Contracts with Framing*

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Abstract

We study a model of contracts in which a profit-maximizing seller uses framing to influence buyers' behavior. Framing affects how buyers compare different products, but does not change buyers' willingness to pay. We provide conditions under which framing is profit-enhancing, and analyze the welfare properties of optimal contracts. Framing that is not too strong reduces total welfare in regulated markets with homogenous buyers, but increases total welfare in markets with heterogenous buyers when the proportion of buyers with low willingness to pay is small.

1 Introduction

Sellers commonly use framing to influence buyers' behavior. When presenting a product menu to buyers, for example, sellers often visually highlight a particular product by placing it in a prominent position, by coloring it differently from other products, or by other means.[1] Sellers also tend to use captions that emphasize product attributes buyers consider desirable, such as

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[1] The Brookfield zoo membership brochure at http://www.brookfieldzoo.org/CZS/Membership highlights the "family plus" membership in at least three different ways.

the percentage of a dairy product that is "fat-free" rather than the actual fat content of the product. More subtle cues like the type of background music played in the store also influence buyers' behavior.[2] In all these cases, framing seems to influence how buyers evaluate products by increasing the attractiveness of some product or product attribute.

Such increased attractiveness may not be persistent. For example, while background music may affect the buyer in the store, this effect is likely to disappear once the buyer exits the store. Reevaluating his purchase at this point, the buyer may return the product if he overpaid for it. Similarly, the effect of highlighting a product depends on the presence of other products, so the effect is likely to disappear if the buyer reassesses the highlighted product in isolation before paying for it. This naturally limits the extent to which sellers can manipulate buyers' behavior via framing.

This paper studies the optimal design of product menus with frames. A profit-maximizing seller chooses menu of bundles, where a bundle is a product and its price, and a frame. The frame affects how buyers compare bundles in the menu but does not persistently change buyers' willingness to pay.

Buyers are characterized by their preferences U that describe how they evaluate bundles absent framing, and by a collection of functions $\{U^f\}$, one for each feasible frame f, that describe how buyers evaluate bundles in the presence of framing. Given a menu of bundles and a frame f, a buyer chooses a U^f-maximal bundle from the menu if it is U-superior to not making a purchase, and otherwise does not purchase anything. The maximization stage, in which the buyer identifies the U^f-maximal bundle in the menu, captures the idea that framing influences how buyers make comparisons. The verification stage, in which the buyer purchases this bundle if it is U-superior to not buying anything, captures the idea that framing is not persistent. As indicated above, the verification stage may take place when the buyer reevaluates his purchase decision after the framing effect wears off. The verification stage may also take place ex-ante: experienced buyers may learn to anticipate the framing effect and avoid interacting with the seller if they anticipate they will over-pay.

Welfare in the model is evaluated with respect to buyers' preferences. This follows the view that frames are details that are irrelevant to buyers' intrinsic valuation of goods, and that their effect is not persistent.[3]

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[2]The effect of background music on purchasing behavior has been studied extensively in the marketing literature. Some examples include Areni and Kim (1993), who showed that classical music led to more expensive wine purchases relative to top-40 music, North and Hargreaves (1998), who showed that classical music increased students' purchase intentions in a cafeteria by approximately 20 percent, and North, Shilcock, and Hargreaves (2003), who showed that spending in a restaurant increased in the presence of classical music relative to pop music or no music.

[3]See Rubinstein and Salant (2008, 2012) for a detailed discussion of this approach to welfare analysis in the

Our analysis focuses on the profit and welfare implications of frames that increase the attractiveness of a particular product attribute. Captions that emphasize a desirable attribute may have this effect, as well as background classical music, which seems to trigger buyers to value quality more highly.[4] We consider two settings in which the seller offers buyers a menu with more than one bundle, so framing has the potential to affect purchasing behavior by influencing how buyers make comparisons.

The first setting is a regulated market in which the seller is required to offer buyers a specific basic bundle in addition to offering them other bundles of his choice. This is often the case in the cable-TV market, where regional cable providers have to offer customers a basic package of channels at a low rate in addition to other packages of their choice.[5] One rationale for this regulation in the absence of framing is that with homogenous buyers it shifts surplus from the seller to buyers without creating efficiency distortions. This is because by offering an additional bundle, the seller can extract from buyers the entire social surplus from this bundle, up to a constant that makes them U-indifferent to the basic bundle. The seller therefore offers buyers the socially efficient product at a lower price than without the regulation.

Frames that increase attractiveness are profit-enhancing in this case because they enable the seller to charge a higher price for the socially efficient product than without framing. The regulation is therefore less effective in shifting surplus from the seller to buyers. More importantly, the regulation creates efficiency distortions: the additional product offered by the seller and purchased by buyers is socially inefficient whenever some surplus is shifted to buyers.

The second setting is a market with heterogenous buyers. Framing that increases attractiveness is not necessarily profit-enhancing in this case. This is because framing triggers buyers with low willingness to pay to perceive premium products, which are targeted at buyers with high willingness to pay, as more attractive than without framing. Since premium products may optimally be priced above low-type buyers' willingness to pay for them, framing may cause these buyers to forgo purchasing altogether, leading to an overall decrease in profit. We show that this can indeed happen when framing is "sufficiently strong."

When framing is not too strong, it is profit-enhancing. The welfare implications of framing in this case are different for high- and low-type buyers. The product purchased by high-type buyers is always less efficient than in the standard model without framing, while the product purchased by low-type buyers is more efficient than in the standard model when the proportion of these buyers is not large. Overall, framing increases total welfare when the proportion of low-type buyers is not large, which implies that framing may assist in mitigating the social inefficiencies created by profit maximization in the presence of private information.

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presence of framing.

[4]See, for example, the findings of Areni and Kim (1993).

[5]See, for example, http://www.fcc.gov/guides/regulation-cable-tv-rates for cable-TV regulation in the US.

Many of these results extend to frames that highlights a particular product. To illustrate this, we consider an insurance setting a-la Stiglitz (1977), in which a risk neutral insurance provider offers a menu of insurance bundles to a population of risk-averse buyers, and can choose to highlight one of the bundles. Buyers anticipate that they will experience regret in case of an accident if they purchase less coverage than in the highlighted bundle. We show that the highlighted bundle optimally coincides with the one targeted at high-risk buyers. This is in line with the real-world phenomenon that sellers tend to highlight premium bundles.[6] Moreover, low-risk buyers are always partially insured, while high-risk buyers are either over-insured or do not purchase any insurance. Insuring low-risk individuals but not high-risk buyers is impossible in the standard model, and is in line with the phenomenon of advantageous selection identified in the empirical literature (See Einav, Finkelstein, and Levin (2012) for a recent survey).

The relevance of frames as a design parameter may extend beyond contracting environments. For example, in the experimental auction of Delgado et al. (2008), frames that highlight the possibility of losing lead to aggressive bidding and higher revenue. We conclude by analyzing an example of an efficient auction, in which framing increases the degree of bidders' anticipated disappointment from losing and thus influences their bidding behavior. As in our contracting environment, the framing effect is not persistent. We observe that the revenue in an efficient auction with an appropriately chosen frame is larger than in a revenue-maximizing (inefficient) frameless auction.

The paper proceeds as follows. Section 1.1 discusses the related literature. Section 2 introduces the framework. Section 3 analyzes regulated markets with homogenous buyers. Sections 4 and 5 study markets with heterogenous buyers. Section 4 discusses why frames that increase attractiveness may reduce the seller's profit, and section 5 solves for the optimal contract and analyzes its welfare properties. Section 6 studies the application to monopolistic insurance. Section 7 concludes with a discussion of another model of buyers' behavior, and with the application to auction design. The Appendix contains proofs that do not appear in the main text.

1.1 Related literature

The paper is related to several growing literatures. The specification of the buyer builds on the framework of individual choice with frames developed by Salant and Rubinstein (2008) and Rubinstein and Salant (2008, 2012). The primitives that describe the buyer in our model correspond to their framework, but our specification of the buyer's two-stage choice procedure is different. Other two-stage choice procedures were studied in the context of individual choice without framing. See, for example, Manzini and Mariotti (2007). In contrast to all these papers, we study the effect of frame-dependent behavior on the outcomes of strategic interactions.

