# Financing Start-Ups*

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**Abstract**

This paper provides a framework for understanding the relation between investor's involvement and the financial claims used to finance the establishment of the firm. Optimal contracts are derived for passive investors, involved investors, and acquirers that take over the management. The main results are that passive investors use debt, involved investors use convertible securities and acquirers use cash or option contracts. The paper also studies how entrepreneurs choose among these three types of investors depending on the characteristics of their projects. The result is that entrepreneurs with small safe projects get financing from passive investors, entrepreneurs with big, risky projects get financing from involved investors and entrepreneurs in industries with weak property rights protection get financing from acquirers.

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## 1 Introduction

Entrepreneurs generally need financing to start a new firm. Different investors are likely to seek different levels of involvement in the start-ups they finance. First, investors such as banks rarely get actively involved in the projects they finance, second investors such as venture capitalists get actively involved in the start-up partnering with the entrepreneur, and third investors such as private equity partnerships or existing firms acquire the project and take over the management, replacing the entrepreneur. This paper provides a framework for understanding the relation between the involvement of the investor and the financial claims used to provide capital. The framework is also used to understand how the entrepreneur chooses between different types of investors.

When investors get involved, the value of the project is enhanced by their advice and cooperation, but the provision of incentives becomes more difficult since a moral hazard in teams problem is created. [1] The financial claims used by involved investors are therefore chosen to deal with an agency problem fundamentally different from the one faced by the standard outside investor.

We consider contracts used by passive investors, involved investors and acquirers. The main finding of the paper is that the financial claims depend on two variables, the cost of the investment and the involvement of the investor in the firm. Passive investors that don't get involved use debt contracts. Acquirers use call option contracts or simply buy firms with cash. The contract used by involved investors depends on the cost of the investment layout. If the cost is high they use debt contracts or redeemables. If the cost is low they use call option contracts. For intermediate cost levels the optimal contract can be achieved by a convertible security where the "convertible" part of the contract depends positively on the level of involvement and negatively on the investment cost .

The analysis allows a characterization of those projects that get funding from each type of investors. Large projects with high start-up costs and high risk get funding from involved investors. Small and safe projects with low investment costs get funding from passive investors. We also consider an environment with weak property rights protection and show that the weaker the property rights protection is, the less inclined entrepreneurs are to seek financing from passive investors.

To solve this problem we use a moral hazard model that incorporates risk explicitly, as opposed to the standard reduced form approach. The explicit representation approach introduced in the paper has two advantages. First, it

[1] This problem has also been called a double moral hazard problem. See for example [Casamatta (2003)].

allows us to relax standard conditions in moral hazard problems such as the monotone likelihood ratio property, difficult to satisfy in applied problems. Second, it allows us to use techniques familiar from the adverse selection literature to identify the states that are best suited to provide incentives and derive the optimal contracts in a simple manner, providing a clear intuition for the results. Intuitively, residual returns provide agents with incentives and with an expected payoff. Residual returns in high revenue realizations provide more incentives per unit of expected payoff than low output realizations. The optimal contract assigns the residual returns in low and high states depending on how important it is to incentivize and compensate each agent.

The predictions of the model are broadly consistent with the behavior of passive investors, acquirers and involved investors. The most common passive investors are banks and similar financial institutions. According to the Federal Survey of Small Businesses Finance, within firms of two years of existence or less; 20.6% of the external financing comes from commercial banks and 15.8% is debt with similar financial institutions. [2] This is more important than financing by trade creditors (17%) and by far more important than money borrowed from family and friends, two sources that are usually claimed to be fundamental in fostering entrepreneurship. [3] Almost all the financing provided by banks is in the form of debt contracts.

Venture Capital is the most documented form of involved investment. Venture capitalists monitor [Lerner (1995)], provide technical and commercial advice [Bygrave and Tymmons (1992)]; meet with suppliers and customers [Gorman and Sahlman (1989)], help in designing strategy and human resources policies [Hellman and Puri (2002)] and visit the firms they finance often [Gorman and Sahlman (1989)]. The number of start ups financed with VC is relatively small. According to the American Association of Venture Capitalists [4], in 2007 the industry was involved in 3813 deals, less than 1000 involved financing seed stage projects. The firms financed with venture capital are different from the average start-up in the United States. The average deal in venture capital requires an annual investment of over 7 million dollars, while the average start-up requires around $25,000. [Stouder and Kirchoff (2004)]. Another difference is that while most start ups are in the service industry, and few are technologically sophisticated [Reynolds (2005)] the venture capital industry is concentrated in a few industries very intensive in technology . In fact close to 80% of all the deals in VC are concentrated in four industries: Biotechnology, Software and Internet, Clean Energy and Telecommunications. Empirical evidence shows that convertible securities are the main source of financing used by venture capitalists. [Gompers (1997)] examine 28 rounds of

[2] See [Berger and Udell (1998)]

[3] For more on this point see [Shane (2008)].

[4] See 2007 Money tree report at https://www.pwcmoneytree.com/MTPublic/ns/index.jsp

financing by a VC to 5 different firms, and found that convertible securities are the main claim used in 23 rounds, and [Kaplan and Stromberg (2003)] found that convertible securities are used in 189 out of 200 rounds.

Acquirers are usually private equity partnerships or well established firms in the industry. While most acquirers simply buy start ups with a fixed payment, anecdotal evidence suggests that some acquirers use call options. Sahlman (1990) discuss the case of Parenting Magazine. The creator of the magazine Robin Wolanner needed to raise 5 million dollars to pay the start up cost. Time Inc, a well established firm in the industry offered to finance the start up using a call option contract for all the equity of the firm.

The paper is closely related to the literature that considers the managerial incentives of investors and entrepreneurs as a double moral hazard problem. Notably [Casamatta (2003)] compares the performance of equity, convertibles and bonds in a model where the firm can either succeed or fail. This paper differs from her analysis by allowing for a continuum of outcomes which allows us to identify the precise set of optimal financial claims and how they depend on the degree of involvement of the investor in the start-up. Other authors study optimal contracting in more complex settings, [Repullo and Suarez (2004)] study contracting with multi-stage financing and [Inderst and Muller (2004)] in a setting with search. These papers restrict attention to linear incentive schemes.

Other papers consider contracts between entrepreneurs and involved investors in different settings, [Schmidt (2003)] shows the optimality of convertible securities in an environment where investment is sequential and observable but not contractible and [Bergeman and Hege (1998)] develop a dynamic model where the investor decides when to get out of the financial agreement. Other authors consider contracts that provide incentives to innovate [Aghion and Tirole (1994)] and contracting when there are weak property rights protection [Anton and Yao (1994)] and [Ueda (2004)]. There is also a vast literature that considers contracting between an entrepreneur and an investor when there are private benefits derived from control rights (see [Marx (1998)] and [Hellman (1998)]).

The paper is also related to the literature that studies moral hazard with limited liability. The standard approach in this literature is due to [Innes (1990)]. This paper takes an alternative approach by modeling risk explicitly. This approach is more tractable and it provides a clear intuition for most of the results. We also extend the results to allow the principal to exert effort.

The rest of the paper is organized as follows. Section 2 presents a simple model of agency where the entrepreneur and investor are risk neutral, but the entrepreneur is wealth constrained. Section 3 explains the dual role of contracts section 4 derives the optimal contracts as a function of the involvement of the investor in the management of the firm. Section 5 discusses which projects in terms of size and start up cost are better suited to get funding from involved investors. Section 6 relaxes the assumption that the entrepreneur is wealth constrained and discusses how the entrepreneur and investor share the investment costs. Section 7 concludes. The proofs of the propositions are presented in the text.

## 2 The Model

There are two risk-neutral agents, an entrepreneur and an investor. The entrepreneur has no wealth and owns a project that requires a fixed investment layout $K$. The project generates a revenue $\Pi$ given by

$$\Pi(\theta, z_E, z_I) = \theta Q(z_E, z_I), \tag{1}$$

where $\theta$ is a random variable with density $f(\theta)$ and cumulative $F(\theta)$ in $[0, \infty)$ with non-decreasing hazard rate.[5] The random variable $\theta$ is the state of nature and represents uncertainty such as technology shocks or stochastic prices. The revenue also depends on the function $Q(z_E, z_I)$ which is increasing, differentiable and strictly concave in the efforts exerted by the entrepreneur and investor, $z_E$ and $z_I$. The multiplicative specification implies that $\Pi(z_E, z_I, \theta)$ satisfies increasing differences in $\{\theta, z_E\}$ and $\{\theta, z_I\}$. Intuitively, this means that the marginal return of effort is increasing in the state of nature $\theta$.

The function $Q(z_E, z_I)$ satisfies the following standard Inada conditions, $\lim_{z_E \to 0} \partial Q/\partial z_E = \infty$, $\lim_{z_E \to \infty} \partial Q/\partial z_E = 0$ for every positive $z_I$, and $\lim_{z_I \to 0} \partial Q/\partial z_I = \infty$, $\lim_{z_I \to \infty} \partial Q/\partial z_I = 0$ for every positive $z_E$. We further assume that $Q$ is additively separable, $\partial^2 Q(z_E, z_I)/\partial z_E \partial z_I = 0$.

