**KELLOGG SCHOOL OF MANAGEMENT**

**Strategic Decision-Making**

DECS-452 Professor Bob Weber
Week #7

1. Please fill out the response sheet at the end of the Maxco-Gambit case, and bring it to class on Thursday/Friday of Week 8. In doing this exercise, **please** remember that the "bid-a-half" strategy we derived for the earlier auction experiment was for a *very* different situation than Gambit is facing here. (I'm not asking you to do any calculations. Just fill in what seems to you like a reasonable strategy.)

2. Read the attached material. In particular, the next page contains the one-page course summary that I used to save for Week 10.

3. Begin working on the final project.

**Strategic Decision-Making: A Capsule Summary**

"All the world's a game, and all the people in it merely players."

In any strategic setting, try your best to anticipate the others' choices of actions. Optimize against those anticipated actions. In order to anticipate the actions of others, look at the strategic problem from the others' perspectives.

When trapped in a disadvantageous strategic situation, consider the value of grabbing the initiative, i.e., making a precommitment.

When holding private information, give consideration to what you *would* be doing, if you held other information than that which you *do* hold. (This is the only way to predict how others will interpret your observed actions, and whether a bluff or signal will work.)

When others are ineffectively trying to give you a signal (i.e., when you can't be certain that the ineffective signal is not a bluff), look at the problem from their perspective, find an effective signal, and challenge them to give it.

In ex ante symmetric settings (i.e., where you differ from the others only in the private information you hold, and where those others also hold private information), test your planned strategy against itself.

Don't lose sight of your best alternative to resolution of the immediate conflict; at times, threats (e.g., "walking away") are justified even if there is some chance that you will have to carry them out (and suffer a small prospective loss).

When the actions of others determine whether or not you will achieve some objective (buy the car, acquire the firm, win the auction), and when those others hold information concerning the value of the objective to you, recognize (and adjust your actions to protect against) adverse selection / the "Winner's Curse."

When short-term strategic considerations work against both your long-term interests and the interests of the others (i.e., when caught in a repeated Prisoners' Dilemma), look for ways to make short-term cooperation verifiable, so "Tit-for-Tat" can be played.

When negotiating with others, look for differences (in time preferences, or risk attitudes, or beliefs, or capabilities, or tastes) through which value can be created.

Don't fall prey to your own psychological biases (anchoring, updating beliefs incorrectly, behaving as if you are more risk-averse than you really are); exploit those biases in others to the extent you consider appropriate.

Recognize that in (rational) markets for objectively-valued commodities, if everyone has equal standing (i.e., if the market is "symmetric") then only those with private information can expect to profit. When facing a "War of Attrition," look for ways to "de-symmetrize" the situation.

Beware the opponent who may do the "unthinkable." Others may view ethical and/or legal constraints differently than you.

**Strategic Decision-Making (DECS-452 Course Outline)**

Multi-issue negotiations (the Riverside/DEC case)

1. Value-creation vs. value-claiming: the "Negotiator's Dilemma"

2. Value is created through differences
   a) In relative valuations: Trades can be arranged
   b) In beliefs: Contingent agreements (i.e., mutually-desired gambles) can be arranged
   c) In risk attitudes: Insurance can be arranged
   d) In time preferences: Intertemporal agreements (e.g., loans) can be arranged
   e) In capabilities: Positive synergies can be jointly exploited

3. The importance of contingent offers
   a) Giving information in a way that guarantees receiving information
   b) Breaking the single negotiation into a series of small steps, in an attempt to avoid the Negotiator's Dilemma through repetition.

4. Third-party intervention
   a) Mediation takes away the "curse" of having to move first
      1) The single negotiating text as a mediation tool
      2) Post-settlement settlements (lock in some gains early, to limit exposure of each party to opposing claiming tactics later)
      3) Mediators care about getting *a* deal, and are therefore exploitable
   b) Joint audit agreements can eliminate some sources of incomplete information
   c) Contingent contracts (e.g., usage charges) can defer specifics until information becomes observable
   d) Arbitration
      1) Classical "binding" arbitration
         i) Arbitrators tend to split differences, encouraging initial extreme positions
      2) Final-offer arbitration: Can lessen the need to go to arbitration
         i) In Major League Baseball in 2012, 142 players filed for salary arbitration early in the offseason. Ultimately, most players avoided hearings with one-year deals, 17 signed contract extensions, and only seven went to arbitration hearings.
   e) The Paradox of Truth: The only way to get parties to fully reveal themselves is to assure them that the revealed information will not be fully used.

**Preferences and Utility**

Do you agree that -

For any two occurrences A and B, you will either prefer A to B, or prefer B to A, or be indifferent between them ?

If you prefer A to B and B to C, then you'll prefer A to C ?

If you're indifferent between A and B, and between B and C, then you'll be indifferent between A and C ?

If you prefer A to B and B to C, then there is **some** probability p such that you'd be indifferent between B, and a "lottery" that gives you a chance p of getting A and a chance 1-p of getting C ?

If you're indifferent between occurrences A and B, and if A is one possible outcome of a lottery, then substituting B for A as an outcome of the lottery will not affect your preferences ?

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The preferences of a rational decision-maker can be represented by a **utility function** (the choice of origin and scale is arbitrary - all else is then determinate); decisions should be evaluated in terms of the expected utility of the outcome. If the utility curve is linear over the range of possible outcomes, this is equivalent to evaluating decisions in terms of expected payoff; if the curve is upward-sloping but downward-bending over the range of outcomes, the decision-maker is “risk-averse.” (Over a wide range of possible outcomes this is typically the case; e.g., for most people, the increase in utility in moving from their current wealth level to a level $1,000,000 higher is greater than the increase in moving from +$1,000,000 to +$2,000,000.)

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Would you rather make a personal investment (A) with a guaranteed profit of $3000, or an investment (B) with an 80% chance of yielding $4000 profit (and a 20% chance of returning no profit)?

Would you rather make a personal investment (C) with a 25% chance of returning $3000 (and a 75% chance of returning no profit), or an investment (D) with a 20% chance of returning $4000 (and an 80% chance of returning no profit)?

To choose A and D is inconsistent with the "substitution" property stated above: C is equivalent a 25% chance of receiving A, and D is equivalent to a 25% chance of receiving B. The fact that many people *do* choose A and D is known as the "Allais Paradox."

Lesson: The way we *actually* make decisions in the face of uncertainty is not necessarily the way we would *like* to be making those choices.

## The Nash Arbitration Scheme

Given complete information about (1) the available alternatives, (2) the preferences of the two parties, and (3) both parties' best alternatives to agreement, should an arbitrator

- make a decision only on the basis of this information?
- choose an alternative which is Pareto-undominated (i.e., which cannot be improved upon for one of the parties at no cost to the other)?
- not force upon the parties an agreement which leaves one worse off than if no agreement had been reached?
- choose symmetrically when the original problem is symmetric?
- not change his mind if he learns that an agreement which he was not going to choose anyway was in fact not feasible?

If so, then the arbitrator *must* always choose the agreement which maximizes the product of the parties' utility gains.

## Arrow's Impossibility Theorem

A **social choice procedure** associates a preference ordering (for society) with each collection of individual preference orderings.

Assume there are at least three alternatives, and at least two individuals. If a social choice procedure is monotonic (i.e., if raising an alternative in some individual's ranking never lowers its social ranking), if the individuals as a group can put any alternative "on top", and if the social ranking of any two alternatives depends only on the individual rankings of those two, then one of the individuals must be a dictator.

