Executive Compensation, Incentives, and the Role for Corporate Governance Regulation[*]

David L. Dicks
Kellogg School of Management

Job Market Paper

October 2, 2007

Abstract

High executive compensation has been criticized in the business press as well as academic papers. Many attribute generous executive compensation to a corporate governance failure. In this paper I model governance and compensation as substitutes for providing incentives. Firms choose governance and compensation optimally. Large firms choose to govern closely and thus economize on executive compensation, while small firms choose lower levels of governance and thus pay their executives more (relative to the size of the firm) to align incentives. However, because large firms must pay enough to keep their managers, who could threaten to leave and go to a smaller firm, small firms impose a pecuniary externality on large firms.

This externality leads to a role for governance regulation. If firms are forced to exercise more governance, these firms will pay their CEOs less, and this in turn will allow the largest firms to pay their CEOs less. From the perspective of investors, optimal governance is strictly increasing in the size of the firm for the regulated firms. Therefore, investors can benefit from corporate governance laws, though this will be at the expense of CEOs.

---

[*] I thank Michael Fishman for his patience and support. I also thank Andrew Hertzberg, Arvind Krishnamurthy, Michael Faulkander and Zhiguo He for their helpful comments.

1 Introduction

This paper examines the link between executive compensation and corporate governance. Many point to the large levels of executive compensation granted by companies and claim that they are due to poor governance. They argue that the CEO is able to extract more from the shareholders due to their lack of control. I argue in this paper that there is an indirect link between compensation and governance. Specifically, I show that in a superstar market for CEOs with agency problems that small firms will govern too little and pay too much, forcing larger firms to pay their CEO inefficiently large amounts of money.

Corporate governance is a popular topic in the business press. Also, much academic work has been done on governance. One of the great questions is why there is a need for regulation. In the current literature, there are two competing reasons why regulation might be necessary: suboptimal contracting and information externalities. I propose in this paper that pecuniary externalities should be considered when setting corporate governance regulation.

With rational parties and optimal contracting, regulation will only improve welfare if there is an externality[1]. Much research has been done concerning information externalities, such as Admati-Pfleiderer(2000). The idea behind this literature is that information about the firm's cash flows has benefits for those outside the firm – either for portfolio choice or real investment decisions. However, it is difficult to measure this benefit, and thus it is unfeasible to use this approach to set optimal levels of governance. Further, with technological advances it is reasonable to suggest that the importance of information externalities may be diminishing. Instead, I endogenously develop a pecuniary externality that leads to a role for corporate governance regulation.

In this paper, I model a general equilibrium in the market for executives. Each firm hires a CEO who adds value to the company, but has the opportunity to divert resources

---

[1] If firms do not use optimal contracts, then it may be possible to improve welfare by regulating firms. However, even if firms are not offering the best contracts to their executives, there is no reason to suggest that the government could do any better. Further, the threat of a takeover could lead to more efficient contracts, provided that anyone knows how to improve contracts.

to personal uses. The company can deal with this moral hazard problem either by paying the CEO enough to discourage diversion, or can exercise governance to make diversion more costly to the CEO. Governance, however, is costly to the firm. Thus, firms face a trade-off between executive compensation and corporate governance.

I use a superstar model similar to Rosen (1992), Tervio (2007), and Gabaix-Landier (2008). Each manager has a level of talent, and the value of a manager's talent is proportional to the size of the firm where the manager works. Further, the level of each CEO's talent is common knowledge, and talent is movable across firms, so each firm needs to pay their CEO enough to keep the manager from going to another firm. However, this leads to an explosion in pay, because the better managers work at larger firms where their talents are used more efficiently[2].

I find a role for corporate governance regulation due to a pecuniary externality. Small firms will handle the moral hazard problem by paying the CEO enough to report truthfully, while large firms would prefer to exercise governance. Because each firm must pay their CEO enough to stay, the small firms force the large firms to pay their CEOs more. The importance of this type of externality has likely increased, due to the widely perceived rise in importance of general purpose human capital for CEO, as documented in Frydman (2005).

I focus on governance that reduces the effectiveness of diverting company resources to personal use. Thus, this corresponds to financial disclosure and accounting standards. Note that governance here does not refer to the GIM index from Gompers et al (2003). The GIM index measures the strengths of shareholder rights, while ignoring reporting requirements. In this paper, I assume that directors maximize the value of their stock, so they are not biased in favor of management. This allows me to study a general equilibrium setting without managerial power, and I still find a role for governance regulation.

---

[2] This model will not work in markets that tend to promote from within, like historically has been true in Japan. However, it seems that this is the kind of market that is dominant in American firms, since many firms hire their CEO from another firm, not from within.

1.1 Related Literature

A related topic that is also important to this paper is executive compensation. Generous executive compensation packages are granted to CEOs, leading some to suggest the presence of wrongdoing. Others suggest that the observed high pay is merely a market price for their labor.

On the wrongdoing side, there is the Managerial Power Perspective, championed in Bebchuk-Fried (2004) which suggests that this recent rise in executive compensation is due to slack governance. Specifically, since the board is often very friendly with the CEO, they might be hesitant to disagree on a matter of pay. Supporting this view, Westphal-Stein (2007) finds that directors who are agreeable are more likely to be placed on other boards, suggesting that directors may not have their incentives aligned with shareholders.

However, on the market price side of the issue, Gabaix-Landier (2008) find that the rise of CEO pay can be attributed to the increase in size of companies. They use a superstar model of the market for executives originally conceived in Rosen (1992), and calibrate the model so it fits the data. Thus, Gabaix-Landier (2008) attribute the recent rise in executive pay to the market working correctly.

This paper contributes to this literature, because I show that, even though the pay for CEOs is driven by a competitive market, not by imposing undue influence on their own pay. However, I still find a role for corporate governance regulation, due to the pecuniary externality that firms pose on each other in the way they handle agency problems.

This paper also relates to Hermalin-Weisbach (1998). In that paper, the shareholders face a trade-off between paying the CEO and giving him job security. Corporate governance, though increasing the value of the company, hurts the CEO's job security. Thus, the company is forced to compensate the CEO for the loss of job security associated with corporate governance. What drives this result, however, is that there is uncertainty over the ability of the CEO, and there is no asymmetric information in their paper. In my paper, talent is observable, but there is a moral hazard problem, resulting in the substitutability of governance and compensation.

The remainder of the paper is structured as follows. In Section 2, I present the basic model, a general equilibrium in the market for executives, with optimal executive compensation and levels of corporate governance. I examine governance regulation in Section 3. Section 4 contains extensions and applications, and I conclude in Section 5.

2 Model

2.1 Single-Firm Model

Consider the following principal-agent problem. A firm's cash flow equals STz, where S is the size of the firm, T is the talent of the manager, and z is random. Assume $z \geq 0$, $z \sim F$, $0 \in \text{support}(F)$, and $Ez = 1$. The firm can choose a level of governance $g \in [0, 1]$, at cost $\kappa g S$. Only the agent observes the cash flow, $y = STz$. The agent can report $\widehat{y} \leq y$, and enjoys private benefit of $\lambda(1-g)(y-\widehat{y})$.[3] Governance, thus, is costly actions taken by the principal to decrease the attractiveness of diverting cash flows to personal uses. Both the principal and agent[4] are risk-neutral, the reported cash flows are contractible, and $0 < \lambda < 1$. The agent has an outside option with $U_0 > 0$. Let $C(\widehat{y})$ be the payment to agent when the agent reports and delivers cash flow $\widehat{y}$.

The manager thus chooses $\widehat{y}$ so that

$$\widehat{y}(y; g) \in \arg\max_{y' \leq y} \{C(y') + \lambda(1-g)(y-y')\}$$

The principal maximizes the objective, which is:

$$E\{\widehat{y}(STz; g) - C(\widehat{y}(STz; g))\} - \kappa g S$$

subject to the agent's incentive compatibility constraint, the agent's limited liability con-

---

[3] The cash-diversion model used here is a linear case of the problem in Diamond(1984) and Lacker-Weinberg(1989), as used in DeMarzo-Fishman(2007). Solutions to this type of problem are isomorphic to solutions for effort problems with binomial effort and binomial output.

[4] Principal and firm are used equivalently throughout this paper, as are manager and agent.

straint, and the agent's participation constraint, which is

$$E\{C(\widehat{y}) + \lambda(1-g)(STz - \widehat{y})\} \geq U_0$$

Lemma 1 *There will be no diversion in equilibrium. Formally,* $\widehat{y}(y; g) = y$ *in equilibrium.*

Intuitively, $1 - \lambda(1-g) > 0$ for any level of g, so the principal is better off paying the manager their private benefit of diversion, and having manager report truthfully. Thus, the problem simplifies to the following:

$$\begin{aligned} \max_{C,g} \quad & E\{STz - C(STz)\} - \kappa g S & & \\ s.t. \quad & C(y) \geq C(\widehat{y}) + \lambda(1-g)(y - \widehat{y}) \qquad \forall \widehat{y} \leq y & & \text{(IC)} \\ & E[C(y)] \geq U_0 & & \text{(IR)} \\ & C(\cdot) \geq 0, \;\; g \in [0,1] & & \text{(LL), (F)} \end{aligned}$$

Theorem 2 *For very large levels of reservation utility ($U_0 > ST$), the project is not carried out. For moderate levels of reservation utility ($\lambda ST \leq U_0 \leq ST$), the participation constraint is tight enough so that the IC constraint is lax, so*

$$C(y) = \frac{U_0}{ST} y, \;\; g = 0$$

For small levels of reservation utility, ($U_0 < \lambda ST$), the optimal contract depends on the level of talent. For $T \leq \frac{\kappa}{\lambda}$, the principal will pay agent enough to satisfy IC, setting

$$C(y) = \lambda y, \;\; g = 0$$

For $T > \frac{\kappa}{\lambda}$, principal governs closely, so participation constraint will bind, so

$$C(y) = \frac{U_0}{ST} y, \; g = 1 - \frac{U_0}{\lambda ST}$$

Note that the compensation to the agent is linear in the cash flow y. Thus, the solution to this problem is to give the agent a share of equity. For very large levels of reservation utility, it is inefficient to take the agent from other pursuits and have him run the project, because the project will produce less than the agent would have produced elsewhere. For moderate levels of reservation utility, there is no agency problem because the agent requires enough compensation to run the project that the principal can give him sufficient pay-performance sensitivity without paying more than is necessary.[5] Finally, for small levels of reservation utility, the principal gets to choose how to induce the agent to report truthfully. If the agent is less talented, the principal chooses to pay the agent enough to report truthfully. Thus, she grants the agent a share λ of the firm. However, if the agent is more talented, the principal finds it optimal to govern the agent as closely as possible and pay as little as possible. It is this small level of reservation utility that will be relevant for the general equilibrium model presented in Section 2.2.

Consider the effect of governance regulation on this single firm. It is obvious that any regulation, either that requires an increase or decrease in governance, will hurt investor welfare, because the investors could have exercised the required level of governance without regulation. This logic extends to a case where we have a cross-section of firms and the participation constraint, $U(T)$, is an *exogenous* function of T. As I will show, however, this result is not robust to the setting where the principals compete for the agents, which is discussed next.

2.2 General Equilibrium Model

Now suppose that there is a continuum of firms, q to 1, where $q > 0$. Suppose $S' < 0$ and $T' < 0$, so that firm q is the largest, and firm 1 is the smallest, and manager q is most talented, and manager 1 is the least talented. In this section, I consider a market equilibrium for managerial talent. The key assumption is that talent is freely movable from firm to firm, and that it is equally useful at any firm. Also, talent is fully observable. Together, these assumptions allow firms to compete for CEOs. Random shocks, z, are independent across firms. Each firm must pick their level of governance prior to hiring their manager,

---

[5] This works by the same intuition that if the principal sells the company to the agent there ceases to be an agency problem. The agent has a sufficient stake (λ) in the company that he will behave.

but firms pick this level of governance optimally (in equilibrium)[6]. Finally, I assume that $T(1) < \frac{\kappa}{\lambda} < T(q)$ so that, by Theorem 2, some firms pay to satisfy the moral hazard problems, while others exercise governance.

Define $W(T)$ to be the equilibrium expected wage for manager with talent T. I will endogenously derive this $W(T)$ later. By Lemma 1, the manager will report truthfully in equilibrium, so firms solve the following problem:

$$\begin{aligned} \max_{C,g,T} \quad & E\left[STz\right] - EC_T(y) - \kappa g S \\ s.t. \quad & C_T(y) \geq C_T(\widehat{y}) + \lambda(1-g(x))(y-\widehat{y}) \qquad \forall \widehat{y} < y && \text{(IC)} \\ & E[C_T(y)] \geq W(T) && \text{(IR)} \end{aligned}$$

To solve the equilibrium, I need to find which managers works at which firm. Because any competitive equilibrium is Pareto optimal, the allocation of managers must be efficient, that is, it must maximize $\int_q^1 S(x)T(x)dx$. This will result in assortive matching of managers to firms. To see this, consider x and x', where $x < x'$.

$$\begin{aligned} S(x) \quad > \quad & S(x') \qquad T(x) > T(x') \\ \Rightarrow \quad & \left[S(x) - S(x')\right]\left[T(x) - T(x')\right] > 0 \\ \Rightarrow \quad & S(x)T(x) + S(x')T(x') > S(x')T(x) + S(x)T(x') \end{aligned}$$

Therefore, assortive matching will result from market equilibrium, so manager x will work for firm x.[7]

By Theorem 2, we can restrict attention to linear contracts. Thus, define $\beta(x)$ so that

---

[6] I need the assumption that firms commit to their level of governance before hiring the agent because if I do not make this assumption, the differential equation for equity share granted to the CEO explodes at x^*. When this assumption is not made for discrete distributions, the contagion of CEO pay is stronger, because off-equilibrium, firms are willing to pay a better CEO not only for their superior talent but also to compensate them for lowered cost of governance, since less governance is necessary due to the higher equity share. In the continuous version, however, this results in a singular point at x^* in the differential equation that defines β.