The state of nature $\theta$ and effort levels $z_E, z_I$ are non contractible. Contracts are contingent only on the revenue $\Pi$. A contract is defined as a pair $\{t, w\}$ that gives the investor a contingent payoff $-t + w(\Pi)$ where $w(\Pi)$ is an arbitrary function. Without loss of generality we set $w(0) = 0$, by defining payoffs in this way $t$ represents the transfer when there is no revenue. In the rest of the paper we use the convention that whenever more than one contract is optimal agents choose the contract that minimizes $t$. In the appendix we present the results under the restriction $t = 0$.[6]

We restrict attention to contracts that satisfy three requirements. First,

[5] The hazard rate is defined as $f(\theta)/(1 - F(\theta))$.
[6] This assumption has previously been use in this litarture (see [Casamatta (2003)] and [Innes (1990)]).

the contract must be robust to renegotiation, and therefore the entire revenue is shared by the agents. Second, because the entrepreneur has no wealth we require the payoff to the entrepreneur, $t + \Pi - w(\Pi)$ to be positive for all revenue levels. Third, we require the payoff to both agents to be non-decreasing in revenue. This assumption prevents agents from sabotaging each other and it also prevents the investor from artificially increasing the revenue to obtain a higher payoff.[7] A contract $\{t, w\}$ is said to be feasible if it satisfies these three requirements. Three examples of feasible contracts that will turn out to be important in the paper are debt, call option and convertible security contracts.

A **debt** contract $\{0, d_r\}$ gives the investor a contingent payoff $d_r = \min\{\Pi, r\}$ for some $r \in \mathbb{R}_0^+$. In a debt contract the entrepreneur is the residual claimant if the revenue is sufficiently high ($\Pi > r$).

A **call option** contract $\{0, c_r\}$ gives the investor a contingent payoff $o_r = \max\{\Pi - r, 0\}$ for some $r \in \mathbb{R}_0^+$. In a call option contract the entrepreneur is the residual claimant for low revenue realizations ($\Pi < r$).

A **convertible security** contract $\{0, c_{r_0,r_1}\}$ gives the investor a contingent payoff that corresponds to the maximum between a debt contract and a call option contract $c_{r_0,r_1}(\Pi) = \max\left[d_{r_0}(\Pi), o_{r_1}(\Pi)\right]$ for some $r_1 \geq r_0$.

For convenience, we assume that there exists a unique effort level optimum for each agent when the contract is a convertible security. Formally we make the regularity assumption that $\int_0^\infty \left[c_{r_0,r_1}(\Pi)\right] f(\theta) d\theta$ is strictly concave in $z_I$ and $\int_0^\infty \left[\Pi - c_{r_0,r_1}(\Pi)\right] f(\theta) d\theta$ is strictly concave in $z_E$.[8]

The timing of the game is the following. First, the entrepreneur offers the investor a feasible contract. If accepts, the investor pays the fixed investment layout $K$. Then agents exert effort simultaneously. Finally, the state of nature $\theta$ is realized and payments are made. The contract offered by the entrepreneur must give the investor an expected payoff at least equal to the investor's outside opportunity $\overline{v}_I$.

We consider three regimes. In the passive investment regime, only the entrepreneur exerts effort. In the involved investment regime, both agents exert effort. In the acquisition regime, only the investor exerts effort. Section 4 de-

[7] This assumption is also commonly used in this literature, see for example [Innes (1990)] and [Casamatta (2003)].

[8] A sufficient but not necessary condition is that the induced distribution $F(\Pi|z_j) = F(\Pi/Q(z_{-j}, z_j)$ is convex in $z_j$ where $j = E, I$. This is known as the cumulatitive distribution function condition or CDFC. Poblete and Spulber (2008) show how the assumption can be relaxed. In the proofs we refer to this assumption as the regularuty assumption.

rives the optimal financial contract under each regime, and section 5 discusses the conditions under which the entrepreneur chooses to finance under each regime.

## 3 Dual Role of Contracts

The key insight of the paper is that contracts play a dual role, they provide agents with incentives to exert effort, and they compensate each agent for their contributions to the firm. The objective of this section is to provide an informal analysis of the trade-off between incentives and compensation in the design of contracts. We begin by examining the incentives and compensation that a contract $\{t, w\}$ provides the investor.

Observe that given a contract $\{t, w\}$ the investor's expected payoff net of investment cost and outside opportunity is

$$u_I(t, w, z_E, z_I) = -t + \int_0^\infty w(\Pi(\theta, z_E, z_I))f(\theta)d\theta - z_I - K - \overline{v}_I. \qquad (2)$$

The first role of the contract is to provide the investor with incentives to exert effort. If the investor exerts a positive effort, then the level is determined by the first order condition

$$\int_0^\infty w_\Pi(\Pi(\theta, z_E, z_I))\Pi_{Z_I} f(\theta)d\theta = 1, \qquad (3)$$

where $w_\Pi = \partial w(\Pi)/\partial\Pi$ is well defined since feasible contracts are continuous and monotonic and therefore differentiable almost everywhere. The derivative of the left-hand side of equation (3) with respect to the slope of the compensation ($w_\Pi$) is $\Pi_Z f(\theta)$. Provided the first-order condition approach is valid, the higher $\Pi_Z f(\theta)$ is, the less we need to increase the payoff to induce a given effort level. The term $\Pi_Z f(\theta)$ represents how powerful a given state is in providing incentives. Intuitively, the higher the likelihood of a state and the marginal return to effort are, the more efficient the state is in providing incentives.

The second role of the contract is to compensate the investor. The level of compensation can be measured by the expected payoff of the contract. We can rewrite the expected payoff (2) as a function of $w_\Pi$ using integration by parts

$$u_I(t, w, z_E, z_I) = -t + \int_0^\infty w_\Pi(\Pi(\theta, z_E, z_I))(1 - F(\theta))\Pi_\theta d\theta - z_I - K - \overline{v}_I. \quad (4)$$

The derivative of the expected payoff with respect to $w_\Pi$ is $(1 - F(\theta))\Pi_\theta$. The higher the term $(1 - F(\theta))\Pi_\theta$ is, the less we need to increase the slope of the contract $w_\Pi$ to provide a given compensation level. The term $(1 - F(\theta))\Pi_\theta$ represents how efficient a state is in providing compensation. Intuitively the lower the revenue and the faster the revenue increases in the state of nature $\theta$, the more efficient the state is in providing a compensation.

The ratio of incentives per unit of compensation

$$\rho_I = \frac{f(\theta)}{1 - F(\theta)} \frac{\Pi_{Z_I}}{\Pi_\theta},$$

measures the relation between incentives and expected payoff when the slope of the contract $w_\Pi$ changes. Because we have assumed that $\theta$ has non-decreasing hazard rate and since $\Pi_z/\Pi_\theta = \theta Q_z/Q$ is increasing in $\theta$, the ratio $\rho_I$ is also increasing in the state of nature $\theta$. Intuitively, contracts that are steeper at high revenue realizations provide more incentives and less compensation than contracts steeper at low revenue realizations.

The same procedure can be used to compare the incentives and compensation that a contract $\{t, w\}$ provides the entrepreneur. The ratio of incentives per unit of compensation for the entrepreneur is

$$\rho_E = \frac{f(\theta)}{1 - F(\theta)} \frac{\Pi_{Z_E}}{\Pi_\theta}.$$

The interpretation is analogous, if the entrepreneur's payoff $t + \Pi - w(\Pi)$ is steeper at high revenue realizations, the entrepreneur receives high incentives and low compensation. If the entrepreneur's payoff is steeper at low revenue realizations the entrepreneur receives a high compensation but low incentives. The intuition can also be stated in terms of the residual returns. Residual returns at low states provide a high compensation and low incentives, while residual returns at high states provide high incentives but a low compensation.

Because agents share the revenue, at any given state if the payoff of the investor $-t + w$ is steeper, then the payoff of the entrepreneur $t + \Pi - w$ is less steep and vice versa. If at any given level we increase the slope of the contract $w_\Pi$, the investor's payoff becomes steeper and therefore the investor's incentives and compensation increase according to the ratio $\rho_I$. At the same time the entrepreneur's payoff becomes less steep and therefore the entrepreneur's incentives and compensation decrease according to the ratio $\rho_E$.

To understand how this trade-off affects the design of the optimal contract, consider for example a debt contract. The payoffs for the entrepreneur $[\Pi - d_r(\Pi)]$ and the investor $[d_r(\Pi)]$ with a debt contract $\{0, d_r\}$ are depicted in figure 1.

Observe that the investor's payoff $d_r(\Pi)$ is steep at low states $(\Pi < r)$, this

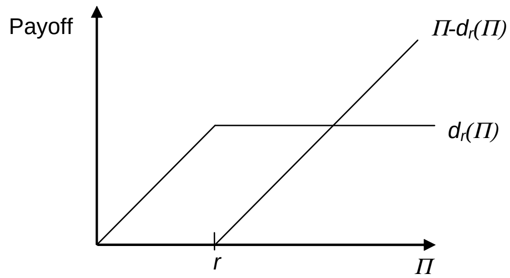


Fig. 1. Debt Contract

implies that the contract provides a high compensation to the investor. Also observe that the entrepreneur's payoff is steep at high states ($\Pi > r$), this implies that the contract provides high incentives to the entrepreneur. As a result, debt contracts are desirable when it is necessary to compensate the investor for a large investment cost, or when the revenue depends mostly on the entrepreneur's effort. On the contrary if the technology owned by the entrepreneur is inexpensive to implement and it requires high effort from the investor, then a debt contracts is a poor instrument because it fails to provide the investor with incentives and it gives the investor too much compensation.

