Capital Taxation with Entrepreneurial Risk

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Abstract

This paper studies the effects of capital taxation in a dynamic heterogeneous-agent economy with uninsurable entrepreneurial risk. Although it allows for rich general-equilibrium effects and a stationary distribution of wealth, the model is highly tractable. This permits a clear analysis, not only of the steady state, but also of the entire transitional dynamics following any change in tax policies. Unlike either the complete-markets paradigm or Bewley-type models where idiosyncratic risk impacts only labor income, here it is shown that capital taxation may actually stimulate capital accumulation. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. In particular, for the preferred calibrated version of the model, when the tax on capital is 25%, aggregate output is 2.5% higher than what it would have been had the tax rate been zero. Turning to the welfare effects of a reform in capital taxation, it is shown how these effects depend on whether one focuses on the steady state or also takes into account transitional dynamics, as well as how they vary in the cross-section of the population (rich versus poor, entrepreneurs versus non-entrepreneurs).

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1 Introduction

This paper studies the macroeconomic and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic risk to the return of their investment choices. Such risk is empirically important for entrepreneurs and wealthy agents, who, even though they represent a small fraction of the population, yet they hold most of an economy's wealth. In this context, capital taxation raises an interesting tradeoff between the distortion of investment incentives and the provision of insurance against idiosyncratic capital-income risk. On the one hand, capital taxation comes at a cost, since it distorts agents' saving decisions. On the other hand, it has benefits, since it provides agents with partial insurance against idiosyncratic investment risk. This suggests that a positive tax on capital income could be welfare-improving, even if it reduced capital accumulation.

Most surprisingly though, it is shown that a positive tax on capital income may actually stimulate capital accumulation. Indeed, the steady-state levels of the capital stock, output and employment may all be maximized at a positive value of the capital-income tax. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. This result stands in stark contrast to the effect of capital taxation both under complete-markets models, and under incomplete-markets models with uninsurable labor-income risk alone. In these models, capital-income taxation, irrespectively of whether it is welfare-improving or not, necessarily discourages capital accumulation.

Model. This paper represents a first attempt to study the effects of capital-income taxation in a general-equilibrium incomplete-markets economy, where agents are exposed to uninsurable idiosyncratic investment risk. The framework builds on Angeletos (2007), who develops a variant of the neoclassical growth model that allows for idiosyncratic investment risk, and studies the effects of such risk on macroeconomic aggregates. Here, as in Angeletos's model, agents own privately held businesses that operate under constant returns to scale. Agents are not exposed to labor-income risk, and they can freely borrow and lend in a riskless bond, but they cannot diversify the idiosyncratic risk in their private business investments. Abstracting from labor-income risk, borrowing constraints, and other market frictions, isolates the impact of the idiosyncratic investment risk and preserves the tractability of the general-equilibrium dynamics. The present model extends Angeletos's model in the following ways. First, a government is introduced, imposing proportional taxes on capital and labor income, along with a non-contingent lump-sum tax or transfer. Second, agents have finite lives, which ensures the existence of a stationary wealth distribution. Third, there is stochastic, though exogenous, transition in and out of entrepreneurship, which helps capture the observed heterogeneity between entrepreneurs and non-entrepreneurs without the complexity of endogenizing occupational choice. Fourth, labor supply is endogenous. Clearly the first element is essential for the novel contribution of the paper. The other three improve the quantitative performance of the model and demonstrate the robustness of the main result.

Preview of results. The main result of the paper is that an increase in capital-income taxation may in fact increase capital accumulation. The intuition behind this result comes from recognizing that the overall effect of the capital-income tax on capital accumulation can be decomposed in two parts. The first part captures the response of capital to the tax in a setting with endogenous saving but exogenously fixed interest rate. This is isomorphic to examining the effects of the capital tax in a "small open economy." In this context, it is shown that an increase in the capital-income tax unambiguously decreases the steady-state capital stock. The second part, which is the core result of this paper, captures the importance of the general-equilibrium adjustment of the interest rate for wealth and capital accumulation. Here, an increase in the tax reduces the effective variance of the risk agents are exposed to. This reduces the demand for precautionary saving, and therefore increases the interest rate, which in turn increases steady-state wealth. With decreasing absolute risk aversion, wealthier agents are willing to undertake more risk, and hence they will increase their investment in capital. In other words, the general-equilibrium effect of the interest rate adjustment is a force that tends to increase investment and the steady-state capital stock.

For plausible parameterizations of the closed economy, the general equilibrium effect dominates for low levels of the capital-income tax, so that steady-state capital at first increases with the tax. In particular, for the preferred calibrated version of the model, the steady-state capital stock is maximized when the tax on capital is 40%. When the tax on capital is 25%, aggregate output is 2.5% higher than what it would have been had the tax rate been zero. The result that the steady-state capital stock is inversely U-shaped with respect to the capital-income tax is robust for a wide range of empirically plausible parameter values. Furthermore, the tax that maximizes steady-state capital is increasing in risk aversion or the volatility of the idiosyncratic risk. This finding reinforces the insurance interpretation of the tax system.

Subsequently, the paper examines the aggregate and welfare effects of eliminating the capital-income tax. This is because an extensive discussion has been conducted within the context of the complete-markets neoclassical growth model about the welfare benefits of setting the capital-income tax to zero. In light of the main result here, revisiting this discussion is worthwhile. In particular, the aggregate and welfare effects are presented from two different perspectives. On the one hand, one might be interested in examining the welfare of the current generation immediately after the policy reform, taking into account the entire transitional dynamics of the economy towards the new steady state with the zero tax. On the other hand, one might be interested in examining the welfare of the generations that will be alive in the distant future, i.e. at the new steady state. For convenience, the first exercise will be referred to as the study of the short-run welfare implications of eliminating the capital-income tax, whereas the second exercise will be referred to as the study of the long-run welfare implications of eliminating the capital-income tax.

First, consider the macroeconomic effects of eliminating the capital-income tax. When markets are complete, investment increases in the short run, and it is also higher at the new long-run steady state with the zero tax, relative to the old steady state with the positive tax. By contrast,in the present model of incomplete markets, investment falls in the short run, as well as in the long run.

Second, consider the welfare effects of eliminating the capital-income tax. These vary across the different types of agents, the different levels of wealth, and the short-run and long-run perspectives. In the short run, poor agents, whether entrepreneurs or non-entrepreneurs, prefer the zero tax. This is because most of their wealth comes from wage income, and, with capital fixed, the present value of wages increases due to a fall in the interest rate. Rich agents, on the other hand, prefer a positive tax, since they benefit more from insurance provision.

In the long run, all types of agents, and at all levels of wealth, prefer a positive tax on capital income. However, the cost of switching to a zero-tax regime is much higher for poorer than for wealthier agents of all types. This is because, in the long run, the elimination of the tax decreases the steady-state capital stock, thereby decreasing the present value of wages. Therefore poorer agents will suffer the most, since human wealth constitutes a big part of their total wealth.

Literature review. This paper focuses on entrepreneurial risk, because such risk is in fact empirically relevant, even in a financially developed country like the United States. For example, Moskowitz and Vissing-Jørgensen (2002) find that 75% of all private equity is owned by agents for whom such investment constitutes at least half of their total net worth. Furthermore, 85% of private equity is held by owners who are actively involved in the management of their own firm. Given this evidence about the United States, one expects that entrepreneurial risk must be even more prevalent in less developed economies, where a large part of production takes place in small unincorporated businesses and where risk-sharing arrangements are much more limited.

This paper relates to the strand of the macroeconomic literature discussing optimal taxation and the effects of taxation. However, most of this literature has focused on labor income risk. Chamley (1986) and Judd (1985) first established the result of zero optimal capital taxation in the long run when markets are complete. Atkeson, Chari and Kehoe (1999) generalized this result to most of the short run for an interesting class of preferences, and to the case of finitely lived agents. Aiyagari (1995) extended the complete-markets framework to include uninsurable labor income risk and borrowing constraints. In this context, when only a limited set of policy instruments are available, it becomes optimal to tax capital in the long run: a positive capital tax increases welfare, but it unambiguously lowers the level of the capital stock.

A related but different normative exercise is conducted by Davila et al. (2007). They examine constrained efficiency, in the spirit of Geanakoplos-Polemarchakis, within a version of Aiyagari's model. This exercise does not allow for risk-sharing through taxes or any other instruments, and instead considers an efficiency concept where the planner directly dictates to the agents how much to invest and to trade. Angeletos and Werning (2006) examine a similar constrained efficiency problem in a two-period version of a model with idiosyncratic investment risk. Albanesi (2006) considers optimal taxation in a two-period model of entrepreneurial activity, in a constrained efficiency setting, and following the Mirrlees optimal policy tradition. The benefit of her approach is that the source of incomplete risk-sharing is endogenously specified as the result of a private information (moral hazard) problem, and that there are no ad hoc restrictions placed on the tax instruments. However, her model does not allow for dynamics, for long-run considerations, or for general-equilibrium effects like those studied in the present paper.

The growing literature on the effects of borrowing constraints on entrepreneurial choices has examined policy questions, and especially the implications of replacing a progressive with a proportional income tax schedule, in an Aiyagari-type framework, i.e. with decreasing returns to scale at the individual level, borrowing constraints, and undiversifiable labor income risk. These policy exercises have been conducted from a long-run perspective, without taking into account transitional dynamics. Some examples in this area include Cagetti and DeNardi (2004), Meh (2005), and Li (2002). Benabou (2002) develops a tractable dynamic general-equilibrium model of human capital accumulation with endogenous effort and missing credit and insurance markets. Within this framework he examines the long-run tradeoffs of progressive taxation and education finance. Finally, Erosa and Koreshkova (2007) examine the long-run effects of switching from progressive to proportional income taxation in a quantitative dynastic model of human capital.

This paper also relates to the branch of the public finance literature that considers the effects of capital taxation on portfolio allocation and risk-taking. Domar and Musgrave (1944) first proposed the idea that proportional income taxation may increase risk-taking, by having the government bear part of the risk facing the agents. This idea was formalized by Stiglitz (1969), within a two-period single-agent model, where asset returns and the level of saving are exogenously given, but where the agent optimally chooses the allocation of his fixed amount of saving between a risky and a riskless asset. Ahsan (1974) extends Stiglitz by endogenizing the intertemporal consumption-saving decision in a two-period model. He shows that the partial-equilibrium effect of capital-income taxation on risk-taking is in general ambiguous. By contrast, in the "small open economy" version of the present model, which allows for wealth accumulation over time but takes the interest rate as exogenously fixed, it is shown that the steady-state capital stock is decreasing in the capital-income tax. This finding highlights that the results here are driven by general-equilibrium effects, which is novel to the literature.

As already mentioned, the present model builds on Angeletos (2007), who abstracted from policy questions and considered instead the effect of investment risk on macroeconomic aggregates. The contribution of the present paper is to study the effects of capital-income taxation on aggregates and welfare. Angeletos and Panousi (2007), in a framework like the one in Angeletos (2007), examine the effects of government spending on macroeconomic aggregates, but for the case where government spending is financed exclusively through lump-sum taxation.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes individual behavior and the aggregate equilibrium dynamics. Section 4 characterizes the steady state in terms of aggregates and distributions. Section 5 presents and discusses the main theoretical result. Section 6 presents the calibration methodology and the parameter choices, along with the implications of the preferred calibrated model for aggregates and distributions. Section 7 quantifies the steady-state effects of capital taxation, as well as the short-run and long-run effects of eliminating the capital-income tax. Section 8 examines the robustness of the results to the availability of a safe asset in positive net supply. Section 9 concludes. All proofs are delegated to the appendix.

2 The Model

Time is continuous and indexed by $t \in [0, \infty)$. There is a continuum of agents distributed uniformly over $[0, 1]$. At each point in time an agent can be either an entrepreneur, denoted by E, or a laborer, denoted by L. The probabilities of switching between these two types are exogenous. In particular, the probability that an agent will switch from being an entrepreneur to being a laborer is $p_{EL}\, dt$, and the probability that he will switch from being a laborer to being an entrepreneur is $p_{LE}\, dt$. The measure of entrepreneurs in the economy at time t is denoted by χ_t.

In what follows, and for expositional simplicity, labor is taken to be exogenous. All proofs in the appendix consider the general case of endogenous labor, where preferences are homothetic between consumption and leisure, i.e. they are of the King-Plosser-Rebelo (1988) specification.

2.1 Preferences

All agents are endowed with one unit of time. Preferences are Epstein-Zin over consumption, c, and they are defined as the limit, for $\Delta t \to 0$, of:

$$U_t = \{ \; (1 - e^{-\beta \Delta t}) \; c_t^{1-1/\theta} \; + \; e^{-\beta \Delta t} \; (E_t \; [\; U_{t+\Delta t}^{1-\gamma} \;] \;)^{\frac{1-1/\theta}{1-\gamma}} \; \}^{\frac{1}{1-1/\theta}} \; , \tag{1}$$

where $\beta > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\theta > 0$ is the elasticity of intertemporal substitution. For $\theta = 1/\gamma$, this reduces to the case of standard expected utility, $U_t = E_t \int_t^\infty e^{-\beta\, s}\, U(c_s)\, ds$, where $U(c_t) = \frac{c_t^{1-1/\theta}}{1-1/\theta}$.

2.2 Entrepreneurs

When an agent is an entrepreneur, he owns and runs a firm operating a constant-returns-to-scale neoclassical production function $F(k, l)$, where k is capital input and l is labor input. An entrepreneur can only invest in his own firm's capital, although he supplies and employs labor in the competitive labor market. Capital investment in his firm is subject to uninsurable risk. The idiosyncratic shocks are i.i.d., hence there is no aggregate uncertainty. An entrepreneur can also save in a riskless bond.

The financial wealth of an entrepreneur i, denoted by x_t^i, is the sum of his holdings in private capital, k_t^i, and the riskless bond, b_t^i:

$$x_t^i = k_t^i + b_t^i\,. \tag{2}$$

The evolution of x_t^i is given by:

$$dx_t^i = (1-\tau_t^K)\; d\pi_t^i + [\; (1-\tau_t^K)\, R_t\, b_t^i + (1-\tau_t^L)\,\omega_t + T_t - c_t^i\;]\, dt\,, \tag{3}$$

where $d\pi_t^i$ are firm profits (capital income), R_t is the interest rate on the riskless bond, τ_t^K is the proportional capital-income tax, ω_t is the wage rate in the aggregate economy, τ_t^L is the proportional labor-income tax, T_t are the transfers received from the government, and c_t^i is consumption. Finally, a no-Ponzi game condition is imposed.