[7] Specifically, I am assuming here that the set of CEOs hired by a corresponding set of firms must have equivalent measure. This is the continuous analogue of each firm hiring one manager.

$C(STz) = \beta(x)S(x)T(x)z$. Given this change of variables, the problem simplifies to

$$\begin{aligned} \max_{\beta,g} \quad & S(x)T(x)(1-\beta(x)) - \kappa g(x)S(x) \\ s.t. \quad & \beta(x) \geq \lambda(1-g(x)) \qquad \text{(IC)} \\ & \beta(x)S(x)T(x) \geq W(T(x)) \qquad \text{(IR)} \end{aligned}$$

Define x^* such that $T(x^*) = \frac{\kappa}{\lambda}$. By Theorem 2, for $x < x^*$, $T(x) > \frac{\kappa}{\lambda}$, the firm would like to govern the manager and pay him nothing, so the IR binds. For $x \geq x^*$, however, $T(x) < \frac{\kappa}{\lambda}$, so the firm will set $\beta(x) = \lambda$ even when IR is nonbinding. The firm does not consider the effect of the pay granted on other firms, so this is the source of the pecuniary externality.

Suppose manager x was not getting paid enough by firm x. The manager's outside option would be to go to another firm. Suppose the manager wished to go to firm x', where $x' > x$. Define $\beta(x, x')$ to be the most that the manager could demand there, so that the firm would be willing to fire manager x'. For firm x' to be willing to replace manager x' with manager x, the profit of firm x' would need to increase by hiring manager x. Thus, it must be the case that

$$\begin{aligned} S(x')T(x)(1-\beta(x,x')) - \kappa g(x')S(x') &\geq S(x')T(x')(1-\beta(x')) - \kappa g(x')S(x') \\ \beta(x,x')S(x')T(x) &\leq S(x')[T(x)-T(x')] + \beta(x')S(x')T(x') \end{aligned}$$

Therefore, this will hold with equality (because $\beta(x, x')$ is the most that firm x' is willing to pay). Thus, endogenously,

$$W(T(x)) = \sup_{x'>x}[S(x')[T(x)-T(x')] + \beta(x')S(x')T(x')]$$

because a manager's outside option is to go to a smaller firm.[8]

Lemma 3 *Managers will prefer to move to the next largest firm, rather than jump down firms. That is, in threatening to leave their firm, managers find small moves preferable to*

---

[8] Off-equilibrium, a manager would not be able to induce a larger firm to hire him without taking less pay than from his firm in equilibrium. Thus, a manager would never go to a larger firm.

large moves. Formally,

$$W(T(x)) = \lim_{x' \to x^+} \{S(x')[T(x) - T(x')] + \beta(x')S(x')T(x')\}$$

So participation constraint becomes

$$\begin{aligned} \beta(x)S(x)T(x) - \beta(x+dx)S(x+dx)T(x+dx) &\geq S(x+dx)[T(x) - T(x+dx)] \\ \Leftrightarrow -(\beta ST)' &\geq -ST' \end{aligned}$$

This constraint shows that executive compensation must increase across firms at least as fast as the product of size and marginal talent. Thus, in general, each firm will set their pay in the following way. For $x \geq x^*$, $T(x) \leq \frac{\kappa}{\lambda}$, so the firm will always set $g = 0$, $\beta \geq \lambda$, and $-(\beta ST) \geq -ST'$, and by complementary slackness one of these will bind. For $x < x^*$, $T(x) > \frac{\kappa}{\lambda}$, so the participation constraint will bind, so $-(\beta ST)' = -ST'$, and $g = \max\{1 - \frac{\beta}{\lambda}, 0\}$. Thus, for $x < x^*$,

$$\beta(x)S(x)T(x) = \beta(x^*)S(x^*)T(x^*) + \int_x^{x^*} S(x)\left(-T'(x)\right) dx$$

Recall that $T' < 0$, so the integral is positive and pay is increasing across firms.

2.3 Specification

Following Gabaix-Landier (2008), I will assume the following specification:

$$\begin{aligned} S(x) &= Ax^{-a} \\ T(x) &= T_{Max} - \frac{B}{b}x^b \end{aligned}$$

where $b < a$. Gabaix-Landier (2008) estimate $b \approx \frac{2}{3}$ and $a \approx 1$ in their empirical section. This implies

$$\begin{aligned} S'(x) &= -aAx^{-a-1} \\ \text{and} \quad T'(x) &= -Bx^{b-1} \end{aligned}$$

Consider the equilibrium in this context. When the endogenous participation constraint binds for all $x \in (x_1, x_2)$,

$$\beta(x_1)S(x_1)T(x_1) = \beta(x_2)S(x_2)T(x_2) + \int_{x_1}^{x_2} S(x)\left(-T'(x)\right)dx$$

so under this specification,

$$\begin{aligned}
\int_{x_1}^{x_2} S(z)(-T'(z))dz &= \int_{x_1}^{x_2} Az^{-a}(Bz^{b-1})dz \\
&= AB\int_{x_1}^{x_2} z^{-a+b-1}dz \\
&= \frac{AB}{a-b}\left(x_1^{b-a} - x_2^{b-a}\right)
\end{aligned}$$

The first relevant cutoff is x^*: as demonstrated above, for all $x < x^*$, the participation constraint will bind. Therefore, for $x < x^*$,

$$\beta(x)S(x)T(x) = \beta(x^*)S(x^*)T(x^*) + \frac{AB}{a-b}\left(x^{b-a} - (x^*)^{b-a}\right)$$

The key question is what happens for $x > x^*$, and thus also, what is $\beta(x^*)$? Specifically, at a point $x > x^*$, which constraint will bind: the incentive compatibility constraint, or the limited liability constraint. For $x = 1$, there is no participation constraint by assumption, since the worst manager has nowhere else to go. However, since $x^* < 1$ by assumption, firm 1 will still grant $\beta(1) = \lambda$ to manager 1. For x near 1, is it enough to grant manager x share $\beta(x) = \lambda$, or is more necessary? The following lemma answers this question.

Lemma 4 *In any equilibrium, there exists x^c such that IR is binding for $x > x^c$, IC is binding for $x \in [x^*, x^c]$, and IR is binding for $x < x^*$. Further, $x^c = 1$ iff*

$$\lambda a T_{Max} \geq B\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right)$$

Thus, for ease of exposition, I make the assumption stated below. Under this assumption on parameters, when companies choose to pay their managers enough to solve the moral hazard problem without governance, the endogenous participation constraint will be non-binding, because managers will be strictly better off staying at their current firms than going to smaller firms. This assumption will hold throughout the remainder of the paper.

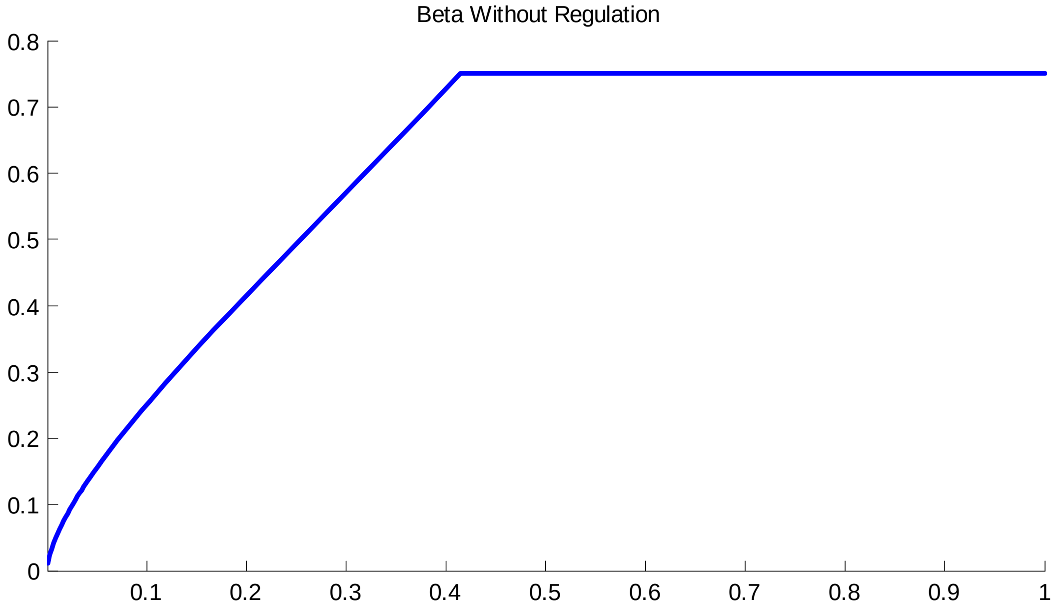

Figure 1: The share of inside equity across firms. Note that the size of firms is decreasing in the index, so the smallest firm is firm 1.

Assumption: $\lambda a T_{Max} \geq B\left(1+\lambda\left(\frac{a}{b}-1\right)\right)$

The equilibrium is summarized by the following proposition.

Proposition 5 *For* $x > x^*$, $\beta(x) = \lambda$ *and* $g(x) = 0$. *For* $x \leq x^*$,

$$\begin{aligned} \beta(x) &= \frac{1}{S(x)T(x)}\left[A(x^*)^{-a}\kappa + \frac{AB}{a-b}(x^{b-a} - (x^*)^{b-a})\right] \\ g(x) &= 1 - \frac{\beta(x)}{\lambda} \end{aligned}$$

Pay-performance sensitivity across firms can be seen in Figure 1. Note that, in small companies ($x > x^*$) the manager's participation constraint is nonbinding, because the shareholders find it too expensive to govern the manager sufficiently. These small companies thus pay sufficiently to induce the CEO to report truthfully. However, these small companies do not take into account the effect that this has on larger companies: that the larger firms must pay their CEO more. This is a pecuniary externality, and thus there is a potential for regulation to improve investor welfare.

Observe the cross-sectional behavior of size, compensation, and governance: as firm size increases, compensation and governance increase. However, pay-performance sensitivity decreases. The properties are consistent with Murphy (1999).

Corollary 6 *Denote the objective function of each principal* $\Pi(x)$*. For* $x > x^*$*,*

$$\Pi(x) = (1-\lambda)Ax^{-a}\left(T_{Max} - \frac{B}{b}x^b\right)$$

For $x \leq x^*$*,*

$$\begin{aligned} \Pi(x) &= Ax^{-a}\left(T_{Max} - \frac{B}{b}x^b - \kappa\right) \\ &\quad - \left(\frac{AB}{a-b}\left[x^{b-a} - (x^*)^{b-a}\right] + A\kappa\,(x^*)^{-a}\right)\left(1 - \frac{\kappa}{\lambda\left(T_{Max} - \frac{B}{b}x^b\right)}\right) \end{aligned}$$

The corollary follows by substitution of $\beta(x)$ and $g(x)$ into $\Pi(x) = S(x)T(x) - \beta(x)S(x)T(x) - \kappa g(x)S(x)$. Note that large firms would like to set $g(x) = 1$, resulting in the $-\kappa$ term in the first coefficient. However, they must pay the manager some share of the company, which results in not only a cost to them, but also a savings of governance cost, which has a net cost of $\left(1 - \frac{\kappa}{\lambda\left(T_{Max} - \frac{B}{b}x^b\right)}\right)$, which is positive on this region.

Now, consider time-series behavior of this equilibrium. Consider the share of firms in an industry that choose to exercise more than minimal governance. This is captured by x^*. When x^* increases, more firms exercise voluntary governance, while when x^* decreases, less firms exercise voluntary governance.

Corollary 7 *The following comparative statics hold for* x^**, the cutoff of firms that prefer to exercise governance rather than pay:* x^* *is*

$$\begin{aligned} \textit{increasing in the talent of CEOs} &: \frac{\partial x^*}{\partial T_{Max}} > 0 \\ \textit{increasing in the severity of moral hazard problems} &: \frac{\partial x^*}{\partial \lambda} > 0 \\ \textit{decreasing in cost of governance} &: \frac{\partial x^*}{\partial \kappa} < 0 \end{aligned}$$

Finally, I examine the impact of a change in governance costs on the equilibrium levels of wages and governance.

Corollary 8 *When monitoring costs increase, the change in wages and governance levels depends on the level of the monitoring costs. For small levels of monitoring costs,* ($\kappa\left(\frac{a}{b(1-\lambda)}+\frac{1}{\lambda}\right) < T_{Max}$)*, wages go down and governance levels go up. For large levels of monitoring costs,* ($\kappa\left(\frac{a}{b(1-\lambda)}+\frac{1}{\lambda}\right) > T_{Max}$)*, wages go up and governance levels go down.*

3 Governance Regulation

In this section, I examine optimal governance regulation. Note that the compensation a CEO receives depends not only on his characteristics, but also on the amount paid to other CEOs. Because of this, the choice a company makes in their compensation and governance decision affects not only that company, but also larger companies. This creates a pecuniary externality. Thus, it is reasonable to consider whether governance regulation can improve welfare.

3.1 Optimal Governance Regulation

In this section, I assume that the regulator can force each firm to carry out any level of governance, and seeks to maximize investor welfare. Also, I assume that the regulator can observe the size of firms, and knows the distribution of talent, but does not know the talent of an individual manager, though the companies do. I assume this so the equilibrium still has a competitive talent market, rather than the regulator assigning managers to firms.

Thus, the regulator solves the following to maximize investor welfare.[9]

$$\begin{aligned}
\max_{g_r(\cdot)} \quad & \int_q^1 \left\{ \left(1-\beta_r(x)\right) T(x)S(x) - \kappa g_r(x)S(x) \right\} dx \\
s.t. \quad & \beta_r(x) \geq \lambda(1-g_r(x)) \\
& -(\beta_r ST)' \geq -ST' \\
& \beta_r \geq 0, g_r \in [0,1]
\end{aligned}$$

The first constraint is the reporting incentive compatibility constraint, the second is the endogenous participation constraint, and the third is limited liability and feasibility. Clearly, $g_r(x) = \max\{1 - \frac{\beta_r(x)}{\lambda}, 0\}$, because governance is costly.