## 4 Characterization of Contracts

This section characterizes the optimal contracts under the passive investment, involved investment and the acquisition regimes. In the passive investment regime only the entrepreneur exerts effort, we model this by setting $z_I = 0$. In the involved investment regime, $z_E$ and $z_I$ are chosen by the entrepreneur and investor respectively. In the acquisition regime we set $z_I = 0$. We assume than in each regime there exists at least one contract $\{t, w\}$ and effort levels $z_E, z_I$ that satisfy all the restrictions, such that the entreprener obtains a positive net expected payoff. This assumption rules out the case when the project is not financed.

### *4.1 Passive Investment*

In the passive investment regime, only the entrepreneur exerts effort. The expected payoff net of the outside opportunity for the entrepreneur and investor are

$$u_E(t, w, z_E) = t + \int_0^\infty [\Pi(\theta, z_E) - w(\Pi(\theta, z_E))] f(\theta) d\theta - z_E - \overline{v}_E, \qquad (5)$$
$$u_I(t, w, z_E) = -t + \int_0^\infty w(\Pi(\theta, z_E)) f(\theta) d\theta - \overline{v}_I - K, \qquad (6)$$

where $\overline{v}_E$ is the entrepreneur's outside opportunity. We do not include the investor's effort $z_I$ as an argument since in this regime we exogenously set $z_I = 0$. The entrepreneur offers the investor a contract to solve

$$\max_{t,w,z_E} \; u_E(t, w, z_E) \qquad \text{OPT}$$

subject to the following constraints

$$z_E \in \arg\max_{z_E} \; u_E(t, w, z_E), \qquad (\text{IC}_E)$$
$$u_I(t, w, z_I) \geq 0, \qquad (\text{IR}_I)$$
$$w(\Pi) \text{ is non decreasing in } \Pi, \qquad (\text{MON}_I)$$
$$\Pi - w(\Pi) \text{ is non decreasing in } \Pi, \qquad (\text{MON}_E)$$
$$\Pi - w(\Pi) + t \geq 0. \qquad (\text{LL})$$

($\text{IC}_E$) is the entrepreneur's incentive compatibility constraint, ($\text{IR}_I$) is the investor's individual rationality constraint, ($\text{MON}_E$) and ($\text{MON}_I$) are the monotonicity constraints for the entrepreneur and investor, and (LL) is the entrepreneur's limited liability constraint. An effort level $z_E$ is implementable if there exists a feasible contract $\{t, w\}$ such that $z_E$ satisfies the incentive compatibility ($\text{IC}_E$) and individual rationality ($\text{IR}_I$) constraints. The main result of this section is that the optimal contract is debt. The optimality of debt is a consequence of three important properties of debt contracts. The first property states that the effort exerted by the entrepreneur in a debt contract is continuous and decreasing in the face value of the debt $r$.

**Property P1** *There exists a continuous decreasing function $z_E(r)$, such that given any debt contract $d_r$, $z_E(r)$ is the unique effort level that satisfies $z_E \in \arg\max_{z_E} u_E(0, d_r, z_E)$. Moreover if a given effort level $z_E^0$ is implementable, then $z_E^0 = z_E(r)$ for some $r > 0$.*

Intuitively, with a debt contract the entrepreneur is the residual claimant whenever $\Pi(\theta, z_E) > r$. The higher the face value of the debt is, the less states at which the entrepreneur is the resisual claimant, and the less incentives the entrepreneur has to exert effort. Moreover, by choosing the appropriate face value $r$, the debt contract can provide as much incentive to exert effort as any other feasible contract $\{t, w\}$. The second property shows that debt contracts maximize the investor's expected payoff.

**Property P2** *If a feasible contract* $\{t, w\}$ different from debt implements $z_E^0$ *and* $z_E^0 \in \arg\max u_E(0, d_r, z_E)$ *for some* $r > 0$, *then* $u_I(0, d_r, z_E^0) > u_I(t, w, z_E^0)$.

Property P2 shows that among all the contracts that implement a given effort level $z_E^0$, the expected payoff of the investor is maximized with the debt contract. Intuitively, debt contracts make the investor the residual claimant in low states, and given a level of incentives, residual claims in low states provide a higher expected payoff.

**Property P3** *The joint surplus* $(u_I + u_E)$ *obtained with a debt contract* $d_r$ *is decreasing in* $r$.

Intuitively, to satisfy the investor's individual rationality constraint ($\text{IR}_I$), the investor must receive a share of the revenue, this makes the entrepreneur to exert a suboptimal effort level. Reducing the face value of the debt increases the entrepreneur's effort, increasing the joint surplus. These three properties allows us to state the first proposition of the paper.

**Proposition 1** *Under the passive investment regime, the optimal contract is debt.*

**Proof.** Let $\{t, w\}$ be a contract different from debt. If $\{t, w\}$ implements $z_E^0$ the investor's net expected payoff is $u_I(t, w, z_E^0) \geq 0$. By Property P1 there exists a debt contract $d_{r_0}$ such that $z_E^0$=argmax$_{z_E} u_E(0, d_{r_0}, z_E)$ moreover by Property P2 $u_I(0, d_{r_0}, z_E^0) > u_I(t, w, z_E^0)$ and therefore $d_{r_0}$ implements $z_E^0$. Observe that at $r = 0$, $u_I(d_0, z(0)) < 0$ and by Property P1 $u_I$ is continuous in $r$, thus there exists a debt contract with face value $r_1 < r_0$ such that $u_I(0, d_{r_1}, z(r_1)) = u_I(t, w, z_E^0)$. Finally notice that $z(r_1) > z(r_0) = z_E^0$ and therefore by Property P3 the joint surplus $(u_I + u_E)$ obtained with $d_{r_1}$ is higher than the joint surplus obtained with $d_{r_0}$ which is equal to the joint surplus obtained with $\{t, w\}$. Finally since by construction $u_I(0, d_{r_1}, z(r_1)) = u_I(t, w, z_E^0)$, and the joint surplus with $d_{r_1}$ is higher, it must be the case that $u_E(0, d_{r_1}, z(r_1)) > u_E(t, w, z_E^0)$. Therefore if there exists an optimal contract it has to be a debt contract. The proof of existence is presented in appendix B. □

Proposition 1 establishes the first important result of the paper. When start-ups are financed by passive investors the optimal contract is debt. This result is consistent with the behavior of banks and other financial institutions that use almost only debt contracts to finance start ups.

Observe that given any effort level $z_E$ the expected payoff of the entrepreneur under a debt contract and $u_E(0, d_r, z_E)$ is decreasing in $r$. Therefore the optimal debt contract will have the lowest possible face value such that the investor's individual rationality constraint is satisfied. At the optimal debt contract the investor's rationality constraint is binding, and the entrepreneur keeps all the surplus from the project.

The optimality of debt was first discussed by [Innes (1990)]. The result presented in this section differs from [Innes (1990)] in the structure of the model. Concretely, [Innes (1990)] assumes that the distribution function satisfies the Monotone Likelihood Ratio Property (MLRP), in contrast this model assumes an explicit multiplicative source of uncertainty that is distributed with non-decreasing hazard rate. This specification allows extending the model to include an active investor and it provides a simple economic intuition for the results.

### *4.2 Involved Investment*

Under the involved investment regime, both agents exert effort and the expected net payoff of the entrepreneur and investor are respectively

$$u_E(t, w, z_E, z_I) = t + \int_0^\infty (\Pi(\theta, z_E) - w(\Pi(\theta, z_E)))f(\theta)d\theta - z_E - \overline{v}_E, \quad (7)$$

$$u_I(t, w, z_E, z_I) = -t + \int_0^\infty w(\Pi(\theta, z_E)))f(\theta)d\theta - z_I - \overline{v}_I - K. \quad (8)$$

Formally, the problem for the entrepreneur is to find a vector level of efforts $\mathbf{z} = (z_E, z_I)$ to solve the following problem

$$\max_{w,t,z_E,z_I} \; u_E(t, w, z_E, z_I) \quad \text{(OPT)}$$

subject to the following constraints

$$z_E \in \arg\max_{z_E} \; u_E(t, w, z_E, z_I), \quad (\text{IC}_E)$$

$$z_I \in \arg\max_{z_I} \; u_I(t, w, z_E, z_I), \quad (\text{IC}_I)$$

$$u_I(t, w, z_E, z_I) \geq 0, \quad (\text{IR}_I)$$

$$w(\Pi) \text{ is non decreasing in } \Pi, \quad (\text{MON}_I)$$

$$\Pi - w(\Pi) \text{ is non decreasing in } \Pi, \quad (\text{MON}_E)$$

$$\Pi - w(\Pi) + t \geq 0. \quad (\text{LL})$$

The same problem without the investor's incentive rationality constrained is called the relaxed problem (RPT).

The main result in this section is that the optimal contract is a combination of debt and options, where the relative importance of the debt component depends on the cost of the investment layout $K$. To show this result we generalize the three properties we present in the previous section.

**Property N1** *There exists a continuous function* $\mathbf{z}(r) = (z_E(r), z_I(r))$*, such that given any debt (option) contract* $d_r$ $(o_r)$*,* $\mathbf{z}(r)$ *is the unique vector of effort levels that satisfy the* $IC_E$ and $IC_I$ *constraints. Moreover if a given effort* $\mathbf{z}^0$ *is implementable, then* $\mathbf{z}^0 = \mathbf{z}(r)$ for some $r > 0$.