[6] For example, the online car rental insurance seller "Insure My Rental Car" highlights its premium policy over the basic one by coloring it in a darker color.

In the context of strategic interactions with frame-dependent behavior, Piccione and Spiegler (2012) and Spiegler (2014) study competition between two firms in a complete-information setting in which frames influence consumers' ability to compare the firms' actions, such as prices. Firms choose "marketing messages," in addition to actions, and these messages jointly determine the frame. The frame and the actions determine how the market is split between the firms. We study a different question, namely the optimal design of product menus with frames by a monopolistic seller in a regulated market or a market with incomplete information. Our model of consumer behavior is also different, since framing is not persistent.

Another related literature is the literature on behavioral contract theory (see Kőszegi (2013) for an excellent survey), and in particular the literature on screening agents with non-standard preferences. In this literature, the agent has at the outset some private information, either on his degree of inconsistency (see Eliaz and Spiegler (2006), Esteban and Miyagawa (2006), Esteban, Miyagawa, and Shum (2007), and Galperti (2013)), or on some payoff-relevant parameter, such as his willingness to pay (see Esteban and Miyagawa (2006) and Carbajal and Ely (2012)). The focus is on the design of an optimal product menu or menus from which the agent makes choices. In our framework the principal has an additional tool, frames, which he uses to temporarily influence how consumers evaluate different products. Our focus is on the optimal use of profit-enhancing frames, and product menus that complement them, to screen agents with payoff-relevant private information.

There are also papers that study implementation with boundedly-rational agents. De Clippel (2013) studies implementation with general choice functions. Glazer and Rubinstein (2012) study a persuasion model in which agents are limited in their ability to find arguments that satisfy a set of rules specified by a principal in order to screen agents. We focus on framing as the cause for boundedly-rational behavior, and study the effect of frame-dependent behavior on the design of profit-maximizing contracts.

2 Framework

A profit-maximizing seller offers a contract (M, f) to buyers. The menu M includes bundles (x, t), where $x \in [0, d] \subset \mathbb{R}$ is a product and $t \in \mathbb{R}_+$ is a price. The frame f belongs to a set F of feasible frames. Frames affect how buyers compare bundles in the menu without influencing buyers' willingness to pay.

To capture this formally, let $U(x, t, \theta) = u(x, \theta) - t$ describe the quasi-linear preferences of a type θ buyer over bundles, where $\theta \in \Theta$ is the buyer's privately-known "taste" parameter, and let $U^f(x, t, \theta) = u^f(x, \theta) - t$ describe how the buyer evaluates bundles in the frame f. The functions u and u^f are differentiable and strictly increasing in x. Let *stayout* $= (0, 0)$ denote the buyer's bundle if he does not purchase anything.

Given a contract (M, f), the set $C^\theta(M, f)$ of possible choices of a type θ buyer consists of:

(1) all the U^f-maximal bundles in M that are weakly U-superior to *stayout*, and

(2) *stayout* if it is weakly U-superior to some U^f-maximal bundle in M.

The correspondence C^θ summarizes a two-stage choice procedure that includes a maximization stage and a verification stage. In the maximization stage, the buyer identifies a U^f-maximal bundle in the menu M. That is, the frame affects how the buyer compares bundles in the menu. In the verification stage, the buyer purchases this U^f-maximal bundle only if it is superior, according to his U-preferences, to not buying anything.

The verification stage reflects the buyer's partial sophistication in that framing does not affect his willingness to pay. One possibility is that after choosing a product from the menu (according to U^f), the buyer may realize that the product is too expensive (according to U), and will not buy it. Another possibility is that the buyer purchases the product, but reassesses his purchase after the framing effect wears off, and returns the product to the store if he over-paid for it. The buyer does not make another purchase because he believes that the product he just returned is the best among the available ones. (Section 7 establishes that most of our results extend to the case in which the buyer goes back to the aisle to make another purchase.)

The verification stage may also take place ex-ante. When buyers are involved in similar interactions or communicate with other buyers, they may learn to anticipate the effect of framing on their behavior, while still not being able to resist this effect at the point of sale. In such cases, they may choose not to interact with the seller if they anticipate they will over-pay.

Turning to the seller, he has a convex, continuous, and strictly increasing cost $c(x)$ of providing the product x, with $c(0) = 0$.[7] His full-information profit maximization problem subject to type θ buyers obtaining a U-utility of $U(0, 0, \theta)$ is strictly concave in x and has a unique "first-best" solution (x^*_θ, t^*_θ). Note that the product x^*_θ is socially efficient in the sense that it maximizes the social surplus $u(x, \theta) - c(x)$ with respect to type θ buyers.

The seller can offer a frameless contract to buyers by choosing the "null" frame $\phi \in F$. In the null frame, $U^\phi = U$, so for any frameless contract (M, ϕ), the set $C^\theta(M, \phi)$ is the set of U-maximal bundles.

Implementation. An allocation rule g assigns to each $\theta \in \Theta$ a bundle $g(\theta)$. A contract (M, f) (partially) implements g if $g(\theta) \in C^\theta(M, f)$ for every $\theta \in \Theta$. In this case, we say that g is implementable with the frame f. An allocation rule is implementable if it is implementable with some frame. Finally, a contract is profit maximizing (or optimal) if it implements an allocation rule that maximizes the seller's profit among all implementable allocation rules.

[7] Section 6 studies the implications of type-dependent cost.

3 Complete information

We begin by considering a setting in which all buyers have the same type θ, which the seller knows. We distinguish between the case in which the seller has full discretion over the menu, and the case in which he is required to include a specific bundle in the menu.

When the seller has full discretion over the menu, framing does not change the predictions of the standard model. This is because if buyers choose the bundle (x, t) with framing, then this bundle is weakly U-superior to *stayout*, so buyers will also choose this bundle without framing if it is the only available one. Thus, whether the seller uses framing or not, he will offer buyers the first-best bundle (x_θ^*, t_θ^*) and capture the entire surplus in excess of $U(0, 0, \theta)$.

The seller, however, may be required by a regulator to offer a specific bundle in the menu. This is the case, for example, in the cable-TV market, in which cable providers often have to offer customers a basic package at a low rate in addition to other packages of their choice. The rest of this section studies how framing changes the effectiveness of such regulation.

Consider a regulation that requires the seller to include in the menu the bundle $(\bar{x}, \bar{t})$ that buyers strictly U-prefer to (x_θ^*, t_θ^*). The product $\bar{x}$ is basic in the sense that $\bar{x} < x_\theta^*$. The seller is allowed to add to the menu other bundles of his choice.

In the standard model, this regulation changes the division of surplus between the seller and buyers without reducing social surplus. This is because the seller can charge for any product x the entire social surplus $u(x, \theta) - c(x)$ up to a constant that makes buyers U-indifferent between this bundle and $(\bar{x}, \bar{t})$. He therefore optimally offers the socially efficient product x_θ^* at a price that makes buyers U-indifferent to $(\bar{x}, \bar{t})$. Thus, the regulation increases consumer surplus at the expense of producer surplus.

The same regulation either reduces social surplus or is ineffective in redistributing it when the seller can use framing that increases attractiveness.

Assumption A1. Framing increases attractiveness: For every frame $f \neq \phi$ and every product x, $u_x^f(x, \theta) > u_x(x, \theta)$.

Frames that increase attractiveness are profit-enhancing because the seller can charge more for the product x_θ^* than in the optimal frameless contract. The optimal contract will therefore involve framing.

Other characteristics of the optimal contract depend on whether there is a frame f that is sufficiently strong so that buyers U^f-prefer the first-best bundle (x_θ^*, t_θ^*) to $(\bar{x}, \bar{t})$. If such a frame is available, then every optimal contract includes the first-best bundle, which buyers choose over $(\bar{x}, \bar{t})$ and other bundles. In this case, efficiency is maintained but the regulation does not redistribute any surplus from the seller to buyers. If such a frame is not available, then

Proposition 1 *The product bought in every optimal contract is strictly above the socially efficient level whenever this is feasible (i.e., when $x_\theta^* < d$), and is efficient otherwise.*

Thus, when framing is not too strong and it is feasible to produce above the efficient level, framing reduces social surplus relative to the optimal frameless contract. Consumer surplus also goes down, because the seller's profit goes up while the social surplus goes down, so redistribution of surplus is less effective than without framing.