Firm profits are given by:

$$d\pi_t^i = [\; F(k_t^i, l_t^i) - \omega_t\, l_t^i - \delta\, k_t^i\;]\, dt \;+\; \sigma\, k_t^i\, dz_t^i\,, \tag{4}$$

where $F(k,l) = k^\alpha l^{1-\alpha}$ with $\alpha \in (0,1)$, and δ is the mean depreciation rate in the aggregate economy. Idiosyncratic risk is introduced through dz_t^i, a standard Wiener process that is i.i.d. across agents and across time[1]. The scalar σ measures the amount of undiversified idiosyncratic risk, and is an index of market incompleteness, with higher σ corresponding to a lower degree of risk-sharing, and $\sigma = 0$ corresponding to complete markets.

2.3 Laborers

When an agent is a laborer, he cannot invest in capital, and he can only hold the riskless bond. He also supplies labor in the competitive labor market. Financial wealth for a laborer i is therefore:

$$x_t^i = b_t^i\,, \tag{5}$$

and its evolution is given by:

$$dx_t^i = [\; (1-\tau_t^K)\, R_t\, b_t^i + (1-\tau_t^L)\,\omega_t + T_t - c_t^i\;]\, dt\,. \tag{6}$$

[1]Idiosyncratic risk is modeled here as coming from uninsurable i.i.d. depreciation shocks. However these shocks could also be modeled as or interpreted as i.i.d. productivity shocks.

2.4 Government

At each point in time the government taxes capital and bond income at the rate τ_t^K, and labor income at the rate τ_t^L. Part of the tax proceeds is used by the government for own consumption at the deterministic rate G_t. Government spending does not affect the utility from private consumption or the production technology. The remaining tax proceeds are then distributed back to the households in the form of non-contingent lump-sum transfers. The government budget constraint is therefore:

$$0 = [\ \tau_t^L\, F_L(\int_i k_t^i, 1) + \tau_t^K\ (\ F_{K_t}(\int_i k_t^i, 1) - \delta\ )\ \int_i k_t^i - G_t\ - T_t\ ]\ dt\,, \tag{7}$$

where $F_{K_t}(\int_i k_t^i, 1)$ is the marginal product of capital in the aggregate economy, and where $\int_i l_t^i = 1$.

2.5 Finite lives and annuities

All households face a constant probability of death, with Poisson arrival rate $v\, dt$ at every instant in time[2]. There is no intergenerational altruism linking a household to its descendants, and utility is zero after death. The discount rate in preferences can then be reinterpreted as $\beta = \tilde{\beta} + v$, where $\tilde{\beta}$ is the psychological or subjective discount rate and v is the probability of death.

To simplify the analysis, it is assumed that there exist annuity firms permitting all agents to get insurance against mortality risk, by freely borrowing the entire net present value of their future labor income. As a result, all agents have human wealth, denoted by h_t, and defined as the present discounted value of their net-of-taxes labor endowment plus government transfers:

$$h_t = \int_t^\infty e^{-\int_t^s (\,(1-\tau_j^K) R_j + v\,)\, dj} (\,(1-\tau_s^L)\omega_s + T_s\,)\, ds\,. \tag{8}$$

Then, total effective wealth for an agent, denoted by w_t^i, is defined as the sum of his financial and human wealth, $w_t^i \equiv x_t^i + h_t$. Hence, effective wealth for an entrepreneur is given by:

$$w_t^i = k_t^i + b_t^i + h_t\,, \tag{9}$$

and effective wealth for a laborer is given by:

$$w_t^i = b_t^i + h_t\,. \tag{10}$$

[2] In general, with finite lives and no altruism, Ricardian equivalence might fail, since some of the tax burden associated with the current issue of a bond is borne by agents who are not alive when the bond is issued. For $v = 0$, Ricardian equivalence holds in the model, because all agents can freely borrow in the riskless bond. However, none of the results of the paper hinge on Ricardian equivalence, hence the government budget constraint will be written as in (7) for v positive but small.

3 Equilibrium

3.1 Individual Behavior

Because entrepreneurs choose employment after their capital stock has been installed and their idiosyncratic shock has been observed, and because their production function, F, exhibits constant returns to scale, optimal firm employment and optimal profits are linear in own capital:

$$l_t^i = l(\omega_t)\, k_t^i \qquad \text{and} \qquad d\pi_t^i = r(\omega_t)\, k_t^i\, dt + \sigma\, k_t^i\, dz_t^i\,, \tag{11}$$

where $l(\omega_t) \equiv \arg\max_l [\, F(1,l) - \omega_t\, l\,]$ and $r(\omega_t) \equiv \max_l [\, F(1,l) - \omega_t\, l\,] - \delta$. Here, $r_t \equiv r(\omega_t)$ is an entrepreneur's expectation of the return to his capital prior to the realization of his idiosyncratic shock, as well as the mean of the realized returns in the cross-section of firms. The key result here is that entrepreneurs face risky, but linear, returns to their investment.

The evolution of effective wealth for an entrepreneur is described by:

$$dw_t^i = [\, (1-\tau_t^K)\, r_t\, k_t^i + (1-\tau_t^K)\, R_t\, (b_t^i + h_t) - c_t^i\,]\, dt + \sigma\, (1-\tau_t^K)\, k_t^i\, dz_t^i\,. \tag{12}$$

The first term captures the expected rate of growth of effective wealth, showing that wealth grows when the total return to saving for an entrepreneur exceeds consumption expenditures. The second term captures the impact of idiosyncratic risk. The evolution of effective wealth for a laborer is described by:

$$dw_t^i = [\, (1-\tau_t^K)\, R_t\, (b_t^i + h_t) - c_t^i\,]\, dt\,. \tag{13}$$

Let the fraction of effective wealth an agent saves in the risky asset be:

$$\phi_t^i \equiv \frac{k_t^i}{w_t^i}\,. \tag{14}$$

Let an agent's marginal propensity to consume out of effective wealth be:

$$m_t^i \equiv \frac{c_t^i}{w_t^i}\,. \tag{15}$$

Let $\mu_t = (1-\tau_t^K) r_t - (1-\tau_t^K) R_t$ be the risk premium, $\rho_t \equiv \phi_t\, (1-\tau_t^K)\, r_t + (1-\phi_t)\, (1-\tau_t^K)\, R_t$ the net-of-tax mean return to saving for an entrepreneur, and $\hat{\rho}_t \equiv \rho_t - 1/2\, \gamma\, \phi_t^2\, \sigma^2\, (1-\tau_t^K)^2$ the net-of-tax risk-adjusted return to saving for an entrepreneur. The net-of-tax return to saving for a laborer is simply $(1-\tau_t^K)\, R_t$.

Because of the linearity of the budget constraints (12) and (13) in assets, and the homotheticity of the preferences, the optimal individual policy rules will be linear in total effective wealth, for given prices and government policies. I.e., for given prices and policies, an agent's consumption-saving problem reduces to a tractable homothetic problem as in Samuelson's and Merton's classic portfolio analysis. Optimal individual behavior is characterized by the following proposition.

Proposition 1. *Let* $\{\omega_t, R_t, r_t\}_{t\in[0,\infty)}$ *and* $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t\in[0,\infty)}$ *be equilibrium price and policy sequences. If an agent i is an entrepreneur, his optimal consumption, investment, portfolio, and bond holding choices respectively are given by*

$$c_t^i = m_t^E\, w_t^i, \quad k_t^i = \phi_t\, w_t^i, \quad \phi_t = \frac{(1-\tau_t^K)\, r_t - (1-\tau_t^K)\, R_t}{\gamma\, \sigma^2\, (1-\tau_t^K)^2}, \quad b_t^i = (1-\phi_t)\, w_t^i - h_t\,. \tag{16}$$

If an agent i is a laborer, his optimal consumption, investment, and bond holding choices respectively are given by

$$c_t^i = m_t^L\, w_t^i, \quad k_t^i = 0, \quad b_t^i = w_t^i - h_t\,. \tag{17}$$

The marginal propensities to consume satisfy the following system of ordinary differential equations:

$$\frac{\dot{m}_t{}^E}{m_t^E} = m_t^E - \theta\beta + (\theta-1)\,\hat{\rho}_t + \frac{\theta-1}{1-\gamma}\, p_{EL}\, [\, (\frac{m^L}{m^E})^{\frac{1-\gamma}{1-\theta}} - 1\,] \tag{18}$$

$$\frac{\dot{m}_t{}^L}{m_t^L} = m_t^L - \theta\beta + (\theta-1)(1-\tau_t^K)\, R_t + \frac{\theta-1}{1-\gamma}\, p_{LE}\, [\, (\frac{m^E}{m^L})^{\frac{1-\gamma}{1-\theta}} - 1\,]\,. \tag{19}$$

From (16) and (17) it is clear that optimal consumption is a linear function of total effective wealth, where the marginal propensities to consume depend only on the type of the agent and not on the level of wealth. In other words, all entrepreneurs share a common marginal propensity to consume, m_t^E, and all laborers share a common marginal propensity to consume, m_t^L. The fraction ϕ_t of wealth invested in the risky asset by an agent who happens to be an entrepreneur is increasing in the risk premium, decreasing in risk aversion, and decreasing in the effective variance of risk, $\sigma(1-\tau_t^K)$. Because of homotheticity and linearity, ϕ_t is the same across all entrepreneurs, and independent of the level of wealth. The policy for optimal bond holdings follows from (9), (10), and (14). The system of (18) and (19) is a system of two Euler equations. It shows that the marginal propensities to consume, conditional on being an entrepreneur or a laborer, depend on the process of the corresponding net-of-tax anticipated (risk-adjusted) returns to saving. The last terms in the Euler equations indicate that the marginal propensity to consume for an agent is affected by the probability that he might switch between being an entrepreneur and being a laborer.

3.2 Equilibrium definition

The initial position of the economy is given by the distribution of (k_0^i, b_0^i) across households. An equilibrium is a deterministic sequence of prices $\{\omega_t, R_t, r_t\}_{t\in[0,\infty)}$, a deterministic sequence of policies $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t\in[0,\infty)}$, a deterministic macroeconomic path $\{C_t, K_t, Y_t, L_t, W_t, W_t^E, W_t^L\}_{t\in[0,\infty)}$, and a collection of individual contingent plans $(\{c_t^i, l_t^i, k_t^i, b_t^i, w_t^i\}_{t\in[0,\infty)})$ for $i \in [0,1]$, such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears, $\int_i l_t^i = 1$, in all t; (iii) the bond market clears, $\int_t b_t^i = 0$, in all t; (iv) the government budget constraint (7) is satisfied in all t; and (v) the aggregates are consistent with individual behavior, $C_t = \int_i c_t^i$, $L_t = \int_i l_t^i = 1$, $K_t = \int_i k_t^i$, $Y_t = \int_i F(k_t^i, l_t^i)$, $W_t = \int_i w_t^i$, $W_t^E = \int_{i,\,E} w_t^i$, and $W_t^L = \int_{i,\,L} w_t^i$, in all t.

3.3 General equilibrium

Because individual consumption and investment are linear in individual wealth, aggregates at any point in time do not depend on the extend of wealth inequality at that time. Therefore here, in contrast to other incomplete-markets models, it is not the case that the entire wealth distribution is a relevant state variable for aggregate dynamics. In fact, for the determination of aggregate dynamics, it suffices to keep track of the mean of aggregate wealth, and of the allocation of total wealth between the two groups of agents. To that end, the fraction of total effective wealth held by entrepreneurs in the economy is defined as:

$$\lambda_t \equiv \frac{W_t^E}{W_t} \,. \tag{20}$$

The aggregate equilibrium dynamics can then be described by the following recursive system.

Proposition 2. *In equilibrium, the aggregate dynamics satisfy*

$$\dot{W}_t/W_t = \lambda_t(\rho_t - m_t^E) + (1-\lambda_t)(\,(1-\tau_t^K)R_t - m_t^L\,) \tag{21}$$

$$\dot{\lambda}_t/\lambda_t = (1-\lambda_t)\phi_t\mu_t + (1-\lambda_t)(m_t^L - m_t^E) + p_{LE}(\frac{1}{\lambda_t} - 1) - p_{EL} \tag{22}$$

$$\dot{H}_t = (\,(1-\tau_t^K)R_t + v\,)H_t - (1-\tau_t^L)\,\omega_t - (\,\tau_t^L\,\omega_t + \tau_t^K\,(\,F_{K_t} - \delta\,)\,K_t - G_t\,) \tag{23}$$

$$K_t = \frac{\phi_t\,\lambda_t}{1-\phi_t\,\lambda_t}\,H_t\;, \tag{24}$$

along with (18) *and* (19).

Equation (21) shows that the evolution of total effective wealth is a weighted average of two terms. The first term is positive when the mean net-of-tax return to saving for entrepreneurs exceeds their marginal propensity to consume, and is weighted by the fraction of total wealth the entrepreneurs hold in the economy. The second term is positive when the net-of-tax return to saving for laborers exceeds their marginal propensity to consume, and is weighted by the fraction of total wealth the laborers hold in the economy. Equation (22) shows the endogenous evolution of the relative distribution of wealth between the two groups of agents. The evolution of λ depends first, on the differential excess return, $\phi_t\mu_t$, the entrepreneurs face on their saving, second, on the difference in the level of saving between entrepreneurs and laborers, $m_t^L - m_t^E$, and third, on the adjustment made for the transition probabilities. Note here that the evolution of consumption can be recovered by aggregating across individual optimal policies, so that $C_t^E = m_t^E W_t^E$ and $C_t^L = m_t^L W_t^L$. Equation (23) shows the evolution of total human wealth, using the government budget constraint $T_t = \tau_t^L \omega_t + \tau_t^K (F_{K_t} - \delta) K_t - G_t$, where F_{K_t} is the marginal product of capital in the aggregate production function $F(K, 1)$. Equation (24) is the bond market clearing condition. It comes from aggregating across individual capital and bond choices as given in (16) and (17), adding up, using $B_t^E + B_t^L = 0$, and using (20). From (24) it follows that, for given prices and human wealth, a decrease in λ decreases K. A fall in λ indicates that the entrepreneurs on average now borrow more from the laborers, hence their wealth will on average be lower. With decreasing absolute risk aversion, this will negatively affect their willingness to take risk, and hence investment and the capital stock will fall.

4 Steady State

A steady state is a competitive equilibrium as defined in section 3.2, where prices, policies, and aggregates are time-invariant. In what follows, the steady state will be characterized, first in terms of aggregates, and then in terms of the invariant wealth distributions.