Intuitively, the lower the face value of the debt $r$, the higher the effort level exerted by the entrepreneur $z_E$ relative to the effort level of the investor $z_I$. Property N1 states that changes in effort levels are smooth. By choosing the appropriate face value $r$, the debt contract can provide as much incentives to exert effort as any other feasible contract $\{t, w\}$.

**Property N2** *If a feasible contract* $\{t, w\}$ *different from a debt or option contract implements* $\mathbf{z}^0$ *and if* $d_r, o_r$ *are a debt and an option contracts that satisfy* $IC_E$ *and* $IC_I$ *at* $\mathbf{z}^0$*, then* $u_I(0, d_r, \mathbf{z}^0) > u_I(0, w, \mathbf{z}^0)$ *and* $u_E(0, o_r, \mathbf{z}^0) > u_E(0, w, \mathbf{z}^0)$.

Intuitively, because debt contracts make the investor the residual claimant in low states, debt contracts maximize the investor's expected payoff. Conversely, option contracts make the entrepreneur the residual claimant in low states, and maximize the entrepreneur's expected payoff.

**Property N3** *The joint surplus* $(u_I + u_E)$ *obtained with a debt (option) contract* $d_r$ $(o_r)$ *in RPT, is quasiconcave in the face value of the debt (option)* $r$ *with a unique maximum at* $r^*$.

Property N3 states that there is a debt contract $r^*$ that implements the best possible combination of efforts $\mathbf{z}^* = (z_E^*, z_I^*)$. From now on we call $\mathbf{z}^*$ the second best level of efforts. The closer is the face value $r$ to $r^*$, the closer will be the implemented effort levels to $\mathbf{z}^*$, and the higher will be the joint surplus. A consequence of Property N3 is that if there exists a feasible contract $\{t, w\}$ that implements $\mathbf{z}^*$ and gives the investor an expected net payoff $u_I(t, w, \mathbf{z}^*) = 0$, then the contract is optimal in OPT. Intuitively if the contract maximizes the joint surplus, and the entrepreneur is able to extract all the surplus, then the contract must be optimal for the entrepreneur.

The next proposition use these three properties of debt contracts to characterize the set of investment costs for which the second best level of efforts $\mathbf{z}^*$ together with $u_I = 0$ and $t = 0$ can be implemented.

**Proposition 2** *There exist cost levels* $K_o < K_d$ *such that iff* $K \in [K_o, K_d]$ *, the second best effort levels* $z_E^*$*,* $z_I^*$ *with* $u_I = 0$ *and* $t = 0$ *are implementable* *in OPT. This can be achieved with convertible security contracts.*

**Proof.** See the appendix.□

Intuitively, if $K$ is small, the entrepreneur offers the investor a low expected payoff. The best way of giving the investor a low expected payoff and provide incentives is by making the investor the residual claimant in high states, this is done with a call option. If $K$ is bigger, the investor will need to own residual returns in low states to increase his expected payoff, this is achieved by adding a debt component to the contract. If $K$ is sufficiently big, the contract converges to debt.

A relevant question at this point is whether a simpler contract can be optimal. If a contract is to be optimal in all the range $[K_o, K_d]$, the contract needs to be flexible enough to resemble debt (for high levels of $K$) and a call option (for low levels of $K$), the simplest contract that achieves this is a convertible security. However, for a particular level of $K$, a simpler contract might achieve the optimal outcome. This result is consistent with the fact that even though convertible securities are the most common claim used in venture capital financing, other forms of financing are also observed. This diversity contrasts with financial institutions like banks that virtually always invest using debt. The next proposition derives the optimal contracts for all levels of $K$.

**Proposition 3** *Under the involved investment regime, the optimal contract depends on the investment cost $K$. There exists levels $K_0 \leq K_d$ such that:*

*a) If $K \geq K_d$, debt is the optimal contract.*

*b) If $K \in (K_o, K_d)$ convertible securities are optimal.*

*c) If $K \leq K_o$, a call option plus a transfer $t$ is optimal.*

**Proof.** a) Assume $K > K_d$ . Let $\mathbf{z}^0 = (z_E^0, z_I^0)$ be the effort levels implemented with a contract $\{t, w\}$ and let $d_{r^*}$ be the debt contract that implements $\mathbf{z}^*$ in RPT. By Property N2 the debt contract $d_{r_0}$ that implements $\mathbf{z}^0$ gives the investor a higher expected payoff than $\{t, w\}$ and therefore $u_I(0, d_{r_0}, \mathbf{z}^0) > 0$. By proposition 2 and because $K > K_d$, $u_I(0, d_{r^*}, \mathbf{z}^*) < 0$. By continuity (Property N1) there exists $\widehat{r} \in (r_0, r^*)$ such that $u_I(0, d_{\widehat{r}}, \mathbf{z}(\widehat{r})) = 0$. Since $\widehat{r}$ is closer to $r^*$ than $r_0$ and because surplus is quasiconvave in $r$ (Property N3) the joint surplus with $d_{\widehat{r}}$ is higher than with $d_{r_0}$ and thus also higher than under $\{t, w\}$. Since $u_I(0, d_{\widehat{r}}, \mathbf{z}(\widehat{r})) = 0$ under the contract $d_{\widehat{r}}$ the entrepreneur gets all the surplus, and therefore $u_E(0, d_{\widehat{r}}, \mathbf{z}(\widehat{r})) > u_E(t, w, \mathbf{z}^0)$. Therefore without loss of optimality we can restrict attention to debt contracts. The existance proof is presented in appendix B. *b*) It follows from Proposition 2. *c*) Assume $K < K_o$ , let $\mathbf{z} = (z_E, z_I)$ be the effort level implemented with a contract

$\{t, w\}$. Let $o_r$ be the call option contract that implements $\mathbf{z}$, and let $t_O$ be the transfer that satisfies $u_I(t, w, \mathbf{z}) = u_I(t_0, o_r, \mathbf{z})$. By property N2 the call contract $o_r$ that implements $\mathbf{z}$ gives the investor a lower expected payoff than $w$ and therefore $t_0 < t$. Since $u_I(t, w, \mathbf{z}) = u_I(t_0, o_r, \mathbf{z})$ and the same effort is implemented, $u_E(t, w, \mathbf{z}) = u_E(t_0, o_r, \mathbf{z})$. This means that the call option is no worse than $\{t, w\}$ for the entrepreneur and because we restrict attention to the optimal contract that minimize the transfer $t$, without loss of optimality we restrict attention to call options. The proof of existence is analogous to $a$). □

Intuitively, if $K > K_d$ the investor's IR constraint prevents the entrepreneur from exerting enough effort. Given the expected payoff of the investor, the contract will maximize incentives to the entrepreneur by making him the residual claimant in high states. The contract that makes the entrepreneur the residual claimant in high states is debt. In the interval $[K_o, K_d]$ the second best efforts $\mathbf{z}^*$ , $u_I = 0$ and $t = 0$ can be implemented with a convertible security, maximizing the entrepreneur's expected payoff. If $K < K_o$, whenever $t = 0$ and the second best level of effort $\mathbf{z}^*$ is implemented, the investor gets a positive net expected payoff. In order to extract as much surplus as possible the entrepreneur uses the contract that minimizes the investor's expected payoff, that is an option contract.

In the appendix we derive the optimal contract under the assumption $t = 0$. The main difference under this assumption is that the optimal contract might leave the investor with a positive net expected payoff. Intuitively, if the investment is low but the contribution of the investor as an advisor is important, the entrepreneur will offer the investor a high expected payoff to induce more effort and increase the value of the project. This contrasts with the case of passive investment, where the investor always gets zero expected payoffs under the optimal contract.

### *4.3 Acquisition*

Under the acquisition regime, only the investor exerts effort. The expected payoffs net of the outside opportunity for each agent are

$$u_E(t, w, z_I) = t + \int_0^\infty (\Pi(\theta, z_I) - w(\Pi(\theta, z_I)))f(\theta)d\theta - \overline{v}_E \tag{9}$$

$$u_I(t, w, z_I) = -t + \int_0^\infty w(\Pi(\theta, z_I)))f(\theta)d\theta - \overline{v}_I - z_I - K. \tag{10}$$

The problem for the entrepreneur is to find a contract to solve

$$\max_{t,w,z_I} u_E(t,w,z_I) \qquad \text{OPT}$$

subject to the constraints ($IC_I$), ($MON_E$), ($MON_I$), ($IR_I$) and (LL). The next proposition shows that it is optimal for the investor to buy all the residual rights from the project.

**Proposition 4** *Under the acquisition regime, the optimal contract is a transfer $t$ and $w(\Pi) = \Pi$.*

**Proof.** If only the acquirer exerts effort the entrepreneur's net expected payoff is

$$u_E(t,w,z_I) = E(\theta)Q(z_I) - z_I - \overline{v}_E - \overline{v}_I - K - u_I(t,w,z_I).$$

Consider the contract $\{t^*, \Pi\}$, where $t^* = \max_{z_I} E(\theta)Q(z_I) - z_I - \overline{v}_I - K$ then $u_E(t^*) = \max_{z_I} E(\theta)Q(z_I) - z_I - \overline{v}_I - K \geq u_E(t,w,z_I)$ whenever $u_I \geq 0$. Moreover the investor's IR constraint is satisfied with equality and therefore $\{t^*, \Pi\}$ is an optimal contract.$\square$

The result is standard in the literature. By buying the firm the investor becomes the owner of all the residual rights and exerts the first best effort level. If we restrict attention to contracts without transfers $t = 0$, then the optimal contract is a call option. [9] Intuitively, option contracts give the investor the residual rights in high states providing better incentives and thus maximizing the value of the project,

In practice while most acquirers simply buy firms with cash, anecdotal evidence such as the case of Parenting Magazine suggests that call options are also used. In this model investors would use option contracts if they face budget constraints that limit the size of the non-contingent transfer $t$ they can pay the entrepreneur.