The intuition for the upward distortion is that similarly to the optimal frameless contract, the seller will not offer a product below the socially efficient level x^*_θ. And at x^*_θ, the seller's marginal production cost is equal to buyers' marginal U-willingness to pay, which in turn is strictly smaller than their marginal U^f-willingness to pay, by increased attractiveness. Thus, the seller can increase his profit by increasing x slightly above x^*_θ and increasing the price by the marginal U^f-willingness to pay.

4 Incomplete information

When buyers are privately informed, the seller in the standard model often optimally offers them a menu with more than one product. Framing that increases attractiveness is not necessarily profit-enhancing in this case, because it may cause buyers with low willingness to pay to perceive products for which they are not willing to pay, targeted at buyers with high willingness to pay, as more attractive than products they would have bought without framing. We now demonstrate this point, and in the next section we characterize the optimal contract and its welfare properties.

There are two types of buyers, *L*ow and *H*igh, with the interpretation that high-type buyers are U-willing to pay more than low-type buyers for an increase in the product, i.e., $u_x(x,H) > u_x(x,L)$ for any product x. The proportion of buyers of type $\theta \in \{L,H\}$ is π_θ, with $\pi_L + \pi_H = 1$.

In addition to increasing attractiveness (Assumption (A1)), we also assume that framing does not "reverse" the ranking of types: with framing high-type buyers still perceive increases in the product as more desirable than low-type buyers.

Assumption A2. For any frame f and any product x, $u^f_x(x,H) > u^f_x(x,L)$.

Assumption (A2) implies that when buyers of both types make a purchase, high-type buyers purchase a weakly larger product than low-type buyers.

A frame that increases attractiveness (Assumption (A1)) without reversing the ranking of types (Assumption (A2)) has two effects on the seller's profit. First, if a buyer chooses a product x from a menu M without the frame, then with the frame this product becomes more attractive relative to smaller products. Thus, with the frame the buyer will continue to choose x over smaller products in the menu even if the price of x is increased slightly (and the prices of the smaller products are not decreased). This is why framing is profit-enhancing in the complete information setting.

Second, the product x becomes less attractive relative to larger products in the menu, whose prices may exceed the buyer's willingness to pay. This implies that a buyer who made a purchase without the frame may not make a purchase with the frame because the bundle he finds most attractive is over-priced. This effect is irrelevant in the complete information setting because the bundle intended for buyers was optimally larger than the regulator's bundle. But with incomplete information, the bundle intended for low-type buyers is often optimally smaller than the bundle intended for high-type buyers.

The potential adverse impact of this effect on the seller's profit does not arise in a contract (M, f), where M is part of a profit-maximizing frameless contract (M, ϕ) and f increases attractiveness, if each buyer weakly U^f-prefers his chosen bundle in the frameless contract to larger bundles in (M, f). Intuitively, this reflects a situation in which the frame is not "too strong". In this case, every profit-maximizing contract involves framing. This is because the first effect above implies that, similarly to the complete information setting, the seller can increase the price of the largest chosen product in M slightly so that every buyer will continue to purchase from the modified menu with the frame f the same product he purchased in (M, ϕ).

On the other hand, when some buyers strictly U^f-prefer larger bundles in (M, f) to their chosen bundle in (M, ϕ), an optimal contract with the frame f may generate strictly lower profit than the optimal frameless contract, despite the increased attractiveness.

Such profit reduction may arise when the seller's ability to vary the products in the menu is limited, e.g., due to regulatory or technological constraints. In this case, the optimal frameless contract may involve selling a "basic" product to low-type buyers and a "premium" product to high-type buyers. With the frame, the seller may be forced to reduce the price of the basic product to make sure that low-type buyers do not find the premium product more attractive than the basic product. The following example illustrates this.[8]

Example 1 (Price discrimination with linear frames) *There are only two available products, a basic product x_L and a premium product $x_H > x_L$, whose production is costless. Buyers' utility $u(x, \theta)$ satisfies $u(0, \theta) = 0$. There is a single frame $f \in \mathbb{R}_+$ that increases attractiveness with $u^f(x, \theta) = u(x, \theta) + xf$, i.e., the frame interacts linearly with the product and does not interact with the type.*

Suppose that $\pi_H \in \left(\frac{u(x_H,L)-u(x_L,L)}{u(x_H,H)-u(x_L,H)}, \frac{u(x_L,L)}{u(x_H,H)}\right)$, so the optimal frameless contract offers both products.[9] The basic product in this contract is bought by low-type buyers, and its price $u(x_L, L)$ is determined by their binding participation constraint. The premium product is bought by high-type buyers and its price $u(x_H, H)-(u(x_L, H)-u(x_L, L))$ is determined by their binding incentive compatibility constraint.

[8]We thank Andrew Rhodes for developing this example.

[9]The condition on π_H is derived by comparing the profit in the optimal contract in which both products are bought to the optimal pooling contract and to the optimal contract in which low-type buyers do not purchase anything.

For the optimal contract with the frame f to generate more profit than the optimal frameless contract, it has to be a separating contract in which low-type buyers buy the basic product and high-type buyers buy the premium product. The maximal price the seller can charge for the premium product is $t_H = u(x_H, H)$, which is the high-type buyers' willingness to pay for this product. Therefore, the maximal price that the seller can charge for the basic product so that in the frame f low type buyers will find this product more attractive than the premium product is $t_L = u(x_H, H) - (u(x_H, L) - u(x_L, L)) - f(x_H - x_L)$. This price should not exceed the low type buyers' willingness to pay for the basic product $u(x_L, L)$, which is guaranteed when $f \geq \frac{u(x_H,H)-u(x_H,L)}{x_H-x_L}$, so the optimal contract with frame f is pinned down in this case.

In this optimal contract, the seller gains $\pi_H(u(x_H, H)-(u(x_H, H)-(u(x_L, H)-u(x_L, L)))) = \pi_H(u(x_L, H) - u(x_L, L))$ on high type buyers relative to the optimal frameless contract, but loses $(1 - \pi_H)(u(x_L, L) - (u(x_H, H) - (u(x_H, L) - u(x_L, L)) - f(x_H - x_L)))$ on low-type buyers. For a large enough f, the loss is larger than the gain.$\diamondsuit$

Profit reduction may also arise when the seller is able to change the products he offers. In this case, with framing he will offer low-type buyers better products than in the optimal frameless contract, rather then reducing prices as in Example 1. The prices of these better products will be relatively low, because the U-willingness to pay of low-type buyers is low. In a setting similar to that of Example 1, this will imply that framing increases the seller's profit because production is costless. But when producing better products is costly, offering them at relatively low prices may decrease the seller's profit more than the gain due to increased attractiveness. Example 3 in the Appendix illustrates this channel for profit reduction. To summarize,

Observation 1 *If there exists an optimal frameless contract (M, ϕ) and a frame f that increases attractiveness such that every type weakly U^f-prefers in (M, f) the product he chooses in (M, ϕ) to larger products, then every optimal contract involves framing. If this is not the case, then it may be that every optimal contract is frameless.*

To rule out situations in which framing decreases profit, our third assumption on framing limits the distortion that framing creates. It states that, similarly to the standard model with incomplete information, high-type buyers want to mimic low-type buyers in the first-best solution to the seller's profit maximization problem.

Assumption A3. For any frame f, $U^f(x_L^*, t_L^*, H) > U^f(x_H^*, t_H^*, H)$.

We now proceed to characterize the set of optimal contracts and their welfare properties under Assumptions (A1)-(A3).