Lemma 9 *Any regulation that lowers governance hurts investor welfare.*

The intuition behind this lemma is that lowering governance forces large firms to pay even more than they would under the no regulation equilibrium. Thus, the no-regulation equilibrium dominates an equilibrium with regulation that lowers governance. Therefore, it is sufficient to consider regulation that increases governance.

Lemma 10 *In the optimal regulation equilibrium, the participation constraint will bind for all $x < x^*$. Thus, for $x < x^*$,*

$$\beta_r(x)T(x)S(x) = \frac{AB}{a-b}\left[x^{b-a} - (x^*)^{b-a}\right] + \beta_r(x^*)T(x^*)S(x^*)$$

Not only is it privately optimal for firm $x < x^*$ to pay as little as possible, but also paying any more than required has a negative effect on larger firms. Thus, it is optimal to set $g_r(x)$ so that the participation constraint and incentive compatibility constraints bind for large firms ($x < x^*$). Thus, for $x < x^*$,

$$\begin{aligned}
g_r(x) &= 1 - \frac{\beta_r(x)}{\lambda} \\
\Pi_r(x) &= \left(1-\beta_r(x)\right) T(x)S(x) - \kappa g_r(x)S(x) \\
&= \left[T(x) - \kappa\right] S(x) - \left\{ \frac{AB}{a-b}\left[x^{b-a} - (x^*)^{b-a}\right] + \beta_r(x^*)T(x^*)S(x^*) \right\} \left(1 - \frac{\kappa}{\lambda T(x)}\right)
\end{aligned}$$

---

[9] In this model, the regulator maximizes investor welfare because if investment is endogenized, improving the investors' payoff will increase investment and thus increase efficiency. See Section 4.2 for an illustration of this.

Note that $\Pi_r(x)$ is strictly decreasing in $\beta_r(x^*)$, and that $\Pi_r(x)$ is a function of only $\beta_r(x^*)$ and parameters. Thus, define $L(\cdot)$ so that

$$L(\beta_r(x^*)) = \int_q^{x^*} \Pi_r(x)dx$$

So, the objective becomes

$$\begin{aligned} \max_{g_r(\cdot)} \quad & L(\beta_r(x^*)) + \int_{x^*}^1 \{(1-\beta_r(x))\,T(x)S(x) - \kappa g_r(x)S(x)\}\,dx \\ s.t \quad & \beta_r(x) \geq \lambda(1-g_r(x)) \\ & -(\beta_r ST)' \geq -ST' \\ & \beta_r \geq 0, g_r \in [0,1] \end{aligned}$$

In order to be a solution to this problem, it is necessary that $g_r(\cdot)$ solves

$$\begin{aligned} \max_{g_r(\cdot)} \quad & \int_{x^*}^1 \{(1-\beta_r(x))\,T(x)S(x) - \kappa g_r(x)S(x)\}\,dx \\ s.t \quad & \beta_r(x) \geq \lambda(1-g_r(x)) \\ & -(\beta_r ST)' \geq -ST' \\ & \beta_r(x^*) = \beta^* \\ & \beta_r \geq 0, g_r \in [0,1] \end{aligned}$$

Proposition 11 *Any solution to the previous problem will satisfy the following. For $x \geq x_1$, $\beta_r(x) = \lambda$, $g_r(x) = 0$. For $x < x_1$, the participation constraint binds everywhere, and $g_r(\cdot)$ is picked to support this, so*

$$\begin{aligned} \beta_r(x)S(x)T(x) &= \frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right) + \lambda S(x_1)T(x_1) \\ g_r(x) &= 1 - \frac{\frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right) + \lambda S(x_1)T(x_1)}{\lambda S(x)T(x)} \end{aligned}$$

The intuition behind this proposition is that the planner would like to have a discontinuity at x^*, so that $\beta_r(x) = \lambda$ for all $x > x^*$, and $\beta_r(x) = 0$ for all $x < x^*$. However, this is not feasible by the participation constraint, so that constraint binds. What remains is to solve the optimal choice of x_1, which I present in the following theorem.

Theorem 12 *x_1 is chosen so that*

$$\int_q^{x_1} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx = 0$$

Thus, for $x \geq x_1$, $\beta_r(x) = \lambda$, $g_r(x) = 0$, and for $x < x_1$, β_r follows the participation constraint, and $g_r(x) = 1 - \frac{\beta_r(x)}{\lambda}$. Further, if

$$\int_q^{1} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx > 0$$

the optimal governance policy is to set $g_r(1) = 1$, $\beta_r(1) = 0$, and allow β_r and g_r to follow participation constraint.

In summary, suppose $\int_q^1 \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx < 0$. To find the optimal governance regulation, pick x_1 so that $\int_q^{x_1} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx = 0$. Because for all $x > x_1$, $\beta_r(x) = \lambda$ and $g_r(x) = 0$, by continuity $\beta_r(x_1) = \lambda$ and $g_r(x_1) = 0$. For all $x < x_1$, the endogenous participation constraint binds, so

$$\beta_r(x)T(x)S(x) = \beta_r(x_1)T(x_1)S(x_1) + \frac{AB}{a-b}\left[x^{b-a} - x_1^{b-a}\right]$$

Also, the IC constraint will bind, so $\beta_r(x) = \lambda(1 - g_r(x))$, so by substitution,

$$\lambda(1 - g_r(x))T(x)S(x) = \lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left[x^{b-a} - x_1^{b-a}\right]$$

To implement this, the regulator needs to require that, for firms $x \in [x^*, x_1]$, that each of those firms sets their governance at least as large as $g_r(x)$, where $g_r(\cdot)$ satisfies the above equation. This leads to the following Corollary.

Corollary 13 *When $\int_q^1 \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx < 0$, the optimal governance regulation is monotonic in firm size. Further, for the region of firms such that governance is required, the regulated governance minimum is strictly increasing in the size of the firm.*

Therefore, the optimal governance regulation has the following structure. For the smallest firms, the regulator should leave them alone. For the middle firms ($x \in [x^*, x_1]$), the regulator

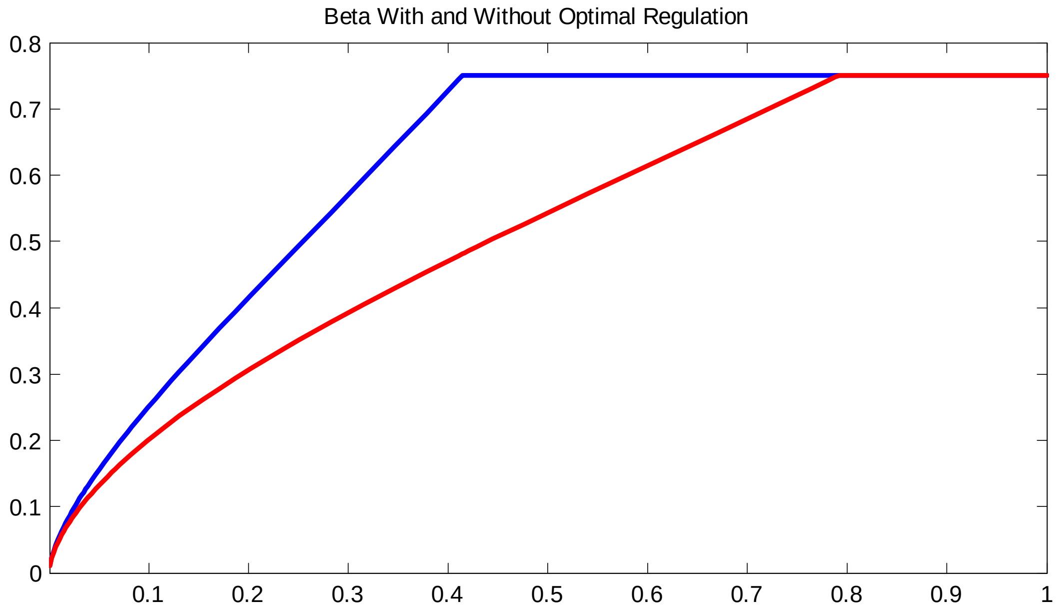

Figure 2: The share of inside equity across firms. The upper line is β in the absence of regulation, while the lower line is β_r in the presence of optimal regulation.

enforces strictly increasing governance requirements. For the largest firms, the regulator can still enforce the above governance schedule. However, she does not need to enforce regulation requirements at those firms, since those firms will govern as closely as possible. x_1 has the following comparative statics.

Corollary 14 *The following comparative statics hold for x_1, the cutoff of firms will be forced to exercise governance under optimal regulation: x_1 is*

$$\begin{aligned} \textit{increasing in the talent of CEOs} &: \frac{\partial x_1}{\partial T_{Max}} > 0 \\ \textit{increasing in the severity of moral hazard problems} &: \frac{\partial x_1}{\partial \lambda} > 0 \\ \textit{decreasing in cost of governance} &: \frac{\partial x_1}{\partial \kappa} < 0 \end{aligned}$$

Because of the implementation of optimal governance, the share of inside equity is changed as shown in Figure 2.

3.2 Implementation of Optimal Governance Regulation

In the Section 3.1, I derived the optimal governance regulation. The optimal regulation was chosen based on the assumption that the regulator knows the size of each firm, knows the distribution of talent, and is able to regulate the level of governance at each firm. It may be difficult to enforce governance requirements at all, and very difficult to regulate at the firm level. Instead, in this section I will examine implementation through an alternate mechanism: a subsidy.

A subsidy has the advantage that, rather than forcing firms to do exercise governance, it changes their incentives so that they will exercise governance. The disadvantage of a subsidy is that it is costly for the regulator.

Suppose that the government subsides a share δ of governance costs. Thus, when a firm chooses g, it will cost them $\kappa(1-\delta)gS$. Because this lowers the cost of governance, it induces more firms to exercise governance. Define $x^*(\delta)$ so that

$$T\left(x^*\left(\delta\right)\right) = \frac{\kappa(1-\delta)}{\lambda}$$

Note that the arguments of Section 2 apply here, only substituting in $\kappa' = \kappa(1-\delta)$ for κ. The objective of the regulator is to maximize aggregate profit less aggregate cost of the subsidy:

$$\begin{aligned}
& \int_q^1 \Pi(x)dx - \int_q^1 \delta\kappa g(x)S(x)dx \\
= & \int_q^1 \left\{\left(1-\beta(x)\right)T(x)S(x) - \kappa(1-\delta)g(x)S(x)\right\}dx - \int_q^1 \delta\kappa g(x)S(x)dx \\
= & \int_q^1 \left\{\left(1-\beta(x)\right)T(x)S(x) - \kappa g(x)S(x)\right\}dx
\end{aligned}$$

Therefore, picking δ_1 so that $x^*(\delta_1) = x_1$, the optimal governance is implemented, because firms $[q, x_1]$ choose to exercise governance and firms $[x_1, 1]$ choose not to. This is summarized in the following corollary.

Corollary 15 *Suppose that the government can raise funds from investors efficiently (for example, a head tax). Then, the optimal governance can be implemented by a subsidy of* *governance costs. The percentage of governance costs paid by the regulator is* δ_1*, where* $T(x_1) = \frac{\kappa(1-\delta_1)}{\lambda}$ *and* x_1 *satisfies* $\int_q^{x_1} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx = 0$.

This result is interesting because the optimal governance regulation can be implemented by a subsidy. This means that, even with all flexibility granted to the regulator, they cannot do any better than they could by this subsidy. It will take some sophistication in order to set the optimal level of the subsidy, but as the next corollary shows, a small subsidy will always improve welfare.

Corollary 16 *If the regulator knows nothing, welfare can still be improved by a small subsidy of accounting costs.*

This corollary follows from the fact that, for $x^*(\delta) < x_1$, $\int_q^{x^*(\delta)} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx > 0$, so the subsidy improves investor welfare. Again, this assumes that the investors efficiently pay for the subsidy. However, this may seem unreasonable, because tax collections are never collected efficiently. Thus, it is reasonable to ask if there is a self-financing subsidy & tax scheme that implements the optimal governance.

Theorem 17 *There exists a self-financing tax and subsidy system that implements the optimal governance regulation and has zero cost to the regulator.*

The proof of this theorem consists of the following structure. Suppose that the regulator provides a subsidy to some firms and taxes others so that, for all firms larger than firm x_1, each firm is indifferent to choosing governance and paying the manager, but suppose that they exercise optimal governance. Under this scheme, the regulator collects a very large amount of revenue. Thus, there are many zero-cost tax-subsidy schemes that implement the optimal governance regulation.

However, there are problems with the notion of a subsidy. First, if the subsidy is not self-financing, it may not be politically feasible, since some would consider it a gift to big business. Second, it is not entirely clear that the subsidy would really lower the cost of governance. I discuss this in more detail in Section 4.5.

3.3 Optimal Governance Floor

In the Section 3.1, I derived the optimal governance regulation. However, due to practical or legal limitations, the regulator may not be able to regulate different levels of governance for different firms. This can be a real constraint, because in the United States, for example, Sarbanes-Oxley does not have different provisions for small firms. Suppose that the regulator is constrained, so that she must treat all firms the same, though the regulator can force all firms to maintain a minimum level of governance, γ. Further, suppose that the regulator is not allowed to use a subsidy as in Section 3.2. Thus, each firm solves the same problem as before, with the additional constraint that $g(x) \geq \gamma$. Thus, each firm solves

$$\begin{aligned} \max_{\beta_\gamma, g_\gamma} \quad & S(x)T(x)(1-\beta_\gamma(x)) - \kappa g_\gamma(x)S(x) \\ s.t. \quad & \beta_\gamma(x) \geq \lambda(1-g_\gamma(x)) \\ & \beta_\gamma(x)S(x)T(x) \geq W(T(x)) \\ & g_\gamma(x) \geq \gamma \end{aligned}$$

which leads us to the next theorem, a generalization of Theorem 2.