## 5. **The role of time**

In the previous chapter, we explored the range of settlements which could be obtained at equilibrium through *any* mechanism for negotiations, and we found that all such settlements could be obtained through the use of a one-stage simultaneous-type-revelation procedure.

However, in actuality negotiations typically take place over time, with the parties gradually revealing themselves through their actions. We shall look in turn at negotiations over time when there is no information to be revealed, at protracted negotiations during which the parties accrue payoffs which depend on their stage-to-stage actions, and finally at negotiations in which information is revealed over time, but the only payoff comes at the conclusion.

### 5.1 **Complete information**

We have already seen that there is no impediment to parties reaching a Pareto-efficient outcome when there is no uncertainty. If we assume that bargaining takes place over time, and that there is a cost associated with delay in reaching an agreement, then the only efficient behavior must involve agreement being reached essentially immediately.

**Example 11 (the Rubinstein offer-counteroffer model)**. Consider a seller, holding an object worth nothing to him, and a buyer who values the object at $300. Both valuations are known to both parties. They negotiate through the exchange of offers: First the seller proposes a sale price, and the buyer either accepts that price, or rejects it. If he rejects it, then he follows with a counter-proposal, stating another price. The seller can either accept this new price, or follow with yet another proposal. And so on...

Assume that one unit of time passes between any rejection and the subsequent proposal. One model of the cost of delay in reaching an agreement involves discounting the payoffs of both parties in the final agreement. Specifically, let $d_S$ and $d_B$ be discount rates between 0 and 1, and assume the payoff to the seller if the final agreement is a price of p in the t-th stage is $d_S^{t-1} \cdot p$ , while the payoff to the buyer is $d_B^{t-1} \cdot (300-p)$.

There are many equilibrium pairings of strategies in this bargaining game. For example, the seller can ask for $290 in every stage, accepting a counter-offer only if it is at least $290, and the buyer can accept any price at or below $290, making a counter-offer of $1 whenever the seller asks for more than $290. (Recall that a strategy for a party must specify his action in any situation he might face.) This pairing leads to a sale at $290 in the first stage; neither can do any better, as long as the other holds to his own specified strategy. Similarly, there are other equilibria which yield immediate sales at *any* price between $0 and $300.

However, the specified strategies call for foolish actions from the parties in certain circumstances. For example, suppose the seller opens with a proposal of $290, and the buyer rejects this offer, making a counter-offer of $289. (While this will not happen if they follow the specified strategies, the seller must be prepared for this possibility.) If $289 is greater than $d_S \cdot$ $300, it would be foolish for the seller to reject this counter-offer: He cannot expect to improve his lot by continuing the game through another stage. Hence, the specified pair of strategies calls for non-optimal actions in some "subgames" which arise off the equilibrium path.

An equilibrium point is said to be *perfect* if the parties' strategies call for optimal behavior in every subgame which might arise. Remarkably, if we restrict our attention to perfect equilibria in this sequential game, we find that there is only one. Let

$$p_S = \frac{300(1 - d_B)}{1 - d_B d_S}, \text{ and } p_B = p_S d_S .$$

The unique perfect equilibrium has the seller asking for $p_S$ initially, and at every subsequent stage accepting any price of $p_B$ or more. He rejects any lower counter-offer, again asking for $p_S$. The buyer accepts any price of $p_S$ or less, rejecting higher prices and counter-offering a price of $p_B$. In this equilibrium, the sale takes place in the first stage, at a price of $p_S$.

If the discount factors are equal, and very close to 1 (i.e., if the interval between successive stages is quite short, so the cost of delayed agreement is small), then $p_S$ will be very near $150. More generally, if the parties face any bargaining problem without uncertainty, and the proposals and counter-proposals consist of feasible agreements, then the unique perfect equilibrium outcome is immediate settlement on an agreement which, when $p_S = p_B$ and both are near 1, is near the Nash solution to the bargaining problem. (This is a "limit" result.) One might view this as further validation of the Nash solution as a "natural" result of negotiations.

Short of the limit, the sale price is somewhat above $150. This is because it is to the seller's advantage to move first; if the buyer were given the first move, the price would be somewhat below $150.

Note that $p_S$ is increasing in $d_S$, and decreasing in $d_B$. The less costly it is to either party to wait, the better off he is in the perfect equilibrium outcome. This accords well with common perception: In negotiations, patience is a virtue.

The parameters $d_S$ and $d_B$ can be given an alternative interpretation. Assume that there is no cost to waiting, but that there is a probability of $1\text{-}d_B$ that the seller will walk away from the negotiations any time he makes an offer which is rejected, and a probability of $1\text{-}d_S$ that the buyer will walk away any time one of *his* offers is rejected. The same, unique perfect equilibrium persists in this new setting: The more likely a party is to walk away if one of his offers is rejected (more precisely, the more likely his opponent *perceives* his departure to be), the better off he is at the equilibrium outcome.

## 5.2 Repeated games with incomplete information

Over the past twenty years, many researchers have studied the repeated play of a game, when the players' interests are strictly opposed and each holds private information. The most-commonly-studied model is one in which the parties are uncertain of the payoff structure of the game; the actions of both parties are publicly revealed at the end of each period, but neither side learns the payoffs; payoffs accumulate over time. While this is not too accurate a model of bargaining (where there usually is an end to the negotiations, and the terminal payoff is much more important than intermediate payoffs), still, investigation of this model provides insight into the way a party's opponent can learn about him from observation of his actions, and therefore, about how the party's actions should be chosen.

A principal result is that optimal strategies typically involve a single initial reference to the information a party holds, followed by period-to-period moves which depend only on the outcome of that single reference. An analogue of this in actual negotiations is the initial briefing of a representative, at which time he is given only the information the party he represents is prepared to reveal in the course of the negotiations; subsequently, no further information is revealed to him (and hence, his choice of actions during the negotiations can reveal no more than the information he is given at the briefing).

Recently, Hart has extended this analysis to games with private information on one side, and gains available to the players through cooperative actions. Hart's result is that, when mutual gains are available, it is frequently necessary to partially brief a representative, send him to the negotiating table, and (depending on the course of the negotiations) to periodically recall the agent for further briefings. This work provides some insight into the process observed in international arms control negotiations.

### 5.3 **Bargaining with incomplete information**

Attempts to extend the Rubinstein offer-counteroffer analysis (with time-discounted payoffs) to bargaining problems with incomplete information have met with difficulties. The most successful approach to date is that of Grossman and Perry; the difficulty they encounter is that such games have many equilibria, and the natural analogue of the "perfection" argument in the Rubinstein model is not clear. An active area of current research is the development of equilibrium selection techniques, to be applied in order to obtain a single "special" equilibrium of such games. However, the predictive appeal of such models is questionable, given the numerous ways in which individuals could attempt to affect the selection process.

In general, negotiations which unfold over time vary in many dimensions: the nature of the information initially held by the parties, the channels of communication, the costs of delayed agreement or conflict, the types of settlements which are feasible - differences in these dimensions lead to problems requiring substantially different analytical approaches. We give here a simple example which has been used by various authors as a model of courtship behavior, primitive tribal customs, military escalation, and strikes. In this example, there is competition for an indivisible prize. Each party knows his own valuation of the prize, communication is limited to observation of the other's intransigence, costs of delayed settlement are opportunity losses, which accrue to both parties linearly over time, and the only possible agreements require total concession by one party.