4.1 Characterization of aggregates

In this section, and for expositional purposes, the steady state is characterized for the case of $\theta = 1$. The more general cases are treated in the appendix. When $\theta = 1$, $m_t^E = m_t^L = \beta$ for all t, and hence aggregate consumption is given by $C_t = \beta W_t$, where $W_t = W_t^E + W_t^L$. The steady state is the fixed point of the dynamic system in Proposition 2. Let government spending, G, be parameterized as a fraction g of tax revenue. The following proposition characterizes the steady state.

Proposition 3. *(i) The steady state always exists. (ii) In steady state, the capital stock, K, and the interest rate, R, are the unique solution to*

$$\lambda = \frac{\beta - (1 - \tau^K) R + p_{LE}}{\beta - (1 - \tau^K) R + p_{LE} + p_{EL}} \tag{25}$$

$$F_K(K) - \delta = R + \sqrt{\frac{1}{\lambda(R)} \gamma \sigma^2 (\beta - (1 - \tau^K) R)} \tag{26}$$

$$K = \frac{\phi(K, R) \lambda(R)}{1 - \phi(K, R) \lambda(R)} \frac{(1 - \tau^L) \omega(K) + (1 - g) (\tau^L \omega(K) + \tau^K (F_K(K) - \delta) K)}{(1 - \tau^K) R + v} \tag{27}$$

Here $F_K(K)$ and $\omega(K)$ are, respectively, the marginal product of capital and the wage rate in the aggregate economy. The proof of proposition 3 is left for the appendix[3]. Equation (25) captures

[3] In simulations the steady state is unique for all parameter values. The same is true for the general case of

the relative wealth inequality between the two groups of agents, as a function of the interest rate, R, and model parameters. Equation (26) can be interpreted as describing the behavior of the capital stock in an open economy, where the interest rate is exogenously given. On the one hand, an increase in the interest rate increases the opportunity cost of capital, and thus tends to lower the capital stock. This would be the only effect present under complete markets. On the other hand, an increase in the interest rate might also increase the steady-state wealth of entrepreneurs. With decreasing absolute risk aversion, this increases entrepreneurs' willingness to take risk, and hence it is a force that tends to increase the steady-state capital stock. Overall, one can show that K increases with R if and only if $\theta > \phi/(1-\phi)$. Also note from (26) that, for given prices, K will be lower the lower is λ, regardless of the level of τ^K. Finally, equation (27) would be irrelevant in the open economy. In the closed economy, it captures bond market clearing and it determines the equilibrium interest rate.

4.2 Characterization of invariant distributions

At each point in time, agents die and are replaced by newborn agents, who are endowed with the wealth of the exiting agents. This force generates mean reversion and guarantees the existence of an invariant wealth distribution. Let $\xi_t^i \equiv w_t^i/W_t$ be the distance between individual and aggregate effective wealth. Let Φ_L and Φ_E be the conditional invariant distributions for laborers and entrepreneurs respectively. The following proposition characterizes the invariant distributions.

Proposition 4. *The conditional invariant distributions Φ_L and Φ_E are characterized by the following second order linear differential system of two equations*

$$0 = \kappa_1\, \xi\, \frac{\partial \Phi_L}{\partial \xi} \;+\; \kappa_2\, \Phi_L \;+\; p_{EL}\, \Phi_E,$$

$$0 = \kappa_3\, \xi^2\, \frac{\partial^2 \Phi_E}{\partial \xi^2} \;+\; \kappa_4\, \xi\, \frac{\partial \Phi_E}{\partial \xi} \;+\; \kappa_5\, \Phi_E \;+\; p_{LE}\, \Phi_L\,,$$

where $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$ are determined by steady-state aggregates.

The point to note here is that the tractability of the model allows for a very detailed characterization of the invariant distributions. This is particularly useful for the case of entrepreneurs, since it is reasonable to expect that the distribution of wealth over entrepreneurs will be to a large extent determined by the realization of entrepreneurial returns[4].

endogenous labor and $\theta \neq 1$, although neither existence nor uniqueness have been formally shown for that case.

[4]Whereas the tractability of the aggregates follows from Angeletos (2007), the result about the tractability of the invariant distributions is novel to the present paper.

5 Steady-State Effects of Proportional Capital Taxation

This section proceeds to develop the core of the contribution of this paper, which is the study of the steady-state effects of capital-income taxation. In particular, the main result here is that an increase in the capital-income tax may actually increase investment and the steady-state capital stock. This possibility arises because of the general-equilibrium effects of the insurance aspect of capital taxation. These effects operate mainly through the endogenous adjustment of the interest rate. For simplicity, and to demonstrate that the qualitative effects do not depend on the finite horizon or the existence of two types of agents, the analysis that follows will assume that $\lambda = 1$ and $v = 0$.

To facilitate the analysis, it is useful to decompose the impact of the capital-income tax on capital in two parts. The first part describes how the steady-state capital stock changes with the tax, τ^K, when the interest rate, R, is kept constant. This effect corresponds to the case of the "small open economy" version of the model. To this end, let $K^o(\tau^K, R)$ be the steady-state level of capital in the open economy, where both τ^K and R are parameters. The second part describes the general-equilibrium adjustment of the interest rate in the closed economy, and the subsequent effects of this adjustment on capital accumulation. To this end, let $R^c(\tau^K)$ be the steady-state interest rate in the closed economy, and let $K^c(\tau^K) \equiv K^o(\tau^K,\, R^c(\tau^K))$ be the steady-state capital stock in the closed economy. Then, the total effect of the capital-income tax on steady-state capital can be decomposed as follows:

$$\frac{dK^c}{d\tau^K} = \frac{\partial K^o}{\partial \tau^K} + \frac{\partial K^o}{\partial R}\frac{dR^c}{d\tau^K}, \tag{28}$$

where the first term is the open-economy effect, and the second term is the closed-economy or general-equilibrium effect. These effects will next be examined in turn.

The first term on the right-hand-side of (28) is the open-economy effect. The following proposition shows that in the open-economy version of the model, an increase in capital-income taxation induces the usual negative incentive effect on capital accumulation.

Proposition 5. *In the open-economy version of the model, an increase in the capital-income tax unambiguously decreases the steady-state capital stock.*

Therefore, in the open economy capital falls with the tax, despite the direct insurance aspect of the tax system that is still present. This aspect of the tax system is that the government, through taxation, reduces the variance of net-of-tax returns. Here the government has effectively become a shareholder in private businesses, thereby improving the allocation of risk bearing in the economy and allowing for more risk taking. However, in the open economy this channel is not strong enough to outweigh the distortionary effect of capital taxation on investment. This result stands in contrast to the findings of Ahsan (1974). Ahsan considers the simultaneous determination of the size and the composition of the optimal portfolio, in a two-period model with exogenous returns.

He shows that the partial equilibrium effect of an increase in capital-income taxation on risk-taking is in general ambiguous. Ahsan's result is, in turn, a generalization of Stiglitz (1969), who examines the effects of proportional capital-income taxation in a two-period single-agent model, taking not only returns, but also the level of saving as exogenously given. Hence, once Ahsan's setting is extended to incorporate endogenous capital return and infinite horizon, the result that the government can increase risk taking and investment in the risky asset by becoming a shareholder in private businesses no longer holds. It is then clear that, on top of the direct insurance role of the government, the endogenous adjustment of the interest rate is also required for the effect of capital taxation on capital to become ambiguous.

The second term on the right-hand-side of (28) is the general-equilibrium effect capturing the fact that in the closed economy the interest rate endogenously adjusts to clear the bond market. This term is further decomposed in two forces.

First, an increase in the capital-income tax reduces the effective volatility of risk for entrepreneurs, $\sigma(1-\tau^K)$; this is the direct insurance effect. Hence the interest rate increases, essentially because of a reduction in the demand for precautionary saving[5], [6].

Second, this increase in the interest rate will generate opposing effects on saving and wealth accumulation[7]. On the one hand, the increase in the interest rate increases the opportunity cost of capital, and hence it tends to decrease the steady-state capital stock. This would be the only effect present if markets were complete. On the other hand, in the present model of incomplete markets, the increase in the interest rate increases steady-state wealth, if the substitution effect of the increase in the saving return it induces is strong enough. With decreasing absolute risk aversion, this increases entrepreneurs' willingness to undertake risk, and it therefore tends to increase investment and the steady-state capital stock. The overall effect of R on K is summarized in the following proposition.

Proposition 6. *In the open-economy version of the model, the steady-state capital stock will be increasing in the interest rate, if and only if* $\theta > \phi/(1-\phi)$.

Hence the product of the two terms on the right-hand-side of (28) might be positive. This means that the general-equilibrium effect of insurance provision on the adjustment of the interest rate, and the subsequent effect of this adjustment on wealth accumulation, is crucial for overthrowing the negative open-economy effect of the capital-income tax on capital. The next sections of the paper will demonstrate how, for empirically plausible parameter values, this general-equilibrium

[5] Note here that in steady-state the interest rate has to be lower than the discount rate in preferences, otherwise saving and wealth would explode. The reasoning is similar to the reasoning in Aiyagari (1994). By contrast, under complete markets, the steady-state interest rate equals the discount rate.

[6] This intuitive result has not been proven in the context of the infinite horizon model, although a proof is available for the two period version of the closed economy, for small τ^K. However, simulations show that in the infinite horizon model, the net interest rate is always increasing in the tax.

[7] This has already been mentioned in section 4.1.

effect will produce the counter-intuitive result that increases in the capital-income tax will at first increase steady-state capital, even with the open-economy effect working in the opposite direction.

6 Simulations, Parameter Choice, and Benchmark Model

For the quantitative part of the paper, the benchmark model analyzed so far is extended to include endogenous labor. Preferences are assumed homothetic between consumption, c and leisure, n, according to the King-Plosser-Rebelo (1988) specification, and they are defined as the limit, for $\Delta t \to 0$, of:

$$U_t = \{ \, (1 - e^{-\beta \Delta t}) \, (c_t^{1-\psi} n_t^{\psi})^{1-1/\theta} + e^{-\beta \Delta t} \, (\, E_t[\, U_{t+\Delta t}^{1-\gamma} \,] \,)^{\frac{1-1/\theta}{1-\gamma}} \, \}^{\frac{1}{1-1/\theta}} \, . \tag{29}$$

The appendix presents all proofs for the general case of endogenous labor.

6.1 Simulations

The dynamic system described in Proposition 2, and generalized to the case of endogenous labor, is highly tractable compared to other incomplete-markets models, where the entire wealth distribution is a relevant state variable for aggregate equilibrium dynamics. The steady state of the system is found by setting the dynamics of all equations to zero. The algorithm first solves for the steady-state aggregates, which are deterministic and characterized by Proposition 3. Subsequently, for any historically given (K_0, χ_0, X_0^E), where χ_0 is the initial measure of entrepreneurs in the economy, and X_0^E is the historically given financial wealth of the entrepreneur group, and using as boundary conditions the steady state values of (H, m^E, m^L), it integrates backward until the path of $(K_t, \lambda_t, H_t, m_t^E, m_t^L)$ is close enough to its steady-state value.

The method of finite differences is used on the general version of the system in Proposition 4. The first and second derivatives of the invariant distributions are replaced by their discrete time approximations. The only conditions imposed are that the probability density functions integrate to one, and that they do not explode to the right. The emerging functions Φ_L and Φ_E are well-behaved and stable.

Subsequently, Monte-Carlo simulations are performed. The processes of dying, of type-switching, and of the idiosyncratic capital-income shocks, are simulated using random number generators for series of 200,000 households and 100,000 years. The wealth distributions generated converge to those produced by the finite differences method, and their variances are stable as time increases. Finally, using these distributions, welfare calculations are performed.

6.2 Parameter choice

The economy is parameterized by (α, β, γ, δ, θ, σ, ψ, v, p_{EL}, p_{LE}, τ^K, τ^L, G). Table 1 presents the parameter choices for the preferred benchmark model calibration.

Parameters	Values
Preferences	
β	0.022
γ	10
θ	1
ψ	0.75
Technology	
α	0.4
δ	0.07
Probabilities	
v	0.0067
p_{EL}	0.18
p_{LE}	0.025
Government	
τ^K	0.25
τ^L	0.35
G/GDP	0.2
Risk	
σ	0.15

Table 1. Benchmark Calibration Values.

The parameter values chosen refer to annual data from the United States. The discount rate is $\beta = 0.022$. The preference parameter is $\psi = 0.75$, which is standard in the macro literature[8]. The income share of capital is $\alpha = 0.4$. The depreciation rate is $\delta = 0.07$. The probability of death is chosen to be $v = 1/150$, a compromise between having an empirically relevant probability of death and allowing for some altruism across generations. The probability of exiting entrepreneurship is $p_{EL} = 0.18$. The probability of entering entrepreneurship is $p_{EL} = 0.025$. These two values were estimated from the PSID and SCF data, and subsequently used for calibrations, by Quadrini (2000). In Quadrini's model, as well as here, they imply a fraction of entrepreneurs in the total population of 12%[9], which is in line with the data, as Quadrini and Cagetti and DeNardi (2006) show.

[8]King, Plosser, and Rebelo (1988), and Christiano and Eichenbaum (1992).
[9]The proof can be found in the appendix.

The elasticity of intertemporal substitution is chosen to be $\theta = 1$. The empirical estimates of the elasticity of intertemporal substitution vary a lot. Using aggregate British data and correcting for aggregation bias, Attanasio and Weber (1993) estimate θ to be about 0.7. Although the exact estimates from micro data vary across studies and specifications, in most cases they are around 1, especially for agents at the top layers of wealth and asset holdings. For example, using data from the Consumer Expenditure Survey (CEX) and an Epstein-Zin specification, as in the present paper, Vissing-Jørgensen and Attanasio (2003) report baseline estimates between 1 and 1.4 for stockholders.

The proportional tax on capital income is $\tau^K = 0.25$. The Congressional Budget Office Background Paper (December 2006) reports that the average marginal rate at which corporate profits are taxed is 35%, whereas the average marginal rate at which non-corporate business income is taxed is around $26\% - 27\%$. The CRS Report for Congress (October 2003) details the capital income tax revisions and effective tax rates due to provisions granted through bonus depreciations of 30% or 50%. If these provisions are taken into account, the average marginal capital income tax is between $20\% - 25\%$ for non-corporate businesses and between $25\% - 30\%$ for corporate businesses. The value of $\tau^K = 0.25$ is chosen to be in the middle of these estimates[10]. The proportional tax on labor income is $\tau^L = 0.35$. The Congressional Budget Office Background Paper (December 2006) reports that the median effective marginal tax rate on labor income is 32%, inclusive of federal, state and payroll taxes[11]. Incorporating the distortionary effect of social security taxes would further increase this number, hence the choice made here. The level of government spending, G, is chosen so that the steady-state government-spending-to-GDP ratio is 20%.