5 Optimal contract and welfare

The set of optimal contracts may include pooling and separating contracts. If some optimal contract is pooling, then it implements the allocation rule $g(\theta) = (x_L^*, t_L^*)$, because the bundle

(x^*_L, t^*_L) is the profit-maximizing bundle subject to low-type buyers being U-indifferent between making and not making a purchase. In particular, framing does not influence the seller's profit in this case. On the other hand,

Proposition 2 *Any optimal contract that is separating involves framing.*

An immediate implication of Proposition 2 is that framing is profit-enhancing whenever the optimal pooling contract is dominated by some separating contract. This happens when $x^*_L < d$, or when $x^*_L = d$ and there is a frame f such that $u_x(d, L) < \pi_L c_x(d) + \pi_H u^f_x(d, H)$. In both cases, the optimal pooling contract is dominated by a separating contract in which low-type buyers are offered a product that is slightly lower than x^*_L at a price that equals their U-willingness to pay, and high-type buyers are offered the product x^*_L at a price that makes them U^f-indifferent to the low-type buyers' bundle. Every optimal contract is therefore separating, so by Proposition 2 framing is profit-enhancing.

Another implication of Proposition 2 is that both types of buyers purchase positive products in any optimal contract. This is in contrast to the standard model, in which the optimal frameless contract excludes low-type buyers when their proportion in the population is small in order to eliminate the information rents of high-type buyers. To see why excluding low-type buyers is never optimal in the model with framing, consider a frameless contract that excludes them and extracts the maximum surplus from high-type buyers by offering them the first-best bundle (x^*_H, t^*_H). Proposition 2 implies that this contract generates strictly less profit than any optimal contract with framing, so such an optimal contract must offer a positive product to low-type buyers. The seller can also exclude high-type buyers in the model with framing by offering them a bundle that is U^f-superior to the other bundles in the menu but is U-inferior to stayout, but this is dominated by the optimal pooling contract when the production cost is type-independent.

Because buyers of both types purchase positive products in an optimal contract, it suffices to focus on contracts with two-product menus $\{(x_L, t_L), (x_H, t_H)\}$, where (x_θ, t_θ) is the bundle purchased by type θ. By Assumption (A2), we have that $(x_H, t_H) \geq (x_L, t_L)$, so we refer to (x_L, t_L) as the basic bundle and to (x_H, t_H) as the premium bundle. Our next proposition identifies the binding constraints in the seller's profit maximization problem.

Proposition 3 *In an optimal contract with a frame f, low-type buyers are U-indifferent between buying the basic bundle and not buying anything, and high-type buyers are U^f-indifferent between buying the premium bundle and the basic bundle.*

The binding constraints in Proposition 3 are similar to the binding constraints in the standard model. But in contrast to the standard model, the constraints do not imply that whenever the basic product is positive high-type buyers strictly U-prefer the premium bundle to not making a purchase.

5.1 Welfare implications

Framing that is profit-enhancing has several welfare implications that differ from those of the standard model. The first relates to the efficiency of the basic product. The basic product with framing is more efficient than in the standard model in the sense that it generates a larger social surplus $\pi_L(u(x_L, L) - c(x_L))$ with respect to low-type buyers when π_L is small. This is because the optimal frameless contract excludes low-type buyers when π_L is small in order to eliminate the information rents of high-type buyers, while the optimal contract with framing always offers low-type buyers a positive product. This positive product is smaller than x_L^*, and thus generates positive social surplus. By Proposition 3, the entire surplus gain goes to the seller.

A second welfare difference relates to the efficiency of the premium product. In the standard model, this product is efficient in the sense that it maximizes the social surplus with respect to high-type buyers, $\pi_H(u(x, H) - c(x))$. This is because for any basic bundle, the seller can extract from high-type buyers the entire social surplus generated from the premium bundle, up to a constant that makes high-type buyers U-indifferent between the premium bundle and the basic bundle. In contrast,

Proposition 4 *The premium product in an optimal separating contract is strictly above the efficient level x_H^* when $x_H^* < d$, and is efficient when $x_H^* = d$.*

The reason for this efficiency distortion is that for any non-trivial basic bundle $(x_L, t_L) < (x_L^*, t_L^*)$ such that low-type buyers are U-indifferent between this bundle and not purchasing anything, increasing the premium product slightly above the efficient level along the high-type's U-indifference curve does not decrease the seller's profit to a first-order. But such an increase makes the premium product strictly more U^f-attractive to high-type buyers relative to the basic bundle, so the basic bundle can be increased along the low-type's U-indifference curve through it without reducing the price of the premium product, which results in a first-order gain to the seller.[10]

A third difference relates to the information rents of high-type buyers. In the standard model, high-type buyers always obtain a strictly positive surplus whenever the basic product is positive. This is because they can mimic low-type buyers, so by choosing the premium bundle they must obtain the surplus they would obtain from choosing the basic bundle. The most the seller can therefore charge for the premium product is a high-type buyer's U-willingness to pay for it minus his U-willingness to pay for the basic bundle. But with framing, the seller can charge for the premium product the high-type buyer's U^f-willingness to pay for it it minus his U^f-willingness to pay for the basic bundle, subject to not exceeding high-type buyers' U-willingness to pay for the premium product. When this last constraint binds, high-type buyers do not obtain any

[10]Over-consumption arises for other reasons in Carbajal and Ely (2012) and Galperti (2013).

surplus. In fact, even a frame the creates a small distortion can eliminate the entire surplus of high-type buyers, as the following example illustrates.

Example 2 (Price discrimination with linear utility) *Suppose that production is costless, that* $L < H \in \mathbb{R}_+$*, that* $u^f(x,\theta) = u(x,\theta) + xf$ *as in the previous example, and that* $u(x,\theta) = x\theta$*. The first-best product is then* $x^* = d$ *independently of buyers' types. Fix some frame* $f > 0$ *and assume that* $\pi_H \in (\frac{L}{H+f}, \frac{L}{H})$*.*

Using well-known properties of the optimal contract in the standard setting,[11] *one can show that because* $\pi_H < \frac{L}{H}$*, the optimal frameless contract is a pooling contract with the bundle* (d, dL)*. The surplus of high-type buyers in this contract is* $d(H-L) > 0$*.*

In the optimal contract with framing, we have that $x_H = d$*, because the optimal pooling contract includes the bundle* (d, dL) *and Proposition 4 implies that* $x_H = x_H^* = d$ *in an optimal separating contract. By Proposition 3, the price of the basic product is* $x_L L$*, and the price of the premium product satisfies* $d(H+f) - t_H = x_L(H+f) - t_L$*, so* $t_H = d(H+f) - x_L(H+f-L)$*. In addition, the price of the premium bundle cannot exceed the* U*-willingness to pay of high-type buyers, i.e.,* $t_H \leq dH$*. The minimal* x_L *that satisfies these conditions is* $\frac{df}{H+f-L}$*, and it is straightforward to verify that because* $\pi_H > \frac{L}{H+f}$*, this* x_L *is profit-maximizing.*

We thus obtain that the uniquely optimal contract is $(\{(\frac{df}{H+f-L}, \frac{dLf}{H+f-L}), (d, dH)\}, f)$*. In contrast to the optimal frameless contract, the surplus of high type buyers in this contract is 0.*$\diamondsuit$

A fourth welfare difference is that framing increases total surplus, i.e., $\pi_L(u(x_L, L) - c(x_L)) + \pi_H(u(x_H, H) - c(x_H))$, relative to the standard model when the proportion of low-type buyers is small. To see why, suppose that the proportion of low-type buyers is small, so that the optimal frameless contract excludes them and offers the first-best bundle (x_H^*, t_H^*) to high-type buyers. Fix a frame f, and let (x_L, t_L) denote a basic bundle such that high-type buyers are U^f-indifferent between this bundle and (x_H^*, t_H^*) and low-type buyers are U-indifferent between this basic bundle and not making a purchase. By increased attractiveness for high-type buyers, $(x_L, t_L) > (0,0)$, and by assumptions (A2) and (A3), $(x_L, t_L) < (x_L^*, t_L^*)$.[12] The total welfare in the contract $(\{(x_L, t_L), (x_H^*, t_H^*)\}, f)$ is higher than in the optimal frameless contract, because the premium product is unchanged and the basic product is more efficient. The profit-maximizing contract with framing further increases welfare: it weakly increases the seller's profit by definition, and it gives buyers a weakly higher U-utility than what they get in the above contract, in which they are U-indifferent to not buying anything.

[11] See, for example, Fudenberg and Tirole (1992, Chapter 7.1.1).