**Example 12 (The War of Attrition)**. Two parties compete for possession of a prize. The two private valuations are independent draws from a commonly-known distribution, and each party knows only his own valuation. The parties face one another passively, suffering a constant loss per unit time. Competition ends when one party withdraws; each pays his accrued loss (the same amount for each), and the remaining party claims the prize.

Milgrom and Weber have studied this situation, and obtained the following results. (1) There is a unique symmetric equilibrium point for this game. (2) At equilibrium, there is a positive probability that competition continues for so long that both the loser *and* the winner suffer net losses. (Of course, at equilibrium each has a non-negative *expected* profit; otherwise, one of them could gain in expectation by withdrawing immediately.) (3) If it is commonly known that both parties have exactly the same valuation, then at equilibrium both have expected payoffs of 0 (i.e., on average, they will "compete away" the entire value of the prize). (4) The greater the likely difference in valuations, the greater the expected payoffs to both. (Or, as the French say, "Vive la difference!") (5) The longer competition endures, the more likely it is to continue. (The outlook for settlement grows steadily bleaker over time. In the strike interpretation, this result provides some justification for a policy of delayed government intervention.)

## 6. **Summary**

Game-theoretic studies of bargaining problems (as well as of other types of conflict situations) have helped to clarify our understanding of the role of private information and the nature of strategies in competitive environments. These studies have also provided a formal structure for the investigation of issues involving efficiency and equity, and have helped to delineate the roles played by intervenors (and the limitations intervenors must acknowledge).

### 6.1 **Research prospects**

One active area of research deals with equilibrium point selection and classification; a goal of this research is to explain why some types of equilibrium behavior are observed more frequently than others. Another area concerns the evaluation and comparison of alternative frameworks for dispute resolution: For example, one might ask how different rules for the allocation of court costs and legal fees in civil suits affect pre-trial settlement behavior. Several technical problems remain to be solved before current analytical techniques can be extended to cover problems involving multiple dimensions of uncertainty, or multiple negotiation stages.

Most of this paper has discussed problems of two-party bargaining. There is a rich history of game-theoretic research into multi-party issues, but most of this work has focused on issues of stability (the core, bargaining set, von Neumann-Morgenstern solution, and the like) and equity (the Shapley value). Little is yet known about the dynamics of coalition formation (and dissolution) in multi-party negotiations.

## 6.2 **Lessons to be remembered**

In conclusion, what are the principal messages game theory has to deliver to practitioners?

For negotiators, there are two: Realize that, when you hold private information, it is important to consider what actions you *would* have taken, had your information been different than it actually is. Carefully consider what strategy you expect your opponent to follow, and whether this expectation is justified. (In particular, if your own strategy, together with your opponent's, does not form an equilibrium point, ask yourself why. Are you expecting him to be less clever than you?)

For intervenors: Be aware of incentive constraints, and their role in occasionally leaving the parties in an irreconcilable position of conflict. (Strikes, for example, are always non-optimal *ex post*; still, the threat of a strike is an essential component of the labor-management bargaining process. The intensity of the threat can reveal useful information, but only if there is a chance that it will have to be carried out.) Provide opportunities for "safe" revelation of information to you; seek means for auditing statements made to you, or for conditioning the contract on future information or behavior.

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**Bibliographic Notes**

Primary sources for general information on the tools and models of game theory are Luce and Raiffa [1957] and Shubik [1982]; Shelling [1960] provides interesting commentary and additional perspectives.

Raiffa [1982] presents a mixture of decision-analytic and game-theoretic views on negotiation; Roth [1985] collects a number of survey and research papers covering the focus of current research activity.

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# MAXCO, INC. AND THE GAMBIT COMPANY

## Part I

Maxco, Inc. and the Gambit Company were fully-integrated, major oil companies each with annual sales over $1 billion and exploration and development budgets over $100 million. Both firms were preparing sealed bids for an oil rights lease on block A-512 off the Louisiana Gulf coast. Although the deadline for the submission of bids was only three weeks away, neither firm was very close to a final determination of its bid. Indeed, management at Maxco had yet to decide whether to bid at all, let alone how much to bid. Although Gambit was virtually certain to submit a bid, the level of Gambit's bid was far from settled. This uncharacteristic hesitancy in the preparation of both firm's bids was a direct result of certain peculiarities in the situation surrounding the bidding for block A-512.

Block A-512 lay in the Alligator Reef area immediately to the south of a known oil-producing region (see Exhibit 1). Just to the north were blocks A-497 and A-498, both of which were already under lease to the Gambit Company. On its leasehold Gambit had two completed wells that had been in production for some time. In addition Gambit had an offset control well in progress near the boundary between its leasehold and block A-512. When this well was completed, Gambit would have access to direct information concerning the value of any oil reserves lying beneath block A-512. Maxco's nearest leasehold, on the other hand, was some seven miles to the southeast. Any bid submitted by Maxco, therefore, would necessarily be based solely on indirect information.

### The Role of Information in Bidding for Oil Rights Leases

In any bidding situation, information concerning either the object of the bidding or the notions of competing bidders is highly prized. This is even more the case in bidding for the rights to oil reserves lying, perhaps, thousands of feet below the surface. There are, of course, various kinds of information available to bidders for oil rights. To summarize these various types of information briefly, two categories—direct and indirect—may be established.

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[T]his case was prepared by Mr. Donald L. Wallace, Research Associate under the direction of Associate Professor John S. Hammond as a basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation. It has developed from a study by Professor Donald H. Woods, Georgia State University.

Information obtained by drilling on a parcel of land is called direct information. Obviously this is the most precise information obtainable concerning the subsurface structure. From core samples taken up during the drilling operation, and from careful laboratory analysis of these samples, considerable information may be accumulated not only about the presence or absence of oil, but also about the type, thickness, composition, and physical properties of each of the various geological strata encountered. Such information then provides the driller with a relatively accurate estimate of the oil reserves lying beneath the parcel. Direct information concerning adjacent parcels may be obtained by drilling offset control wells. These wells are offset from the principal producing areas and are located near the boundaries of the leased parcel. Such wells may then provide a particular lessee with precise and valuable information about adjacent parcels.

Indirect information is obtained from sources other than drilling and may be roughly divided into two kinds: scouting and nonscouting. Scouting information is gained by observing the operations of other drillers. By counting the sections of drill pipe—each of known length—introduced into a hole, an observer may infer the depth of the hole. By observing the quantity of cement—required by law—used to plug the various porous strata that are encountered, the thicknesses of these strata may be determined. Normally, however, this type of scouting information will not yield nearly the precision available to the driller himself. It can help in the determination of whether or not oil reserves exist at a particular location, but it is much less useful in determining the size of the reserves.

More definite scouting information may sometimes be obtained by more clandestine means. Eavesdropping on informal conversations in public places, subtle forms of bribery and interrogation, even forcible entry onto a competitor's drilling site may provide much more detailed—and more valuable—information. An extreme anecdote tells of two men caught while inspecting a competitor's drilling log—the source document of a driller's direct information. The men were reportedly held at gunpoint for several days in anticipation of the approaching deadline for the submission of bids. Managing to escape, the day before the deadline, the two men were able to report back what they had seen in the log. As a result, the operator whose log had been compromised was forced to raise his bid by $7 million.