## 5 Regime Choice

This section studies how the entrepreneur chooses among the three investment regimes. It shows how the decision depends on four characteristics of the project. The investment cost $K$, the size of the project, the risk of the project and property rights protection. We define the entrepreneur's expected payoff net of the outside opportunity as $u_E^P$, $u_E^N$ and $u_E^A$ under the passive investment, involved investment and the acquirer regime respectively. The outside

[9] The proof of this is very similar to the optimality of debt under passive investment and is therefore omitted.

opportunity of the entrepreneur and investor are defined as $\overline{v}_E^P, \overline{v}_I^P$ for passive investment, $\overline{v}_E^N, \overline{v}_I^N$ for involved investment and $\overline{v}_E^A, \overline{v}_I^A$ for the acquisition regime. The outside opportunities under each regime might differ because of learning costs of becoming involved in the project, because investors might be scarce or because entrepreneurs obtain private benefits from being involved in the project.

Throughout this section we focus our attention in projects with investment costs $K \in (K_o, K_d)$. This is the most interesting case, because in this range the optimal financial claim depends on the involvement of the investor.

### *5.1 Investment Cost*

We first analyze the impact of the investment layout cost $K$ in the regime choice. The main result is that the higher the investment cost is, the more important is the involvement of the investor in the project.

**Proposition 5** *The performance of involved investment relative to passive investment* $(u_E^N - u_E^P)$*, is increasing in* $K$*.*

**Proof.** Take any $K' > K$. Let $r'$ and $r$ be the face value of the optimal debt contract under passive investment for $K'$ and $K$. Because $IR_I$ is binding at the optimal contract, it is the case that $u_E^p(K) = E(\theta)Q(z_E(r)) - z_E(r) - \overline{v}_I - \overline{v}_E - K$ and $u_E^p(K') = E(\theta)Q(z_E(r')) - z_E(r') - \overline{v}_I - \overline{v}_E - K'$. Moreover by proposition 2, $u_E^N(K) = E(\theta)Q(\mathbf{z}^*) - z_E^* - \overline{v}_I - \overline{v}_E - K$ and $u_E^N(K') = E(\theta)Q(\mathbf{z}^*) - z_E^* - \overline{v}_I - \overline{v}_E - K'$ . To prove the proposition by contradiction suppose that $(u_E^N(K') - u_E^P(K')) < (u_E^N(K) - u_E^P(K))$ then it must be the case that $E(\theta)Q(z_E(r')) - z_E(r') > E(\theta)Q(z_E(r)) - z_E(r)$. By property P1 and P3 this means that $r' < r$. But then $r'$ satisfy $u_I(0, d_{r'}, z(r')) \geq 0$ with investment $K'$ and it must also satisfy it with investment $K$ and this violates the optimality of the debt contract $d_r$ when the cost is $K$ since $u_E(0, d_{r'}, z_E) > u_E(0, d_r, z_E)$ for every $z_E$ and the entrepreneur would prefer the contract $d_{r'}$ when the investment cost is $K$, violating the optimality of $r$. $\square$

Intuitively, if the investment cost is high, the investor needs to receive a high share of the revenue in order to get compensated for the investment. Because the investor gets a higher compensation, it is easier to provide the investor with incentives, and it is more efficient to have the investor involved in the project.

Because it is easier to provide the investor with incentives, acquisition also becomes more efficient than passive investment as the start up costs increase. Moreover under some circumstances the performance of acquisition relative to involved investment increases in cost of the investment layout. A proof of this is presented in appendix A. The last result reinforces the idea that the higher the investment cost the more important is the involvement of the investor.

The results presented in this section also apply to the cost of capital. The investment cost $K$ can be interpreted as the layout cost $K$ times the cost of capital $K(1+\rho)$. As the cost of capital increases involved investment becomes a relatively better financing alternative.

In this section we changed the investment cost keeping the expected revenue constant and therefore, changing the return over investment of the project. The next section deals with changes in the investment cost keeping the return over investment constant.

### *5.2 Size*

In this section we analyze the impact of the size of the project in the regime choice. Bigger projects have a larger investment cost and larger expected revenue. We assume the revenue of the project is $\lambda\Pi(\theta, \mathbf{z})$, and the investment layout cost is $\lambda K$. The parameter $\lambda$ represents the size of the project. Observe that the model has four parameters; the revenue function $\lambda\Pi(\theta, \mathbf{z})$, the investment cost $\lambda K$ and the outside opportunity of agents $\overline{v}_E$ and $\overline{v}_I$. Increasing the first two parameters in $\lambda$ is equivalent to reducing the outside opportunities. Intuitively, in a larger project the outside opportunity become less relevant relative to the investment cost and the expected revenue.

The main result is that if the project is large enough, then involved investment is the best financing alternative.

**Proposition 6** *If the project* ($\lambda$) *is big enough; then involved investment is the best financing alternative* ( $u_E^N > u_E^P$ *and* $u_E^N > u_E^A$).

**Proof.** We need to show that if $\overline{v}_E, \overline{v}_I$ are zero, then Involved investment is strictly better than other financial regimes. Let $d_{r_0}$ be the optimal debt contract under passive investment. This is the best possible contract by Proposition 1. The net expected payoff of the entrepreneur and investor are given by

$$u_E^P = \int_0^\infty \max\{\theta Q(z_E^P) - r_0, 0\} f(\theta) d\theta - z_E^P \tag{11}$$

$$u_I^P = \int_0^\infty \min\{\theta Q(z_E^P), r_0\} f(\theta) d\theta - K \tag{12}$$

Let $z_E^N, z_I^N > 0$ be the effort levels exerted under an involved investment regime and contract $d_{r_0}$.Observe that $u_E(d_{r_0}, z_E, z_I) = \int_0^\infty \max\{\theta Q(z_E, z_I) - r\}^+ - z_E$ has increasing differences in $\{z_E, z_I\}$ and therefore by [Topkis (1998)] Theorem 2.8.5 $z_E^N > z_E^P$ . Notice that by revealed preferences $u_I^N(d_{r_0}, z_E^N, z_I^N) \geq u_I^N(d_{r_0}, z_E^N, 0)$ and by monotonicity $u_I^N(d_{r_0}, z_E^N, 0) > u_I^N(d_{r_0}, z_E^P, 0) = u_I^P$. Therefore the investor IR constraint is satisfied. Moreover $u_E^N(d_{r_0}, z_E^N, z_I^N) > u_E^N(d_{r_0}, z_E^P, z_I^N)$ and also by monotonicity $u_E^N(d_{r_0}, z_E^P, z_I^N) > u_E^N(d_{r_0}, z_E^P, 0)$ therefore the entrepreneur prefers the involved investment regime rather than passive investment. The argument for the acquisition regime is analogous and therefore omitted.□

The result is intuitive, if the project is large enough the cost of a failure is big and the revenue is potentially big, therefore it is possible to provide both agents with good incentives. A similar result can be derived with respect to the return over investment of the project. If the expected rate of return is high enough then involved investment is the best financing alternative.

### *5.3 Risk*

Empirical evidence shows that projects financed by venture capital are risky. For example in a sample of 110 investments, Huntsman and Hoban (1980) found that 17 percent of the projects generated a complete loss. In a larger sample with 383 investments Sahlman (1990) reports that 35% of the projects yielded a total loss or were unable to repay the initial investment. [10]

In this section we analyze the impact of risk in the investment regime. We represent risk in a simple way. Assume that the revenue of the project is

$$(1 - \widetilde{p})\theta Q(z_E, z_I) + \widetilde{p}E(\theta)Q(z_E, z_I)$$

Where $\widetilde{p}$ is a random variable that takes value 1 with probability $p$, and 0 otherwise. The parameter $p$ reflects the risk of the project. After the entrepreneur chooses the investor, the entrepreneur and investor observe the realization of $\widetilde{p}$ and renegotiate the contract. The main result of this section is that as risk increases involved investment becomes a relatively better alternative than passive investment.

**Proposition 7** *The performance of involved investment relative to passive investment* $(u_E^N - u_E^P)$, *is increasing in risk* $p$.