The coefficient of relative risk aversion is chosen to be $\gamma = 10$. The empirical estimation of γ is a complicated task, because, as Vissing-Jørgensen and Attanasio (2003) detail, it requires making additional assumptions about the covariance of consumption growth with stock returns, the share of stocks in the financial wealth portfolio, the properties of the expected returns to human capital, and the share of human capital in overall wealth. Using the Consumer Expenditure Survey (CEX), Vissing-Jørgensen and Attanasio find estimates of risk aversion for stockholders in the range of $5 - 10$, but with a broader sample and under different assumptions these estimates go up to $20 - 30$. They also compare their results to Campbell (1996), who estimates γ in the range of $17 - 25$, using data on monthly and annual returns, and assuming that the entire financial portfolio is held in stocks. Dohmen et el. (2005) present evidence on the distribution of risk attitudes in the population, using survey questions and a representative sample of $22,000$ individuals living in Germany. The behavioral relevance of their survey is tested by conducting a complementary field experiment, based on a representative sample of 450 subjects. The conclusion is that the survey measure is a good predictor of actual risk-taking behavior. They find that the bulk of the mass in

[10] Altig et al. (2001) report a proportional capital income tax of 20% at the federal level, but they also subject capital income to a 3.7% state tax.

[11] This number is also reported by Jokisch and Kotlikoff (2006).

the γ-distribution is located between $1 - 10$. There is, however, a non-negligible mass of estimates in the range of higher values, up to 20. Barsky et al. (1997) measure risk aversion based on survey responses by participants in the Health and Retirement Study to hypothetical situations. These situations were constructed using an economic theorist's concept of the underlying parameter. They find that most individuals fall in the category that has mean relative risk aversion of 15.8. Cohen and Einav (2005) use a data set of 100,000 individuals' deductible choices in auto insurance contracts, to estimate the distribution of risk preferences. They find that the 82nd percentile in the distribution of the coefficient of relative risk aversion is about $13 - 15$.

The standard deviation of the idiosyncratic entrepreneurial returns is chosen to be $\sigma = 0.15$. The empirical estimation of the level of idiosyncratic risk facing an entrepreneur is a very difficult task, and has not as yet received much attention in the literature. So far, the most thorough, if not the only, attempt to measure idiosyncratic risk is by Moskowitz and Vissing-Jørgensen (2002). They document poor diversification and extreme concentration of entrepreneurial investment, significant heterogeneity in individual investment choices, and high risk at the individual level due to high bankruptcy rates. However, because of the problems arising when imputing labor income, and because of the lack of sufficient time dimension in the Survey of Consumer Finances (SCF) data, they cannot provide an accurate estimate of the volatility of entrepreneurial returns for unincorporated businesses. In the end they conjecture that the volatility of returns for private firms cannot be lower than the corresponding volatility of publicly held firms, which the find to be about 0.5 per annum. Davis et al. (2006) use the Longitudinal Business Database (LBD), which contains annual observations on employment and payroll for all establishments and firms in the private sector, to estimate the volatility of employment growth rates. They find that in 2001 the ratio of private to public volatility was in the range $1.43 - 1.75$. Given that the average annual standard deviation for public firms in the period 1990-1997 was 0.11[12], and that there is, at least in the context of the present model, a close relationship between volatility of profits and volatility of labor demand, the choice of $\sigma = 0.15$ could also be justified from this perspective. Finally, this choice generates an annual variance for steady-state consumption growth in the range indicated by the micro data, once consumer heterogeneity is taken into account[13].

Parameters γ and σ are especially important for the calibrated model, for two reasons. First, they directly influence λ, the fraction of wealth held by entrepreneurs in the economy. In light of the discussions in sections 3.3 and 4.1 about the dynamic and steady-state effects of agent heterogeneity on capital accumulation, the calibrated model's implications about λ are a good criterion of model performance. As will be shown in the next section, the choices $\gamma = 10$ and $\sigma = 0.15$, which seem empirically relevant given the discussion above, produce, without an attempt to match it, a value for λ that is not far from the values documented in the data. Second, parameters γ and σ relate to

[12] Campbell et al. (2001).

[13] Aït-Sahalia et al. (2001), and Malloy et al. (2006).

the interpretation of the capital-income tax as providing insurance. For this reason, comparative statics will also be performed to show how the tax that maximizes the steady-state capital stock varies with risk aversion and the volatility of risk. The main result, that steady-state capital is inversely U-shaped with respect to the capital-income tax, is preserved qualitatively for $\sigma \in (0, 1)$ and for $\gamma \in [1, 20]$.

6.3 Steady-state aggregates and distributions

This section examines the quantitative performance of the model in terms of aggregates and wealth distributions. This exercise is interesting for three reasons. First, it indicates how wealth inequality is influenced by the random-walk component introduced in wealth by the idiosyncratic investment risk. Second, it shows how wealth inequality depends on the excess returns to entrepreneurship, which is an interesting question in it own right, but also in view of the impact of agent heterogeneity on capital accumulation and the steady-state capital stock. Third, if the performance of the model can match relevant aspects of the data, this should give some additional confidence in the main quantitative results presented in the next section, about the effects of capital-income taxation on capital accumulation and welfare.

Table 2 presents the implications of the model for steady-state aggregates, and compares them to the data from the United States economy. The model's capital-output ratio is 2.6. Investment is 18% of GDP. The safe rate is 1.9%. The steady-state fraction of entrepreneurs, χ^{ss}, matches the data by choice of the transition probabilities, as explained in the previous section. Entrepreneurs hold 32% of total wealth in the economy, where the equivalent of λ in the data is the ratio X^E/X. This is because in the data wealth is defined as total net worth, i.e. it is financial wealth, X, as defined in the present model, plus housing. The share of total wealth held by entrepreneurs in the data ranges between 35% – 55%. The fraction of entrepreneurs in the top 10% of the population is 22% in the model, whereas in the data this number ranges between 32% – 54%[14].

	K/Y	I/Y	G/Y	R	χ^{ss}	X^E/X	χ^{ss} in top 10%
US Data	2.65	17%	20%	2%	10 – 19%	35 – 55%	32 – 54%
Model	2.6	18%	20%	2.5%	12%	32%	22%

Table 2. Steady-State Aggregates.

As mentioned earlier, the choices of γ and σ were made without any attempt to match λ to the data. This is a good indication of the model's performance, given the significance of agent heterogneity for capital accumulation.

Next, the wealth distribution generated by the model is examined. Compared to the data, the model generates a much larger fraction of agents at negative levels of wealth, most likely because of

[14]The data on entrepreneurs and wealth concentration are as reported in Cagetti and De Nardi (2006).

the absence of borrowing constraints. The model's conditional wealth distribution, however, does a better job at matching the observed data. The first two rows of Table 3 present the percentiles for wealth computed by Quadrini (2000), using the PSID and SCF samples for 1994 and 1992 respectively. The last row is the conditional wealth distribution generated by the benchmark calibrated model.

	Top Percentiles				
	30%	20%	10%	5%	1%
SCF	87.6	79.5	66.1	53.5	29.5
PSID	85.9	75.9	59.1	44.8	22.6
Model	75.85	63.13	43.77	29.16	10.12

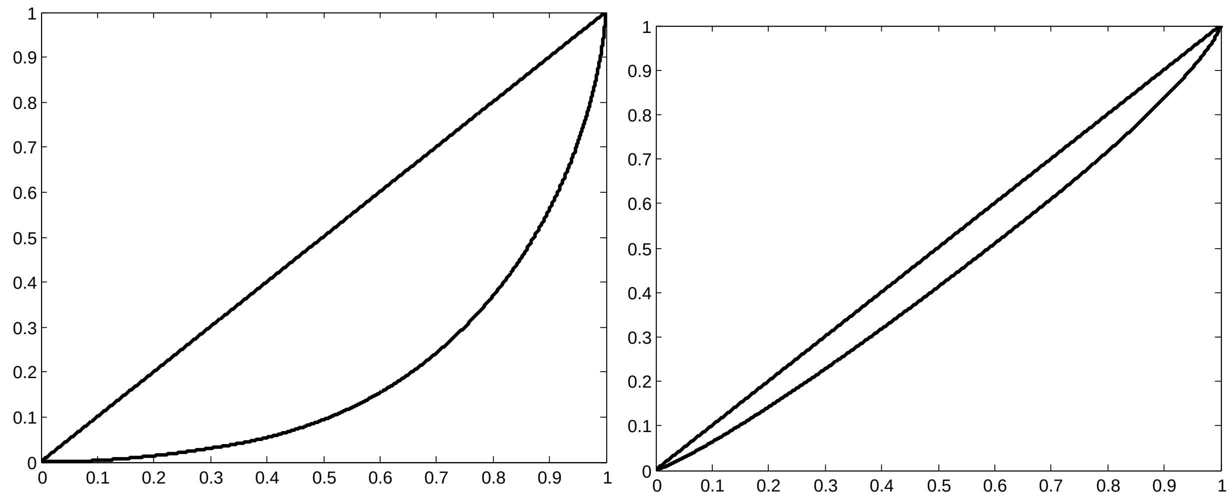
Table 3. Distribution of Wealth in the US and in the Model.

Aiyagari's (1994) benchmark calibration predictions for the wealth holdings of the top 5% and the top 1% of the population are 13.1% and 3.2% respectively. Hence the present model highlights how the random-walk component introduced in wealth by entrepreneurial risk helps generate a fatter right tail in the wealth distribution[15].

Next, Figure 1 plots the Lorenz curves for the model's wealth and consumption distributions. The model produces results in the right direction, in that the distribution of wealth over the population is much more unequal than the distribution of consumption. The model's Gini coefficient for wealth, conditional on wealth being positive, is 0.61. The model's Gini coefficient for consumption is 0.14[16]. In the data, the Gini coefficient for total net worth is 0.8, and the Gini coefficient for consumption is 0.32.

[15] A tractable extension that could improve the model's prediction about wealth concentration at the top would be to introduce a third state, in which an agent gets to be an entrepreneur operating a very high return or very low risk production function. Then the transition probabilities between the three states can be freely chosen to match desired moments of the wealth distribution. In particular, making the good entrepreneurial state the least persistent and the most likely to transition to the state of being a laborer would increase the precautionary saving, and therefore the wealth concentration, of the very rich agents.

[16] The differences in the Gini coefficients are due to the presence of human wealth: since poorer agents have higher human to financial wealth ratios, they can sustain relatively high consumption. This would not be the case in the presence of borrowing constraints.



(a) Wealth

(b) Consumption

Figure 1: *Lorenz Curves for Wealth and Consumption*

Finally, the implications of the model for the wealth distributions over entrepreneurs and laborers are presented. Figure 2 plots the conditional distributions of wealth for the two groups. On the horizontal axis is wealth normalized by mean annual income in the economy. On the vertical axis are frequencies. The solid line represents entrepreneurs, and the dashed line laborers. Consistent with the data, the distribution of wealth for the population of entrepreneurs displays a fatter tail than the one for laborers. This is due to the random-walk component that the uninsurable investment risk introduces into entrepreneurial wealth. Furthermore, the entrepreneurial wealth distribution is shifted to the right, and it has lower frequencies at lower levels of wealth. This is due to the higher mean return of the total entrepreneurial portfolio. Finally, the distributions of wealth for both groups have significant mass of people with wealth higher than fifty times mean income. In the model, the laborers at the right tail of the wealth distribution are former successful entrepreneurs.

