum_{h \neq g} \omega^k_g \frac{\int_{i \in g} -(p^k_i - x^k)^2 dF(p_i)}{\int_{i \in g} dF(p_i)} - \omega^k_h \frac{\int_{i \in h} -(p^k_i - x^k)^2 dF(p_i)}{\int_{i \in h} dF(p_i)} \right].
\]

Unlike in the previous section, relative status is no longer linear in \(x\). When an individual chooses to identify with group \(g\) his induced ideal point is now

\[
\arg\max_{x \in X} v_i(x | g) = \left( \frac{\omega^k_g p^k_i + \gamma \sum_{h \neq g} \omega^k_g \omega^k_h - \omega^k_h p^k_h}{\omega^k_g + \gamma \sum_{h \neq g} \omega^k_h - \omega^k_h} \right)_{k=1,2}.
\]

Example 3  Possible equilibria under a social planner \((s_{plan})\).

In this example we will plot out potential equilibrium outcomes when there are two policy dimensions, and in particular, \(X = [0, 1] \times [0, 1]\). It will be assumed that there are two disjoint groups, \(A\) and \(B\), and that all members of a group have the same policy preferences. For \(i \in A\), \(p_i = (\frac{1}{3}, 1)\). For \(i \in B\), \(p_i = (\frac{2}{3}, 0)\). Thus the groups are more similar along the first policy dimension. It is also assumed that the status of no group identification is \(\nu = \frac{1}{3}\). The following graphs depict the equilibrium actions of group members under different policy scenarios. Issue salience terms are varied across graphs.
In Figure 7 the groups each weight each dimension equally. The groups are more similar along the first policy dimension than along the second. In this example it is always possible to induce a non-ethnic equilibrium when policy along the more contested dimension is fixed at a compromise point $x^2 = \frac{1}{2}$. A non-ethnic equilibrium can be attained for any possible policy outcome along the first, less contested, dimension, even when both groups care equally about the two dimensions. This result has implications for institutional design. It implies that it may be possible to reduce factional conflict by taking policy discretion away from a legislature along policy dimensions that are more highly polarized, possibly through constitutional design. An example is the first amendment of the U.S. Constitution’s proclamation that:

Congress shall make no law respecting an establishment of religion, or prohibiting the free exercise thereof...
References