Theorem 18 *For $U_0 > S(T-\kappa\gamma)$, project is not carried out. For $\lambda(1-\gamma)ST \leq U_0 \leq S(T-\kappa\gamma)$, the participation constraint is tight enough so that the IC constraint is lax, so*

$$\beta_\gamma(x) = \frac{U_0}{ST}, \; g_\gamma(x) = \gamma$$

For $U_0 < \lambda(1-\gamma)ST$, the optimal contract depends on the level of talent. For $T \leq \frac{\kappa}{\lambda}$, the principal will pay agent sufficiently to satisfy incentive problems, setting

$$\beta_\gamma(x) = \lambda(1-\gamma), \; g_\gamma(x) = \gamma$$

For $T > \frac{\kappa}{\lambda}$, principal governs closely, so participation constraint will bind, so

$$\beta_\gamma(x) = \frac{U_0}{ST}, \; g_\gamma(x) = 1 - \frac{U_0}{\lambda ST}$$

Note that the governance floor not only imposes a cost on firms, but also affect real investment decisions, because it makes an otherwise profitable project be rejected, because when $S\left(T-\kappa\gamma\right)\leq U_0<ST$, the incentive constraint would have been lax, and the project would have been profitable for the firm, but the governance regulation is too costly.

Again, suppose the same specification from the main section, supposing that $S(x)=Ax^{-a}$ and $T(x)=T_{Max}-\frac{B}{b}x^b$. Further, I suppose that $T(1)\geq\kappa$, so that all firms will find it optimal to stay open, no matter how harsh the regulation. Note that Lemma 3 holds in this case, so a CEOs' preferred outside option will be to go to a slightly smaller firm rather than to a much smaller firm. This leads us to a generalization of Lemma 4, which leads us to a generalization of Proposition 5.

Lemma 19 *In any equilibrium, there exists* x^c *such that IR is binding for* $x>x^c$*, IC is binding for* $x\in[x^*,x^c]$*, and IR is binding for* $x<x^*$*. Further,* $x^c=1$ *iff*

$$\lambda\left(1-\gamma\right)aT_{Max}\geq B\left(1+\lambda\left(1-\gamma\right)\left(\frac{a}{b}-1\right)\right)$$

Finally, x^c *is decreasing in* γ*.*

Proposition 20 *If* $x^*<x^c$*, the three relevant regions are* $[q,x^*)$*,* $[x^*,x^c)$*, and* $[x^c,1]$*. For* $x\in[x^c,1]$*,*

$$\begin{aligned}\beta_\gamma(x)S(x)T(x) &= \lambda(1-\gamma)A\left(T-\frac{B}{b}\right)+\frac{AB}{a-b}\left(x^{b-a}-1\right)\\ g_\gamma(x) &= \gamma\end{aligned}$$

For $x\in[x^*,x^c)$*,*

$$\begin{aligned}\beta_\gamma(x) &= \lambda(1-\gamma)\\ g_\gamma(x) &= \gamma\end{aligned}$$

For $x\in[q,x^*)$*,*

$$\begin{aligned}\beta_\gamma(x)S(x)T(x) &= \kappa(1-\gamma)S(x^*)+\frac{AB}{a-b}\left(x^{b-a}-\left(x^*\right)^{b-a}\right)\\ g_\gamma(x) &= 1-\frac{\beta_\gamma(x)}{\lambda}\end{aligned}$$

If $x^* > x^c$, the two relevant regions are $[q, x^c)$ and $[x^c, 1]$. For all x,

$$\begin{aligned}\beta_\gamma(x)S(x)T(x) &= \lambda(1-\gamma)A\left(T - \frac{B}{b}\right) + \frac{AB}{a-b}\left(x^{b-a} - 1\right) \\ g_\gamma(x) &= \max\left\{1 - \frac{\beta_\gamma(x)}{\lambda}, \gamma\right\}\end{aligned}$$

Therefore, I have found the general equilibrium behavior of all firms when in the presence of a governance floor. Now, to find the optimal governance floor, I merely have to optimize over γ. Note that, for $x \in [x^c, 1]$, these firms are exercising inefficiently large amounts of governance, because $\beta_\gamma(x) > \lambda(1 - g_\gamma(x))$.

The profit of a firm x is given by Π_γ, which is given by

$$\Pi_\gamma(x) = (1 - \beta_\gamma(x))S(x)T(x) - \kappa g_\gamma(x)S(x)$$

where β_γ and g_γ are given by the previous proposition. Thus, the regulator's objective function is

$$I(\gamma) = \int_q^1 \Pi_\gamma(x)dx$$

Thus, the optimal regulation floor is solved by maximizing I over γ.

Proposition 21 *$I(\gamma)$ is continuous, differentiable, and weakly concave in γ. Further, $I(\gamma)$ is strictly concave for interior solutions. Thus, there exists a unique solution, γ^*, to the regulator's choice of floor. Also, if it is optimal to enforce a floor, there will be inefficiently large levels of governance exercised at some firms. Formally, $\gamma > 0$ implies that $x^c < 1$, so that for $x \in (x^c, 1)$, $g_\gamma(x) > 1 - \frac{\beta_\gamma(x)}{\lambda}$.*

Corollary 22 *It is optimal to impose a positive governance floor iff*

$$\kappa S(x^*) \int_q^{x^*} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx > \kappa \int_{x^*}^1 S(x)dx - \lambda \int_{x^*}^1 S(x)T(x)dx$$

Corollary 16 shows that the optimal floor can be very unstable. For a very small change in parameters, from a set such that the previous inequality fails to a set of parameters such that it holds, the optimal γ^* will jump from 0 to above γ_0, where γ_0 satisfies

$$\lambda\left(1 - \gamma_0\right) a T_{Max} = B\left(1 + \lambda\left(1 - \gamma_0\right)\left(\frac{a}{b} - 1\right)\right)$$

This also implies that in some scenarios, the optimal floor is 0, which means that it may be optimal to not impose governance regulation if it must floor. This contrasts the previous section, where I found that a little governance will always improve welfare. The difference is that the floor is a very blunt instrument to regulate governance, while I supposed before that the regulation could be fine-tuned to target the middle firms which impose the pecuniary externality.

Also, note that the previous analysis holds because I assumed that $T(1) > \kappa$. This assumption guarantees that all firms will stay open regardless of the choice of regulation. However, if I allow $T(1) < \kappa$, then firms can endogenously choose to close if the governance regulation is too severe. For a governance floor γ, a firm will stay open only if

$$\begin{aligned}(1-\lambda(1-\gamma))S(x)T(x)-\kappa\gamma S(x) &\geq 0\\ T(x) &\geq \frac{\kappa\gamma}{1-\lambda(1-\gamma)}\end{aligned}$$

Thus, define x_0 such that $T(x_0)=\frac{\kappa\gamma}{1-\lambda(1-\gamma)}$. Endogenizing x_0, however, makes the problem intractable. Though the FOCs can still be found, concavity is no longer satisfied, because we are no longer guaranteed that $\frac{dx^c}{d\gamma}<0$. It is important to note, however, that the optimal choice of governance floor may force some firms to close.

Thus, it greatly improves welfare for the governance not to be required to take the form of a floor. A governance floor not only causes inefficiently high levels of governance, but it can also force some firms to close.

4 Discussion of Results and Policy Implications

4.1 Sarbanes-Oxley

Sarbanes-Oxley (SOX) was passed in 2002 following the fall of Enron as well as many other accounting scandals in 2001. Zhang (2005) documents a negative stock market response to the passage of SOX. Further, she finds that the market responded negatively to news that the law was more likely to pass or was going to be more severe. Thus, she concludes that the regulation was harmful.

Contrasting this is Hochberg, Sapienza, and Vissing-Jorgensen (2007) (HSVJ), which finds that SOX has a positive effect on some firms. They sort firms based on their lobbying decisions and find that the firms that lobbied against strict implementation of SOX had a positive return relative to other firms.

This model can explain why we see these two facts. CEOs at large firms would be the first to oppose governance regulation. This is because they would receive the most severe drop in compensation. This is supported by the fact that the lobbying firms tend to be much larger in HSVJ than those that do not lobby. Further, these are the firms who have the best response to any regulation, so they have a positive return when matched with other firms. This needs to be tempered with the negative stock market reaction to the passage of SOX, as found in Zhang.

This paper shows that in equilibrium, an assignment model will lead to a pecuniary externality. Thus, investor welfare can be improved by increasing governance that lowers private benefits of diversion. Supporting this view, HSVJ finds that the effect is concentrated in those who lobbied against enhanced financial disclosure, since these measures are the most likely to improve transparency and thus decrease private benefits of diversion.

One of the biggest problems with SOX is that it treats all firms the same, forcing all firms to meet stringent governance requirements (Though the SEC has considered lowering requirements for small firms, to date they have decided against doing so). Their only outside option is to delist from their exchange. However, this is very inefficient, since it hurts diversifiability, and thus investment. This model implies that the required level of governance should be increasing in firm size. Note that this is not because of fixed costs, but because the pecuniary externality that small firms impose on owners of large firms is small, while the externality imposed by medium firms is much larger. By enforcing an increasing governance requirement on firms, this will lessen the damage done to small firms, while still helping large firms.

In the model I assumed that there is a single market for CEO talent, and that this talent is transferable to other companies. However, it is unreasonable to suppose that a CEO of a manufacturing firm and a CEO of a financial firm could switch companies without any loss. What is more reasonable is that the market for CEOs is segmented – that there is a market for CEOs of financial firms, a market for CEOs of manufacturing firms, a market for CEOs of retail firms, and so on. Thus, one can interpret this model as expressing an equilibrium in a specific industry. This interpretation is supported by the fact that the positive returns in HSVJ are dampened when controlling for the 1-digit industry category. My model suggests that most of the positive returns come from size and industry, because industry proxies for labor market conditions for CEOs.

Also, since each industry likely has a different market for CEOs, different governance laws for different industries will make sense. For example, in industries that are heavily regulated, such as utilities, it seems reasonable to believe that moral hazard problems are smaller, i.e. λ is smaller there.[10] Thus, it will be inefficient to force firms to carry out costly governance measures when there is smaller moral hazard problems, and thus less of a pecuniary externality.

Thus, the optimal regulation should treat different industries differently. This may be difficult to implement, however. First, the traditional challenge that regulation leads to corruption is particularly relevant when the regulator has discretion. Second, there are legal boundaries to focusing laws on one specific industry, evidenced by the legal challenges against elevated minimum wage laws for big box retailers. However, regulators will likely get close to optimal regulation by conditioning governance regulation on size and 1-digit industry codes.

Finally, there is evidence that regulators underestimated the cost of these governance measures. There are two likely reasons for this. First, because of the fall of Arthur Andersen, the "Big Five" became the "Big Four," so past accounting fees were no longer a good prediction of future accounting fees. Further, regulation on auditor independence likely

---

[10] For example, because regulators are already paying very close attention to these firms, it would be more difficult to divert resources from these firms.

decreased firms' bargaining power. Thus, these together caused accounting costs to increase. Note that this is the same as the regulator underestimating κ in my model, which will lead to excessive levels of required governance.

Therefore, the SEC should make the following changes:

1. Lower governance requirements on the smallest firms. The positive externalities that these firms can spread to other firms is small, and the cost to small firms of constrained financing, because they delisted, is large.

2. Make governance requirements to be increasing in firm size. This curtails the contagion of executive compensation in an efficient manner. Some firms will be forced to regulate more than they wish to, because they would rather pay their CEO than to monitor him. However, the positive effect this has on the large firms outweighs the loss to the medium firms.

3. Condition required governance on industry. If an industry had no moral hazard problem, then there would be no pecuniary externality, and thus the role for governance regulation would be seriously diminished.

4.2 Impact on Real Investment Decisions

In my model, I assumed that the objective of the regulator is to maximize investor welfare. A reasonable question is why is investor welfare important? In this section, I will show that expected return is critical in determining the level of investment. Specifically, I will endogenize A, where the size of the firm $S(x) = Ax^{-a}$.

First, for a level of A, define the profit of firm x without regulation as $\Pi_{nr}(x; A)$, so

$$\Pi_{nr}(x; A) = (1 - \beta(x))S(x)T(x) - \kappa g(x)S(x)$$

Thus, for $x \geq x^*$, $\Pi_{nr}(x;A) = (1-\lambda)S(x)T(x)$, and for $x < x^*$,

$$\begin{aligned}\Pi_{nr}(x;A) &= Ax^{-a}\left(T_{Max} - \frac{B}{b}x^b - \kappa\right) \\ &\quad - \left(\frac{AB}{a-b}\left[x^{b-a} - (x^*)^{b-a}\right] + A\kappa\,(x^*)^{-a}\right)\left(1 - \frac{\kappa}{\lambda\left(T_{Max} - \frac{B}{b}x^b\right)}\right)\end{aligned}$$

Next, define the profit of firm x with optimal regulation as $\Pi_r(x;A)$, so

$$\Pi_r(x;A) = (1-\beta_r(x))S(x)T(x) - \kappa g_r(x)S(x)$$

Thus, for $x > x_1$, $\Pi_r(x;A) = (1-\lambda)S(x)T(x)$, and for $x < x_1$,

$$\begin{aligned}\Pi_r(x;A) &= Ax^{-a}\left(T_{Max} - \frac{B}{b}x^b - \kappa\right) \\ &\quad - \left(\frac{AB}{a-b}\left[x^{b-a} - (x_1)^{b-a}\right] + \lambda A x_1^{-a}\left(T_{Max} - \frac{B}{b}x_1^b\right)\right)\left(1 - \frac{\kappa}{\lambda\left(T_{Max} - \frac{B}{b}x^b\right)}\right)\end{aligned}$$

Note that $\Pi_{nr}(x;A) = A\Pi_{nr}(x;1)$, and $\Pi_r(x;A) = A\Pi_r(x;1)$, so thus profit scales in the parameter A.