Less melodramatic, but highly significant, sources of indirect information are available through means other than scouting. Nonscouting information is obtained, first, from published sources such as government geological and geophysical surveys, and reports of previous explorations. Second, nonscouting information may be obtained from local seismic surveys conducted either by in-house personnel or private contractors. A third source of nonscouting information is found in the trading of dry hole information. The tradition among drilling operators is to reveal their dry hole experiences. The feeling seems to be that there is far more to be gained from the reciprocal exchange of dry hole information than could be gained from watching a competitor pour a considerable investment into a site that in known to be barren. Finally, nonscouting information may also be obtained from independent prospectors, promoters, and traders who may have become familiar with certain tracts in the past and are willing to trade this information, again on a reciprocal basis.

As might be suspected in an environment where information has such a high—and immediate—value, internal security presents a clear and ever-present problem. Bank-type vaults, armed guards, and electrified fences are, commonplace. On occasion, entire drilling rigs have been encased in canvas to thwart the efforts of prying eyes. Substantial slowdowns in operations, however, under almost unbearable working conditions have also resulted. Furthermore, a blanket of security must also be placed over the derivation and submission of bids. Information on the level of a particular bid can be even more valuable than information on the value of reserves.

When bids were being prepared for the tracts surrounding Prudhoe Bay on Alaska's North Slope, one company packed its entire bidding organization onto a railroad train and ran it back and forth over the same stretch of track until bids had been prepared and submitted and the bidding deadline had passed.

Finally, with information such a prime concern, circulation of false information is often attempted. If an operator is successful in leaking false negative information about a particular parcel, he may be able to later "steal" the parcel with a relatively low bid. On the other hand, to divert attention from a particular parcel, an operator may feign interest in another one by seeming to conduct tests there.

**Maxco's Bidding Problem**

Mr. E. P. Buchanan, Vice President for Exploration and Development had primary responsibility for preparing Maxco's bid. Mr. Buchanan's information of block A-512 was, as indicated previously, indirect in nature. Although some scouting information on Gambit's offset-control well was available to him, the primary basis of his information was a private seismic survey, together with published government geological maps and reports. Maxco had acquired the survey data, in a jointly-financed effort with Gambit, through the use of a private contractor. The contractor, Noble and Stevens, had prepared a detailed survey of the entire Alligator Reef area several years previously when blocks A-497 and A-498 were up for bid. Under the joint financing arrangement, identical copies of the completed report had then been submitted to both Maxco and Gambit. Such an arrangement, while unusual, was not without precedent in known oil-producing areas. Exhibit 1 represents an updated version of a subsurface map included in Noble and Stevens' report.

Based on all of the information available to him, Mr. Buchanan's judgment concerning the monetary value of the oil reserves under block A-512 was essentially captured by the probability mass function given in Exhibit 2. Furthermore, Mr. Buchanan held that Maxco's bid should be based solely on this monetary value of the oil reserves. Since it was known that no nearby blocks were to be put up for bid for at least 10 years, Mr. Buchanan did not ascribe any informational value to owning a lease on block A-512.

Mr. Buchanan also felt—for the present at least—that Gambit's uncertainty was virtually identical to his own. He was sure, however, that Gambit's well would be completed by the deadline for the submission of bids. At that time Gambit would know the value of the reserves up to perhaps, plus or minus 5% or 10%.

For the past several years, Mr. Buchanan had refused to bid on any parcels of land where he felt he was at a distinct disadvantage to a competing bidder. If a competitor had superior (direct) information about a parcel while Maxco had only indirect information, then Mr. Buchanan preferred not to bid at all.

Less than five months ago, however, in an area not far from Alligator Reef, Mr. Buchanan had lost a bid on a block adjacent to a Maxco leasehold. Maxco had gone to the expense of drilling an offset control well on its own block and had found a reasonably large oil reserve. Maxco had then lost the bid, however, to a competitor who was operating solely on the basis of indirect information. In addition, the competitor's winning bid had still been low enough to provide for a substantial profit on the venture.

Thus Mr. Buchanan was considering a change in his policy. While he very such doubted that anyone else would enter the bidding for block A-512, he was beginning to feel that he himself should do so. If he did decide to bid, he then wondered what sort of bid might be reasonable.

## Part II

### Gambit's Bidding Problem

Mr. Buchanan's counterpart in the Gambit Company was a Mr. K. R. Mason; primary responsibility for preparing Gambit's bid thus rested with him.

Until Gambit's well on the Alligator Reef leasehold was completed, Mr. Mason's information concerning block A-512 would be indirect in nature. The primary basis of that information was still the private seismic survey, for which Gambit had contracted jointly with Maxco, together with published government geological maps and reports.

Although Mr. Mason also had detailed production logs on the two producing wells on Gambit's leasehold, he felt that this information was not relevant to the problem of assessing the potential value of block A-512. There was almost certainly some cross-faulting in the Alligator Reef area (see Exhibit 1). Since this cross-faulting would probably terminate the producing area, the principal uncertainty surrounding the value of block A-512 was the precise location of the northernmost cross-fault. Thus, Mr. Mason's judgment was also essentially captured by the probability mass function given in Exhibit 2. Although Mr. Mason's judgment certainty did not coincide precisely with Mr. Buchanan's, the facts available to the two men and the economics in the two companies were largely similar. Neither man's estimate of the situation, therefore, differed significantly from Exhibit 2.

This would, of course, change dramatically when Gambit's offset control wall was completed. At that time Mr. Mason would be able to reevaluate the property with a much higher degree of precision.

Normally Mr. Mason would then be in a position to submit a bid relatively close to the true value of the block while still allowing a generous margin for profit. Other bidders, not knowing the true value of the block, would be unable to adopt such a strategy. If they bid at all, they would have to either bid relatively low or risk the possibility "buying in high" to a disastrously unprofitable situation.

Over the past year, however, several operators in the Louisiana Gulf Coast had narrowly lost out when bidding for blocks on which they had direct information. Granted chat in no case were extremely large reserves lost; nevertheless, operators bidding with nothing but indirect information had been able to "steal away" substantial reserves from operators who were basing their bids on direct information.

With a view toward reassessing his approach to this kind of situation, Mr. Mason thought that it might be useful to prepare a whole schedule of bids. For each possible "true value" of the reserves, Mr. Mason felt that he should be able to establish an appropriate bid — given that value of the reserves. Thus, Mr. Mason felt that he ought to be able to complete a bid schedule similar to that given in Exhibit 3. He was wondering, however, what a reasonable schedule of bids might be like.