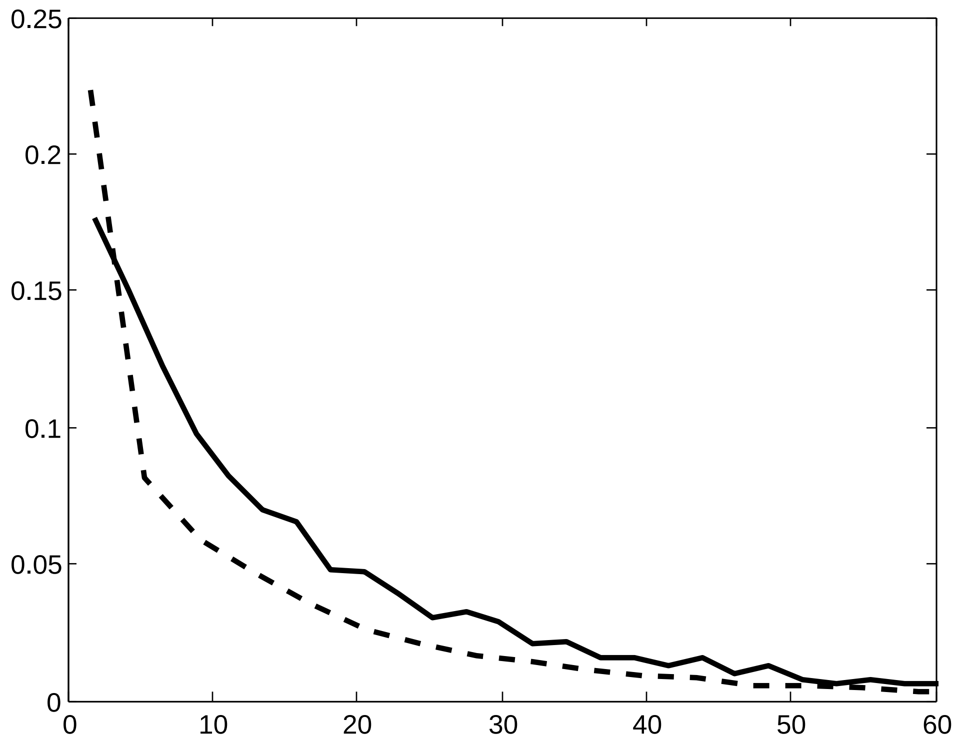
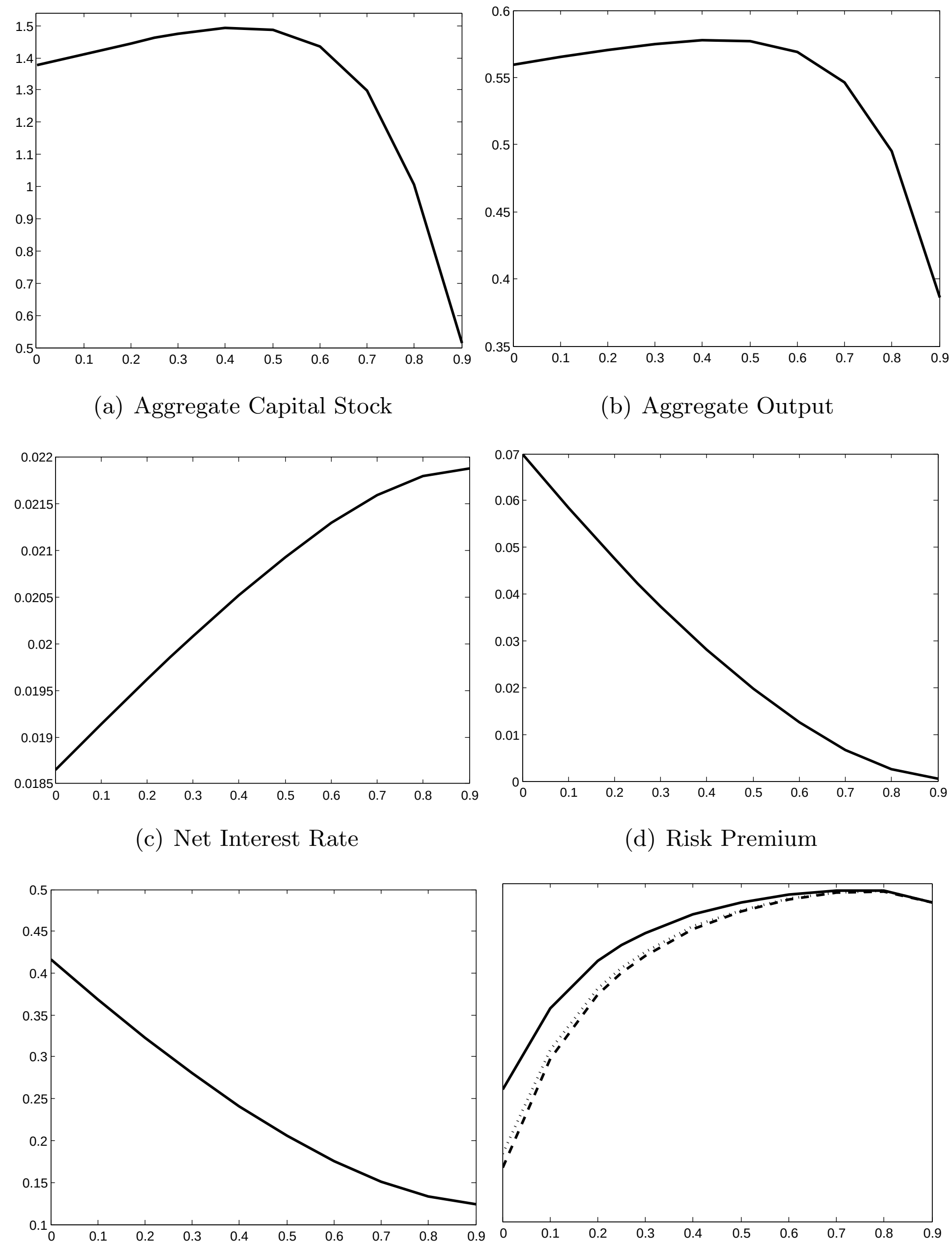


Figure 2: *Wealth Distribution for Entrepreneurs and Laborers*

7 Effects of Capital-Income Taxation

7.1 Steady state

Having gained some confidence about the overall quantitative performance of the model, this section quantifies the main theoretical result of the paper, which is that an increase in the capital-income tax increases the steady-state capital stock, when the tax is low enough. This result is due to the general-equilibrium effect of the insurance aspect of the capital-income tax, and it operates mainly through the endogenous adjustment of the interest rate.



(a) Aggregate Capital Stock (b) Aggregate Output

(c) Net Interest Rate (d) Risk Premium

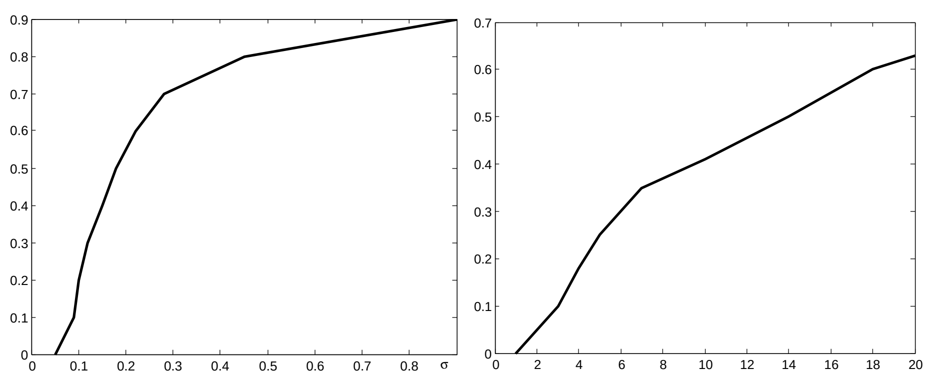
(e) Fraction of Wealth Held by E (f) Welfare

Figure 3: *Steady State and Capital-Income Taxation*

Figure 3 shows the behavior of the steady-state aggregates, and of welfare, with respect to the capital-income tax, for the benchmark calibrated version of the model. Output (panel (a)), capital (panel (b)), employment, the capital-labor (capital per work-hour) ratio, and output per work-hour are all inversely-U shaped with respect to the capital-income tax, and they reach a maximum when $\tau^K = 0.4$. At this point, steady-state output is 3.3% higher than when $\tau^K = 0$, capital per work-hour is 8.3% higher, and output per work-hour is 3.2% higher. At $\tau^K = 0.4$, the capital-labor ratio and output per work-hour under complete markets are 21% and 9% lower than when $\tau^K = 0$. So for output per work-hour there is a 12% difference between complete and incomplete markets. As shown in Figure 3(f), aggregate welfare is maximized at $\tau^K = 0.7$, whether for entrepreneurs (solid line), laborers (dashed line), or the economy as a whole (dotted line). Naturally, entrepreneur welfare is higher than laborer welfare for all tax levels[17].

When the capital-income tax increases, the effective volatility of risk facing an entrepreneur, $\sigma(1-\tau^K)$, decreases. This reduces the demand for precautionary saving, and it therefore increases the interest rate. Figure 3(c) shows that the (net) interest rate tends to the discount rate, $\beta = 0.022$, as $\tau^K \to 1$. Figure 3(d) reinforces this interpretation of the capital-income tax as providing insurance: when the tax increases, the precautionary saving motive becomes weaker, and therefore entrepreneurs are satisfied with a lower risk premium. Figure 3(e) shows that the fraction of wealth held by entrepreneurs in the economy is decreasing in the capital-income tax. This result comes from the combination of the weaker precautionary saving motive, the fall in the risk premium, and the increase in the cost of borrowing due to the increase in the interest rate.

Figure 4 presents robustness checks with respect to volatility, σ, and risk aversion, γ. On the vertical axis is the tax that maximizes the steady-state capital stock. When either the volatility of risk increases or risk aversion increases, the tax that maximizes the steady-state capital stock increases. These comparative statics reinforce the insurance interpretation of the tax system. They also indicate that the main result of the paper is robust to the wide range of empirically plausible values of $\sigma \in (0, 1)$ and of $\gamma \in [1, 20]$. In particular, for the low value of $\sigma = 0.15$, the capital-income tax that maximizes the steady-state capital stock is positive for all $\gamma > 1$.



(a) Robustness Check: Volatility of Risk (b) Robustness Check: Risk Aversion

Figure 4: *Robustness Checks*

[17] Entrepreneur welfare is also higher than laborer welfare for all levels of wealth, since entrepreneurs are unconstrained in their investment choices.

7.2 Dynamics of eliminating the capital-income tax

The paper now proceeds to examine the aggregate and welfare implications of eliminating the capital-income tax. In the standard complete-markets neoclassical growth model, the optimal capital-income tax is zero in the long run, as well as in most of the short run for an interesting class of preferences. Steady-state welfare is also decreasing in the level of the capital-income tax. These findings have initiated an extensive debate as to the possible benefits of eliminating the tax on capital income. By contrast, the main result of the present paper is that an increase in the capital-income tax may actually increase the steady-state capital stock. In light of this result, it is worthwhile to revisit the discussion on the implications of setting the capital-income tax to zero.

The effects on aggregates and welfare when the capital-income tax is eliminated will be examined from two perspectives. On the one hand, one might be interested in examining the welfare of the current generation immediately after the policy reform, taking into account the entire transitional dynamics of the economy towards the new steady state with the zero tax. On the other hand, one might be interested in examining the welfare of the generations that will be alive in the distant future, i.e. at the new steady state. For convenience, the first exercise will be referred to as the study of the short-run implications of eliminating the capital-income tax, whereas the second exercise will be referred to as the study of the long-run implications of eliminating the capital-income tax.

The present model can in fact examine the short-run implications of policy reforms, because it is very tractable, compared to other incomplete-markets models, where the entire wealth distribution is a relevant state variable. Here only the mean of the wealth distribution is relevant for aggregate dynamics, which constitutes a significant gain in tractability, and allows for the entire dynamic response of the economy, after a policy change, to be considered. This is important, because it has long been recognized that the short-run effects of policy may well be very different from the long-run effects.

Here the economy starts from the steady state described by the benchmark calibration parameters, where the capital-income tax is $\tau^K = 0.25$. Subsequently, the tax is set to zero, ceteris paribus.

7.2.1 Aggregate effects

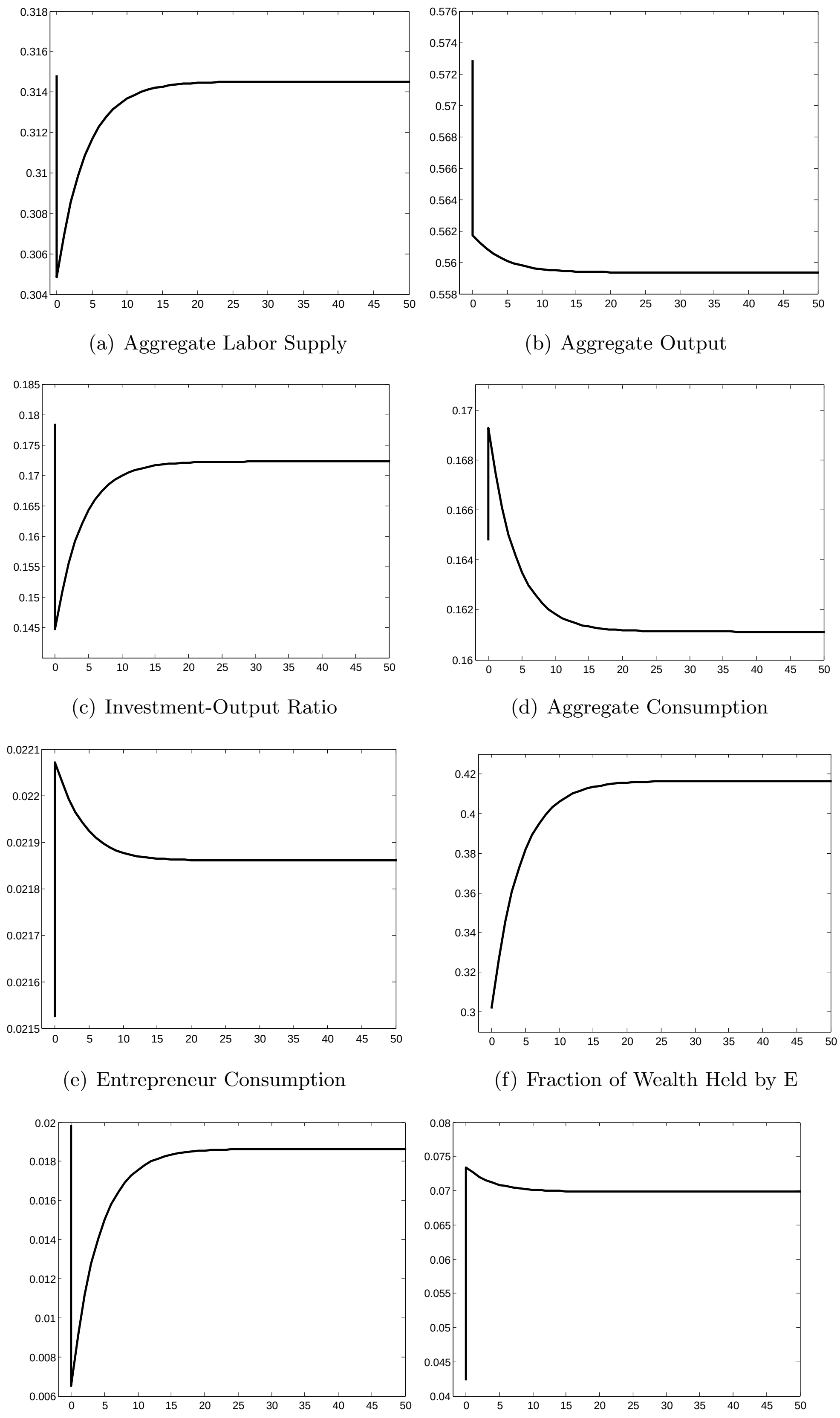
This section presents the short-run and long-run responses of the aggregate variables to the policy reform that eliminates the capital-income tax. The results are compared to the case of complete markets[18]. Table 4 shows the response of the aggregates on the impact of the policy reform, as well as at the new steady state, under both complete markets and the present model of incomplete markets. The effects on the interest rate, R, the risk premium, μ, and the investment-output ratio, I/Y, are in percentage units. The rest of the numbers denote percentage changes.

[18]The complete-markets calibration uses the relevant parameter values from the benchmark Table 1.

	Short Run		**Long Run**	
	Incomplete	Complete	Incomplete	Complete
L	−3.17	8.35	−0.1	1.27
Y	−1.95	5.001	−2.36	6.96
C	2.74	−5.12	−2.13	5.27
I/Y	−3.39	5.96	−0.63	2.34
R_{net}	−1.33	1.18	−0.12	0
μ	3.11	0	2.75	0
CE	2.54	NA	1.56	NA
X^E/X	0	NA	11.49	NA

Table 4. Dynamics of Eliminating the Capital-Income Tax.