Suppose that a continuum of identical investors have the choice between investing in an index fund[11] and consuming. So, by investing I_i, agent i will receive a share of $\frac{I_i}{\int_0^1 I_i di}$ of all companies. In the absence of regulation, the investor i will receive investment income

$$D_{nr}(I_i) = \frac{I_i}{\int_0^1 I_i di}\int_q^1 \Pi_{nr}(x;A)dx$$

Let the market for capital clear, so

$$\begin{aligned}\int_q^1 Ax^{-a}dx &= \int_0^1 I_i di \\ A &= \frac{\int_0^1 I_i di}{\int_q^1 x^{-a}dx}\end{aligned}$$

Thus, by scale invariance of profit, the investment income that the investor receives will be

$$\begin{aligned}D_{nr}(I_i) &= \frac{I_i}{\int_q^1 x^{-a}dx}\int_q^1 \Pi_{nr}(x;1)dx \\ D_{nr}(I_i) &= I_i R_{nr}\end{aligned}$$

---

[11] I need to assume that investors can only invest in a mutual fund for tractability.

where $R_{nr} = \frac{\int_q^1 \Pi_{nr}(x;1)dx}{\int_q^1 x^{-a}dx}$. Similarly, under optimal governance regulation, $D_r(I_i) = I_i R_r$, where $R_r = \frac{\int_q^1 \Pi_r(x;1)dx}{\int_q^1 x^{-a}dx}$. Note that $R_r > R_{nr}$. Further, investment income will be risk-free by the law of large numbers.

Thus, suppose that each investor maximizes

$$\begin{aligned} U &= u(c_0) + \beta u(c_1) \\ c_0 &= w - I_i \\ c_1 &= I_i R_g \end{aligned}$$

where R_g is the return under the prevailing governance regulation, $u' > 0$, $u'' < 0$, and u has relative risk aversion of less than 1. Substituting in, the investor's problem becomes

$$\max_{I_i} \{u(w - I_i) + \beta u(I_i R_g)\}$$

The first order condition for this problem is

$$u'(w - I_i) = \beta R_g u'(I_i R_g)$$

By differentiating both sides with respect to R_g, we find that I_i is increasing in R_g.

So, because $R_{nr} < R_r$, not only will returns under optimal investment be higher, but also that investment will be higher, because the agency costs are lower. Further, by the envelope theorem, when R_g increases by ε each investor is better off by $\beta u'(I_i R_g)\varepsilon$.

Therefore, when the regulator steps in and enforces optimal corporate governance regulation, investment increases. This is because investors make their investment decisions based upon how much they expect to get in return. Because the optimal governance regulation improves the payout to investors, more is invested and the economy is improved. This increase in efficiency is the result of reducing aggregate agency costs. Also, because R_f, the return from the choice of an optimal floor, satisfies $R_{nr} \leq R_f < R_r$, the optimal governance regulation will result in more investment than under the best governance floor, which results in more investment than the equilibrium without governance. Therefore, corporate governance regulation can have real effects, because if it is done efficiently it will increase investment by lowering agency costs. In the next section, I will discuss how this agency cost can also explain hurdle rates that seem too high.

4.3 Endogenous Hurdle Rates Premiums

A hurdle rate is often used in capital budgeting decisions. For example, Graham-Harvey (2001) finds that 56.94% of CFOs surveyed used a hurdle rate, making it the third most used technique behind net present value and internal rate of return. A hurdle rate criterion is similar to an internal rate of return, except that the project is accepted iff the internal rate of return is greater than the hurdle rate (the internal rate of return criterion uses the correct risk-adjusted rate).

The common criticism of the use of a hurdle rate is that the rates used are too high. Indeed, Meier-Tarhan (2007) documents that the average hurdle rate that firms reported in a survey was 14.1%, which is equivalent to a real rate of 11.6%. This is a premium over cost of capital of between 5% and 7.5%, depending on the level of the equity premium. Thus, firms appear to be using too large of a rate. Under the usual assumptions, this hurts firm value by biasing projects against long-term projects and by causing firms to reject profitable projects that have a smaller internal rate of return than the hurdle rate. Meier-Tarhan (2007) attribute the large hurdle rates to a number of frictions, including financial flexibility of the firm.

In this section, I will show how agency costs will lead to hurdle rates that exceed the cost of capital, and that the hurdle rate is correlated with the size of the firm, level of corporate governance, and the CEO's pay-performance sensitivity.

Suppose that after each firm enters into a contract with their CEO, which specifies the share of the firm granted to the CEO, $\beta(x)$, and the level of governance, $g(x)$, that the company unexpectedly finds the opportunity for a new project, independent of existing projects. Suppose that the project can either succeed or fail, and succeeds with probability

p. In case of success, the project pays R, but pays 0 if it fails. However, the CEO can report failure on the project and divert the cash flow R to personal consumption, which gives him private benefit $\lambda(1-g(x))$. In order to prevent him from doing this, the company must grant him a share of the new project at least as large as $\lambda(1-g(x))$. Thus, $\beta(x) = \lambda(1-g(x))$. Thus, supposing a unit discount factor, the project is taken iff it has positive payoff to investors after agency costs, which holds iff

$$\begin{aligned} p\left(R-\beta(x)R\right) &\geq I \\ \frac{pR}{(1+h(x))} &\geq I \end{aligned}$$

where

$$\frac{1}{1+h(x)} = 1-\beta(x)$$

Thus, $h(x)$ is the hurdle rate when the interest rate is 0.

First, note that $h(x) > 0$ for all x, so that all firms have a positive hurdle rate premium above the cost of capital. The reason for this is that I assume that the manager is unable to invest enough to "buy" his own share of the project, and thus the project must earn enough to not only pay back the investors' their money, but also enough to cover the agency costs. Thus, I have endogenously derived a role for the hurdle rate premium, because of agency costs. Further, this shows that h will be large when β is large and small when β is small. Therefore, we would expect that the hurdle rates will be larger at smaller companies, adjusted for risk, because smaller companies will need to devote a larger share of cash flows to agency problems. Finally, hurdle rates will be decreasing in the quality of governance and increasing in the manager's pay-performance sensitivity.

4.4 Mutual Funds and Governance

Governance is also important in mutual funds. It is commonly observed that mutual funds have poor governance, and that consumers are taken advantage of by them. This view is supported by Kuhnen (2007), which finds favoritism is seen in the relationships of fund directors and advisory firms. Many attribute poor governance to entrenchment of fund managers. Because funds are easily transferable between mutual funds, it is difficult to imagine that investors do not have other outside options.

This paper shed light on this issue. This model works for mutual funds and fund managers as well as it does for firms and executives. As small funds, it may not be worth it to exercise governance, but rather to just pay the manager generous fees. This affects large funds, however, because they must pay their managers large enough to induce him not to leave for the smaller firm. Because governance and compensation are substitutes, large funds will find it optimal to exercise less governance as well, resulting in an industry-wide lack of meaningful governance.

I should note that fund families do not alleviate this problem. Suppose that Fidelity decided to increase governance in one of their smaller funds, suboptimally, hoping to improve the profitability of the large funds. This would not work because the other small funds would still pay instead of exercise governance, still imposing the pecuniary externality on the larger funds. Thus, because there are many fund families, it is unlikely that these large families will be able to coordinate among themselves sufficiently to solve the governance problem.

4.5 Problems with Subsidies and Endogenous Governance Costs

In Section 3.2, I show that the optimal governance regulation can be implemented by granting a subsidy for governance costs. However, this might not work out quite as well in practice, because I assume that the cost of governance is $\kappa g S$, for a constant κ across firms and various regulatory environments. If we interpret the cost of corporate governance as accounting fees, the accounting firms may extract some of this benefit of this subsidy in higher rates.

Further, under governance schemes where corporate governance is required by the government, the parameter κ could change. Though the market for governance could be perfectly competitive, it very likely would have an upward sloping aggregate supply curve. Supposing that κ was a function of $\int_q^1 g(x)S(x)dx$, this would impact the optimal governance regulation. Though the optimal regulation would still take the same form, the government would just require less of it. However, matters could get even worse if the regulation gives the accounting firms market power, because then they could extract rents from firms. There is some evidence that this sort of event happened after Sarbanes-Oxley passed.

5 Conclusion

In this paper, I model a general equilibrium in the market for executives. At each firm, there is a moral hazard problem, which can be dealt with by paying the manager enough to induce truth telling or by governing the company very closely. Thus, pay and governance are substitutes. Because there is a pecuniary externality in this setting, governance regulation is optimal.

This model predicts that the compensation of CEOs will be increasing in firm size, while the pay-performance sensitivity will decrease in firm size. These are documented in Murphy (1999). The empirical finding that pay-performance sensitivity is decreasing in firm size is usually attributed to risk aversion. All parties are risk-neutral in my paper, but I find this because the largest firms choose to exercise more governance, which substitutes for pay.

Also, I find that the larger firms should exercise more governance than smaller firms, and that the optimal governance regulation forces some of the middle firms to govern more closely than it would like to in order to dampen the contagion of executive pay. Thus, the optimal governance policy should ignore the smallest firms. This regulation will be strictly increasing in the size of the firm.

If the regulator is restricted to a floor on governance, the optimal floor may be zero. Further, this will cause inefficient levels of governance to be used at some firms, because the endogenous participation constraint combined with the required level of governance will cause the incentive compatibility constraint to hold with strict inequality, implying that these firms could lower their levels of governance, improve their profit, and hurt no other firms.

Finally, I show endogenously that marginal hurdle rates will be higher at smaller, less governed firms than at larger, better governed firms, because marginal agency costs are higher at the smaller firms than at the larger firms. This explains why there are financial constraints limiting capital to smaller firms.

Two important issues that this paper does not address is endogenous governance costs and delisting. In this paper, I assume that the cost of governance is independent of regulation. However, following the passage of SOX, accounting fees have grown. Because firms are required to carry out these measures, they lose bargaining power over price. Thus, it would be interesting to incorporate this into the model, perhaps by making κ, the cost of governance, increase when regulation increases (for example, by making κ an increasing function of aggregate regulated governance expenditures). Further, in this paper, I assume that if a firm does not want to follow regulation, their only alternative is to close. Under SOX, however, firms have the option to delist, yet still operate. Thus, another interesting extension would be to model this decision, and the impact that this option has on other firms.

References

[1] Admati, Anat and Paul Pfleiderer (2000) “Forcing Firms to Talk: Financial Disclosure Regulation and Externalities,” *Review of Financial Studies*, **13**, 479-519.

[2] Bebchuk, Lucian and Jesse Fried (2004), *Pay Without Performance: The Unfulfilled Promise of Executive Compensation* (Harvard University Press).

[3] Connolly, Marie and Alan B. Krueger (2005) “Rockonomics: The Economics of Popular Music,” Princeton Working Paper.

[4] DeMarzo, Peter M. and Michael J. Fishman (2007), “Agency and Optimal Investment Dynamics” *Review of Financial Studies*, **20**, 151-188.

[5] Diamond, Douglas W. (1984) “Financial Intermediation and Delegated Monitoring,” *The Review of Economic Studies* **51**, 393-414.

[6] Dye, Ronald (1990) “Mandatory versus Voluntary Disclosure: The Cases of Financial and Real Externalities” *The Accounting Review,* **65**, 1-24

[7] Edmans, Alex, Xavier Gabaix and Augustin Landier (2007) “A Calibratable Model of Optimal CEO Incentives in Market Equilibrium,” Available at SSRN: http://ssrn.com/abstract=999096.

[8] Fishman, Michael J. and Kathleen M. Hagerty (2003) “Mandatory Versus Voluntary Disclosure in Markets with Informed and Uninformed Customers,” *The Journal of Law, Economics, & Organization*, **19**, 45-63.

[9] Frydman, Carola (2005) "Rising Through the Ranks: The Evolution of the Market for Corporate Executives, 1936-2003," MIT Working Paper.

[10] Gabaix, Xavier and Augustin Landier (2008) “Why Has CEO Pay Increased So Much?” *Quarterly Journal of Economics*, forthcoming.

[11] Gompers, Paul A., Joy Ishii, and Andrew Metrick (2003) "Corporate Governance and Equity Prices," *Quarterly Journal of Economics*, **118**, 107-155.

[12] Graham, John and Campbell Harvey (2001) "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics*, **60**, 187-243.

[13] Hermalin, Benjamin E. and Michael S. Weisbach (1998) "Endogenously Chosen Boards of Directors and Their Monitoring of the CEO," *American Economic Review*, **88**, 96-118.

[14] Hochberg, Yael, Paola Sapienza, and Annette Vissing-Jorgensen (2007) "A Lobbying Approach to Evaluating the Sarbanes-Oxley Act of 2002," NBER Working Paper 12952.

[15] Krueger, Alan (2005) "The Economics of Real Superstars: The Market for Rock Concerts in the Material World" *Journal of Labor Economics,* **23**, 1-30.

[16] Kuhnen, Camelia (2007), "Social Networks, Corporate Governance and Contracting in the Mutual Fund Industry," Kellogg School of Management Working Paper.

[17] Lacker, Jeffrey M. and John A. Weinberg (1989) "Optimal Contracts Under Costly State Falsification," *The Journal of Political Economy*, **97**, 1345-1363.

[18] Meier, Iwan and Vefa Tarhan (2007) "Corporate Investment Decision Practice and the Hurdle Rate Premium Puzzle," Loyola University Chicago Working Paper.