[12] If $(x_L, t_L) \geq (x_L^*, t_L^*)$, then low-type buyers U^f-prefer (x_L, t_L) to (x_L^*, t_L^*) because they are U-indifferent between these two bundles. Because high-type buyers have the same U^f-ranking of these two bundles, and because they are U^f-indifferent between (x_H^*, t_H^*) and (x_L, t_L), we obtain that they U^f-prefer (x_H^*, t_H^*) to (x_L^*, t_L^*) contradicting Assumption (A3).

6 An application to reference point framing

Sellers often highlight a particular product in the menu, and the highlighted product frequently includes premium features that are not included in other products. For example, Costco lists its executive membership first on its website, and emphasizes that it includes a 2% annual rebate that is not included in its basic membership. Similarly, the online car rental insurance seller "Insure My Rental Car" highlights its premium policy over the basic one by coloring it in a darker color. The premium policy covers personal property and hotel burglary, which are not included in the basic policy.

In the spirit of Kahneman and Tversky's (1991) model of reference-dependent choice, the highlighted product may serve as a reference point to which buyers compare other products. Buyers anticipate that they will experience a loss or regret if they purchase a product that is inferior to the highlighted product, and the model stipulates that their marginal sensitivity to losses decreases in the size of the loss.

This section studies the effect of such reference-point framing in the context of insurance when individuals have private information on their risk level. The analysis extends to other settings in which buyers treat the highlighted bundle as a reference point, and have decreasing marginal sensitivity to losses.

Model. We follow Stiglitz's (1977) monopolistic insurance setting. A risk-neutral profit-maximizing insurance provider offers a menu of insurance bundles to a population of risk-averse individuals. Each individual has initial wealth w, and may suffer an accident of size $A > 0$. An individual's privately-known probability of an accident is $\theta \in \{L, H\}$, with $0 < L < H < 1$. The proportion of Low-risk individuals in the population is $\pi_L > 0$ and of High-risk individuals is $\pi_H = 1 - \pi_L > 0$. Each individual's preferences over wealth are summarized by a strictly increasing, strictly concave, and continuously differentiable function u.

An insurance bundle is a pair (x, t), where t is the premium paid by the individual to the insurance provider upfront and $x \geq 0$ is the amount paid by the provider to the individual if the accident occurs. The expected utility of an individual with risk level θ from the bundle (x, t) is

$$U(x, t, \theta) = \theta u(w - t - A + x) + (1 - \theta)u(w - t).$$

We depart from Stiglitz's setting by assuming that in addition to offering a menu of insurance bundles, the provider can also highlight one bundle in the menu. We denote the highlighted bundle by $f = (x_f, t_f)$, and identify no highlighting with $f = (0, 0)$. A contract is thus a pair (M, f), where $f \in M$. Thus, there is a dependency between the menu and the frame. The set F of frames is the set of all possible bundles.

In a frame f, an individual treats the highlighted bundle as a reference point. He anticipates that if he purchases an insurance bundle (x, t) with coverage $x \leq x_f$, he will experience regret of $r(x_f - x)$ if the accident occurs, in addition to the effect of the accident on his wealth. That is, in the frame f an individual chooses from the menu a bundle (x,t) that maximizes

$$U^f(x,t,\theta) = \theta\left(u\left(w-t-A+x\right) - 1_{x\le x_f} r(x_f - x)\right) + (1-\theta)\,u\left(w-t\right),$$

where the regret function r satisfies the following properties:

- $r'(\Delta) > 0$ for $\Delta \ge 0$: Regret is increasing in the difference in coverage $\Delta = x_f - x$ between the reference coverage and the chosen coverage,
- $r''(\Delta) < 0$ for $\Delta > 0$: Marginal regret is decreasing, and
- $r(0) = 0$: There is no regret if the chosen coverage is equal to the reference coverage.[13]

This model departs from the assumptions of Section 2 in several respects. First, the seller's cost $c(x,\theta) = x\theta$ is type-dependent and is increasing in type. This changes the analysis of the optimal contract because the seller may wish to exclude high-risk individuals, who they are more costly to serve. This can in fact happen, as we show below.

Second, there is a dependency between the menu and the frame because the reference bundle has to be offered in the menu. Thus, an optimal contract may in principal require three bundles: a basic one targeted at low-risk individuals, a premium one targeted at high-risk individuals, and a reference bundle. The seller's profit-maximization problem then has the additional constraints that type-θ buyers U^f-prefer the bundle (x_θ, t_θ) to the reference bundle. We will omit these constraints in solving for the optimal contract, and then verify that the resulting optimal reference bundle coincides with the premium bundle.

Third, the specification of buyers' preferences and frame-dependent behavior is not quasi-linear, and does not satisfy Assumptions (A1) and (A2).[14] But the following two properties hold and are sufficient to establish Proposition 3 and characterize the optimal contract. First, a weaker version of increased attractiveness holds: fixing a reference bundle, the attractiveness of buying an additional unit of coverage below the reference coverage is larger with framing than without framing.[15] Second, the natural extension of Assumption (A2) to a non quasi-linear environment holds: a high-risk individual is willing to pay more than a low-risk individual for an additional unit of coverage, regardless of the reference bundle, because a high-risk individual is more likely to have an accident.[16]

Optimal contract. Because the optimal frameless contract is separating (see Stiglitz (1977))

[13]Note that r does not depend on the premium in the reference bundle. Our characterization of the optimal contract extends to cases in which r decreases in the reference premium, as long as r's dependency on the premium satisfies conditions that parallel those in the first two bullet points.

[14]Assumption (A3) holds, because $x_H^* = x_L^* = A$ and $t_H^* > t_L^*$.

[15]Formally, $\frac{\partial U^f(x,t,\theta)/\partial x}{\partial U^f(x,t,\theta)/\partial t} > \frac{\partial U(x,t,\theta)/\partial x}{\partial U(x,t,\theta)/\partial t}$ for any $f \neq (0,0)$ and $x < x_f$.

[16]Formally, $\frac{\partial U^f(x,t,H)/\partial x}{\partial U^f(x,t,H)/\partial t} > \frac{\partial U^f(x,t,L)/\partial x}{\partial U^f(x,t,L)/\partial t}$ for any frame f.

and framing does not increase the profit from pooling contracts, any optimal contract is separating. Any optimal contract also has the following properties, which are proved in the Appendix.

Property 1 *The reference coverage is identical to the premium coverage in any optimal contract in which high-risk individuals purchase insurance.*

Property 1 is in line with the real-world phenomenon that the reference product often coincides with the premium product. This property arises because the marginal sensitivity to losses is decreasing, so by setting the reference coverage to be equal to the premium coverage, the seller minimizes the attractiveness of the basic insurance bundle to high-risk individuals.

Property 2 *In an optimal contract, low-risk individuals purchase insurance regardless of the distribution of types.*

Property 2 implies that reference-point framing increases the social surplus with respect to low-risk individuals when their proportion in the population is small. This is because in this case the optimal frameless contract excludes low-risk individuals. By Proposition 3, the entire surplus gain goes to the seller.

Property 3 *In an optimal contract, high-risk individuals are either strictly over-insured, or do not purchase insurance.*

Property 3 implies that framing reduces the social surplus with respect to high-risk individuals. One channel for inefficiency is over-consumption of high-risk individuals, which enables the seller to increase the coverage of low-risk individuals, similarly to the setting of Section 5. A new channel for inefficiency is exclusion. Because high-risk individuals are more costly to serve than low-risk individuals, the insurance provider may want to exclude them, and only serve low-risk individuals. This is impossible in the standard model, but can be done with framing by offering high-risk individuals a premium insurance bundle that they U^f-prefer to the basic bundle but that is U-inferior to not purchasing insurance.

Taken together, Properties 2 and 3 imply that with framing the seller may optimally choose to serve only low-risk individuals. This is in line with the phenomenon of advantageous selection identified in the empirical literature. Such advantageous selection will arise in our setting when high-risk individuals are at a very high risk of having an accident, so the profit from fully insuring them is low, whereas low-risk individuals are at intermediate risk, so the profit from fully insuring them is high. Insuring both types then generates less profit than excluding high-risk individuals and fully insuring low-risk individuals, because whenever low-risk individuals are substantially insured, any insurance bought by high-risk individuals leads to a loss.

