Knowledge (Getting Down to "Basics")

Plato 427-347 BC *The Republic*

If we attempt to understand why things are as they are, and what general categories can be used to understand various particulars around us, without reference to any forms (universals) we will fail completely.

Aristotle 384-322 BC *Posterior Analytics*

All demonstration must be founded on principles already known. The principles on which it is founded must either themselves be demonstrable, or be "first principles," which cannot be demonstrated, nor need to be, being evident in themselves.

First principles are derived by induction, from the sense-perception implanting the true universals in the human mind.

The Axiomatic Method (Logical Deduction)

Euclid c.300 BC The Elements

- 1. Any two points can be joined by a straight line.
- 2. Any straight line segment can be extended indefinitely in a straight line.
- 3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
- 4. All right angles are congruent.
- 5. Parallel postulate. If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

Digression: The Scientific Method (Experiential Induction)

Ibn al-Haytham 1021 Book of Optics

- 1. Explicit statement of a problem, tied to observation and to proof by experiment
- 2. Testing and/or criticism of a hypothesis using experimentation
- 3. Interpretation of data and formulation of a conclusion using mathematics
- 4. The publication of the findings

Roger Bacon 1266 Opus

The four causes of error:

- 1. Authority ("The Boss/Leader/Book says so!")
- 2. Custom ("We've always done it this way.")
- 3. The opinion of the unskilled many ("Everybody says so!")
- 4. The concealment of real ignorance with the pretence of knowledge ("Based on my credentials and vast experience, ..." or "Well, our consultants say ...")

The Scientific Method: Observe, hypothesize, (infer), verify or falsify.

Back to the Axiomatic Method: In Mathematics ...

The traditional approach, in which axioms were supposed to be self-evident and so indisputable, was swept away during the course of the nineteenth century, by the development of non-Euclidean geometry, the foundations of real analysis, Cantor's set theory and Frege's work on foundations, and Hilbert's 'new' use of the axiomatic method as a research tool.

John von Neumann 1925 The Axiom of Foundation:

Every non-empty set *A* contains an element *B* which is disjoint from *A*.

Two results which follow from the axiom are that "no set is an element of itself", and that "there is no infinite sequence (a_n) such that a_{i+1} is an element of a_i for all i".

But in the Social Sciences ...

Thomas Jefferson et al 1776 The Declaration of Independence

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. --That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, --That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness.

John von Neumann and Oskar Morgenstern 1944 *Theory of Games and Economic Behavior*, (following Bernoulli 1738): Utility

John Nash 1950 The Bargaining Problem: Arbitration

Kenneth Arrow 1951: Social choice

Lloyd Shapley 1953: Fair allocation and (with Martin Shubik) political power

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