[19] Murphy, Kevin (1999) "Executive Compensation," *Handbook of Labor Economics.*

[20] Rosen, Sherwin (1981) "The Economics of Superstars," *American Economic Review*, **71**, 845-858.

[21] Rosen, Sherwin (1982) "Authority, Control and the Distribution of Earnings," *Bell Journal of Economics*,**13**, 311—323.

[22] Rosen, Sherwin (1990) "Contracts and Market for Executives," University of Chicago - Economics Research Center 90-12.

[23] Tervio, Marko (2007) "The Difference That CEOs Make: An Assignment Model Approach," University of California-Berkeley Working Paper.

[24] Westphal, J.D. & Stern, I. (2007) "Flattery will get you everywhere (Especially if you are a male Caucasian): How ingratiation, boardroom behavior, and demographic minority status affect the likelihood of gaining board appointments at US Companies." *Academy of Management Journal*, Forthcoming.

[25] Zhang, Ivy (2005) "Economic Consequences of the Sarbanes-Oxley Act of 2002," University of Rochester Working Paper.

A Appendix A: Proofs

Proof of Lemma 1. Pick an optimal contract, and pick a set of positive measure $A \subset Y$ where the manager would report $\widehat{y}(y;\lambda) < y$. Set $C(y;g) = \lambda(1-g)(y-\widehat{y}) + C(\widehat{y};g)$ for all $y \in A$, which induces manager to tell the truth. This improves objective by at least $[1-\lambda(1-g)]\int_A (y-\widehat{y}(y,\lambda))\,dF > 0$. This is at least, because manager might report y instead of $\widehat{y}$ when $y' > y$ is observed by manager. Because this change improves objective, the contract cannot be optimal. Further, because this argument works for $\forall A \subseteq Y$, principal would always find it optimal to inducing truth-telling. ■

Proof of Theorem 2. When $U_0 > ST$ project ceases to be profitable, since efficiency is improved by letting agent accept his outside option. When $\lambda ST \leq U_0 \leq ST$, setting $C(y) = \frac{U_0}{ST}y$, clearly satisfies the IC (with $g = 0$) and LL. This is optimal because principal cannot lower agent's expected payoff, since the IR binds.

Lastly, consider $U_0 < \lambda ST$, and suppose g is optimal level of governance. Note WLOG $C(0) = 0$, since IC is slack at 0, so LL binds. Suppose, at the solution to the above problem, $\exists z \in$support(F) such that $C(STz) = \lambda(1-g)STz + \delta$, for $\delta > 0$. The IC Constraint implies that $C(STz') \geq \lambda(1-g)STz' + \delta$ for all $z' \in \overline{Z}(z) = \{z' | z' \in \text{support}(F), z' \geq z\}$. Thus, by setting $C(STz) = \lambda(1-g)STz$, we can also decrease $C(STz')$ by δ as well, and thus improve objective by $\delta[1 - \lim_{z \to y^-} F(z)]$. Thus, $C(STz) = \lambda(1-g)STz$ almost surely in any optimal contract. The problem simplifies to

$$\begin{aligned} \max_{g} \quad & ST(1-\lambda(1-g)) - \kappa g S \\ s.t. \quad & E[C(y)] \geq U_0, g \in [0,1] \end{aligned}$$

Note that objective becomes $ST(1-\lambda) + S(\lambda T - \kappa)g$, so increasing g is beneficial to principal iff $T > \frac{\kappa}{\lambda}$. If $T \leq \frac{\kappa}{\lambda}$, $g = 0$, and thus $C(STz) = \lambda STz$. If $T > \frac{\kappa}{\lambda}$, IR binds, so $E[C(STz)] = U_0$. Because governance is costly, IC will bind, so $C(\cdot)$ will be linear, and thus $C(STz) = U_0 z$, so $g = 1 - \frac{U_0}{\lambda ST}$. ■

Proof of Lemma 3. Because $W(T(x)) \geq \sup_{x' > x}[S(x')[T(x) - T(x')] + \beta(x')S(x')T(x')]$,

$W(T(x)) \geq [S(x+dx)[T(x) - T(x+dx)] + \beta(x+dx)S(x+dx)T(x+dx)]$. Because $\beta(x)S(x)T(x) \geq W(T(x))$, $\beta(x)S(x)T(x) \geq [S(x+dx)[T(x) - T(x+dx)] + \beta(x+dx)S(x+dx)T(x+dx)]$, which implies $\beta(x)S(x)T(x) - \beta(x+dx)S(x+dx)T(x+dx) \geq S(x+dx)[T(x) - T(x+dx)]$, so $-(\beta ST)' \geq -ST'$. Thus, for $x' > x$, $-[\beta(x')S(x')T(x') - \beta(x)S(x)T(x)] \geq -\int_x^{x'} S(z)T'(z)dz$, so $\beta(x)S(x)T(x) \geq \beta(x')S(x')T(x') - \int_x^{x'} S(z)T'(z)dz > \beta(x')S(x')T(x') + S(x')(T(x) - T(x'))$, where strict inequality holds because S is strictly decreasing. Thus, local moves down are better for the agent than big moves down. Firms with $x' < x$ will not consider manager x because in order to convince firm x' to accept manager x, manager x will have to accept a smaller share than $\beta(x')$ of the new firm, and because the firm cannot increase governance, manager x would not report truthfully. This assumes $\beta(x') \leq \lambda$. If $\beta(x') > \lambda$, then the differential equation holds with equality, so for $x'' = x' + \delta$, $\beta(x')S(x')T(x') = \beta(x'')S(x'')T(x'') - \int_{x'}^{x''} S(z)T'(z)dz$, so $\beta(x')S(x')T(x') - \beta(x'')S(x'')T(x'') = \int_{x'}^{x''} S(z)(-T'(z))dz < S(x')[T(x') - T(x'')]$, so manager x'' could not induce firm x' to replace her manager with him without taking less pay than he originally had at firm x''. This logic extends to all $x > x'$. ■

Proof of Lemma 4. Consider the following function, ϕ:

$$\begin{aligned}\phi(x) &= \beta(1)S(1)T(1) + \frac{AB}{a-b}\left(x^{b-a} - 1\right) - \lambda S(x)T(x) \\ &= \lambda A(T_{Max} - \frac{B}{b}) + \frac{AB}{a-b}(x^{b-a} - 1) - \lambda A x^{-a}(T_{Max} - \frac{B}{b}x^b)\end{aligned}$$

Note that $\phi(\cdot)$ is the difference between pay required to keep manager x from switching to firm $x + dx$, given that such contagion begins at $x = 1$, and the amount firm x must pay manager x for incentive problem. Note that

$$\begin{aligned}\phi'(x) &= -ABx^{b-a-1} + \lambda a A x^{-a-1}(T_{Max} - \frac{B}{b}x^b) + \lambda A x^{-a} B x^{b-1} \\ &= Ax^{-a-1}\left[-B\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right)x^b + \lambda a T_{Max}\right]\end{aligned}$$

Note that $\phi(1) = 0$, and that $\phi'(x) > 0$ iff $B(1+\lambda\left(\frac{a}{b} - 1\right))x^b < \lambda a T_{Max}$, iff $x < \left[\frac{\lambda a T_{Max}}{B\left(1+\lambda\left(\frac{a}{b}-1\right)\right)}\right]^{1/b} \equiv x'$. Thus, for $x < x'$, $\phi'(x) > 0$, and $x > x'$, $\phi'(x) < 0$. If $\phi'(1) < 0$, then $x' < 1$, so

$\phi(x') > \phi(1) = 0$, so there exists a $x^c \in (0, x')$ such that $\phi(x^c) = 0$, so $\phi(x) > 0$ for all $x > x^c$ and $\phi(x) < 0$ for all $x < x^c$. So, when $\phi(x) > 0$, the IR from 1 is binding, so firm x must pay their manager x more than is necessary for IC constraint to keep her from going to firm $x + dx$.

Now suppose $x^* < x^c$, and consider $x \in (x^*, x^c)$. Consider IR constraint, that

$$\begin{aligned} -(\beta ST)' &\geq -ST' \\ (\beta ST)' &\leq ST' \\ \beta' ST + \beta S'T + \beta ST' &\leq ST' \end{aligned}$$

Now consider when this is satisfied by $\beta(x) = \lambda$, $\beta'(x) = 0$. The IR constraint is satisfied iff

$$\begin{aligned} \lambda S'T &\leq (1-\lambda)ST' \\ -\lambda A a x^{-a-1}(T_{Max} - \frac{B}{b}x^b) &\leq (1-\lambda)Ax^{-a}(-Bx^{b-1}) \\ \lambda A a x^{-a-1}T_{Max} - \lambda A a \frac{B}{b} x^{b-a-1} &\geq (1-\lambda)ABx^{b-a-1} \\ \lambda a T_{Max} &\geq \left(1 + \lambda\left(\frac{a}{b} - 1\right)\right) Bx^b \end{aligned}$$

which is identical to the condition on x such that $\phi'(x) > 0$. Thus, because $x < x^c < x'$, $\phi'(x) > 0$, so the IR constraint is slack for $x \in (x^*, x^c)$. ■

Proof of Proposition 5. By previous lemma, IR binds for $x \in [q, x^*)$, and IC binds for $x \in [x^*, 1]$. Thus, for $x \geq x^*$, $\beta(x) = \lambda$ and $g(x) = 0$. Also, by definition, $T(x^*) = \frac{\kappa}{\lambda}$, so for all $x < x^*$, the endogenous IR constraint will bind, so

$$\begin{aligned} \beta(x)S(x)T(x) &= \beta(x^*)S(x^*)T(x^*) + \frac{AB}{a-b}(x^{b-a} - (x^*)^{b-a}) \\ &= \lambda S(x^*)\frac{\kappa}{\lambda} + \frac{AB}{a-b}(x^{b-a} - (x^*)^{b-a}) \\ &= \kappa S(x^*) + \frac{AB}{a-b}\left(x^{b-a} - (x^*)^{b-a}\right) \end{aligned}$$

The results for $g(\cdot)$ holds because $g(x) = \max\{0, 1 - \frac{\beta(x)}{\lambda}\}$. ■

Proof of Corollary 7. Note that x^* is defined so that $T_{Max} - \frac{B}{b}(x^*)^b - \frac{\kappa}{\lambda} = 0$. Thus, differentiating with respect to T_{Max}, I find that

$$\begin{aligned} 1 - B\left(x^*\right)^{b-1}\frac{\partial x^*}{\partial T_{Max}} &= 0 \\ \frac{\partial x^*}{\partial T_{Max}} &= \frac{1}{B\left(x^*\right)^{b-1}} \end{aligned}$$

Note that this is positive. By similar logic,

$$\begin{aligned} \frac{\partial x^*}{\partial \lambda} &= \frac{\kappa}{\lambda^2}\frac{1}{B\left(x^*\right)^{b-1}} \\ \frac{\partial x^*}{\partial \kappa} &= -\frac{1}{\lambda B\left(x^*\right)^{b-1}} \\ \frac{\partial x^*}{\partial B} &= -\frac{x^*}{Bb} \\ \frac{\partial x^*}{\partial b} &= \frac{x^*}{b^2} - \frac{x^*}{b}\log(x^*) \end{aligned}$$

■

Proof of Corollary 8. Recall that, when $g(x) > 0$, $g(x) = 1 - \frac{\beta(x)}{\lambda}$, and β satisfies

$$\begin{aligned} \beta(x)S(x)T(x) &= \beta(x^*)S(x^*)T(x^*) + \frac{AB}{a-b}\left(x^{b-a} - \left(x^*\right)^{b-a}\right) \\ &= \kappa S(x^*) + \frac{AB}{a-b}\left(x^{b-a} - \left(x^*\right)^{b-a}\right) \end{aligned}$$

because $\beta(x^*) = \lambda$ and $T(x^*) = \frac{\kappa}{\lambda}$. Thus,

$$\begin{aligned} \frac{\partial \beta}{\partial \kappa} &= \frac{1}{S(x)T(x)}\left[S(x^*) + \kappa S'(x^*)\frac{\partial x_1}{\partial \kappa} + AB\left(x^*\right)^{b-a-1}\frac{\partial x^*}{\partial \kappa}\right] \\ &= \frac{A\left(x^*\right)^{-a}}{\lambda S(x)T(x)}\left[-\left(1-\lambda\right) + \kappa\frac{a}{B\left(x^*\right)^{b}}\right] \end{aligned}$$

because $\frac{\partial x^*}{\partial \kappa} = -\frac{1}{\lambda B(x^*)^{b-1}}$. Thus, $\frac{\partial \beta}{\partial \kappa} > 0$ iff $\kappa a > \left(1-\lambda\right)B\left(x^*\right)^b = \left(1-\lambda\right)b\left(T_{Max} - \frac{\kappa}{\lambda}\right)$ iff

$$\kappa\left(\frac{a}{b\left(1-\lambda\right)} + \frac{1}{\lambda}\right) > T_{Max}$$

Further, $\frac{\partial g}{\partial \kappa} = -\frac{1}{\lambda}\frac{\partial \beta}{\partial \kappa}$, thus concluding the proof. ■

Proof of Lemma 9. Note that in the no regulation equilibrium, firms $x > x^*$ set $g(x) = 0$, so the regulator cannot force such a firm to decrease governance. Suppose the regulator forces firm $\widetilde{x}$, where $\widetilde{x} < x^*$, to lower optimal governance from $g(\widetilde{x})$ to $g(\widetilde{x}) - \varepsilon$. The IC constraint forces $\beta(\widetilde{x})$ to increase to $\beta(\widetilde{x})+\varepsilon\lambda$. Because the participation constraint is binding for all $x<\widetilde{x}$, so $\beta(x)T(x)S(x)=\frac{AB}{a-b}\left(x^{b-a}-\widetilde{x}^{b-a}\right)+\beta(\widetilde{x})T(\widetilde{x})S(\widetilde{x})$. Thus, the change forces firms to switch from paying $\beta(x)$ to paying $\beta(x)+\varepsilon\lambda\frac{T(\widetilde{x})S(\widetilde{x})}{T(x)S(x)}$. Thus, the governance used by firm x decreases from $g(x)$ to $g(x)-\varepsilon\frac{T(\widetilde{x})S(\widetilde{x})}{T(x)S(x)}$. Thus, the regulation changes the profit of firm x to

$$\begin{aligned}\Pi_r(x) &= \left(1-\beta^r(x)\right)T(x)S(x)-\kappa g^r(x)S(x)\\ &= \Pi_{nr}(x)-\varepsilon\lambda T(\widetilde{x})S(\widetilde{x})\left[1-\frac{\kappa}{\lambda T(x)}\right]\\ &< \Pi_{nr}(x)\end{aligned}$$