Exhibit 1

MAXCO, INC. AND THE GAMBIT COMPANY

Subsurface Map of the Alligator Reef Area

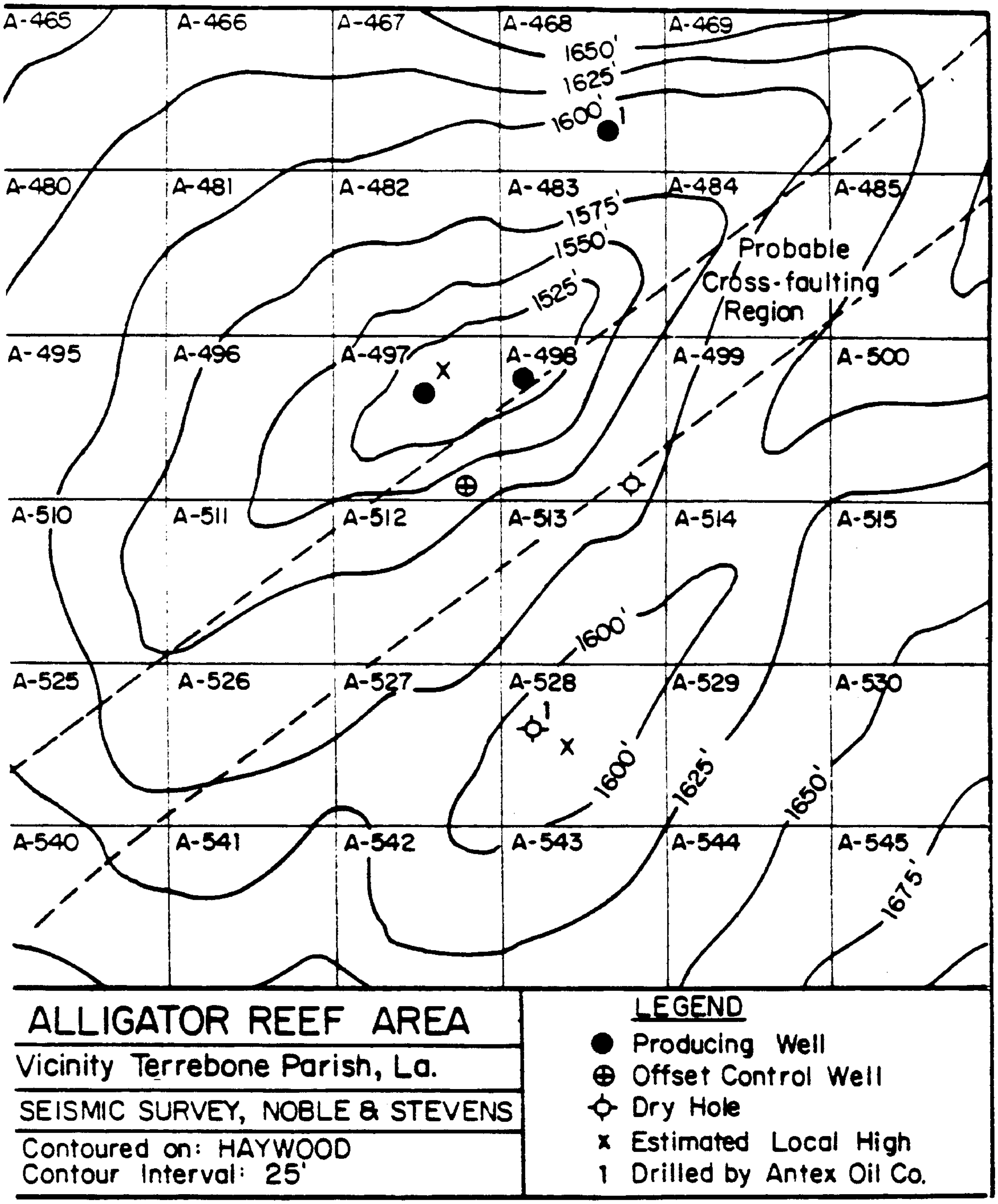

Exhibit 2

MAXCO, INC. AND THE GAMBIT COMPANY

Probability Distribution of Monetary Values

| Monetary Value of Oil Reserves* ($ millions) | Probability |
|---|---|
| 1.7 | 0.03 |
| 2.7 | 0.06 |
| 3.7 | 0.10 |
| 4.7 | 0.17 |
| 5.7 | 0.28 |
| 6.7 | 0.18 |
| 7.7 | 0.08 |
| 8.7 | 0.04 |
| 9.7 | 0.02 |
| 10.7 | 0.01 |
| 11.7 | 0.01 |
| 12.7 | 0.01 |
| 13.7 | 0.01 |
| | 1.00 |

Mean Value = $5.83 million

*Net Present Value at 10%.

Exhibit 3

MAXCO, INC. AND THE GAMBIT COMPANY

Gambit's Bid Schedule

| If the true value of the reserves is | then Gambit's bid should be: |
|---|---|
| $1.7 million | $_______________million |
| $2.7 million | $_______________million |
| $3.7 million | $_______________million |
| $4.7 million | $_______________million |
| $5.7 million | $_______________million |
| $6.7 million | $_______________million |
| $7.7 million | $_______________million |
| $8.7 million | $_______________million |
| $9.7 million | $_______________million |
| $10.7 million | $_______________million |
| $11.7 million | $_______________million |
| $12.7 million | $_______________million |
| $13.7 million | $_______________million |

**KELLOGG SCHOOL OF MANAGEMENT**
**Strategic Decision-Making**

# Final Project

DECS-452 Professor Bob Weber

This assignment may be done in groups of up to five. Groups of one, two, three, four, or five must do at least five, six, seven, eight, or ten problems, respectively. **Every group must do the first three problems.** (Your grade will be based on the **average** score you receive on submitted analyses.) One set of papers should be turned in by each group at our last regular meeting (June 2/June 3). Feel free to be creative (for example, if you want to pull together a group of friends and have them play one of the games, go ahead and report the results). Don't hesitate to call, or to come in and talk with me, if you think you are stuck, or fear that you are going astray. (I might offer smiles and frowns at appropriate points, but of course can't actually push you forward.) Each of the problems can be written up in one to three pages, although further **insightful** discussion is not discouraged. [Honor Code: You may discuss these problems *only* with your group members, and with me. In particular, you may not discuss them with those who've taken the course before, nor access any materials related to previous offerings of this course. Outside references and readings are fine, as long as they're credited when appropriate.] **All members of groups of size three or greater must also send me an email no later than midnight, Friday, June 3, dividing 60 points between all members of their group (except themselves) in proportion to the relative contributions of those group members to the group effort.** (Please use the subject heading "Peer Rating".) Only in extreme cases will these ratings affect any grades.

1. Three individuals, A, B, and C, are trying to jointly decide whether to take action x, y, or z. It is commonly known to all of them that A prefers x to y, and y to z; B prefers y to z, and z to x; C prefers z to x, and x to y. They finally agree to each vote (simultaneously) for an alternative: If at least two vote for the same alternative, it will be selected. If each votes for a different alternative, then A's vote will be implemented. (A is somewhat "senior" to the other two.)

How would you vote, as A? As B? As C? Explain your choices. [Answer this by writing three short memos: Your advice (with justification) to A if you were hired as a consultant to A exclusively, and similarly your advice to B or C if hired by one of them exclusively.]

2. During the 1980 U.S. presidential campaign, Reagan's advisors released a press statement, in which they said, "We concede that, in any tense situation, Ronald Reagan is more likely to react with military force than Jimmy Carter is. Still, we argue that the use of military force will be less likely if Reagan is elected President." The press jumped all over them, and they retracted the statement the next day. Comment on the possible validity of their original (quoted) statement – in particular, list and discuss any assumptions that might play a role in determining its validity.

3. Consolidated Products and their competitor, Regional Brands, have been presented with an opportunity to apply for a license to distribute hand-carved wooden replicas of the 2012 Olympics mascot in the United States. The licensing arrangement is simple: If both apply for the license, they'll pay $4,000,000 apiece, while if only one applies, that firm will pay $6,000,000. On the basis of published market research concerning U.S. consumer interest in the London Olympics, it's considered somewhat more likely (60% chance) that total profits from the U.S. market will be about $6,500,000 than that there will be enough interest (40% chance) to bring total profits up to $10,000,000.

Consolidated is in the process of doing its own detailed (private) study of market prospects, and will soon know which of these two possibilities is actually the case. The fact that they're doing this study is public knowledge.