7 Discussion

This paper presents a model of contracts with framing. The main postulate of the model is that framing influences how buyers make comparisons, but does not persistently change their willingness to pay. The inability to persistently change willingness to pay may arise when buyers are partially sophisticated in the sense that they anticipate the framing effect or reevaluate their purchase decision ex-post.

There are of course alternative models of partial sophistication in which buyers invoke their underlying preferences during the purchase process. One alternative — that is particularly relevant when buyers reevaluate their purchase decision ex-post and return the product if it is too expensive — is that buyers "go back to the aisle" and make another purchase according to U^f, ignoring the product they just returned. Each buyer continues in this fashion until he finds a U^f-maximal bundle, among those he did not eliminate yet, that is U-superior to not buying anything, or he exhausts all the available bundles. Formally, this translates to a choice procedure in which the buyer chooses the U^f-maximal bundle from among those that are weakly U-superior to *stayout*, and does not make a purchase if this set is empty.

The increased sophistication of buyers, who repeatedly apply their underlying preferences to eliminate bundles, does not change the characterization of the optimal contract in Section 5. This is because the optimal allocation rule of Section 5 is implementable in the alternative model. And to verify that this allocation rule is profit-maximizing among all implementable allocation rules, note that in the alternative model (1) a type-θ buyer purchases (x_θ, t_θ) only if it is U-superior to not buying anything, and (2) if low-type buyers make a purchase, then high-type buyers purchase the premium bundle only if it is U^f-superior to the basic bundle, which are the only relevant constraints in the original model. One can use similar reasoning to verify that the predictions of Section 5 are also robust to a specification in which after returning a product, buyers go back to the aisle and make another purchase according to their underlying preferences.

Mechanism design with framing. The potential relevance of frames as a design parameter may extend to multi-agent environments and, more specifically, to auctions. For example, in Filiz-Ozbay and Ozbay's (2007) experimental auction, announcing before the auction starts that the winning bid will be revealed after the auction ends causes bidders to bid more aggressively. In the experimental auction of Delgado et al. (2008), subjects bid more aggressively in an auction with a frame that highlights the possibility of losing than in a baseline frameless auction. It seems that in both experiments, framing triggers bidders to anticipate a larger disappointment from losing than in a frameless setting, which leads to more aggressive bidding behavior conditional on participation. We conclude by illustrating that an efficient auction with a frame that increases bidders' anticipated disappointment from losing can raise more revenue than the profit-maximizing frameless auction.

An auction designer wishes to maximize his profit from selling a non-divisible good to one of N potential bidders, subject to allocating the good to the bidder with the highest value. Bidders' private values for the good are independently and uniformly distributed on $[0, 1]$. The preferences of a bidder with private value θ over lotteries (x, t), where x is the probability of winning the item and t is the monetary cost of obtaining this probability, are summarized by the function $U(x, t, \theta) = x\theta - t$.

To maximize profit, the designer chooses a frame $f \in F$ that affects bidders' anticipated disappointment from losing. In a frame f, with $f : [0, 1] \to \mathbb{R}_+$, a bidder with value θ evaluates outcomes according to the function $U^f(x, t, \theta) = x\theta - (1-x)f(\theta) - t$ if he decides to place a bid. The function f satisfies $f(0) = 0$ (i.e., bidders cannot be triggered to anticipate disappointment if they do not value the good) and $f'(\theta) > 0$ (i.e., the anticipated disappointment increases with the bidder's type). To capture the idea that in competitive settings bidders may be affected by framing only if they decide to actually compete with other bidders by placing a bid, we assume that $U^f(stayout, \theta) = 0 = U(stayout, \theta)$.

In an auction, each bid b is associated with a specific probability of winning x_b and a transfer t_b determined by the allocation rule and the behavior of other bidders. We can therefore think about bids as bundles. In a symmetric equilibrium $b(\theta, f)$ of an auction with a frame f, we thus have that $b(\theta, f) \in C^\theta(M, f)$, where M is the menu of winning probabilities and transfers associated with all possible bids.

Because bidders can avoid disappointment by not placing a bid, inducing participation in an auction with framing is harder than in a frameless one. But conditional on participating, bidders bid more aggressively in an auction with framing because they stand to lose more. The following observation shows that when the auctioneer has sufficient flexibility in designing the frame, the increase in revenue that results from the increased aggressiveness in bidding conditional on participation may be larger than the possible decrease in revenue due to the more demanding participation constraint.

Observation 2 *Consider an auction with a frame f that implements an efficient allocation such that for every type θ, $f(\theta)$ is strictly larger than the expected surplus of type θ in a frameless efficient auction. Then, the revenue in the auction with the frame f is strictly higher than the revenue in a revenue-maximizing (inefficient) frameless auction.*

8 Appendix

Proof of Proposition 1. Denote by (x, t) a bundle chosen by a buyer in an optimal contract. It cannot be that $x < x^*_\theta$, because then replacing (x, t) with the bundle $(x^*_\theta, t + \Delta)$, where $\Delta = u(x^*_\theta, \theta) - u(x, \theta)$, would increase the seller's profit (by increased attractiveness and the concavity of the seller's profit-maximization problem). If $x = x^*_\theta < d$, then $t < t^*_\theta$ (otherwise

(x_θ^*, t_θ^*) is implementable), so $U(x_\theta^*, t, \theta) > U(x_\theta^*, t_\theta^*, \theta) = U(0, 0, \theta)$, and by optimality of the contract $U^f(x_\theta^*, t, \theta) = U^f(\bar{x}, \bar{t}, \theta)$. For small $\varepsilon > 0$, let $\Delta = u^f(x_\theta^* + \varepsilon, \theta) - u^f(x_\theta^*, \theta)$. Thus, $U^f(x_\theta^*, t, \theta) = U^f(x_\theta^* + \varepsilon, t + \Delta, \theta)$. In addition, $u_x^f(x_\theta^*, \theta) > u_x(x_\theta^*, \theta) = c'(x_\theta^*)$ (the equality follows from the definition of $x_\theta^* < d$), so for sufficiently small ε we have that $U^f(x_\theta^* + \varepsilon, t + \Delta, \theta) = U^f(\bar{x}, \bar{t}, \theta)$, $U(x_\theta^* + \varepsilon, t + \Delta, \theta) > U(0, 0, \theta)$, and $c(x_\theta^* + \varepsilon) - c(x_\theta^*) < \Delta$. Thus, replacing the bundle (x, t) with $(x + \varepsilon, t + \Delta)$ increases the seller's profit.

Example 3. Consider the price discrimination setting of Example 1, with $u(x, \theta) = x\theta$, $c'(x) = x$ for $x \leq 1$, and $c'(x) = 1 + (x - 1)/B$ for $x > 1$, where B is large.[17] Suppose that the seller can only increase attractiveness substantially. Specifically, suppose that $F = \{\phi, f\}$, where $f = 9$. Suppose also that high type buyers' U-willingness to pay for quality is much higher than that of low type buyers. Specifically, $L = 1$ and $H = 2$. Finally, suppose that $\pi_L > 1/2$.

We now specify two frameless contracts D and E by describing their menus D and E, and show that the profit that any contract with the frame f generates is strictly lower than the maximum of the profit that these two contracts generate. Let $D = \{(x_L^*, t_L^*), (x_H^*, t_H)\}$, where $x_L^* = 1$, $x_H^* = 1 + B$, $t_L^* = 1$, and $t_H = 2B + 1$. Then, $(x_L^*, t_L^*) \in C^L(D, \phi)$ and $(x_H^*, t_H) \in C^H(D, \phi)$. When buyers choose these bundles, D generates profit $\pi_H(B + 1)/2$ from high type buyers, which is only π_H less than the first-best profit from selling to high type buyers, and generates the first-best profit from low type buyers. Let $E = \{(\varepsilon, \varepsilon), (x_H^*, t_H^* - \varepsilon)\}$ for some small $\varepsilon > 0$. Then, $(\varepsilon, \varepsilon) \in C^L(E, \phi)$ and $(x_H^*, t_H^* - \varepsilon) \in C^H(E, \phi)$. When buyers choose these bundles and ε is sufficiently small, the profit that E generates is strictly higher than the first-best profit from selling to high type buyers, because $\pi_L > \pi_H$.