The final inequality holds because $T(x)>\frac{\kappa}{\lambda}$ for $x<x^*$. ■

Proof of Lemma 10. By similar logic to the proof of the Lemma 7, note that a decrease in governance and increase in pay in these large firms hurts their profit (by Theorem 2) and the profits of all larger firms (by tightening their participation constraint), and leaves small firms unaffected. Thus, it is optimal to allow the firms $x<x^*$ to pay as little as possible, so the endogenous participation constraint will bind for $x<x^*$ in equilibrium under optimal regulation. ■

Proof of Proposition 11. Consider the profit of firm $x>x^*$, when forced to carry out governance $g_0(x)$. Assume that the participation constraint is nonbinding, so the firm will set $\beta(x)=\lambda(1-g_0(x))$, because they prefer to pay rather than govern by Theorem 2. Thus,

$$\begin{aligned}\Pi(x) &= T(x)S(x)-\beta(x)T(x)S(x)-\kappa g_0(x)S(x)\\ &= (1-\lambda)T(x)S(x)-g_0(x)S(x)\left[\kappa-\lambda T(x)\right]\\ &= (1-\lambda)T(x)S(x)-g_0(x)S(x)\left[\kappa-\lambda T(x)\right]\end{aligned}$$

Note that the profit in a small firm is strictly decreasing in required governance, $g_0(x)$.

Next, consider the endogenous participation constraint, $-(\beta TS)\geq -ST'$, or equivalently, $(\beta TS)' \leq ST'$. Thus, for $x \in [x^*, 1]$

$$\begin{aligned} \int_{x^*}^{x} (\beta TS)'\, du &\leq \int_{x^*}^{x} \left(-ABu^{b-a-1}\right) du \\ \beta(x)T(x)S(x) - \beta(x^*)T(x^*)S(x^*) &\leq \frac{AB}{a-b}\left(x^{b-a} - (x^*)^{b-a}\right) \end{aligned}$$

Further, because the firm will set $\beta(x) = \lambda(1 - g_0(x))$,

$$\lambda\left[1 - g_0(x)\right]T(x)S(x) \leq \beta(x^*)T(x^*)S(x^*) - \frac{AB}{a-b}\left[(x^*)^{b-a} - x^{b-a}\right]$$

Note that if this constraint is lax, the social planner could demand less governance from firm x, as well as from firms near x, by switching to the governance plan that makes the participation constraint bind. Thus, holding $\beta(x^*)$ constant at β^*, an deviation from the governance policy that makes participation constraint bind for all x such that $g(x) > 0$ results in making firms exercise inefficiently large amounts of governance without benefit. ■

Proof of Theorem 12. Note that, by the previous theorem, $\exists x_1$ so that $\forall x < x_1$, the participation constraint binds, and for $x > x_1$, $g_r(x) = 0$. First, suppose that $x_1 < 1$, so that $\beta_r(x_1) = \lambda$, and $g_r(x_1) = 0$. Thus, for $x < x_1$,

$$\begin{aligned} \beta_r(x)T(x)S(x) &= \lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left[x^{b-a} - x_1^{b-a}\right] \\ g_r(x) &= 1 - \frac{\beta_r(x)}{\lambda} \end{aligned}$$

The profit of firm $x > x_1$ is given by $\Pi_r(x) = (1-\lambda)\,T(x)S(x)$, and the profit of firm $x < x_1$ is given by

$$\begin{aligned} \Pi_r(x) &= (1 - \beta_r(x))T(x)S(x) - \kappa g_r(x)S(x) \\ &= S(x)\left(T(x) - \kappa\right) - \left[\frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right) + \lambda T(x_1)S(x_1)\right]\left(1 - \frac{\kappa}{\lambda T(x)}\right) \end{aligned}$$

Given a choice of firms, x_1, let the profit of firm x be given by $\Pi(x, x_1)$. Note that, because $\Pi(x, x_1)$ is continuous in x,

$$\frac{d}{dx_1}\int_q^1 \Pi(x, x_1)dx = \int_q^1 \frac{d}{dx_1}\Pi(x, x_1)dx$$

Also, note that $\frac{d}{dx}\Pi(x, x_1) = 0$ for $x > x_1$, because they are unaffected by this change in governance. For $x < x_1$,

$$\frac{d}{dx_1}\Pi(x, x_1) = Ax_1^{-a-1}\left\{\lambda a T_{Max} - Bx_1^b\left(1 + \lambda\frac{a}{b} - \lambda\right)\right\}\left(1 - \frac{\kappa}{\lambda T(x)}\right)$$

Thus, the derivative of the objective function, is given by

$$\frac{d}{dx_1}\int_q^1 \Pi(x, x_1)dx = Ax_1^{-a-1}\left\{\lambda a T_{Max} - Bx_1^b\left(1 + \lambda\frac{a}{b} - \lambda\right)\right\}\int_q^{x_1}\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx$$

Note that, by assumption, $\lambda a T_{Max} \geq B\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right)$, and because $x_1 < 1$, $\lambda a T_{Max} > Bx_1^b\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right)$, so the first order conditions are satisfied iff

$$\int_q^{x_1}\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx = 0$$

Note that, for all $x < x^*$, $T(x) > \frac{\kappa}{\lambda}$, so $1 - \frac{\kappa}{\lambda T(x)} > 0$, and thus $\int_q^{x^*}\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx > 0$. Further, for all $x > x^*$, $T(x) < \frac{\kappa}{\lambda}$, so $1 - \frac{\kappa}{\lambda T(x)} < 0$, thus for $z > x^*$, $\int_q^z\left(1 - \frac{\kappa}{\lambda T(x)}\right)$ is strictly decreasing in z. Thus, suppose that there exists x_1 such that $\int_q^{x_1}\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx = 0$. Note that, for all $z \in [x^*, x_1)$, $\frac{d}{dx_1}\int_q^1 \Pi(x, x_1)dx > 0$, and for all $z \in (x_1, 1]$, $\frac{d}{dx_1}\int_q^1 \Pi(x, x_1)dx < 0$. Therefore, x_1 is the unique solution to the maximization problem.

To this point, I have show that such an x_1 is unique and optimal, if it exists. If $\int_q^1\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx < 0$, such an x_1 must exist, since $\int_q^{x^*}\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx > 0$ and $\int_q^z\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx$ is strictly decreasing in z for $z \in (x^*, 1]$. Finally, allow me to consider what happens when $\int_q^1\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx > 0$.

When $\int_q^1\left(1 - \frac{\kappa}{\lambda T(x)}\right)dx > 0$, it is strictly optimal to make participation constraint bind for all $x < 1$. The only variable left is $g(1)$, the required governance of firm 1. For all $x < 1$,

$$\begin{aligned}\beta(x)T(x)S(x) &= \frac{AB}{a-b}\left(x^{b-a} - 1\right) + \beta(1)T(1)S(1)\\ g(x) &= 1 - \frac{\beta(x)}{\lambda}\end{aligned}$$

Thus, for all x,

$$\begin{aligned}\Pi(x) &= (T(x) - \kappa)S(x) - \left(1 - \frac{\kappa}{\lambda T(x)}\right)\left[\frac{AB}{a-b}\left(x^{b-a} - 1\right) + \beta(1)T(1)S(1)\right]\\ &= (T(x) - \kappa)S(x) - \left(1 - \frac{\kappa}{\lambda T(x)}\right)\left[\frac{AB}{a-b}\left(x^{b-a} - 1\right) + \lambda(1 - g(1))T(1)S(1)\right]\end{aligned}$$

Thus,

$$\frac{d\Pi}{dg(1)} = \left(1 - \frac{\kappa}{\lambda T(x)}\right) \lambda T(1) S(1)$$

Therefore, the FOC for $g(1)$ is

$$\lambda T(1) S(1) \int_q^1 \left(1 - \frac{\kappa}{\lambda T(x)}\right) dx > 0$$

Thus, when $\int_q^1 \left(1 - \frac{\kappa}{\lambda T(x)}\right) dx > 0$, it is optimal to set $g(1) = 1$. ■

Proof of Corollary 14. x_1 satisfies $\psi(x_1) = 0$, where $\psi(z) = \int_q^z \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx = 0$. Thus, by total differentiation with respect to κ at x_1,

$$\begin{aligned} -\int_q^{x_1} \frac{1}{\lambda T(x)} dx + \left[1 - \frac{\kappa}{\lambda T(x_1)}\right] \frac{\partial x_1}{\partial \kappa} &= 0 \\ \frac{\partial x_1}{\partial \kappa} &= \frac{\int_q^{x_1} \frac{1}{\lambda T(x)} dx}{1 - \frac{\kappa}{\lambda T(x_1)}} \end{aligned}$$

Thus, $\frac{\partial x_1}{\partial \kappa} < 0$ because $T(x_1) < \frac{\kappa}{\lambda}$. By similar logic,

$$\begin{aligned} \frac{\partial x_1}{\partial \lambda} &= \frac{\int_q^{x_1} \frac{\kappa}{\lambda T(x)} dx}{\frac{\kappa}{\lambda T(x_1)} - 1} > 0 \\ \frac{\partial x_1}{\partial T_{Max}} &= \frac{\int_q^{x_1} \frac{\kappa}{\lambda [T(x)]^2} dx}{\frac{\kappa}{\lambda T(x_1)} - 1} > 0 \end{aligned}$$

■

Proof of Theorem 17. Consider the tax-subsidy $\delta(x)$, where a positive number is a tax, and a negative number is a subsidy. For a firm to be willing to carry out governance, it is necessary that

$$\begin{aligned} \lambda T(x) - \kappa \left(1 + \delta(x)\right) &\geq 0 \\ \kappa \delta(x) &\leq \lambda T(x) - \kappa \end{aligned}$$

To show that there exists a self-financing tax-subsidy scheme that supports the optimal governance regulation, it is sufficient to show that the tax-subsidy scheme that makes all firms indifferent to carrying out governance makes positive revenue, because then the regulator could make all firms in $[q, x_1)$ strictly prefer governance. Therefore, suppose the regulator enforces tax-subsidy $\delta(x)$ such that $\kappa\delta(x) = \lambda T(x) - \kappa$. The net benefit to the government is

$$\begin{aligned}
\text{TaxRevenue} &= \int_q^{x_1} (\lambda T(x) - \kappa)\, g(x)S(x)dx \\
g(x) &= 1 - \frac{\beta(x)}{\lambda} \\
\beta(x)T(x)S(x) &= \lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right) \\
g(x)S(x) &= S(x) - \frac{\beta(x)S(x)}{\lambda} \\
&= S(x) - \frac{\lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right)}{\lambda T(x)} \\
\text{TaxRevenue} &= \int_q^{x_1} (\lambda T(x) - \kappa) \left(S(x) - \frac{\lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right)}{\lambda T(x)} \right) dx \\
&= \int_q^{x_1} \left(1 - \frac{\kappa}{\lambda T(x)}\right) \left[\lambda T(x)S(x) - \left(\lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right)\right)\right] dx
\end{aligned}$$

Define $\psi(x) = 1 - \frac{\kappa}{\lambda T(x)}$ and $\phi(x) = \lambda T(x)S(x) - \left(\lambda T(x_1)S(x_1) + \frac{AB}{a-b}\left(x^{b-a} - x_1^{b-a}\right)\right)$. Note $\phi(x_1) = 0$, and

$$\begin{aligned}
\phi'(x) &= \lambda T'(x)S(x) + \lambda T(x)S'(x) + ABx^{b-a-1} \\
&= Ax^{-a-1}\left[B\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right) x^b - \lambda a T_{Max} \right] \\
&< 0
\end{aligned}$$

because $\lambda a T_{Max} \geq B\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right) > B\left(1 + \lambda\left(\frac{a}{b} - 1\right)\right)x^b$. Also, note $\psi'(x) < 0$, because $T'(x) < 0$. Therefore, because $\int_q^{x_1} \psi(x)dx = 0$, $\phi' < 0$, $\psi' < 0$, and $\phi(x_1) = 0$, this implies that $\int_q^{x_1} \psi(x)\phi(x)dx > 0$. Therefore, there is strictly positive tax revenue from this tax-subsidy scheme which implements the optimal governance regulation, thus proving the existence of a self-financing tax-subsidy scheme that implements the optimal governance regulation. ■

Proof of Theorem 18. This proof is very similar to Theorem 2, and will borrow heavily from it. Because the regulator requires governance level $g(x) \geq \gamma$, if $U_0 + \kappa\gamma S(x) > S(x)T(x)$, the project is too expensive to carry out. If $\lambda(1-\gamma)S(x)T(x) \leq U_0 \leq S(x)\left(T(x) - \kappa\gamma\right)$, setting $C(y) = \frac{U_0}{ST}y$ and $g(x) = \gamma$ will satisfy the IC constraint. When $U_0 < \lambda(1-\gamma)S(x)T(x)$, by identical logic to proof of Theorem 2, the principal prefers to pay if $T(x) < \frac{\kappa}{\lambda}$ and to govern closely if $T(x) \geq \frac{\kappa}{\lambda}$. Thus, for low talented CEOs, the IC constraint will bind and the IR will be slack, and for high talented CEOs, the firm will govern close enough until both the IC and IR constraints will bind. ■