Regional is thinking about running a very-private study of its own. They estimate that the cost of doing such a study (and keeping Consolidated from learning that they've done so) to be $100,000. The study would give them the same knowledge that Consolidated will have.

Ultimately, each of the two must independently decide whether to apply for a license. (The decisions of both will be revealed simultaneously.) If both apply, they'll end up splitting the available profits.

(a) Represent this game in strategic form. (To be consistent, make Regional the "Row" player. Hint: You should end up with a 6-rows-by-4-columns representation.)

(b) Under what circumstances (i.e., with what beliefs concerning Consolidated's behavior) should Regional spend the $100,000 before making its license-application decision?

(c) If Consolidated could publicly precommit to a strategy, what strategy would you advise them to bind themselves to? What expected profit would you tell them to expect as a result of the precommitment?

4. Two contracts are to be let successively (two months apart) at competitive bid. You will be bidding for both of these contracts, in competition with a single other firm. Both contracts involve the use of a new technology which has two uncertain cost dimensions (it is a two-process technology). Both firms currently have imperfect estimates along the first dimension, and must submit bids for the first contract.

The winning (low) bidder will be awarded the contract, and **both** bids will be made public. The next two months will not provide sufficient time to gain further information about the first cost component. However, both firms expect to have imperfect estimates of the second component (as well as their first-round, first-component estimate, and knowledge of their competitor's first-round bid) by the time they have to bid on the second contract.

At the unique equilibrium in this two-stage game, it can be shown (1) that higher cost estimates in the first round lead to higher first-round bids (i.e., the first-round bidding strategy is a strictly-increasing function of the cost estimate), and (2) that either bidder can increase his first-round expected profit by bidding slightly less than his equilibrium bid.

At first glance, this seems paradoxical. The first-round bidding strategy does *not* serve to conceal your firm's private information, in the sense that, at equilibrium, the other firm can perfectly infer your first-round estimate from your first-round bid. But then, why shouldn't you set your first-round bid to maximize your first-round expected profit? Explain.

5. Your firm is preparing to bid for the mineral rights on a tract of offshore territory. You have asked a team of operations research analysts to recommend a bid.

"There are four other potential bidders," they report. "Each of the five of us, as well as the government, has conducted a geological survey of the area. Although our estimation procedures are all equally good, there are substantial measurement errors to which the estimation process is subject. Indeed, our private estimate was that the tract has a commercial value of $100 million, but the government has just announced that its value estimate is only $60 million. Of course, the lease actually has the same value to all of us. Incorporating the government's estimate with our own, we obtain a revised estimate of $82 million.[1]

"We've scoured the competitive bidding literature (and have encountered several of our competitors' staff members at the library, looking at the same articles). All of the papers we've read suggest that we base our bid on our revised estimate. In fact, the equilibrium strategies for a five-person bidding game very similar to the one we face have been published, and indicate that the equilibrium bid associated with a revised value estimate of $82 million is $58 million.

"We suggest that the firm indeed bid $58 million. We really believe that the others will also be following the published strategies (i.e., that they will analyze the situation the same way we have). Of course, we may be outbid. But if we do win, we'll have an expected profit of $24 million."

(a) What statement (which specific sentence) is most **obviously** wrong here?

(b) Their analysis would have led to the same bid recommendation, had the revised estimate of $82 million come (correctly) from a private estimate of $60 million and a public estimate of $100 million. Does this seem reasonable? If so, explain (why which estimate is private and which public is irrelevant); if not, comment (under which of the two situations would you be tempted to bid more, and why?).

6. You have been named the chairman of a committee, charged to evaluate the predictive abilities of a number of stock analysts. It has been proposed that each analyst be asked to report his belief about the likelihood of a short-term upward price movement in each of ten stocks. Then, the actual price movements will be observed, and on each stock separately, the analyst will receive a score based on the probability he had assigned to upward movement, and the actual direction of movement. Each participating analyst will be compensated in proportion to his final score. [Assume that the analysts are neutral in their attitudes towards risk, for sums of money in the range of potential compensation. Further assume that more-talented analysts can indeed better predict price movements.]

(a) One possibility is to award each analyst, on each stock, either $p$ or $1-p$ points (where $p$ is the probability he assigned to upward price movement, and he receives the first score if the stock indeed moves up in price, and the second score otherwise). Do you foresee any problems in announcing that this scoring procedure will be used?

---

[1]Everyone held a common prior estimate (based on previous production in the area, a matter of public record) of $86 million for the value of the lease. The "revised" estimate is
$1/3 \cdot$ (common prior) + $1/3 \cdot$ (private estimate) + $1/3 \cdot$ (public estimate). You may assume that the underlying statistics in the problem justify this formula.

(b) Finally, a slightly-more-involved scoring procedure is decided upon, wherein an analyst receives $1-(1-p)^2$ or $1-p^2$ points, depending on the direction of price movement. Are you happier with this than with the procedure given in (a)?

(c) At the last minute, you are told that the high-scoring analyst will also receive a lucrative, prestigious new assignment, and that this will be announced prior to the competition. What effect do you see this announcement having?

7. Consider an idealized society, consisting of two types of individuals. Each individual views his life as unfolding over two periods of time: First, he is "young," and later, "mature." Each wishes to maximize his lifetime consumption, and to distribute that consumption as evenly as possible between his two periods of life; specifically, his happiness (utility) over his lifetime is the product of his consumption levels when young, and when mature. However, lifetime income streams are different for the two types of individuals: Persons of the first type have an income stream of (20,20), and those of the second type, (10,60). (Those of the latter type defer their earnings in favor of acquiring extra education when young.)

Clearly, there are gains available from trading arrangements between individuals of the two types. (For example, consumption streams of (19,22) and (11,58) are preferred by both to their original income streams. This corresponds to the first "lending" one unit to the second when young, and being paid back with 100% interest when mature.)

In this society, we might expect some enterprising individual to establish a "bank," which pays interest on deposits and collects interest on loans. Assume the banker announces the following policy: Each young individual submits a slip of paper, listing an amount and an interest rate. Those of the first type (income stream of (20,20)) indicate the amount they are willing to deposit, and the lowest interest rate at which they will make that deposit. Those of the second type (income streams of (10,60)) indicate the amount they wish to borrow, and the highest rate they are willing to pay. The banker will then (having seen all of the slips) announce an interest rate at which the total amount offered for deposit equals the amount asked for in loans. (This might involve some rationing at the margin.) All first-type individuals who offered deposits at that rate or less will have their offers accepted **at the rates they specified**. All second-type individuals who offered to pay the announced rate or more will receive loans, again **at the rates they specified**. (The banker gets to consume the spread.) The government has granted the banker a monopoly on trade; those whose offers are not accepted must simply consume their own income streams.

(a) If there are equal numbers of both types of individuals, what will be the announced price at equilibrium? What will an individual of either type write on his slip?

(b) If individuals are free to choose their type when young (e.g., are free to choose the amount of education acquired), what proportion of the population will choose each type, at equilibrium?

8. You and two roommates have just signed the lease on a new two-bedroom apartment. The obvious question: Who gets to sleep alone? The three of you decide that each will write down a dollar amount. The one who writes the highest amount gets the single bedroom, but must give half of the written amount to each of the others each month. The lease runs $900 per month, and each of the three of you is equally likely to value the right to sleep alone at anywhere between $0 and $100 per month (all values are equally likely). Your own valuation is $70. (Be sure to note that the winner has to pay the losers, so everyone gets *something*.)