**Proof.** The optimal contract with involved investment always implements the second best effort levels $\mathbf{z}^*$ therefore the net expected payoff of the entrepreneur is

$$u_E^N = E(\theta)Q(z_E^*, z_I^*) - z_E^* - z_I^* - \overline{v}_E^N - \overline{v}_I^N - K \tag{13}$$

[10] For more on this point see [Ueda (2004)] and [Schmidt (2003)]

When the project is financed by a passive investor, the optimal contract is debt, if the revenue is $\theta Q(z_E)$ then by Property P3 the effort level implemented is $z_E^D$ strictly less than optimal. If the revenue is $E(\theta)Q(z_E)$ then a debt contract can implement the first best. Since in either case the entrepreneur keeps all the surplus of the project, the net expected payoff is:

$$u_E^P = E(\theta)\left(pQ(z_E^D) + (1-p)Q(z^{FB}\right)) - pz_E^D - (1-p)z_E^{FB} - \overline{v}_E^P - K \quad (14)$$

Observe that $u_E^P$ is decreasing in $p$, while $u_E^N$ is constant; therefore $u_E^N - u_E^P$ is increasing in $p$.□

Intuitively, as risk increase the passive investor becomes the residual claimant in more states, worsening the incentives of the entrepreneur. In the involved investment regime the convertible security contract is flexible enough to accommodate the changes in risk without affecting the incentives of agents.

### *5.4 Property Rights*

Consider now the possibility that before paying for the investment layout $K$, the investor can undertake the project without the entrepreneur. Suppose that if the project is undertaken, the investor succeeds in implementing the project with probability $q$ and obtain a private benefit $B$. We use $q$ as a measure of expropriation risk.

**Proposition 8** *The performance of involved investment relative to passive investment* $(u_E^N - u_E^P)$*, is increasing in the expropriation risk* $q$ *.*

**Proof** To prevent the investor from undertaking the idea, the expected payoff of the investor needs to increase in $qB$. This is mathematically equivalent to increasing $K$, and the proof follows from proposition 5.□

Intuitively, to prevent the investor from undertaking the project, the entrepreneur is forced to increase the expected payoff of the investor in $qB$. This makes it relatively easier to provide incentives to the investor, and thus involved investment becomes a relatively more attractive alternative.

Proposition 8 suggests that involved investment like VC should be observed more often in industries where entrepreneurs face a higher expropriation risk. The result is consistent with the fact that VC financing is very concentrated in industries intensive in technology and research and development where property rights play an important role.

The main assumption underlying proposition 8 is that under the passive and involved investment regime, the investor is equally likely to succeed if decides to undertake the project. Ueda (2004) building on Anton and Yao (1994) develops a model where an involved investor is more likely to succeed than a passive investor and obtains the opposite result. This suggests that the nature of the technology is important in determining how property rights protection affects funding decisions.

## 6 Optimal Investment

In this section we relax the assumption that the entrepreneur is wealth constrained and maximize with respect to the optimal investment level by the entrepreneur and investor. Without loss of generality we restrict attention to contracts without transfers ($t = 0$).[11]

Let $H$ be the amount of investment provided by the investor, then $K - H$, is provided by the entrepreneur. The net expected payoff of the entrepreneur and investor are

$$u_E(w, z_E, z_I, H) = \int_0^\infty (\Pi - w(\Pi))f(\theta)d\theta - z_E - \overline{v}_E - (K - H) \qquad (15)$$

$$u_I(w, z_E, z_I, H) = \int_0^\infty w(\Pi)f(\theta)d\theta - z_I - \overline{v}_I - H \qquad (16)$$

The entrepreneur face the following problem

$$\max_{w,H,z_E,z_I} \; u_E(w, z_E, z_I, H) \qquad (17)$$

Subject to the constraints (IC$_E$), (IC$_I$), (IR$_I$), (MON$_E$) and (MON$_I$).

First, notice that passive investment will never be optimal. Intuitively, if the investor does not get involved, the investor's investment creates an agency problem that reduces the performance of the project. The main result in this section describes the optimal contract with involved investment. It shows that a strictly positive investment level is optimal.

**Proposition 9** *Under the involved investment regime, it is optimal to set* $H \in [K_o, K_d]$.

**Proof.** The problem is equivalent to OPT replacing $K$ by $H$. By proposition 2 only for $H \in [K_o, K_d]$, $\mathbf{z}^*$ is implementable with $u_I = 0$ and $t = 0$ and the expected payoff of the entrepreneur is $\widehat{u}_E^N = E(\theta)Q(\mathbf{z}^*) - z_E^* - z_I^* - \overline{v}_I^N - \overline{v}_E^N$ .

[11] Observe that if the entrepreneur invests $H$ and receives a transfer $t$, this is equivalent to the entrepreneur investing $H - t$ and the investor investing $t$. Therefore without loss of generality we can restrict attention to investments net of transfers.

Suppose that a different investment level is chosen, then either $\mathbf{z}^* \neq \mathbf{z}$ in such case by the definition of $\mathbf{z}^*$

$$u_E^N = E(\theta)Q(\mathbf{z}) - z_E - z_I - u_I - \overline{v}_E - \overline{v}_I < E(\theta)Q(\mathbf{z}^*) - z_E^* - z_I^* - \overline{v}_E - \overline{v}_I = \widehat{u}_E^N$$

or $u_I^N > 0$, in such case

$$u_E^N \leq E(\theta)Q(\mathbf{z}^*) - z_E^* - z_I^* - u_I^N - \overline{v}_E^N - \overline{v}_I^N < \widehat{u}_E^N$$

Therefore no investment level outside $[K_o,\, K_d]$ can be optimal. □

The proposition shows that the optimal investment level is not uniquely determined. The optimal contract varies with the chosen level of investment $H$. When $H = K_d$ the investor requires a high compensation and the optimal contract is debt. When $H = K_o$ the investor gets a lower compensation and the optimal contract is a call option. If $H \in (K_o, K_d)$ the optimal contract can be achieved using convertible securities.

Interestingly [Kaplan and Stromberg (2003)] found that the residual rights of venture capitalists decrease as the outcome of the project increases; this means that contracts are closer to debt than call options. In light of the model this evidence suggests that the level of investment chosen by the investor is close to $K_d$. This might be the case because entrepreneur's face a higher cost of capital or because entrepreneur's don't have access to credit markets. [Gompers and Lerner (1997)] pages 128-29 argues that entrepreneurs that seek money from VCs are likely to be credit constrained.

It is also worth noticing that under an optimal investment level $H$, agents exert the optimal amount of effort and the entrepreneur captures the entire surplus from the project. Intuitively, because the investor invests in the firm the entrepreneur can induce the investor to exert effort without giving up any rents. Therefore from the entrepreneur's perspective involved investment is a better form of obtaining advice than hiring a consultant that needs to be motivated .This dual role of venture capital as investors and advisors was discussed by [Casamatta (2003)].

Under passive investment, the entrepreneur's investment (and therefore wealth) is fundamental in determining the efficiency of the contract, in contrast, under involved investment, changes in the entrepreneur's investment can often be accommodated by changing the contract without affecting the efficiency of the relation. The next proposition shows that if $K \in (K_o, K_d)$ less wealthy entrepreneurs are more likely to seek involved investment.

**Proposition 10** *The performance of passive investment relative to involved investment* $(u_E^P - u_E^N)$, *is increasing in the entrepreneur's investment* $K - H$.

**Proof**. Because we assume that $K \in (K_o, K_d)$, by proposition 9, $u_E^N =$

$E(\theta)Q(\mathbf{z}^*) - z_E^* - z_I^* - u_I - \overline{v}_E - \overline{v}_I$ for any $K - H \in (0, K - K_d)$. Consider now passive investment. Let $d_{r_0}$ be the optimal debt contract, the net expected benefit of the investor is $u_I^P(d_{r_0}, z_E(r_0), H)$. With the same debt contract, but a higher investment by the entrepreneur (with $H' < H$), the investor obtains a net expected benefit $u_I^P(d_{r_0}, z_E(r_0), H') > u_I^P(d_{r_0}, z_E(r_0), H)$, and therefore by continuity (Property P1) there exists $r_1 < r_0$ such that $u_I^P(d_{r_1}, z_E(r_0), H') = u_I^P(d_{r_0}, z_E(r_0), H)$. By Property P1 this debt contract implements a higher effort level $z_E$, and since $u_I$ doesn't change but the joint surplus $(u_E + u_I)$ is increasing in $z_E$, $u_E^P(d_{r_1}, z_E(r_0), H') > u_E^P(d_{r_0}, z_E(r_0), H)$ so $u_E^P$ is decreasing in $H$ and therefore $u_E^P - u_E^N$ is decreasing in $H$. □

The last proposition highlights the difference between outside investment and involved investment. Involved investment work as an asymmetric partnership where the entrepreneur contributes with the technology or idea, and the investor with capital. In involved investment the financial contract seeks to solve a moral hazard in teams problem and doesn't necessarily depend on the liability of the entrepreneur. On the contrary in the passive investment regime, the entrepreneur needs to sell residual rights to raise capital, worsening the incentives. Under passive investment, the less liability the entrepreneur has, the more residual rights he needs to sell and the worse the performance of the project is.

## 7 Conclusion

The paper develops a framework for understanding the relation between the involvement of investors and the financial claims used to finance start-ups. The paper shows that the optimal contract depends on the involvement of investor and in the cost of the investment layout. If the project gets funding from a passive investor the optimal contract is debt. If the project gets funding from an acquirer the optimal contract depends on the cost of the investment layout. For high investment costs, the optimal contract is debt, for low cost the contract is a call option. In other case the optimal contract is a convertible security where the debt component depends positively on the cost of investment layout cost.

The analysis shows that entrepreneurs with small and safe projects seek funding from passive investors, and entrepreneurs with big risky projects seek financing from involved investors or acquirers. It also shows that if there are weak property rights protection the entrepreneur is reluctant to get funding from passive investors.

