Dynamic Capital Budgeting with Risk Constraints

OR

"RISK-BASED VALUATION"

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Capital budgeting in practice

- Most firms do not follow the NPV rule taught in b-school
  - Valuation model issues: Optimism and model risk
  - Resource constraints: People and/or capital
  - Optionality: The value of waiting to invest
  - Capital structure: Debt overhang
- Solutions
  - Ad-hoc discount rate adjustments
  - NPV adjustments
  - Minimum NPV thresholds
  - Constrained optimization when opportunities are known
- Challenges
  - Stochastic opportunity set with constrained resources
  - Supportable discount rates
  - Multiperiod valuations

Statement of the problem and the thesis

- Capital-constrained firms have theoretical and practical problems with CAPM-based capital budgeting
  - Idiosyncratic risk may be costly
  - Cash flows in different periods may be causally related or correlated
    - (this matters if idiosyncratic risk matters)
  - Benchmarks and comparables should be used when available (e.g. oil forwards)
  - Using forwards in a CAPM context can be challenging
  - Options and other nonlinear relationships are difficult to include
  - Some CAPM parameters are unknown
    - (e.g. correlation between project and market *return*)
  - Project data normally occur in prices and levels, not returns
- Firms lack an integrated and consistent framework for valuing multiperiod projects in capital-constrained environments.
  - This presentation uses simulation as a unifying framework to achieve this objective
  - The capital constraint is modeled by putting a price on idiosyncratic risk

Outline of this presentation

- General valuation equation for any given risk measure
  - Risk-neutrality and time-neutrality
- Derive pricing formulae with idiosyncratic risk
- Explain the derivation of the cost of risk
- Appendix
  - Apply consistent framework for CAPM, forwards & options

Adapting a general valuation equation

- Every asset satisfies the general valuation equation GVE
  - Expected return = Required return
  - {Exp capital gain} + Exp cash flow = Cash opportunity cost + Risk compensation
  - $\{E[V_{t+1}] - V_t\} + E[C_{t+1}] = r\,V_t + k\,R_t$ at all times t
- Notes
  - R_t may contain systematic priced risks as well as idiosyncratic risks
  - When R_t is proportional to V_t, we get a risk premium formulation
  - In the CAPM (levels), the risk compensation simplifies to
    - $k\,R_t = \beta(E(r_M) - r_f)\,V_0 = \beta_L[\mu_M - (1+r_f)M_0]$
- If we move the cost of risk (kR_t) to the left side of the GVE equation, we obtain the usual risk-neutrality
  - Often assume residual risk is unpriced

Time neutrality

- The *time-neutral* transformation of cash flows is achieved by discounting all the cash flows and risk measures at the riskless rate and then using an effective riskless rate of 0.
  - Test: Discount cash flows and risk measures at the riskless rate in the GVE and apply a zero discount rate
    - $E[V_{t+1}]/(1+r) - V_t + E[C_{t+1}]/(1+r) = 0 + k\,R_{t+1}/(1+r)$
    - Multiplying by (1+r) and rearranging terms, this produces the original GVE
- This transformation has two important consequences
  - Improves tractability of certain valuation problems
  - Risk charge is proportional to risk rather than project value

Example: Single period model

- Normally distributed cash flow C_1 in one year (μ_C, σ_C)
  - Risk measure σ_C, cost of risk k
- Apply the GVE:
  - $\{\mu_C - V_0\} + 0 = r_f V_0 + k\sigma_C$
  - $V_0 = [\mu_C - k\sigma_C] / (1 + r_f)$
- Expected return equation
  - $E(r_V) = r_f + k\sigma_C / V_0$
- Same cash flow, but now correlated with the market
- Apply the GVE:
  - $\{\mu_C - V_0\} + 0 = r_f V_0 + \beta_L[\mu_M - (1+r_f)M_0] + k\sigma_\varepsilon$
  - $V_0 = [\mu_C - \beta_L[\mu_M - (1+r_f)M_0] - k\sigma_\varepsilon] / (1 + r_f)$
- The expected return equation
  - $E(r_V) = r_f + \beta[E(r_M) - r_f] + k\sigma_\varepsilon / V_0$

Multiperiod models

- Joint normally distributed cash flows $C_1,\ldots,C_N$ with correlation matrix **R**, standard deviation vector $\boldsymbol{\sigma}$ and mean vector $\boldsymbol{\mu}$
  - The Cholesky decomposition of **R** is given by **C**, and **I** is the identity matrix
- Make time-neutral conversion for convenience
  - Convert $C_j^* = C_j/(1+r_f)^j$
  - Replace $\mu_j^* = \mu_j/(1+r_f)^j$ and $\sigma_j^* = \sigma_j/(1+r_f)^j$
- Choose risk measure and value
  - Variance of total value (PVAR) $V_0 = \boldsymbol{\mu}^{*\prime}\mathbf{1} - k\,(\boldsymbol{\sigma}^{*\prime}\mathbf{R}\,\boldsymbol{\sigma}^{*})$
  - Stdev of total value (RPV) $V_0 = \boldsymbol{\mu}^{*\prime}\mathbf{1} - k\,(\boldsymbol{\sigma}^{*\prime}\mathbf{C}\,\mathbf{1})$
  - Stdev of total value, zero corr (CFAR) $V_0 = \boldsymbol{\mu}^{*\prime}\mathbf{1} - k\,(\boldsymbol{\sigma}^{*\prime}\mathbf{I}\,\mathbf{1})$

Properties of these models

- Idiosyncratic risk matters
  - hedging adds value
- Correlations between cash flow periods matter
- Ordering of cash flows matters
- Values are non-additive
  - a negative NPV incremental project can add value
- Easy to add market factors (multifactor risk)
- Easy to include nonlinear payoff functions