Consider a contract with the frame f that excludes buyers of some type, i.e., these buyers choose *stayout*. If the contract excludes high type buyers, then the profit it generates is bounded above by the first-best profit from selling to low type buyers, which is strictly lower than the profit generated by D. If it excludes low type buyers, then the profit it generates is bounded above by the first-best profit from selling to high type buyers, which is strictly lower than the profit generated by E.

Now consider a non-excluding contract G with the frame f, denote by $(x'_\theta, t'_\theta) \neq$ *stayout* the bundle that buyers of type θ choose, and suppose that G generates more profit than any excluding contract. To generate more profit than D, the contract G must generate a profit of at least $\pi_H(B + 1)/2$ from high type buyers, because D already generates the first-best profit from low type buyers. This implies that $x'_H > B/4$, because $H = 2$. Because low type buyers weakly U^f-prefer (x'_L, t'_L) to (x'_H, t'_H), we must also have that $x'_L(1 + f) - t'_L \geq x'_H(1 + f) - t'_H$. Because $t'_L \geq 0$ (otherwise, excluding low type buyers and selling the first-best to high type buyers is profit enhancing), $t'_H \leq 2x'_H$ (otherwise, high type buyers would strictly U-prefer *stayout* to

[17] The cost of producing x units is therefore $c(x) = x^2/2$ for $x \leq 1$ and $c(x) = \left(1 - B + 2(B - 1)x + x^2\right)/2B$ for $x > 1$.

$(x'_H, t'_H))$, and $f = 9$, we obtain that $x'_L \geq 4x'_H/5 > B/5$. But for B large enough, every unit above $B/8$ sold to low type buyers leads to a loss of at least $1/16$, even if low type buyers are charged $L = 1$ per unit. This implies that for a large enough B the loss in G on low type buyers is larger than the possible gain on high type buyers.

Proof of Proposition 2. Suppose to the contrary that there is an optimal separating contract that is frameless, and denote by $g(\theta) = (x_\theta, t_\theta)$ the optimal allocation it implements. The standard theory tells us that $x_L \leq x_L^*$, $x_H = x_H^*$, low-type buyers are U-indifferent between $(0, 0)$ and (x_L, t_L), and high-type buyers are U-indifferent between (x_L, t_L) and (x_H, t_H). Consider a modified contract with the menu $\{(x_L, t_L), (x_H, t_H)\}$ and a frame $f \neq \phi$. By Assumption (A1), high-type buyers strictly U^f-prefer (x_H, t_H) to (x_L, t_L), but low-type buyers may also U^f-prefer (x_H, t_H) to (x_L, t_L). We now modify the bundles in a way that increases profit until $g(\theta)$ with the modified bundles is implemented. First, increase t_H until high-type buyers are either U^f-indifferent between (x_H, t_H) and (x_L, t_L) or are U-indifferent between (x_H, t_H) and $(0, 0)$. If the latter occurs before the former, increase x_L and t_L along the U-indifference curve of low type buyers through $(0, 0)$. By Assumption (A3), high-type buyers will be U^f-indifferent between (x_H, t_H) and (x_L, t_L) before (x_L, t_L) reaches (x_L^*, t_L^*). By Assumption (A2) the modified contract implements g, and by properties of the seller's problem this generates a strictly higher profit than the original contract, a contradiction.

Proof of Proposition 3. If $f = \phi$, then we are in the standard setting, in which these properties are well known. Suppose that $f \neq \phi$. If the contract is a pooling one, then the properties follow immediately. It thus remains to consider a separating contract in which all buyers choose a positive product and $f \neq \phi$. The seller's problem (conditional on f) can therefore be written as follows:

Choose $((x_L, t_L), (x_H, t_H))$ to maximize $\pi_L(t_L - c(x_L)) + \pi_H(t_H - c(x_H))$ subject to:

$$\text{IR}_\theta^U : U(x_\theta, t_\theta, \theta) \geq U(0, 0, \theta) \text{ for } \theta \in \{L, H\},$$

$$\text{IC}_\theta^f : U^f(x_\theta, t_\theta, \theta) \geq U^f(x_{\theta'}, t_{\theta'}, \theta) \text{ for } \theta, \theta' \in \{L, H\} \text{ and } \theta' \neq \theta.$$

Considering an optimal contract, we first note that if IC_θ^f holds strictly, then IR_θ^U binds, otherwise t_θ can be increased slightly without violating any of the constraints. This implies that either IC_H^f or IC_L^f bind, otherwise, because by Assumption (A3) $\{(x_L, t_L), (x_H, t_H)\} \neq \{(x_L^*, t_L^*), (x_H^*, t_H^*)\}$, some x_θ can be increased or decreased slightly along the U-indifference curve of agent θ to decrease $|x_\theta - x_\theta^*|$, which increases the principal's profit, without violating any of the constraints.

In fact, IC_H^f must bind. Indeed, suppose that IC_L^f binds. By Assumption (A2), because $x_L < x_H$, IC_H^f holds strictly, so IR_H^U binds. We now modify the bundles in a series of steps in a way that increases profit, such that either at some point along the sequence all the constraints are satisfied, so the modified bundles generate more profit than the optimum, a contradiction, or the modified bundles are (x^*_θ, t^*_θ) and IC^f_H holds, which contradicts Assumption (A3). The first step applies if $x_H > x^*_H$. In this case, decrease (x_H, t_H) continuously along the high type's U-indifference curve until either IC^f_H binds or $x_H = x^*_H$. In the former case, Assumption (A2) implies that IC^f_L holds,[18] so all the constraints are satisfied and the principal's profit increases, a contradiction. We therefore have that $x_H \leq x^*_H$ and IC^f_H holds strictly. Now increase t_L until IR^U_L binds. This further relaxes IC^f_H. Finally, if $x_L < x^*_L$, increase (x_L, t_L) continuously along the low type's U-indifference curve until either IC^f_H binds or $x_L = x^*_L$. In the former case, we obtain a contradiction as in the first step. We have therefore reached a situation in which (i) $x_L \geq x^*_L$ and IR^U_L binds, (ii) $x_H \leq x^*_H$ and IR^U_H binds, and (iii) IC^f_H holds strictly. Now, (i), Assumption (A1), and Assumption (A2) imply that

$$U(x_L, t_L, L) = U(x^*_L, t^*_L, L) \Rightarrow U(x_L, t_L, H) \geq U(x^*_L, t^*_L, H) \Rightarrow U^f(x_L, t_L, H) \geq U^f(x^*_L, t^*_L, H),$$

and (ii) and Assumption (A1) imply that

$$U(x^*_H, t^*_H, H) = U(x_H, t_H, H) \Rightarrow U^f(x^*_H, t^*_H, H) \geq U^f(x_H, t_H, H),$$

so by (iii) we have $U^f(x^*_H, t^*_H, H) > U^f(x^*_L, t^*_L, H)$, which contradicts Assumption (A3).

Because IC^f_H binds, by (A2) we have that IC^f_L holds strictly, so IR^U_L binds.

Proof of Proposition 4. First observe that $x_L \leq x^*_L$ and $x_H \geq x^*_H$. Indeed, if $x_L > x^*_L$, then decrease (x_L, t_L) slightly along low-type buyers' U-indifference curve so that IC^f_L continues to hold. By Assumption (A1) and (A2) this relaxes IC^f_H, so all constraints hold and the profit increases, a contradiction. If $x_H < x^*_H$, then increase (x_H, t_H) slightly along the high type's U-indifference curve so that IC^f_L continues to hold. By Assumption (A1) this relaxes IC^f_H, so all constraints hold and the profit increases, a contradiction.

Finally, suppose that $x^*_H < d$ and $x_H = x^*_H$. If IR^U_H holds strictly, then $x_L < x^*_L$, similarly to the standard setting.[19] And if IR^U_H binds, then $x_L < x^*_L$, because Assumption (A3) implies that $\{(x_L, t_L), (x_H, t_H)\} \neq \{(x^*_L, t^*_L), (x^*_H, t^*_H)\}$. But $x_L < x^*_L$ implies that the principal's marginal

[18]It must be that $x_H \geq x_L$, because by IR^U_L and (A2) (x_L, t_L) lies below the high type's U-indifference curve through $(0, 0)$, so IC^f_H binds before x_H reaches x_L.