The model predicts simple contracts in the financial agreements. In practice we observe that some contracts like those observe in the VC industry are rather complex. This may be due to the fact that the model abstracts from the intertemporal dimension of these agreements. The model could be extended by incorporating stage financing by involved investors to address for this problem. Another interesting extension would be to include information asymmetries between entrepreneur and investor by assuming private information in the productivity parameter $\theta$. In addition the model can be extended by examining the effects of investor syndicates on investor involvement and on the choice of financial claims.

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# Appendix A

**Proof of Property P1.** The net expected benefit for the entrepreneur with a debt contract is

$$u_E(0, d_r, z_E) = \int_{r/Q(z_E)}^{\infty} (\theta Q(z_E) - r) f(\theta) d\theta - z_E - \overline{v}_E$$

i) Uniqueness of $z_E(r)$ follows directly from the regularity assumption. ii) To prove that $z_E$ is decreasing, observe that $\frac{\partial^2 u_E}{\partial z_E \partial r} = -Q_{z_E} \frac{R}{Q(z_E)^2} < 0$ therefore $u_E$ has strictly increasing differences in $\{-r, z_E\}$ and $z_E(r)$ is decreasing by Theorem 2.8.5 in Topkis (1998) .iii) Continuity follows from uniqueness and the theorem of maximum iv) To see that debt contracts can implement any implementable effort level, consider a feasible contract $\{t, w\}$ that implements $z_E^0$. All contracts are increasing and continuous in $\Pi$, and therefore differentiable *a.e.* As a result the contract must satisfy the first order condition.

$$Q_{z_E^0} \int_0^{\infty} (1 - w_\Pi(\theta Q(z_E^0)) \theta f(\theta) d\theta = 1$$

Because $1 \geq w_\Pi \geq 0$ , there exists an $r$ such that

$$\int_0^{\infty} w_\Pi(\theta Q(z_E^0)) \theta f(\theta) d\theta = \int_0^{r/Q(z_E^0)} \theta f(\theta) d\theta$$

And then it is the case that

$$Q_{z_E^0} \int_{r/Q(z_E^0)}^{\infty} \theta f(\theta) d\theta = 1. \tag{18}$$

Which implies that the debt contract $d_r$ satisfies the first order condition at $z_E^0$, and by the regularity assumption, the debt contract implements $z_E^0$.

**Proof Property P2.** All contracts are increasing and continuous in $\Pi$, and therefore differentiable *a.e.* As a result any contract that implement $z_E^0$ must satisfy the first order condition.

$$Q_{z_E^0} \int_0^{\infty} (1 - w_\Pi(\theta Q(z_E^0)) \theta f(\theta) d\theta = 1. \tag{19}$$

It is also the case that if a debt contract $d_r$ satisfy the first order condition at $z_E^0$, then $z_E^0$ is optimum for the entrepreneur by assumption 1.

Therefore to prove the lemma it is sufficient to show that the debt contract that satisfies the first order condition at $z_E^0$ gives the investor a higher expected payoff than any contract that satisfy the first order condition at $z_E^0$ evaluated at $z_E^0$.

Consider the problem:

$$\max_{w} -t + \int_0^\infty w(\theta Q(z_E^0)))f(\theta)d\theta \quad \text{s/t} \tag{20}$$

$$1 = Q_{z_E^0} \int_0^\infty (1 - w_\Pi(\theta Q(z_E^0))\theta f(\theta)d\theta \tag{21}$$

$$1 \geq w_\Pi \geq 0 \tag{22}$$

By Fubini $\int_0^\infty w(\theta Q(z_E^0))f(\theta)d\theta = w(0) + Q(z_E^0)\int_0^\infty (w_\Pi(\theta Q(z_E^0))(1 - F(\theta))d\theta$ and therefore the problem can be re-written as:

$$\max_{w_\Pi} -t + \int_0^\infty Q(z_E^0)w_\Pi(\theta Q(z_E^0))(1 - F(\theta))d\theta \tag{23}$$

It is clear that is optimal to set $t = 0$. Also since both the objective function and the constraint are linear in $w_\Pi$, the problem can be represented with the Lagrangian

$$\pounds = \int_0^\infty w_\Pi(\theta Q(z_E^0)) \left[(1 - F(\theta))Q(z_E^0) - \lambda(\theta f(\theta)Q_{z_E^0}\right] + \lambda Q_{z_O}\theta f(\theta)d\theta \tag{24}$$

Since the problem is linear in $w_\Pi$ it is optimal to set $w_\Pi = 0$ if $\frac{f(\theta)}{1-F(\theta)}\frac{\theta Q_{z_E}}{Q(z_E^0)} > \frac{1}{\lambda}$ and $w_\Pi = 1$ otherwise. Since $\frac{f(\theta)}{1-F(\theta)}\theta$ is increasing in $\theta$ this corresponds to the debt contract that satisfies the first order condition at $z_E^0$ □

**Proof Property P3.**

The joint surplus with the contract $d_r$ is given by $u_E + u_I = E(\theta)Q(z_E(r)) - z_E(r)$.

Notice that $z_E(r)$ is decreasing and continuous and therefore differentiable almost everywhere. The derivative of the joint surplus with respect to $r$ $\partial(u_E + u_I)/\partial r$ is $(E(\theta)Q_{z_E} - 1)\,\partial z_E/\partial r$. At the effort level $z_E$ the first order condition (18) must be satisfied, replacing it we get

$$\frac{\partial(u_E + u_I)}{\partial r} = \left(\frac{E(\theta)}{\int_{r/Q(z_E)}^\infty \theta f(\theta)} - 1\right)\frac{\partial z_E}{\partial r} \tag{25}$$

This can be rewritten as

$$\frac{\partial(u_E + u_I)}{\partial r} = \left(\frac{\int_0^{r/Q(z_E)} \theta f(\theta)}{\int_{r/Q(z_E)}^\infty \theta f(\theta)}\right)\frac{\partial z_E}{\partial r} \tag{26}$$

Because the term in parenthesis is positive and $\partial z_E/\partial r$ is negative $\partial(u_E + u_I)/\partial r$ is negative. □

**Proof Property N1** Consider a debt contract $d_r$, the agents marginal return to effort is given by

$$\frac{\partial u_E}{\partial z_E} = Q_{z_E} \int_{r/Q(z_E,z_I)}^{\infty} f(\theta)\theta d\theta - 1 \tag{27}$$

$$\frac{\partial u_I}{\partial z_I} = Q_{z_I} \int_0^{r/Q(z_E,z_I)} f(\theta)\theta d\theta - 1 \tag{28}$$

Observe that $\frac{\partial^2 u_E}{\partial z_E \partial z_I} > 0$ and $\frac{\partial^2 u_I}{\partial z_E \partial z_I} < 0$.By standard monotone comparative static results this imply that the entrepreneur's reaction curve $z_E(z_I)$ is increasing in $z_I$ and the investors $z_I(z_E)$ is decreasing in $z_E$ and therefore they cross at only one point, this shows that $\mathbf{z}$ is unique. Continuity in $r$ follows from the fact that both reaction curves are continuous in $r$ and the theorem of maximum.

To prove that debt contracts can implement any implementable effort level, assume that the contract $\{t, w\}$ implements $z_E^0, z_I^0$.Then it must be the case that $Q_{z_E^0} \int_0^\infty [1 - w_\Pi(\theta Q(\mathbf{z}^0))]\, \theta f(\theta) d\theta = 1$ and $Q_{z_I^0} \int_0^\infty [w_\Pi(\theta Q(\mathbf{z}^0)]\, \theta f(\theta) d\theta = 1$ let $r$ be the level that satisfies $\int_0^{r/Q(\mathbf{z}^0)} \theta f(\theta) d\theta = \int_0^\infty [w_\Pi(\theta Q(\mathbf{z}^0)]\, \theta f(\theta) d\theta$. Then the contract $d_r$ satisfies the first order conditions for the entrepreneur and the investor at $z_E^0, z_I^0$ and by our regularity assumption, it implements $z_E^0, z_I^0$. The proof for the option contract is analogous and therefore omitted□

**Proof of Property N2** First observe that whenever (IC$_E$) is satisfied, (IC$_I$) will also be satisfied by Property **N1** .Therefore the proof is almost identical to P2, and will therefore be omitted.□

**Proof of Property N3** To prove this consider first the following problem

$$\begin{gathered} max_S E(\theta) Q(z_E, z_I) - z_E - z_I \text{ s/t} \\ z_E \in \arg\max sQ(z_E, z_I) - z_I \\ z_I \in \arg\max sQ(z_E, z_I) - z_I \end{gathered}$$

The first order conditions $sQ_{z_E} = 1$ and $(1-s)Q_{z_I} = 1$ are sufficient to define $z_E(s)$ and $z_I(s)$ since $Q$ is concave and additively separable. Observe that $s$ is a feasible contract. We call it, equity contract.

We first claim that the joint surplus is quasiconcave in $s$. To prove this note that the problem can be re-written as

$$max_{z_E,z_I} E(\theta)Q(z_E,z_I) - z_E - z_I \text{ s/t}$$
$$\frac{1}{Q_{z_E}} + \frac{1}{Q_{z_I}} = E(\theta)$$

If the border Hessian of the constraint problem has a positive determinant , then there is a unique extremum that corresponds to a local maximum.