Under complete markets, a permanent (unanticipated) tax cut leads to an immediate negative jump in consumption and an immediate positive jump in investment. Capital slowly increases and converges to a higher steady-state value, whereas consumption is initially lower and increases over time. In other words, the long-run increase in investment requires an initial period of lower consumption, which in turn allows for a short-run increase in investment as well. By contrast, under incomplete markets, the exact opposite is the case. In light of the main mechanism of the paper, investment decreases in the long run. This allows for a short-run increase in consumption, and therefore necessitates a short-run fall in investment. In particular, the investment-output ratio falls by more than 3 percentage units. Figures 5 and 6 plot the impulse responses of the variables when the capital-income tax is eliminated, under incomplete and complete markets respectively.



(a) Aggregate Labor Supply

(b) Aggregate Output

(c) Investment-Output Ratio

(d) Aggregate Consumption

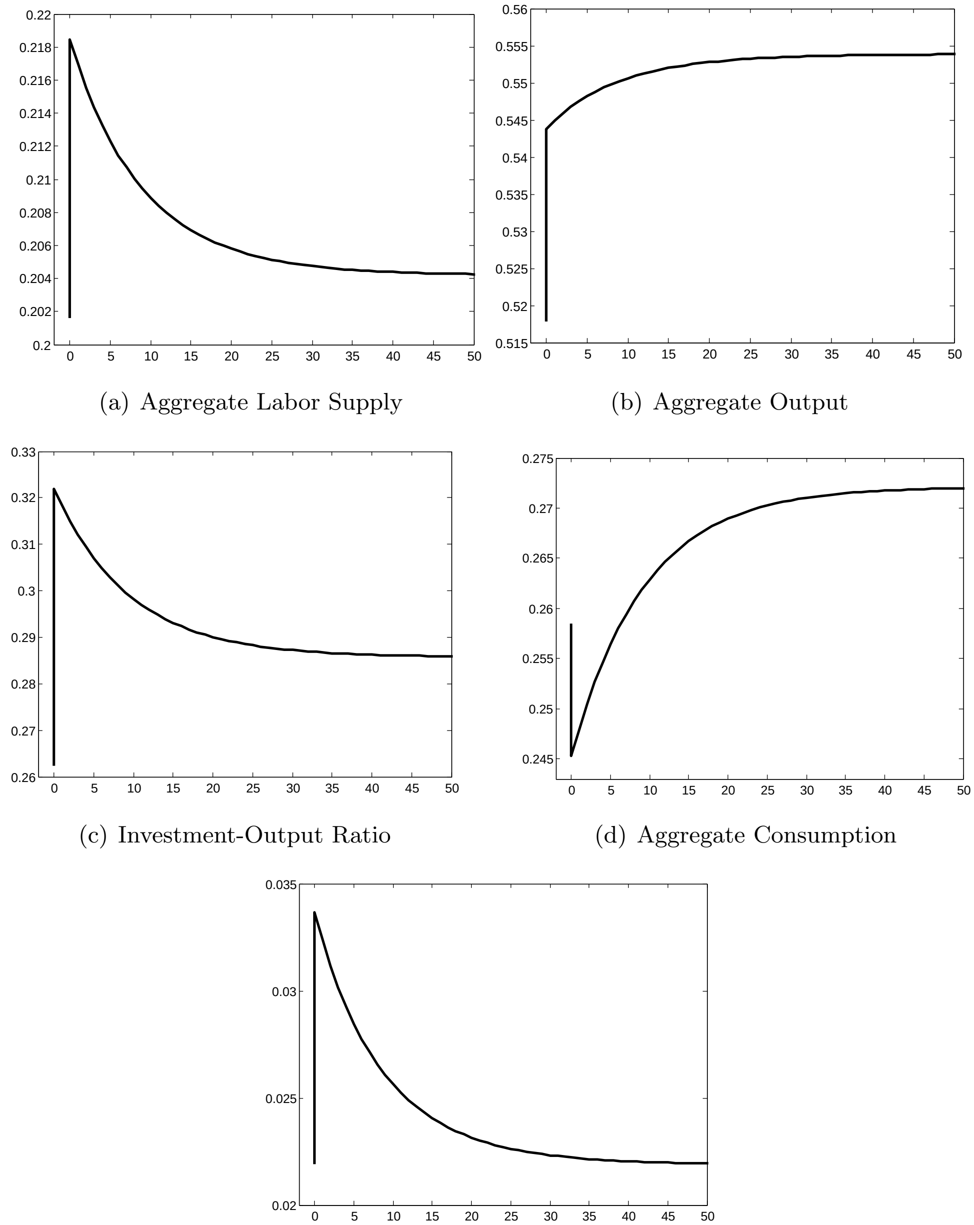
(e) Entrepreneur Consumption

(f) Fraction of Wealth Held by E

(g) Net Interest Rate

(h) Risk Premium

Figure 5: *Dynamics of Incomplete Markets: Eliminating the Capital-Income Tax*



(a) Aggregate Labor Supply

(b) Aggregate Output

(c) Investment-Output Ratio

(d) Aggregate Consumption

(e) Net Interest Rate

Figure 6: *Dynamics of Complete Markets: Eliminating the Capital-Income Tax*

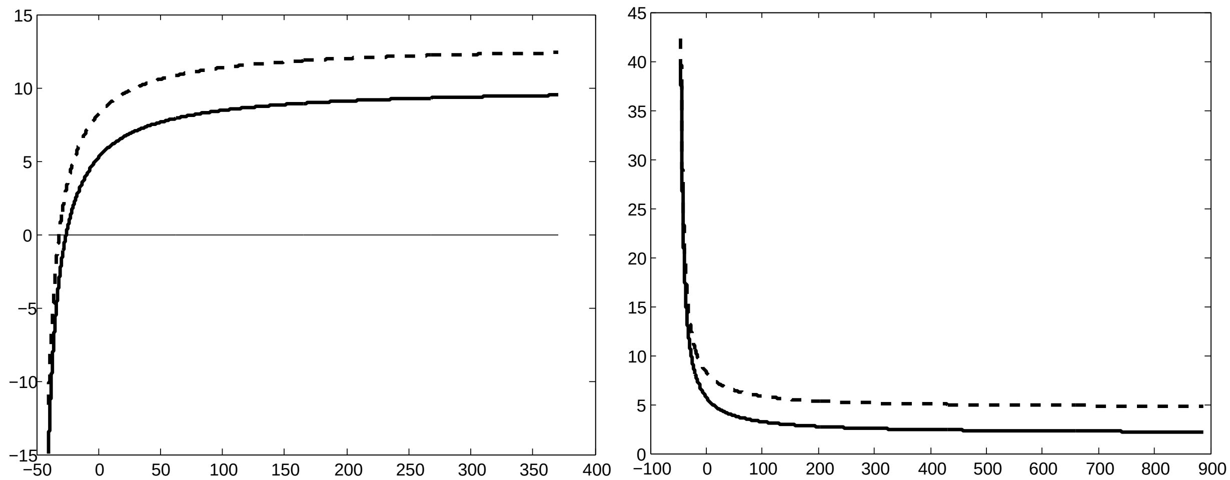
7.2.2 Welfare effects

This section studies the welfare implications of eliminating the capital-income tax. These implications are represented in terms of a compensating differential for each level of wealth and each type of agent, whether entrepreneur or laborer. I.e., starting from the old regime with $\tau^K = 0.25$, the question is what fraction of his financial wealth would an agent be willing to give up in order to avoid the impact of the new regime initiated by the policy change. Depending on the perspective, the new regime is either the short run, when capital is not allowed to adjust and when the welfare of the current generation is examined, or the long run, when the comparison is between steady states, after capital has adjusted and new generations have entered the economy.

Figure 7(a) presents the welfare implications for entrepreneurs (solid line) and laborers (dashed line) in the short run, i.e. at the impact of the policy change. These short-run welfare effects have taken into account the entire transitional dynamics of the economy towards the new steady-state. Financial wealth normalized by annual mean income is on the horizontal axis, and the compensating differentials are on the vertical axis. A negative number on the vertical axis indicates that an agent would have to be paid to be indifferent between the old regime and the regime initiated by the impact of the policy change, and hence he prefers the new regime with the zero capital-income tax. Table 5 presents the short-run mean welfare effects over percentiles for entrepreneurs and laborers. The numbers are in percentages.

	Entrepreneurs	**Laborers**
bottom 1%	−1.4	−0.007
bottom 5%	−2.2	−0.001
bottom 10%	0.004	0.03
1st quintile	8.7	11.93
2nd quintile	20.86	20.62
3rd quintile	22.77	21.98
4th quintile	23.59	22.57
5th quintile	24.06	22.91
top 30%	35.92	34.24
top 10%	12.01	11.48
top 5%	6.04	5.75
top 1%	1.21	1.15

Table 5. Short-Run Welfare Implications of Eliminating the Capital-Income Tax.



(a) Short-Run Welfare Implications

(b) Long-Run Welfare Implications

Figure 7: *Short-Run vs Long-Run Welfare Implications in Cross-Section*

Figure 7(a) and Table 5 show that in the short run, poor agents, whether entrepreneurs or laborers, prefer the zero capital-income tax regime. As wealth increases, both entrepreneurs and laborers prefer the positive capital-income tax regime. Finally, the mean cost of the tax cut is higher for the middle-class agents than for the very rich. These cross-sectional differences can be explained by referring to the first row of Figure 8, which plots the short-run response of human wealth and of the (risk-adjusted) returns to saving for laborers and entrepreneurs respectively, against the tax rate of the policy reform. In the short run, the decrease in the capital-income tax from $\tau^K = 0.25$ to $\tau^K = 0$ increases the demand for precautionary saving, and therefore leads to a fall in the interest rate[19]. Since the capital stock is historically given and cannot change, the fall in the interest rate increases human wealth. For poor agents, whether entrepreneurs or laborers, human wealth constitutes a significant part of their total wealth, and hence they benefit a lot from the elimination of the tax. Furthermore, for poor agents the fall in the saving returns does not carry as much weight as the increase in their human wealth. Finally, poor agents do not benefit much from insurance directly, since they invest little or nothing in the risky asset. Therefore, in the short run, poor agents prefer the zero capital-income tax regime, because the elimination of the tax increases their safe income, and safe income is a big part of their total wealth.

[19]In the short run, the net interest rate may actually fall if the tax of the reform is very high. This possibility, which does not emerge in the long run, is due to the usual distortionary effect of big tax increases on investment.

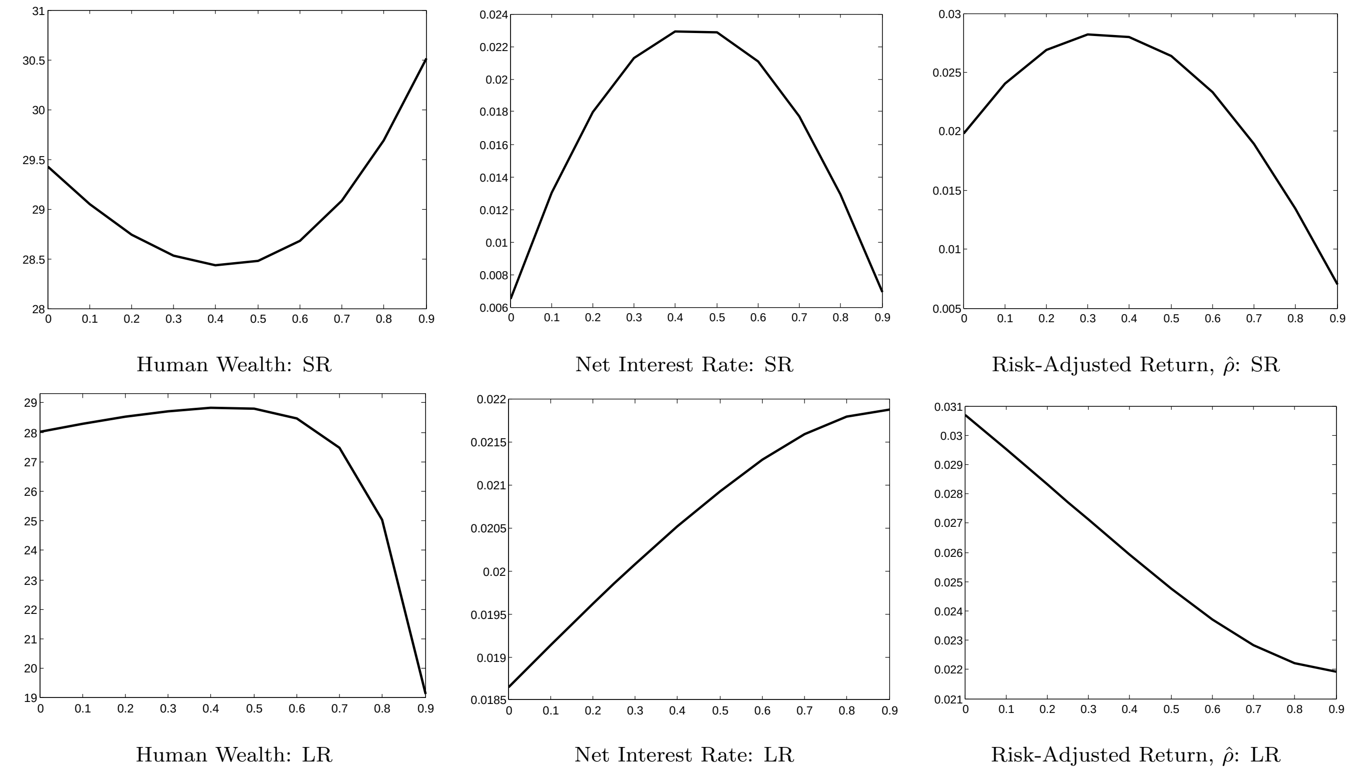


Figure 8. *Short Run (SR) vs Long Run (LR): Human Wealth and Saving Returns*

Figure 7(b) presents the welfare implications for entrepreneurs (solid line) and laborers (dashed line) in the long run, i.e. comparing across steady states. Clearly these long-run effects are different from the short-run effects. In the long run, both types of agents and at all wealth levels prefer the steady state with the positive tax, the rich less so than the poor, and the entrepreneurs less so than the laborers. These cross-sectional differences can be explained by referring to the second row of Figure 8, which plots the long-run response of human wealth and of the (risk-adjusted) returns to saving for laborers and entrepreneurs respectively, against the tax rate of the policy reform. In the long run, the decrease in the capital-income tax from $\tau^K = 0.25$ to $\tau^K = 0$ increases the demand for precautionary saving, and it therefore leads to a fall in the interest rate. This effect is as in the short run. However, in the long run, the general-equilibrium implications of the interest rate adjustment for capital accumulation become relevant. In particular, the fall in the interest rate reduces stead-state wealth and capital accumulation. The fall in the steady-state capital stock dominates the fall in the interest rate, so that steady-state human wealth falls when the capital-income tax is eliminated. This adversely affects poor agents of all types, since human wealth represents a big part of their total wealth. Because the risk-adjusted return for entrepreneurs, $\hat{\rho}$ increases when the capital-income tax is eliminated, the cost of the policy change is not as high for an entrepreneur as it is for a laborer at any given level of wealth.