“New Rules” for Capital Budgeting

- Put a cost on scarce resources (including risk capital)
  - Deduct shadow costs of constrained resources
- Treat risk differently than other scarce resources
  - Not additive
  - Need to address interactions across time
- Simulate and regress cash flows on priced assets/contracts
  - Include nonlinear relationships naturally
  - Price residual risk using the methods in this paper
- Accept a project if the valuation of the firm with the project exceeds the value of the firm without the project

Determining the private cost of risk (k)

- k is a measure of the adverse impact caused by increased risk
- If firm accepts a contract or purchases an asset, the incremental risk will generally
  - Add to the risk of the agent's cash flow
  - Increase the risk of declines in future wealth
  - Increase the likelihood of financial distress or bankruptcy
- The value of k is chosen on the margin so the firm is compensated for the cost to his income statement or balance sheet.
- Most financial institutions have determined an explicit cost of risk which they use in their valuations of financial assets and contracts.
- Other methods to determine k
  - k can be determined implicitly, much like an IRR, if project valuation benchmarks are known
  - k can also be chosen using peer performance analysis
  - k can be determined from an internal auction for risk capital

The consistent framework

- The cash flows of a project along with its traded value drivers can be simulated.
  - Relationships may be linear or nonlinear.
- Time-neutralize cash flows, traded assets and forward prices.
- Regress adjusted cash flows on traded value drivers and compute covariance matrix of residuals.
- Choose the appropriate risk measure.
- Determine the appropriate cost of risk k.
- Value the project using PVAR or RPV.
- Replace NPV criterion:
  - Accept an incremental project if the risk-based valuation of the package exceeds the risk-based valuation of the standalone project.

Example: Value an infinite-lived project

- Algorithm step 1: Value the firm without the project
  - Convert all cash flows (random variables) to their time-neutral values
  - Calculate the sum of the mean cash flows
  - Deduct charges for constrained resources
    - people, working capital
  - Deduct risk charge for risks realized in period 1
    - $\sigma_1, \rho_{12}\sigma_2, \rho_{13}\sigma_3, \ldots$ (Note comparison to Cholesky decomposition)
  - Deduct risk charges for risks realized in period 2
    - $\sqrt{(1-\rho_{12}^2)}\sigma_2, \ldots$
  - Note: If market value is known, k can be chosen to calibrate to market value
- Algorithm step 2: Value the firm with the project
- Algorithm step 3: Compare two values

APPENDIX: Obtaining the CAPM

- One-year project
- Simulated levels of cash flow (C_1) and market index (M_1)
- Regress C_1 on M_1
  - $C_1 = [\mu_C - \beta_L \mu_M] + \beta_L M_1 + \varepsilon$
  - (note β_L is in levels not returns; $\beta_L = cov(C_1, M_1)/var(M_1)$)
- Discount risk-free and market-correlated cash flows assuming residual risk is unpriced
  - $V_0 = [\mu_C - \beta_L \mu_M]/(1+r_f) + \beta_L M_0 + 0$ — Replication pricing
  - $V_0 = [\mu_C - \beta_L \{\mu_M - M_0(1+r_f)\}]/(1 + r_f)$ — Risk-neutral pricing
  - $V_0 = \mu_C/(1+r_f) - \beta_L \{\mu_M/(1+r_f) - M_0\}$ — Time-neutral pricing

  (Replication, risk-neutral and time-neutral pricing: } Equiv)
- Substitute
  - $\beta_L = \beta V_0/M_0$, $C_1 = V_0(1+r_V)$, $M_1 = M_0(1+r_M)$ — Convert levels to returns
  - $E(r_V) = r_f + \beta(E(r_M) - r_f)$ — CAPM expected return eq.
  - $V_0 = \mu_C/(1+E(r_V))$ — CAPM valuation

Valuing a one-year oil project using forwards

- W=WTI (West Texas Intermediate Crude Oil)
- Simulated levels of cash flow (C_1) and oil prices (W_1)
  - Cash flow depends on revenues and costs, both of which are functions of oil prices
- Regress C_1 on W_1
  - $C_1 = [\mu_C - \beta_L \mu_W] + \beta_L W_1 + \varepsilon$
- Discount risk-free and oil-correlated cash flows assuming residual risk is unpriced and F_W= forward price of oil
  - $V_0 = [\mu_C - \beta_L \mu_W]/(1+r_f) + \beta_L F_W/(1+r_f)$ Replication
  - $V_0 = [\mu_C - \beta_L \{\mu_W - F_W\}]/(1+r_f)$ Risk-neutral
  - Equivalent to time-neutral valuation if $F_W \leftarrow F_W/(1+r_f)$
- Q: What if the relationship between C and W is nonlinear?

Options in a discrete-time Black-Scholes framework

- Slight difference: Allow only one rebalancing period, at time zero
- Simulated levels of stock price (S_T) and call option payout (C_T)
  - α = continuous expected growth rate of the stock
- Regress C_T on S_T
  - $C_T = a + bS_T$
    - $b = [E(C_T S_T) - E(C_T)E(S_T)] / [E(S_T^2) - E(S_T)^2]$
    - $a = E(C_T) - b\,E(S_T)$
- Discount risk-free and stock-correlated cash flows assuming residual risk is unpriced
  - $C_0 = S_0 \exp[(\alpha - r_f)T]\,N(d_1(\alpha)) - X\exp(-r_f T)N(d_2(\alpha)) - bS_0(\exp[(\alpha - r_f)T] - 1)$
    - $d_1(\alpha)$ is the Black-Scholes d_1 with α substituted for r_f
  - Simplifies to Black-Scholes when $\alpha = r_f$