[19]If $x^*_L = x_L$, then $x^*_L = x_L < x_H = x^*_H < d$, because the contract is separating. That $x^*_L < d$ implies that the principal's marginal cost at x^*_L is equal to low type buyers' marginal u-utility, whereas $x^*_L < x^*_H$ implies that high type buyers' marginal u-utility at x^*_L is strictly higher (because the profit function is concave along each type's U-indifference curve). Therefore, by Assumption (A2), decreasing x_L by some small ε along the low type's U-indifference curve decreases high type buyers' U^f-utility from the bundle (x_L, t_L) by at least $\delta\varepsilon$ for some $\delta > 0$ that is independent of ε. This decreases means that t_H can be increased by $\delta\varepsilon$ without violating IC^f_H. Thus, for sufficiently small ε this leads to an increase in the principal's profit, because to a first order the change in profit from changing the the low type's bundle is 0, and this change allows an increase in profit from the high type that is positive to a first order.

profit at x_L along the low type's U-indifference curve is positive, while $x_H = x_H^*$ implies that the principal's marginal profit at x_H along the high type's U-indifference curve is 0. Therefore, the profit can be increased by increasing x_H slightly along the high type's U-indifference curve, which relaxes IC_H^f and makes it possible to increase x_L along the low type's U-indifference curve.[20]

Proof of Property 1. Assume to the contrary that there exists an optimal contract in which high-risk individuals buy insurance and in which the reference coverage x_f differs from the high type's coverage x_H. By Proposition 3, whose proof applies to the insurance setting as well, IC_H^f holds with equality in this contract, and because $x_H > x_L > 0$, by Assumption (A2) IC_L^f holds strictly. We now modify this contract by modifying the reference coverage to derive a contradiction to Proposition 3. If $x_f < x_H$, then increase x_f slightly to $x_{\tilde{f}}$ (or slightly above the low-risk individual's coverage x_L if $x_f < x_L$) so $\mathrm{IC}_L^{\tilde{f}}$ still holds. This increases the regret associated with purchasing the low-risk individuals' bundle (but not with purchasing the high-risk individuals' bundle), so $\mathrm{IC}_H^{\tilde{f}}$ holds strictly and all other constraints hold. If $x_f > x_H$, then decrease x_f slightly to $x_{\tilde{f}}$ so $\mathrm{IC}_L^{\tilde{f}}$ still holds. This makes low-risk individuals' bundle less attractive relative to that of high-risk individuals, because r is concave and $x_H > x_L$. Again, this implies that $\mathrm{IC}_H^{\tilde{f}}$ holds strictly and all other constraints hold. In both cases, the new separating contract generates the same profit as the original one, and is therefore optimal, but in contradiction to Proposition 3, the constraint $\mathrm{IC}_H^{\tilde{f}}$ holds strictly.

Proof of Property 2. Letting $f = (x_H^*, t_H^*)$, we obtain the result using the same arguments for non-exclusion of low type buyers in the discussion following Proposition 2, whose proof applies to the insurance setting as well.

Proof of Property 3. Consider an optimal contract in which high-risk individuals buy insurance. First, observe that the contract is separating. To see why, suppose to the contrary that the contract is pooling with the bundle (x, t). Clearly, $0 < x \leq A$ and $t > 0$. If $x = A$, then decrease x by a small $\varepsilon > 0$ and decrease t slightly so that $U(x, t, L)$ remains unchanged. The profit in the new pooling contract is higher because the provider makes essentially the same profit on low-risk individuals but gains approximately $\varepsilon(H - L)$ on high-risk individuals. Now suppose that $0 < x < A$. Since $x > 0$ and IR_L^U holds, IR_H^U holds strictly. Then, add another insurance bundle (x_H, t_H) aimed at the high-risk individual, with $x_H < A$ slightly larger than x, such that IR_H^U continues to hold and IC_H^f holds with equality. By Assumption (A2), IC_L^f

[20]More precisely, increasing x_H by some small ε along the high type buyers' U-indifference curve increases their U^f-utility from the bundle (x_H, t_H) by at least $\delta\varepsilon$ for some $\delta > 0$ that is independent of ε. And increasing x_L by some small γ along the low type buyers' U-indifference curve increases the high type buyers' U^f-utility from the bundle (x_L, t_L) by no more than $\alpha\gamma$ for some $\alpha > 0$. Thus, the increase of x_H by ε allows to increase x_L by at least $\delta\varepsilon/\alpha$. And because the marginal effect on the profit of such an increase in x_H is 0, whereas the marginal effect on the profit of the increase in x_L is positive, for small ε the profit increases.

continues to hold. Profit strictly increases, because high-risk individuals are risk-averse (as are low-risk individuals) and are therefore U-willing to pay more for the additional unit of insurance than the cost to the risk-neutral provider, and their U^f-willingness to pay is weakly higher.

Second, $x_L \leq A$ and $x_H \geq A$, as in the proof of Proposition 4. Thus, to complete the proof it suffices to verify that any contract in which low-risk individuals are partially insured, high-risk individuals are fully insured, and full coverage is highlighted is not optimal. Consider such a contract, and increase the high-risk individuals' coverage and premium slightly along their U-indifference curve. This does not change the provider's profit to a first order, because when high-risk individuals are fully insured their willingness to pay for an additional unit of insurance is identical to the provider's cost of providing this unit. Because the new coverage is larger than the reference coverage, U-indifference implies that a high-risk individual is also U^f-indifferent between his original bundle and the new bundle, so IC_H^f continues to hold; and IC_L^f continues to hold because it held strictly before the change. Now increase x_f to equal the new coverage $x_{\tilde{f}}$ for the high-risk individual. Then $\mathrm{IC}_H^{\tilde{f}}$ holds strictly, and $\mathrm{IC}_L^{\tilde{f}}$ continues to hold if the change in coverage is small enough. Finally, increase the low-risk individuals' coverage and premium slightly along their U-indifference curve, which strictly increases profit to a first order and does not violate any of the constraints.

Proof of Observation 2. It suffices to show that the revenue in an efficient auction with the frame f that assigns to every type θ an anticipated disappointment that equals his expected surplus in an efficient frameless auction is larger than in the optimal frameless auction.

Let N denote the number of bidders in the auction. By Myerson (1981), one optimal frameless auction is a second-price auction with a reserve price of $1/2$. The revenue in this auction is:

$$\left(\frac{1}{2}\right)^N \cdot 0 + N\left(\frac{1}{2}\right)^N \cdot \frac{1}{2} + \sum_{i=2}^{N} \binom{N}{i} \left(\frac{1}{2}\right)^N \left(\frac{1}{2} + \frac{i-1}{i+1}\frac{1}{2}\right) = \frac{N-1}{N+1} + \frac{1}{2^N\,(N+1)}.$$

By the revenue equivalence theorem, the revenue in an efficient auction with the frame f is identical to the revenue in a second price auction with the frame f, where our assumption on f implies that $f(\theta) = \theta^N/N$. Given that a type θ bidder places a bid in this second price auction, it is weakly dominant for him to bid $\theta + f(\theta)$. And if all bidders bid in this way, then a simple calculation shows that bidding in the auction is U^f-superior to not bidding. Therefore, the revenue in this auction is the second-order statistic of the valuations, $(N-1)/(N+1)$, plus the second-order statistic of $f(\theta)$, which a straightforward calculation shows is $(N-1)/(2N\,(2N-1))$. Thus, the revenue is $(N-1)/(N+1) + (N-1)/(2N\,(2N-1))$.

It remains to verify that

$$\frac{N-1}{2N\,(2N-1)} \geq \frac{1}{2^N\,(N+1)}.$$

This holds for $N = 2$. For $N \geq 3$, it suffices to show that $2^N \geq 2\left(2 - \frac{1}{N}\right)/\left(1 - 1/N^2\right)$. This inequality holds, because for $N \geq 3$ we have that

$$2^N \geq 8 > \frac{4}{1-\frac{1}{9}} > \frac{2\left(2-\frac{1}{N}\right)}{1-\frac{1}{N^2}}.$$

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