In conclusion, the elimination of the capital-income tax has welfare implications that differ across time and in the cross-section of the population. These differences are due to the general-equilibrium effects of the interest rate adjustment on capital accumulation. In particular, they operate mainly through the different response of human wealth: in the short run, when the capital stock cannot adjust, human wealth increases after the elimination of the capital-income tax, whereas in the long run, when capital accumulation changes endogenously, human wealth falls. Therefore, poor agents prefer a zero capital-income tax in the short run, but a positive capital-income tax in the long run. Rich agents always prefer a positive tax, but less forcefully in the long-run, because in the long run the elimination of the tax increases the mean entrepreneurial portfolio return[20].

8 Extension: Introducing Publicly Traded Sector

So far it has been assumed that all investment is subject to uninsurable idiosyncratic risk. This might not be an appropriate assumption for a country like the United States, where private equity actually accounts for about 50% of total financial wealth. To this end a second sector of production is formally introduced. In this sector, all firms are publicly traded, and it is assumed that they can perfectly diversify away all idiosyncratic risks[21]. The mean return to capital in the public sector is lower than in the private sector, otherwise no entrepreneur would invest in the private sector. Both entrepreneurs and laborers can invest in the public sector. Public sector capital is taxed at the rate tau_t^K, and public sector labor is taxed at the rate τ_t^L. In equilibrium, the marginal product of capital in the public sector is equal to the risk-free rate. The rest of the equilibrium characterization proceeds as in the benchmark model, with bond holdings now replaced by the sum of bond and public equity holdings.

In the public sector, where there is no scope for insurance, an increase in the capital-income tax unambiguously reduces investment, so that public capital is a negative function of the tax. As a result, the overall effect of the tax on the aggregate capital stock is in general ambiguous. In addition though, the increase in the capital tax might now trigger a reallocation of resources away from the low-risk low-productivity public sector towards the higher-risk higher-productivity private sector, thus increasing total factor productivity. As a result, aggregate output may increase with the tax, even if aggregate capital falls.

The model with the public sector is slightly recalibrated, so as to match the US economy aggregates. In steady state, private capital is maximized for $\tau^K = 0.5$, whereas public capital falls all the way with the tax. Overall, the aggregate capital stock is falling with the tax, but less so

[20]Under complete markets, and starting from the steady state with $\tau_K = 0.25$, the average long-run welfare gain (in terms of consumption equivalent) of eliminating the capital-income tax is 1.7%, whereas the average short-run welfare gain is 0.6%.

[21]This is an extreme assumption made here for analytical convenience. In fact, the data indicates that public firms do not have a perfectly diversified shareholder base. Himmelberg et al. (2002), using the Worldscope database for panel of public firms across 38 countries, find that the median inside equity ownership share is 40%.

than under complete markets. As a result, when $\tau^K = 0.4$, output per work-hour is about 3% lower than it would have been had the tax rate been zero, whereas under complete markets it is about 9% lower. Furthermore, total factor productivity increases by 7%. Steady-state welfare is maximized for $\tau^K = 0.6$, which is lower than in the benchmark model.

To summarize, in steady state and for $\tau^K \simeq 40\%$, output per work-hour is 12% higher than under complete markets when all production takes place in the private sector, and it is 6% higher than under complete markets when the private sector accounts for 50% of financial assets.

9 Conclusions

This paper studies the aggregate and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic entrepreneurial risk. The surprising result emerging is that a positive tax on capital income may actually stimulate steady-state capital accumulation: for empirically plausible calibrated versions of the model, the steady-state levels of the capital stock, output and employment are all maximized for a positive value of the capital-income tax. For the preferred benchmark calibration, when the tax on capital is 25%, the capital stock is 2.5% higher than what it would have been had the tax rate been zero. This result stands in stark contrast to the effect of capital-income taxation in either complete-markets models, or in Bewley-type incomplete-markets models: in these models, capital-income taxation necessarily discourages capital accumulation. The result of the present paper is due to the endogenous general-equilibrium adjustment of the interest rate and of wealth in the long run.

Although the present paper provides some useful guidance about the direction of optimal policy, it does not solve for the fully optimal policy. An interesting direction for future research is the formal study of optimal policy, either in the Ramsey tradition (though allowing for lump-sum taxes, as in the present model), or in the Mirrlees tradition of endogenizing the source of market incompleteness and having no ad hoc restrictions placed on the set of available instruments.

The present model focuses on the effects of uninsurable entrepreneurial risk, and abstracts from labor-income risk, borrowing constraints, and decreasing returns to scale at the individual level. Extending the model to include these relevant aspects of the data and revisiting the effects of capital taxation in this richer setting is important, not only to get a better quantitative evaluation of the implications of capital taxation, but also to examine whether the general-equilibrium effects identified here might interact with other sources of market incompleteness in an interesting way. For example, after an increase in the capital-income tax, the increase in steady-state wealth documented here could make borrowing constraints less binding. At the same time, the increase in the steady-state interest rate could also increase the cost of borrowing. Further investigating these rich general-equilibrium interactions will greatly facilitate a better theoretical and quantitative assessment of the implications of fiscal policy in dynamic heterogeneous-agent environments.

10 Appendix: Proofs

Lemma 1. *Let preferences be described by:*

$$J_t = \{(1-e^{-\beta\Delta t})(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta} + e^{-\beta\Delta t}(E_t[J_{t+\Delta t}^{1-\gamma}])^{\frac{1-1/\theta}{1-\gamma}}\}^{\frac{1}{1-1/\theta}} ,$$

where c is consumption and n is leisure. Then, given the processes for c and n, the utility process is defined as the solution to the following integral equation:

$$U_t = E_t \int_t^{\infty} z(c_s, U_s)\, ds \,, \tag{30}$$

where

$$z(c,U) \equiv \frac{\beta}{1-1/\theta}\left[\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{((1-\gamma)U)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U\right]. \tag{31}$$

Proof of Lemma 1. Define the functions

$$g(x) = \frac{((1-\gamma)x)^{\frac{1-1/\theta}{1-\gamma}}}{1-1/\theta}$$

$$U_t = \frac{J_t^{1-\gamma}}{1-\gamma}\,.$$

Then:

$$g(U_t) = \frac{J_t^{1-1/\theta}}{1-1/\theta} = (1-e^{-\beta\Delta t})\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{1-1/\theta} + e^{-\beta\Delta t} g(E_t[U_{t+\Delta t}])\,.$$

Take a first order Taylor expansion in Δt :

$$g(U_t) = g(U_t) + \beta\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{1-1/\theta}\Delta t - \beta g(U_t)\Delta t + g'(U_t)E_t[\Delta U_t]\,.$$

Then

$$E_t[\Delta U_t] = -\frac{\beta\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{1-1/\theta} - \beta g(U_t)}{g'(U_t)}\Delta t\,,$$

where

$$\frac{g(U_t)}{g'(U_t)} = \frac{(1-\gamma)U_t}{(1-1/\theta)}\,.$$

Hence

$$E_t[\Delta U_t] = -z(c_t, n_t, U_t)\Delta t\,,$$

where

$$z(c_t, n_t, U_t) \equiv \frac{\beta}{1-1/\theta}[\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{((1-\gamma)U_t)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U_t]\,.$$

For a more general proof of the above and for a proof of existence and uniqueness of the solution to the integral equation (30) see Duffie and Epstein (1992).

Proof of Proposition 1. Because of the CRRA/CEIS specification of preferences, guess that the value function for an entrepreneur is

$$J(w^E, t) = B_t^E \, \frac{w^{E\,1-\gamma}}{1-\gamma} \, ,$$

where the term B_t^E captures the time dimension. The Bellman equation for an entrepreneur is

$$0 = \max_{c^E, n^E, \phi} \; z(c^E, n^E, J^E(w^E,t)) + \frac{\partial J^E}{\partial w^E}(w^E,t)[(\phi(1-\tau_t^K)r_t + (1-\phi)(1-\tau_t^K)R_t)w^E - c^E - (1-\tau_t^L)\omega_t n^E]$$

$$+ \frac{\partial J^E}{\partial t}(w^E,t) \; + \frac{1}{2}\frac{\partial^2 J^E}{\partial w^{E\,2}}(w^E,t)\sigma^2(1-\tau_t^K)^2\phi^2 w^2 \; + \; p_{EL}[J(w^L,t) - J(w^E,t)] \, ,$$

where the function z is given by (31), and where the last term shows that the entrepreneur might switch into being a worker with probability p_{EL}. Because of the homogeneity of J^E in w^E, the marginal propensity to consume and the portfolio choice will be the same for all entrepreneurs. The first order condition for the optimal portfolio allocation gives the condition for ϕ_t in (16). Combining the first order conditions for consumption and leisure we get the optimal leisure choice:

$$n_t^i = (\psi/(1-\psi))(1/(1-\tau_t^L)\omega_t)/c_t^i \, . \tag{32}$$

From the envelope condition we get

$$m^E \equiv B^{E\,\frac{1-\theta}{1-\gamma}}\left(\frac{\psi}{1-\psi}\frac{1}{(1-\tau^L)\omega}\right)^{-\psi(1-\theta)}(1-\psi)^\theta \beta^\theta \, .$$

Similarly, guess that the value function for a laborer is

$$J(w^L, t) = B_t^L \frac{w^{L\,1-\gamma}}{1-\gamma} \, ,$$

The Bellman equation for a laborer is

$$0 = \max_{c^L, n^L} \quad z(c^L, n^L, J^L(w^L,t)) \; + \frac{\partial J^L}{\partial w^L}(w^L,t)[R_t w^L - c^L - (1-\tau_t^L)\omega_t n^L]$$

$$+ \frac{\partial J^L}{\partial t}(w^L,t) \; + \; p_{LE}[J(w^E,t) - J(w^L,t)] \, .$$

The consumption and leisure choices for the laborer are made in the exact same way as for the entrepreneur, and from the envelope condition we get:

$$m^L \equiv B^{L\frac{1-\theta}{1-\gamma}}(\frac{\psi}{1-\psi}\frac{1}{(1-\tau^L)\omega})^{-\psi(1-\theta)}(1-\psi)^\theta\beta^\theta\,.$$

It follows that

$$\frac{B^E}{B^L} = (\frac{m^E}{m^L})^{\frac{1-\gamma}{1-\theta}}\,.$$

Using this, the first order conditions, the envelope conditions, and plugging back into the Bellman equation we get (18) and (19).

Proof of Proposition 2. Let $\tilde{R}_t$ be the effective risk-free rate. The human wealth for each individual $i = E, L$ in the economy is $h_t^i = \int_t^\infty e^{-\int_t^s \tilde{R}_j dj}((1-\tau_s^L)\omega_s + T_s)ds$. The human wealth of the measure-χ_t group of entrepreneurs is $H_t^E = \chi_t \int_t^\infty e^{-\int_t^s \tilde{R}_j dj}((1-\tau_s^L)\omega_s + T_s)ds$, and the human wealth of the measure-$(1-\chi_t)$ group of laborers is $H_t^L = (1-\chi_t)\int_t^\infty e^{-\int_t^s \tilde{R}_j dj}((1-\tau_s^L)\omega_s + T_s)ds$. Hence total human wealth is $H_t = H_t^E + H_t^L = \int_t^\infty e^{-\int_t^s \tilde{R}_j dj}((1-\tau_s^L)\omega_s + T_s)ds = h_t^i$. Using the Leibniz rule, and substituting in from the government budget constraint (7), we get that the evolution of total human wealth is described by (23). Since only entrepreneurs invest in capital, the aggregate capital stock in the economy is given by $K_t = \phi_t W_t^E$. For an agent in the E and L group respectively $b_t^E + h_t^E = (1-\phi_t)w_t^E$ and $b_t^L + h_t^L = w_t^L$. Aggregating over each group we get $B_t^E + \chi_t H_t = (1-\phi_t)W_t^E$ and $B_t^L + (1-\chi_t)H_t = W^L$. Adding up and using the fact that $B_t^E + B_t^L = 0$ we get $H_t = (1-\phi_t)W_t^E + W_t^L$. Now using $W_t = W_t^E + W_t^L$ and $K_t = \phi_t W_t^E$ we get $W_t = K_t + H_t$. Combining $H_t = (1-\phi_t)W_t^E + W_t^L$, $K_t = \phi_t W_t^E$, and $\lambda_t = W_t^E/W_t$ we get (24). Aggregating across leisure choices we get $\frac{\psi}{1-\psi}\frac{1}{(1-\tau_t^L)\omega_t}C_t + L_t = 1$, where $C_t = m_t^E W_t^E + m_t^L W_t^L$, $W_t^L = W_t - W_t^E$, and $W_t = K_t + H_t$. Aggregating across (12) and (13), and adding up, using $B_t^E + B_t^L = 0$, $H_t = H_t^E + H_t^L$, and labor market clearing, we get:

$$W_t = [(1-\tau_t^K)r_t K_t + (1-\tau_t^K)R_t H_t - \frac{1}{1-\psi}C_t]dt\,.$$

Using $H_t = (1-\phi_t)W_t^E + W_t^L$, $K_t = \phi_t W_t^E$, $\mu_t = (1-\tau_t^K)r_t - (1-\tau_t^K)R_t$, and $C_t = m_t^E W_t^E + m_t^L W_t^L$, and dividing through with W_t we get

$$\frac{\dot{W}_t}{W_t} = (1-\tau_t^K)r_t\phi_t\mu_t + (1-\tau_t^K)R_t - \frac{1}{1-\psi}(\lambda_t m_t^E + (1-\lambda_t)m_t^L)\,,$$

which gives (21) when we use $\rho_t = \phi_t\mu_t + (1-\tau_t^K)R_t$. Aggregating across (12), and subtracting from (21), we get (22).