The border Hessian of the problem is

$$H = \begin{bmatrix} 0 & -\left(\frac{Q_{11}}{(Q_1)^2}\right) - \left(\frac{Q_{22}}{(Q_2)^2}\right) \\ -\left(\frac{Q_{11}}{(Q_1)^2}\right) & Q_{11} & 0 \\ -\left(+\frac{Q_{22}}{(Q_2)^2}\right) & 0 & Q_{22} \end{bmatrix}$$

The determinant of the Hessian is

$$-Q_{11}\left(\frac{Q_{22}}{(Q_2)^2}\right)^2 - Q_{22}\left(\frac{Q_{11}}{(Q_1)^2}\right)^2 > 0$$

For every value of $s$ in the original problem, there is a unique pair $(z_E, z_I)$ that satisfies the first order conditions (By concavity). Therefore there is a unique extremum in the problem with respect to $s$,and it is a local maximum. This implies that the joint surplus is quasiconcave with respect to $s$.

To prove that the joint surplus is quasiconcave in $r$ , let $s(r)$ be the equity comtract that implements the same effort level as $d_r$ in RPT. $s(r)$ is unique since each equity contract implements a different $\mathbf{z}$. It is easy to check that $s(0) = 1$ and $s(\infty) = 0$. Moreover $s(r)$ is monotonic. To prove monotonicity suppose not. Then there exists $r_1 > r_0$ such that $s(r_1) = s(r_0)$. This means that both debt contracts implement the same effort levels $\mathbf{z}$, and therefore the first order condition $Q_{z_E} \int_{r/Q(z_E,z_I)}^{\infty} f(\theta)\theta d\theta = 1$ holds for $r_1$ and $r_0$ which is a contradiction. Therefore $s(r)$ is monotonic. Because of monotonicity we can define $r(s)$ which is also monotonic. Finally the joint surplus is quasiconcave in $r$ since quasiconcavity is preserved under monotonic transformations. The proof for the option contract is analogous and therefore omitted. □

**Proof of proposition 2** . Let $r_o$ and $r_d$ be the strike price of the call option and face value of the debt contract that implement $\mathbf{z}^*$ in RPT. They exists by property N1.

Define $K_o$ as the capital level that makes $u_I(0, o_{r_o}, \mathbf{z}^*) = 0$ .Then by Property N2 $u_I$ is strictly positive for any $K < K_o$ and contract $\{0, w\}$ that implements $\mathbf{z}^*$.

Define $K_d$ as the capital level that makes $u_I(0, d_{r_d}, \mathbf{z}^*) = 0$. Then by Property N2 $u_I$ is be strictly negative for any $K > K_d$ and contract $\{0, w\}$ that implements $\mathbf{z}^*$.

Assume now that $K \in (\ K_o, K_d)$ and consider a convertible security $c_{r_1,r_2}$ where $r_o \leq r_1 < r_d$. The first order conditions of effort for the entrepreneur is

$$Q_{z_E} \int_{r_1/Q(\mathbf{z})}^{r_2/Q(\mathbf{z})} \theta f(\theta) d\theta = 1 \tag{29}$$

Given $r_1$ and $\mathbf{z} = \mathbf{z}^*$ there exists a unique strike price $r_2$ such that the first order condition is satisfied. Define $\widehat{r_2}(r_1)$ as the level $r_2$ that satisfies 29. It is straightforward to check that the contract $c_{r_1,\widehat{r_2}(r_1)}$ satisfy both agents first order conditions at $\mathbf{z}^*$. Moreover, by the regularity assumption $c_{r_1,\widehat{r_2}(r_1)}$ implements $\mathbf{z}^*$ in RPT. To simplify notation let $c_{r_1,\widehat{r_2}(r_1)} = \widehat{c}_{r_1}$.

Finally notice by the definition of $r_o$ and $r_d$, $r_2(0) = r_o$ and in that case the contract is an option, also and $\lim_{r_1 \to r_d} r_2(r_1) = \infty$ and the contract converges to debt. Therefore

$$u_I(0, o_{r_o}, \mathbf{z}^*) = u_I(0, \widehat{c}_0, \mathbf{z}^*) < 0 < u_I(0, \widehat{c}_{r_d}, \mathbf{z}^*) = u_I(0, d_{r_d}, \mathbf{z}^*) \tag{30}$$

Finally by continuity there exists $r' \in (0, r_d)$ such that $u_I(0, \widehat{c}_{r'}, \mathbf{z}^*) = 0$. The convertible contract $\widehat{c}_{r'}$ implements $\mathbf{z}^*$ and $u_I(0, \widehat{c}_{r'}, \mathbf{z}^*) = 0$, therefore is optimal in OPT $\square$

**Proposition** *$u_E^A - u_E^N$ can be increasing in $K$.*

**Proof.** Assume that $K$ increases to $K'$, where $K < K^D < K'$.

With an acquirer $u_I^A = E(\theta)Q(z^A) - c(z^A) - \overline{v}_I - K$ and therefore $u_I^A(K) - u_I^A(K') = K' - K$.

Because $K < K^D$, the utility of the $VC$ with $K$ is $u_I^{VC}(K) = E(\theta)Q(z_I^*, z_E^*) - c(z_I^*) - c(z_E^*) - \overline{v}_I - \overline{v}_E - K$. Because $K' > K^D$ $z(K) \neq z^*$ and $u_I^{VC}(K') = E(\theta)Q(z_I, z_E) - c(z_I) - c(z_E) - \overline{v}_I - \overline{v}_E - K'$ and since the surplus is maximum at $z^*$, $u_I^{VC}(K) - u_I^{VC}(K') > K' - K$.$\square$

# Appendix B

**Proof of existence of an optimal debt contract in proposition 1** Since $\lim_{r \to \infty} z_E(r) = 0$ we define $\overline{r}$ to be the value that satisfies $E(\theta)Q(z(\overline{r})) = K/2$. A debt contract with face value $r > \overline{r}$ never satisfy the investor IR constraint and therefore we can restrict attention to the set of debts with face value in $[0, \overline{r}]$. Finally observe that the problem of finding the best debt contract can rewritten as $\max_r u_E(d_r, z_E(r))$ subject to $u_I(d_r, z_E(r)) \geq 0$ over the set $[0, \overline{r}]$. Since $u_E$ and $u_I$ are continuous in $r$, the problem has a solution because we are maximizing a continuous function over a compact set.□

**Proof of existence of an optimal debt contract in proposition 3.** Since debt contracts are better than any other contract, without loss of optimality we can rewrite the problem as $\max_r u_E(0, d_r, \mathbf{z}(r))$ subject to $u_I(0, d_r, \mathbf{z}(r)) \geq 0$. By property $N1$ both functions are continuos in $r$.By assumption, there exists a contract {t,w} that implements an effort level $u_E > \varepsilon$. By proposition 3, there exists a debt contract $\{0, d_r\}$ that implements an effort level $u_E > \varepsilon$. Remember that $u_E(0, d_r, \mathbf{z}(r)) < \int_{r/Q(\mathbf{z})}^{\infty} (\theta Q(\mathbf{z}(r)) - r) f(\theta) d\theta$ and from the proof of property $N3$ observe that $Q(\mathbf{z}(r))$ is bounded at $\overline{Q}$, and then $u_E(0, d_r, \mathbf{z}(r)) < \overline{Q}(1 - F(r/\overline{Q}))$ Let $\overline{r}$ be the level that satisfies $\overline{Q}(1 - F(\overline{r}/\overline{Q})) = \varepsilon/2$. Then without loss of optimality we can restrict attention to $r \leq \overline{r}$ and the existence of an optimal contract follows from the fact that we are maximizing a continuous function over a compact set.

**Proposition 2'** *The optimal contract between an entrepreneur and an involved investor if we restrict $t = 0$ depends on the investment $K$. There exists levels $K_o \leq K_d$ such that:*

*a) If $K > K_d$ , debt is the only optimal contract.*

*b) If $K \in (K_o, K_d)$ convertible securities are optimal.*

*c) If $K < K_o$ , a call option is the only optimal contract.*

**Proof.** Let let $o_{r_o^*}$ be the option contract that implements $\mathbf{z}$ in RPT.

a) It follows from proposition 2.

$b$) it follows from proposition 3

$c$) If $K < K_o$ , let let $\mathbf{z} = z_E, z_I$ be the effort level implemented with a contract $\{0, w\}$ . By property N2 the call contract $r_C$ that implements $\mathbf{z}$ gives the investor a lower expected payoff and therefore the entrepreneur a higher expected payoff. If $u_I(0, o_r, \mathbf{z}) \geq 0$; then the call option $o_r$ is better than the contract $\{0, w\}$. If $u_I(0, o_r, \mathbf{z}) < 0$ ,notice that by proposition 2 $u_I(\ 0, d_{r_o^*}, \mathbf{z}^*) > 0$. By continuity there exists $\widehat{r}$ between $r$ and $r_o^*$ such that that $u_I(0, d_{\widehat{r}}, \mathbf{z}(\widehat{r})) = 0$. Also because $\widehat{r}$ is closer to $r_0^*$ than $r$; and by quasi-concavity the surplus is higher than under $w$. The call contract $\widehat{r}_C$ is feasible and and since $u_I(0, d_{\widehat{r}}, \mathbf{z}(\widehat{r})) = 0$, $u_E(0, d_{\widehat{r}}, \mathbf{z}(\widehat{r})) > u_E(0, w, \mathbf{z})$. □