Proof of Lemma 19. This proof is similar to the proof of Lemma 4, except that we substitute $\widetilde{\lambda} = \lambda(1-\gamma)$ into the function ϕ. Thus, consider $\phi(\cdot)$

$$\begin{aligned}\phi(x) &= \beta(1)S(1)T(1) + \frac{AB}{a-b}\left(x^{b-a}-1\right) - \lambda(1-\gamma)S(x)T(x) \\ &= \lambda(1-\gamma)A(T_{Max} - \frac{B}{b}) + \frac{AB}{a-b}(x^{b-a}-1) - \lambda(1-\gamma)Ax^{-a}(T_{Max} - \frac{B}{b}x^{b})\end{aligned}$$

which is the difference between the amount needed to pay CEO x to keep him from switching to firm $x + dx$, when such contagion begins at 1, and the amount needed to pay the CEO to induce him to report truthfully given firm is exercising minimal governance, γ. By identical logic to Lemma 4, the results hold (substituting $\beta(x) = \lambda(1-\gamma)$ for $\beta(x) = \lambda$ when proving that line is flat between x^* and x^c). Finally, I need to prove that x^c is decreasing in γ. First note that $x^c = 1$ iff

$$\lambda\left(1-\gamma\right)aT_{Max} \geq B\left(1 + \lambda\left(1-\gamma\right)\left(\frac{a}{b}-1\right)\right)$$

Note that as we increase γ, the left hand side goes to zero, while the right goes to B, so as we increase γ, there exists a γ_0 so that for $\gamma > \gamma_0$, the above inequality fails, and $x^c < 1$. Next consider interior x^c. x^c is defined as the solution to $\phi(x^c) = 0$. Thus, by implicit function theorem,

$$\frac{dx^c}{d\gamma} = -\frac{\frac{\partial\phi}{\partial\gamma}|_{x=x^c}}{\frac{\partial\phi}{\partial x}|_{x=x^c}}$$

Note that $\frac{\partial\phi}{\partial x}|_{x=x^c} = \phi'(x^c) > 0$ because $x^c < x'$. (Recall that, when $x^c < 1$, x' is the unique point such that $\phi'(x') = 0$, and, by Lemma 4, for all $x > x'$, $\phi'(x) < 0$, and for $x < x'$, $\phi'(x) > 0$) Further,

$$\begin{aligned}\frac{\partial\phi}{\partial\gamma} &= -\lambda A\left(T_{Max} - \frac{B}{b}\right) + \lambda Ax^{-a}\left(T_{Max} - \frac{B}{b}x^{b}\right) \\ &= \lambda\left[S(x)T(x) - S(1)T(1)\right]\end{aligned}$$

Because S and T are strictly decreasing functions, ST is also strictly decreasing, so $S(x)T(x) > S(1)T(1)$, and thus $\frac{\partial \phi}{\partial \gamma}|_{x=x^c} > 0$, and therefore $\frac{dx^c}{d\gamma} < 0$. ■

Proof of Proposition 20. The result follows from Lemma 13 and from noting that $T(x^*) = \frac{\kappa}{\lambda}$, by definition. Everything else follows from substitution. ■

Proof of Proposition 21. β, S, T, and g are continuous, finite-valued functions on the $[q, 1]$. Further, S and T are continuously differentiable, while β and g and differentiable almost everywhere (the exceptions being at x^* and x^c). Thus, by the Fundamental Theorem of Calculus, I will be continuous and differentiable for all x, and twice-differentiable for almost all x. Solving for the optimal floor γ^*, the FOCs are given by $I'(\gamma^*) = 0$. By the Fundamental Theorem of Calculus,

$$I'(\gamma) = \int_q^1 \frac{d}{d\gamma}\Pi_\gamma(x)dx$$

because Π_γ is continuous in x. Thus, I need to split up the cases of when $x^* \leq x^c$ and when $x^* > x^c$. First, suppose that $x^* \leq x^c$. For $x > x^c$,

$$\begin{aligned} \Pi_\gamma(x) &= S(x)\left(T(x) - \kappa\gamma\right) - \lambda\left(1-\gamma\right)T(1)S(1) - \frac{AB}{a-b}\left(x^{b-a} - 1\right) \\ \frac{d}{d\gamma}\Pi_\gamma(x) &= -\kappa S(x) + \lambda T(1)S(1) \end{aligned}$$

For $x \in (x^*, x^c)$,

$$\begin{aligned} \Pi_\gamma(x) &= (1 - \lambda(1-\gamma))S(x)T(x) - \kappa\gamma S(x) \\ \frac{d}{d\gamma}\Pi_\gamma(x) &= \left[\lambda T(x) - \kappa\right]S(x) \end{aligned}$$

For $x < x^*$,

$$\begin{aligned} \Pi_\gamma(x) &= S(x)\left(T(x) - \kappa\right) - \left[\kappa(1-\gamma)S(x^*) + \frac{AB}{a-b}\left(x^{b-a} - (x^*)^{b-a}\right)\right]\left[1 - \frac{\kappa}{\lambda T(x)}\right] \\ \frac{d}{d\gamma}\Pi_\gamma(x) &= \kappa S(x^*)\left[1 - \frac{\kappa}{\lambda T(x)}\right] \end{aligned}$$

Thus, for a solution γ^* such that $x^* \leq x^c$, the FOC is that $I'(\gamma) = \int_q^1 \frac{d}{d\gamma}\Pi_\gamma(x)dx = 0$.

Further, $I'(\gamma) = \int_q^1 \frac{d}{d\gamma}\Pi_\gamma(x)dx = \int_q^{x^*} \frac{d}{d\gamma}\Pi_\gamma(x)dx + \int_{x^*}^{x^c} \frac{d}{d\gamma}\Pi_\gamma(x)dx + \int_{x^c}^1 \frac{d}{d\gamma}\Pi_\gamma(x)dx$. Thus,

$$\begin{aligned} I'(\gamma) &= \kappa S(x^*) \int_q^{x^*} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx + A\lambda T_{Max} \int_{x^*}^{x^c} x^{-a} dx - \lambda \frac{AB}{b} \frac{1}{1+b-a} \left[(x^c)^{b-a+1} - (x^*)^{b-a+1}\right] \\ &\quad + \lambda S(1)T(1)(1-x^c) - \kappa \int_{x^*}^1 S(x)dx \end{aligned}$$

Intuitively, the first element is the marginal benefit to large firms of increasing the floor, because it lowers the pay that they must pay their CEO. The second two terms is the marginal benefit to the middle firms of lowering the executive compensation. The fourth piece is the marginal benefit to the smallest firms of lowering their executive compensation. The final term is the marginal cost of governance to middle and small firms. Note that the large firms benefit from an increase in the floor, while middle and small firms are hurt. Notice also that the $I'(\cdot)$ is a function of parameters and x^c, which I proved in Lemma 13 is decreasing in γ. Thus,

$$\begin{aligned} I''(\gamma) &= A\lambda T_{Max} \left(x^c\right)^{-a} \frac{dx^c}{d\gamma} - \lambda \frac{AB}{b} \left(x^c\right)^{b-a} \frac{dx^c}{d\gamma} - \lambda S(1)T(1) \frac{dx^c}{d\gamma} \\ &= \lambda \left[S(x^c)T(x^c) - S(1)T(1)\right] \frac{dx^c}{d\gamma} \end{aligned}$$

Because $S(x^c)T(x^c) > S(1)T(1)$, $\lambda > 0$, and $\frac{dx^c}{d\gamma} < 0$, this is locally concave on the region that $x^* < x^c < 1$. Note that when $x^c = 1$, $\frac{dx^c}{d\gamma} = 0$, so $I(\cdot)$ is locally linear.

Consider when $x^c < x^*$. Then, the relevant regions are $[q, x^c]$, and $[x^c, 1]$. For $x > x^c$,

$$\begin{aligned} \Pi_\gamma(x) &= S(x)\left(T(x) - \kappa\gamma\right) - \lambda(1-\gamma)S(1)T(1) - \frac{AB}{a-b}\left(x^{b-a} - 1\right) \\ \frac{d}{d\gamma}\Pi_\gamma(x) &= \lambda S(1)T(1) - \kappa S(x) \end{aligned}$$

For $x < x^c$,

$$\begin{aligned} \Pi_\gamma(x) &= S(x)\left(T(x) - \kappa\right) - \left[\lambda(1-\gamma)S(1)T(1) + \frac{AB}{a-b}\left(x^{b-a} - 1\right)\right]\left[1 - \frac{\kappa}{\lambda T(x)}\right] \\ \frac{d}{d\gamma}\Pi_\gamma(x) &= \lambda S(1)T(1)\left[1 - \frac{\kappa}{\lambda T(x)}\right] \end{aligned}$$

Again, because $I'(\gamma) = \int_q^1 \frac{d}{d\gamma}\Pi_\gamma(x)dx = \int_q^{x^c} \frac{d}{d\gamma}\Pi_\gamma(x)dx + \int_{x^c}^1 \frac{d}{d\gamma}\Pi_\gamma(x)dx$,

$$I'(\gamma) = \int_{x^c}^1 \left\{\lambda S(1)T(1) - \kappa S(x)\right\} dx + \lambda S(1)T(1) \int_q^{x^c} \left\{1 - \frac{\kappa}{\lambda T(x)}\right\} dx$$

Again, the first term is the net marginal benefit to smaller firms of increasing the floor, which is negative. The second term is the net marginal benefit to large firms of increasing the floor, which is positive. For the second order conditions,

$$\begin{aligned} I''(\gamma) &= -\left[\lambda S(1)T(1) - \kappa S(x^c)\right]\frac{dx^c}{d\gamma} + \lambda S(1)T(1)\left[1 - \frac{\kappa}{\lambda T(x^c)}\right]\frac{dx^c}{d\gamma} \\ &= \kappa A\left[(x^c)^{-a} - \frac{T(1)}{T(x^c)}\right]\frac{dx^c}{d\gamma} \end{aligned}$$

Note that $x^c < 1$ and $a > 0$, and $T(1) < T(x^c)$, so $(x^c)^{-a} > 1 > \frac{T(1)}{T(x^c)}$, and $\kappa > 0$, $A > 0$, and $\frac{dx^c}{d\gamma} < 0$, so $I''(\gamma) < 0$, and thus I is locally concave on $x^c < x^*$. Finally, note that for $x^c = q$, I is locally linear, because $\frac{dx^c}{d\gamma} = 0$.

To complete the proof, allow me to define γ_0, γ_1, and γ_2. Define γ_0 as the floor of governance such that for all $\gamma < \gamma_0$, $x^c(\gamma) = 1$, and for $\gamma > \gamma_0$, $x^c(\gamma) < 1$. Define γ_1 such that for $x^c(\gamma_1) = x^*$. (Recall that x^* does not depend on γ, because $T(x^*) = \frac{\kappa}{\lambda}$.) Finally, define γ_2 such that for all $\gamma < \gamma_2$, $x^c(\gamma) > q$, and for all $\gamma > \gamma_2$, $x^c(\gamma) = q$. These governance levels, γ_0, γ_1, and γ_2 are unique by Lemma 13. Further, by above section of this proof, I have shown that I is linear on $[0, \gamma_0)$, strictly concave on (γ_0, γ_1), strictly concave on (γ_1, γ_2), and linear on $(\gamma_2, 1]$. Thus, to prove global concavity, all I need to show is that

$$I'_+(\gamma_1) \leq I'_-(\gamma_1)$$

because then $I'(\cdot)$ will be a weakly decreasing function. Recall that $I'_-(\gamma_1) = \lim_{\gamma\to\gamma_1^-} I'(\gamma)$, and $I'_+(\gamma_1) = \lim_{\gamma\to\gamma_1^+} I'(\gamma)$. Note that $\lim_{\gamma\to\gamma_1} x^c(\gamma) = x^*$. Thus,

$$\begin{aligned} I'_-(\gamma_1) &= \kappa S(x^*)\int_q^{x^*}\left\{1 - \frac{\kappa}{\lambda T(x)}\right\}dx + \lambda S(1)T(1)(1 - x^*) - \kappa\int_{x^*}^1 S(x)dx \\ I'_+(\gamma_1) &= \int_{x^*}^1 \left\{\lambda S(1)T(1) - \kappa S(x)\right\}dx + \lambda S(1)T(1)\int_q^{x^*}\left\{1 - \frac{\kappa}{\lambda T(x)}\right\}dx \end{aligned}$$

Therefore, I will be globally concave iff $I'_-(\gamma_1) - I'_+(\gamma_1) \geq 0$.

$$\begin{aligned} I'_-(\gamma_1) - I'_+(\gamma_1) &= \left[\kappa S(x^*) - \lambda S(1)T(1)\right]\int_q^{x^*}\left\{1 - \frac{\kappa}{\lambda T(x)}\right\}dx \\ &= \lambda\left[S(x^*)T(x^*) - S(1)T(1)\right]\int_q^{x^*}\left\{1 - \frac{\kappa}{\lambda T(x)}\right\}dx \end{aligned}$$

The second line holds because $T(x^*) = \frac{\kappa}{\lambda}$. Thus, because $\lambda > 0$, $S(x^*)T(x^*) > S(1)T(1)$, and for all $x < x^*$, $T(x) > \frac{\kappa}{\lambda}$, so $\left\{1 - \frac{\kappa}{\lambda T(x)}\right\} > 0$, $I'_{-}(\gamma_1) - I'_{+}(\gamma_1) > 0$, and thus I will be globally concave.

Therefore, without loss of generality, $\gamma^* \in \{0\} \cup (\gamma_0, \gamma_2] \cup \{1\}$. Thus, whenever the regulator imposes a governance floor $\gamma^* > 0$, $x^c > 0$ and thus inefficient governance is being carried out at the smallest firms. ■