(a) If you expect each of the other two to bid a fraction $f$ of their valuations, what fraction of your own valuation should you bid?

(b) At equilibrium, what fraction $f$ will all three bid?

[Hint: This problem can be approached analytically, or can be spreadsheeted. It might be worth noting that, if $k$ random numbers are drawn uniformly and independently between 0 and 1, the expected values of the smallest, next-smallest, ... largest are $1/(k+1)$, $2/(k+1)$,..., $k/(k+1)$.]

9. A large chain of hardware stores faces a series of individual potential competitors, each of which is thinking of opening a store in one of the towns in which the chain is currently unchallenged. Whenever challenged, the chain has two options: to engage in intensive local price competition against the entrant (which reduces the chain's earnings from the challenged store, but eventually drives the entrant out of business), or to share the local market with the challenger. In each town, the (net present value of all current and future) payoffs to the chain and its potential competitor depend on the competitor's entry decision and the chain's response, as indicated below.

| | | Challenger | |
|---|---|---|---|
| | | enter | stay out |
| Chain | fight | (-1,-5) | (5,0) |
| | share | (2,2) | (5,0) |

There are twenty potential entrants, each of which in turn receives a single chance to enter. All remaining potential entrants observe the outcome (the entry decision, and the chain's response) prior to the next round.

(a) If, instead, there were only one round and one potential entrant, what would you expect to see happen?

(b) What would you expect the chain to do (what would *you* do) in response to the entry of a competitor in the first round of the actual 20-round game?

(c) Discuss your answers to (a) and (b), in the light of the following claim: "Each of the twenty rounds is independent of the others, since each potential entrant only acts in one round; therefore, the potential entrant at any round should not be influenced by the outcomes of the preceding rounds." (If your answers to (a) and (b) are the same, you probably agree; if they're different, you disagree. In either case, explain why, or discuss the assumptions underlying your answer.)

10. In the Ware case, we originally assumed that the true value of future sales was (from National's perspective) equally likely to be anywhere between $15 and $20 million. If, instead, we had made the more realistic assumption that the true value was normally distributed, with mean $17.5 million and standard deviation $1.5 million, National's expected payoffs would have been somewhat different. What would National's expected payoff have been, if they entered the race, and if Ware used a $17 million entry cut-point? A $19.5 million cut-point? (Hint: Figure, in turn, the probability of Ware having entered, given that National successfully completes the project and applies first for a patent, and then the expected value of the market given that Ware entered and the expected value given that Ware stayed out. The "adverse selection" spreadsheet from Week 3 will be useful here.)

11.

(a) Alfred (poor and risk-averse, with utility for money equal to the square-root of the amount he possesses) and Burton (rich and risk-neutral) have met again. This time, Alfred owns only a ticket in the finals of the state lottery. The ticket has a 50% chance of winning $6400, and a 50% chance of being worth nothing. How much would Nash suggest that Burton pay Alfred for the ticket?

(b) In the "used-car" exercise we did in class, the car was worth 50% more to the buyer than to the seller, and we found that the buyer could not justify any positive offer. If, instead, the car were worth $180 more to the buyer than to the seller, and the value to the *seller* was equally likely to be anywhere in the range from $200 to $600, what would be the buyer's optimal "take-it-or-leave-it" offer?

(c) In the Salty Dog case, the buyer had a best alternative source of purchase at a price that was (from the seller's perspective) equally likely to be anywhere between $3000 and $6000. (The buyer, of course, knew exactly what this alternative price was.) If, as the seller, you held the power to set a first-and-final, take-it-or-leave-it price, (which the buyer would accept for sure, as long as it was lower than his best alternative price), and if your own best alternative offer on the "Dog" was $2200, what price would you name?

12.

(a) Two chess-players (Roger and Charles) are preparing for a head-to-head tournament, where a $10,000 prize is at stake. Roger is a somewhat better player, and has (in everyone's estimation, including his own and Charles') a 60% chance of winning the tournament.

The tournament acquires a sponsor: The GreatChess Company, which sells a computer chess-playing program. The rules are changed: Each player may, at a personal cost of $1500, choose to utilize the program during the tournament.

If both use the program, the tournament outcome will become a toss-up (50:50). If only Roger uses it, his chance of winning will rise to 80%. But if only Charles uses it, he'll become the 60:40 favorite to win.

Roger has come to you for advice: Should he buy access to the program? Assuming that both players seek to maximize their expected profit, what do you tell him (and why)?

(b) Managing traffic systems is a complex task due to the self-interested behavior of travelers. Increasing the capacity of a major highway might, for example, encourage drivers to stop using alternate routes or traveling at off-peak hours, or reduce their use of public transportation, leading to no real reduction in congestion.

Consider the following situation: Sleepytown (in western Illinois) has 2000 commuters who must commute to work in Chicago (eastern Illinois) every morning. Currently, there are two routes: One is a 60-mile stretch of state highway that runs from Sleepytown to East Junction, from which a 10-mile street leads into Chicago. The other is a 10-mile street from Sleepytown to West Junction, from which another 60-mile city highway leads right into Chicago. Both highways are designed to carry 1500 cars/morning at the maximum legal speed of 60 miles/hour, and both streets can carry 1000 cars/morning at a maximum speed of 40 miles/hour. However, the actual speed along any road (street or highway) drops if the number of cars exceeds the designed capacity: Specifically, the speed of traffic is the "maximum legal speed" if the number of commuters on the road is no more than the designed capacity, and drops to (maximum legal speed) x (capacity/commuters) otherwise. (To keep things as simple as possible, we'll assume that no one other than the Sleepytown commuters uses these roads.)

Currently, half of the commuters take each of the two routes. Each route takes 1 hour on the highway segment, and 15 minutes on the street segment.

Both of the highways meander a bit. It has been proposed to build a high-speed limited-access connection directly between West Junction and East Junction. It would be 30 miles long, with a capacity of 2000 cars/morning and a maximum legal speed of 90 miles/hour. This would give each driver three route-choices instead of the current two choices.

Assuming that none of the effects of the first paragraph occur (i.e., there are still just 2000 self-interested commuters), what would you expect to be the impact of the addition of this new "express" route?

13. Consider a two-person game which evolves continuously over time, and which one of the players will eventually win. Let p(t) be the probability, at time $t$, that player I will be the eventual winner.

(a) Show that the probability that $p(t)$ will rise to $a$ before it drops to $b$, given that it is currently $p$ (where $a > p > b$), is $(p-b)/(a-b)$. (E.g., if the NU basketball team has only a 20% chance of beating Michigan at the beginning of a game, then there is a 40% chance that the game will reach the point where it's a toss-up (50:50) before the end.)

(b) (Backgammon) Assume that the initial stake (the amount the loser must pay the winner) is $s$. Further assume that at first, either player has the right, at any time, to offer to double the stake. If his offer is rejected, the game ends immediately and he wins the original stake. If his offer is accepted, the game continues with a stake of $2s$. Subsequently, the other player holds the exclusive right to at any time offer to redouble the stake. He wins $2s$ if the offer is refused. If the offer is accepted, the game continues with a stake of $4s$, and the exclusive right to make the next double passes back to the first doubler. The game continues in this manner, with the right to double occasionally passing from one player to the other.