Proof of Proposition 3. The proof starts for the case of exogenous labor and $\theta = 1$. In steady state $\dot{W}/W = 0$, which from (21) yields $\lambda = (\beta - (1-\tau^K)R)/(\phi\mu)$. Combining this with (22) in steady state gives (25), which verifies that $\lambda < 1$. Differentiating with respect to R we get that $\lambda'(R) < 0$. Plugging this back into $\dot{W}/W = 0$ we get

$$\phi\mu = \frac{(\beta - (1-\tau^K)R)(\beta - (1-\tau^K)R + p_{LE} + p_{EL})}{(\beta - (1-\tau^K)R + p_{LE})},$$

from which we get (26) if we use the definition of μ. Differentiating this with respect to R we get that $\mu'(R) < 0$ and $\phi'(R) < 0$ in steady state. Finally, combining $\dot{H} = 0$ from (23) and bond market clearing (24) we get (27).

For uniqueness notice first that we can write (26) as

$$\mu(R) = \sqrt{\frac{1}{\lambda(R)}\gamma\sigma^2(1-\tau^K)^2(\beta - (1-\tau^K)R)} \quad ,$$

from which we get

$$K(R) = (\frac{\mu(R) + (1-\tau^K)R}{\alpha(1-\tau^K)})^{\frac{1}{\alpha-1}}.$$

Define the ratio of the net foreign asset position to domestic capital for an open economy that faces an exogenously given interest rate:

$$D(R) \equiv \frac{(1-g\tau^L)(1-\alpha)K(R)^{\alpha-1} + (1-g)\tau^K f_K(R)}{((1-\tau^K)R + v)} - \frac{1}{\phi(R)\lambda(R)} + 1.$$

Assume $\tau^K \simeq 0$ for simplicity. For existence and uniqueness of the steady state for the closed economy it suffices to prove that there exists a unique R solving $D(R) = 0$, where $R \in (-v, \beta)$. From (25) we have $\lambda(-v) = (\beta + v + p_{LE})/(\beta + v + p_{LE} + p_{EL})$ finite positive, hence so is $\mu(-v)$, $\phi(-v)$, and also $K(-v)$ as long as $\mu(-v) \geq v$. Next, $\lambda(\beta)$ is finite positive, so $\mu(\beta) = 0$, clearly $K(\beta)$ is the capital stock under complete markets, which is finite and positive, and $f_K(\beta) = R = \beta$ when markets are complete. Finally, note that $1 - g\tau^L > 0$. Then

$$\lim_{R\to -v^+} D(R) = [(1-g\tau^L)(1-\alpha)K(-v)^{\alpha-1} + (1-g)\tau^K f_K(-v)] \lim_{R\to -v^+} \frac{1}{R+v} - \frac{1}{\phi(-v)\lambda(-v)} + 1 = +\infty.$$

$$\lim_{R\to\beta^-} D(R) = [(1-g\tau^L)(1-\alpha)K(\beta)^{\alpha-1} + (1-g)\tau^K\beta]\frac{1}{\beta+v} - \frac{1}{\lambda(\beta)}\lim_{R\to\beta^-}\frac{1}{\phi(R)} + 1 = -\infty.$$

Hence the steady state always exists.

When labor is endogenous and $\theta = 1$, then $m^E = m^L = (1-\psi)\beta$, and the proofs above carry through the same way, with $f_K(K/L)$, and $\omega(K/L)$. So for characterization of the steady state we need to add the labor market clearing condition, and the steady state system will be in K, L, R. In particular, labor market clearing, combined with $C = (1-\psi)\beta W$, $\lambda = W^E/W$ and $W^E = K/\phi$ gives

$$L = (\frac{\psi\beta}{(1-\tau^L)\omega(K,L)}\frac{1}{\lambda(R)}\frac{K/L}{\phi(K,L,R)} + 1)^{-1}.$$

Finally, when labor is endogenous and $\theta \neq 1$ then

$$\lambda = \frac{\frac{1}{1-\psi} m^L - (1-\tau^K)R + p_{LE}}{\frac{1}{1-\psi} m^L - (1-\tau^K)R + p_{LE} + p_{EL}},$$

and

$$\mu = \sqrt{\frac{\gamma\sigma^2(1-\tau)^2}{\lambda}(\frac{1}{1-\psi}(m^E\lambda + m^L(1-\lambda)) - (1-\tau^K)R)} \; .$$

Here we need to add two more equations to characterize the steady state, namely the Euler conditions for the marginal propensities to consume. This will be a system of two equations in two unknowns to be solved as a function of steady state prices. The conditions needed for establishing that $\lambda > 0$ are satisfied in simulations.

Proof of Proposition 4. Let d_t be the indicator function, where $d_t = 1$ for entrepreneurs and $d_t = 0$ for laborers. The dynamic system for the state vector (ξ_t^i, d_t) is:

$$\begin{aligned} \dot{\xi}_t^i &= \mu(\xi_t^i, d_t) + \sigma(\xi_t^i, d_t)\, dz_t^i - (\xi_t^i - 1)N_t^1 \\ \dot{d}_t &= s(d_t)\, dN_t^2 \,, \end{aligned}$$

where dN_t^1 is the Poisson process denoting death with arrival rate vdt, and where dN_t^2 is the Poisson switching process with arrival intensity $p(I)\, dt$:

$$\begin{aligned} p(d) &= p_{LE} \quad \text{if } d = 0 \\ p(d) &= p_{EL} \quad \text{if } d = 1 \,, \end{aligned}$$

where:

$$\begin{aligned} s(d) &= 1 \quad\quad \text{if } d = 0 \\ s(d) &= -1 \quad \text{if } d = 1 \,, \end{aligned}$$

and:

$$\begin{aligned} \mu(\xi_t, 1) &= [\frac{1}{1-\psi}(\bar{m}_t - m_t^E) + \phi_t(1-\lambda_t)((1-\tau_t^K)r_t - (1-\tau_t^K)R_t)]\xi_t \\ \mu(\xi_t, 0) &= [\frac{1}{1-\psi}(\bar{m}_t - m_t^L) - \lambda_t\phi_t((1-\tau_t^K)r_t - (1-\tau_t^K)R_t)]\xi_t \\ \sigma(\xi_t, 1) &= \phi_t\sigma_t(1-\tau_t^K)\xi_t \\ \sigma(\xi_t, 0) &= 0 \,. \end{aligned}$$

Let $\Phi_E \equiv \Phi(\xi, 1)$ and $\Phi_L \equiv \Phi(\xi, 0)$ be the conditional distributions for entrepreneurs and laborers respectively. Let the newborn household get a weighted average $aW_t + (1-a)w_t^i$, where $0 < a < 1$, upon birth. In steady state the conditional distribution Φ_L satisfies the forward Kolmogorov equation:

$$0 = -\frac{\partial(\mu(\xi,0)\,\Phi_L)}{\partial\xi} - p(0)\,\Phi_L + (p\Phi_L)(\xi, 0-\eta(0)) - v\Phi_L + \frac{v}{1-a}\Phi_L(\frac{\xi-a}{1-a})\,,$$

and the conditional distribution Φ_E satisfies the forward Kolmogorov equation:

$$0 = \frac{1}{2}\frac{\partial^2(\sigma(\xi,d)^2\Phi_E)}{\partial\xi^2} - \frac{\partial(\mu(\xi,1)\,\Phi_E)}{\partial\xi} - p(1)\,\Phi_E + (p\Phi_E)(\xi, 1-\eta(1)) - v\Phi_E + \frac{v}{1-a}\Phi_E(\frac{\xi-a}{1-a})\,.$$

In the two equations above we need to calculate:

$$(p\Phi)(\xi, d-\eta(d)) = p(d-\eta(d))\Phi(\xi, d-\eta(d))\,.$$

To that end, let the old state be d, and the new state be d'. They are related through $d' = d + s(d)$, and we need to compute $\eta(d') = s(d)$. For $d = 0$, we have $d' = 0 + s(0) = 0 + 1 = 1$, and $\eta(d') = \eta(1) = s(0) = 1$, hence $\eta(1) = 1$. For $d = 1$, we have $d' = 1 + s(1) = 1 - 1 = 0$, and $\eta(d') = \eta(0) = s(1) = -1$, hence $\eta(0) = -1$. Therefore:

$$p(0-\eta(0))\Phi(\xi, 0-\eta(0)) = p(1)\Phi(\xi,1) = p_{EL}\Phi_E\,,$$

and:

$$p(1-\eta(1))\Phi(\xi, 1-\eta(1)) = p(0)\Phi(\xi,0) = p_{LE}\Phi_L\,.$$

Substituting for $\mu(\xi_t, 0)$, $\mu(\xi_t, 1)$, $\sigma(\xi_t, 0)$, $\sigma(\xi_t, 1)$ and using the above, we can write the system of the two Kolmogorov equations as:

$$0 = c_1\xi^2\frac{\partial^2\Phi_E}{\partial\xi^2} + c_2\xi\frac{\partial\Phi_E}{\partial\xi} + c_3\Phi_E + p_{LE}\,\Phi_L + \frac{v}{1-\alpha}\Phi_E(\frac{\xi-a}{1-a})$$

$$0 = c_4\xi\frac{\partial\Phi_L}{\partial\xi} + c_5\Phi_L + p_{EL}\,\Phi_E + \frac{v}{1-a}\Phi_L(\frac{\xi-a}{1-a})\,,$$

where:

$$c_1 = \frac{\phi^2\sigma^2(1-\tau)^2}{2}$$

$$c_2 = 2\phi^2\sigma^2(1-\tau)^2 - [\frac{1}{1-\psi}(\bar{m} - m^E) + \phi\mu(1-\lambda)]$$

$$c_3 = \phi^2\sigma^2(1-\tau)^2 - [\frac{1}{1-\psi}(\bar{m} - m^E) + \phi\mu(1-\lambda)] - p_{EL} - v$$

$$c_4 = \lambda\phi\mu - \frac{1}{1-\psi}(\bar{m} - m^L)$$

$$c_5 = \lambda\phi\mu - \frac{1}{1-\psi}(\bar{m} - m^L) - p_{LE} - v$$

Now, the Laplace transform for any variable y is defined as:

$$Y(s) = \int_0^\infty e^{-st} y(t) dt \,.$$

and therefore:

$$Y'(s) = -\int_0^\infty e^{-st} t y(t) dt = -L[ty(t)] \;\Rightarrow\; L[ty] = -\frac{d}{ds} Y(s) \,.$$

and:

$$Y'(s) = -\int_0^\infty e^{-st} t y(t) dt \;\Rightarrow\; Y''(s) = \int_0^\infty e^{-st} t^2 y(t) dt = L[t^2 y(t)] \,.$$

Hence we find that:

$$L[ty'] = \int_0^\infty e^{-st} t y' dt = -sY'(s) - Y(s) \,,$$

and:

$$L[t^2 y''] = \int_0^\infty e^{-st} t^2 y'' dt = s^2 Y''(s) + 4sY'(s) + 2Y(s) \,.$$

Let $c = \frac{1}{1-a}$, $k = \frac{a}{1-a}$, and $\tau = ct - k$, then $d\tau = cdt$ and $t = \frac{\tau + k}{c}$. So we have:

$$L[y(\frac{t-a}{1-a})] = \int_{k/c}^\infty e^{-st} y(ct-k) dt = \frac{1}{c} e^{-s\frac{k}{c}} L[y(t)]_{s\to s/c} = (1-a) e^{-k(1-a)} L[y(t)]_{s \to s(1-a)} \,.$$

Hence when $a = 1$:

$$L[y(\frac{t-a}{1-a})] = (1-1) e^{-k(1-1)} \int_0^\infty y(t) dt = 0 \cdot 1 \cdot 1 \,,$$

if y is a probability density function. Hence the last term in both Kolmogorov equations will drop out when $a = 1$. After changing variables to $\xi = e^x$, and defining $\partial\Phi_E/\partial x \equiv \Phi_2$ we get:

$$\begin{pmatrix} \Phi_L' \\ \Phi_E' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} 0 & c_5/c_4 - 1 & p_{EL}/c_4 \\ 1 & 0 & 0 \\ c_2/c_1 - 3c_1 & -p_{LE}/c_1 & -2 + c_2/c_1 - c_3/c_1 \end{pmatrix} \begin{pmatrix} \Phi_L \\ \Phi_E \\ \Phi_2 \end{pmatrix}$$

Since all coefficients are constant, and ξ is bounded, a Lipschitz condition is satisfied, hence the solution to the system exists and is unique. The conditional densities can be recovered by inverting the Laplace transforms.

Proof of Proposition 5. Consider the open economy, i.e. $R_t = R$ for all t, with entrepreneurs only, so that $\lambda = 1$, and assume that $v = 0$. Then the equations describing the evolution of the marginal propensity to consume and of total effective wealth ((18) and (21)) become:

$$\frac{\dot{W}_t}{W_t} = \rho_t - m_t = \phi_t \mu_t + (1 - \tau_t^K)R - m_t \tag{33}$$

$$\frac{\dot{m}_t}{m_t} = m_t - \theta\beta + (\theta - 1)\hat{\rho}_t \tag{34}$$

In steady state, equation (33), using $\hat{\rho}_t = 1/2\, \phi_t \mu_t + (1 - \tau_t^K)R$, gives:

$$m_t = \theta\beta - (\theta - 1)(\frac{1}{2}\phi_t \mu_t + (1 - \tau_t^K)R)$$

Plugging this into the steady state version of (34), we get:

$$0 = \frac{\theta + 1}{2}\phi_t \mu_t + \theta(1 - \tau_K)R - \theta\beta$$

where $\phi\mu = (F_{K_t} - \delta - R)^2/\gamma\sigma^2$. Therefore, an increase in τ_t^K reduces the second term on the right-hand-side. Since the interest rate is constant, the only way stationarity can be restored is if K_t falls (this is the only way $\phi_t \mu_t$ can increase). Hence, at the new steady state of the open economy with a higher capital-income tax, steady-state capital is unambiguously lower.

Proof of Proposition 6. Take the open economy with $\lambda = 1$ and $\upsilon = 0$. Then, using (33) and (34) in steady state, using the definition of $\hat{\rho}$, and taking the total differential with respect to K and R gives:

$$\frac{\partial K}{\partial R} = \frac{\phi - \theta(1 - \phi)}{\phi(\theta + 1)} \frac{1}{F_{KK}},$$

which proves that:

$$\frac{\partial K}{\partial R} > 0 \Leftrightarrow \theta > \frac{\phi}{1 - \phi}.$$

Lemma 2. *The steady state measure of entrepreneurs is given by $p_{LE}/(p_{LE} + p_{EL})$.*

Proof of Lemma 2. Call χ the measure of entrepreneurs today, and χ' their measure tomorrow. Then $\chi' = \chi(1 - p_{EL}) + (1 - \chi)p_{LE}$. But in steady state $\chi = \chi'$ hence $\chi = p_{LE}/(p_{LE} + p_{EL})$.

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