At what point (that is, when $p(t)$ reaches what level) should Player I first offer to double? (Hint: The point at which he should double -- or subsequently redouble -- is the same, regardless of whether the right to double is up for grabs or he currently holds the right exclusively. Furthermore,the optimal doubling point must be a point at which his opponent is indifferent between accepting or rejecting the double.)

14. Quoted from an article on strategic thinking in the *Harvard Business Review*:

"Question: Byzantium Gate Corporation holds a patent that gives it an effective monopoly on a certain semiconductor device. Your company, Barbarian Inc., is engaged in an R&D project aimed at obtaining a patent on an equally effective alternative design that would enable you to enter the field and compete with Byzantium. But Byzantium could also pursue a patent on the alternative and, if successful, forestall Barbarian's entry. Assuming that the company that invests the most money in R&D will win the race, will Byzantium spend aggressively to keep you out? In other words, should you forget the whole thing?

"Answer: First, ask yourself how much you expect to gain from the patent. The answer is: one company's profits in a duopoly market — a quantity we'll call D. It follows from this that the most you'll be willing to spend on getting the patent is D. Now ask yourself how much Byzantium stands to lose by not obtaining the patent. The answer is: its current monopoly profits (M) minus D. The question is, which is larger, D or M-D. Since it is generally true that competition reduces total industry profits, we can assert that $M > 2D$, which means that $M-D > D$. So the answer is yes, Byzantium will outspend you. Stay out."

Comment. (What do you think of all this? Does a correct analysis depend on any specific underlying assumption?)

15. Each of 10 farmers has 150 bushels of tomatoes which he must sell at the market Saturday or leave to rot. These farmers, from past experience, all know that the approximate market price per bushel will be $p = 10 - Q/100$ (in dollars), where Q is the total amount shipped to the market. (Actually, the price will hold steady at 10 cents per bushel if more than 990 bushels are delivered.)

(a) First, consider this as a cooperative (cartel) situation in which the farmers agree as a group on a fixed number of bushels which each will ship in order to maximize group profits. If each farmer can be trusted to hold to the agreement, what amount should they agree to each ship?

(b) Next, consider this as a noncooperative situation (binding agreements are not feasible, or are forbidden by law). As a group, they can jointly recommend how much each should ship to market. However, this recommendation must be a symmetric equilibrium point, i.e., no one farmer should be able to gain by unilaterally deviating from the recommended quantity. What is the best recommendation which can be made? (Is there more than one symmetric equilibrium point?)

(c) Finally, assume that one farmer has the ability to make his shipment before the others, and knows that they will all observe the amount he ships. How much would you recommend that he ship? (Answer this question in the context of the noncooperative setting laid out in (b).)

(d) Which of these three situations is most advantageous to the consumers?

16. Consider an "affair of honor" (i.e., a duel) involving three disputants. They position themselves at the corners of a triangle, each twenty paces from the others.

(a) In round after round, a referee chooses one of the three at random (using equal probabilities), and the selected disputant is given a single opportunity to fire at a selected target (for example, one of the other two disputants). Once one is hit (and eliminated), the duel continues, with the referee randomly selecting in each subsequent round between the remaining two, until only one is left standing.

Disputant A has an 80% chance of hitting any selected target; B has a 60% chance, and C only a 40% chance. What are the chances of each of the three emerging as the winner?

(b) Now, consider the same basic situation, but without a referee. Instead, the opportunity to fire cycles from C, to B, to A, to C again, then B again, and so on (skipping over anyone who has been "eliminated"). Obviously, once only two remain, they will alternately fire at each other. But what is the ideal targeting strategy for each of the three, as long as all three are still standing?

Notes:

(1) These models have been applied to a number of situations, ranging from comparative advertising to international relations.

(2) The analysis can be carried out implicitly, without ever requiring that an infinite series be summed. For example, consider situation (b), once only two competitors remain: Let the accuracy of disputant X be x, and the accuracy of Y be y. Then

Prob (X wins when going first) = x + (1-x)(1-Prob (Y wins when going first)), and
Prob (Y wins when going first) = y + (1-y)(1-Prob (X wins when going first)) .

These two relationships can be solved together to yield the solution

Prob (X wins when going first) = x / (x+y-xy) .

(Alternatively, the two duels can be modeled as absorbing Markov chains – if that carries meaning for you.)

17. Random numbers are chosen independently and uniformly from the unit interval (i.e., between zero and one). One number is placed in each of n boxes. Player II writes each number on the outside of its box, with the exception that he may incorrectly label any one box of his choosing. Player I is now allowed to look at the outsides of the boxes. He selects any one box, and receives from Player II an amount equal to the number inside that box. Obviously, Player I can ignore the labels entirely, and therefore the value of the game is at least 1/2.

a) For n = 2, show that the value of the game is 1/2.

b) Show that, for sufficiently large n, the value of the game is strictly greater than 1/2. How large must n be for this result to hold? (Give the best answer you can.)

18.

(a) Recall the "missionary" problem we discussed in class. Assume that there had been only three couples on the island (and that all three husbands' spouses had been unfaithful at some time in the past.) Then each husband (Tom, Dick, and Harry) knew there were unfaithful wives on the island, even before the missionary spoke. And each knew that each of the others knew. Therefore, even after the missionary spoke, each knew that nothing would happen on the first night after the missionary's departure.

Assume that the couples all walk out of their huts at 9:00 AM in the morning after that first night, and see one another. What does Tom know at 9:01 AM that he didn't know at 8:59 AM? (Answer in the form "At 8:59 AM Tom didn't know ______; at 9:01 AM, he did.")

(b) Imagine that you are playing the 20-stage repeated Prisoners' Dilemma, and that you and your partner have cooperated through the first 16 stages. You are rational, and believe your associate to be rational as well. Therefore, you fully intend to not cooperate in stages 19 and 20. What statement are you making if you cooperate at stage 17? Suggestion: Your answer should be of the form "My action would imply that I think there's a reasonably large chance that either ..... or ..... or ..... ." (You might, of course, have more or fewer "or"s.)

19. Following the herd: It is "common wisdom" that, if you are in a strange town and must choose between two restaurants, you should go to the one which has more customers. Let's see if this is true.

Assume that you are in a resort area, where 90% of the people are tourists, and 10% are locals. (This breakdown is common knowledge, although no tourist can tell the difference between a tourist and local just by looking at them.) There are two restaurants: Amy's Appetizers (A) and Bob's Biteables (B). All the locals know which serves better food, and always go there. The tourists have no special information: Each initially thinks it's a 50:50 proposition as to which is better.

As lunchtime approaches, people begin to go to one restaurant or the other. Tourists look in the window of each to see how many folks are already there, *and then go to whichever is more likely to be better, given the numbers of folks already in each.* (Each makes his/her decision based on the assumption that the previous tourists have acted this way as well.) If it's still a tossup, they flip a coin to decide which restaurant to patronize.

Clearly (following these rules), if one restaurant has patrons and the other is empty, a tourist will go to the one with patrons. And, if both have the same numbers of patrons, a tourist will flip a coin.

Prepare a 4-by-4 table showing the probability of Amy's restaurant being the better one, given that it currently has j patrons and Bob's has k (where j and k each take values between 0 and 3). [Hint: The main diagonal of the table will have 50% in each entry. And each entry above the diagonal will be the complement (i.e., they'll add to one) of the corresponding entry below the diagonal. Therefore, you really have to work out just 6 probabilities. ]

20. Pick a topic/book/article/situation of personal interest, and write a short report/analysis which you consider strategically insightful.