KELLOGG SCHOOL OF MANAGEMENT

Game Theory and Strategic Decision-Making

DECS-452 Professor Bob Weber
Week #5

1. Please complete the Salty Dog negotiation with your assigned counterpart, and post your results (both sides should submit their results separately) on the form on the class webpage **no later than 10 PM this Thursday (day section) / 10 PM next Wednesday (evening section)**.

   Through the wonder of modern technology, I've distributed several different versions of the briefing sheets. This much is common knowledge: Each Snowytown representative has an alternative offer in hand, which is equally likely (from the SSC perspective) to be anywhere between $1000 and $4000. Each Smith Sisters representative has found an alternative source of purchase at a fixed price, which is equally likely (from the Snowytown perspective) to be anywhere between $3000 and $6000.

   All other information in the briefing is private knowledge. You can say whatever you wish to about it, but can't actually show the briefing anyone else. Furthermore, you can't condition any negotiated agreement on the contents of your briefing sheet.

2. Over the next week, complete the midterm assignment (which will constitute 20% of the course grade). It may be done individually, or in groups of two or three. **Every group must do the first problem (A), and one of the other two (B1 or B2).** One set of papers should be turned in by each group a week from now (May 3 for the day section / May 5 for the evening section). Each of the problems can be written up in one to four pages, although further **insightful** discussion is not discouraged. *[Honor Code: You may discuss these problems only with your group members, and with me. In particular, you may not discuss them with those who've taken the course before, nor access any materials related to previous offerings of this course.]*

3. The above two assignments are enough for this week. Next week, I'll be pairing you up for a second out-of-class negotiation roleplay. I'll also be giving you a short assignment based on the third problem in the "Common Knowledge" section of the Week-4 handout.

DECS-452 Course Outline

H. The repeated Prisoners' Dilemma

1. Results of out-of-class exercise
2. Tit-for-Tat as an example of a possible strategy
3. Equilibrium outcomes: (D,D) in all stages, by backward induction
4. Properties of strategies:
    a) "Nice": Don't be first to defect.
    b) "Provocable": Strike back immediately any time he hits.
    c) "Forgiving": Respond to cooperation with immediate cooperation.

    d) "Simple": If you want to send a message, do so clearly.
    e) "Extending the hand of friendship": After several mutual hits (early in the game), try precisely two cooperative moves. (This is slightly non-provocable.)
    f) "Sensible": After several alternating moves, stick on cooperation once, in attempt to break cycle. (This is also slightly non-provocable.)

5. Tit-for-Tat: The only nice, provocable, forgiving strategy.
6. The Axelrod experiments: Computer 1, computer 2, and evolutionary
7. Interpretation with respect to maintenance of long-term relationships
8. Common knowledge
    a) The "missionary" problem
    b) Reconciling game theory with experimental results: Among human actors, rationality is never common knowledge.

The commons problem (the multi-person Prisoners' Dilemma)

1. The pollution game
    a) The difficulty of sustaining cooperation
        1) Penalties cannot be directed at specific violators
        2) Penalizing actions may be misinterpreted as defections
    b) Regulation as a solution: The parties desire intervention
2. Cartels

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Game Theory and Strategic Decision-Making: Midterm Project

A. Haydon and National are competitors in the packaging-materials business. A short-term market has developed for a polymerized cellophane which can be used for the packaging of toys. Haydon has established a facility for the production of this product, and during the testing of this facility has learned the unit cost of production. They are now prepared to take on several six-month delivery contracts. (Toy manufacturers traditionally contract on a semi-annual basis.) Haydon must decide whether to announce a sales price of $500 per unit, or of $300 per unit. (To simplify the problem, we will assume that no other pricing strategies are available.) At the higher price, it appears that there will be demand over the next six months for approximately 2000 units of production; at the lower price, for approximately 5000 units.

National does not yet have a production facility for this product; it will cost approximately $120,000, and will take about six months, to establish one. Due to technological uncertainties (concerning the reliability of the production process — it is one which subjects the machinery to an uncontrollable amount of downtime), they are not certain what the unit cost of production will be. Indeed, they think it is equally likely to be either $100 per unit, or $200. Whatever the case, they know that Haydon faces the same cost and has already learned this cost during its testing. (Indeed, both parties know everything laid out here.)

In six months, it will be time for Haydon — and National, should it choose to enter the competition — to seek a second round of contracts. Should both be in the market, they will split evenly the demand for the product, at the higher price ($500/unit) if unit costs are high, and at the lower price otherwise.* (If both are in the market, both will by then know the unit cost of production. The demand and pricing pictures should be about the same at that time as now. If Hayden is alone in the market six months from now, it will price optimally, independently of how it priced in the previous six months.) Given the large changes that take place in the toy industry on an annual basis, both firms expect there to be no further demand for this product after the second six-month period. Because of the specialized nature of the equipment used for the production of this product, the facilities will have negligible salvage value.

Note: For parts (a)-(c) below, the starred statement above simplifies the problem by making the second-stage pricing issue totally non-strategic.

(a) Haydon has learned that the unit cost of production is in fact $100. What price should they announce for the first round of contracts?

(b) If National observes Haydon announcing a high price, should they undertake the establishment of a competing facility?

(c) How would your answers to (a) and (b) change if the cost (to National) of setting up a production facility were $380,000? [Hint: An insightful analysis of part (c) is the heart of this problem. At the very least, you should provide a full representation of the problem in strategic form. You might summarize your analysis in the form of memos you would write as an outside strategic consultant to either party.]

(d) The starred statement in the last paragraph of the problem might be called into question. Assume that National has entered the market. If the two firms were forced to simultaneously announce binding prices (either $300 or $500) in the second round, and if demand is split between them if they both announce the same price, and all demand goes to the low-price firm otherwise, what price-announcement strategy would you expect to see each follow in that round? (Your answer should cover both the high-cost and low-cost cases.)

B1. An expedition leader (Alfred) and his four team members (Blackie, Chuck, Dirk, and Edward, listed from oldest to youngest, with each out-ranking those younger than he) have come upon a stash of 10 gold coins in a mountain cave. It's the leader's prerogative to specify how the coins will be divided amongst the five of them. However, the expedition is run somewhat as a democracy: After he proposes a division, everyone (himself included) gets to vote "Yes" or "No" to the division. (The votes are declared publicly, with the leader announcing his vote first, followed by the team members, from highest to lowest in rank.) If at least half vote "Yes," the division takes place.

If more than half vote "No," the leader is thrown off a cliff. The highest-ranking team member becomes the new leader (and must propose a division of the coins). This process continues until a division is agreed upon by at least half of those still alive.

Everyone assigns the highest priority to survival. Next in importance is wealth (that is, the number of coins received). And, all other things being equal, each likes seeing other people thrown off the cliff. (It's a rather cutthroat group of explorers.) All of this is common knowledge. [Feel free to rewrite the problem into one involving a CEO and four senior VPs if you wish.]

(a) What division of the coins should Alfred propose? Why?

(b) Assume, instead, that a strict majority is required to approve a division. (If half or more vote "No," the leader is thrown off the cliff.) Now, what division should Alfred propose, and why?

(Hint: In deciding how to vote on Alfred's proposal, each of the other four will certainly compare what he will get if Alfred's proposal is accepted with what would happen if Alfred is thrown off the cliff.)

B2. A deck contains three cards, labeled A, B, and C. Each of two players is dealt a card from the deck. Player I looks at his card, and then guesses which of the three cards Player II holds (i.e., says "A", "B", or "C"). II hears I's guess, looks at his own card, and then guesses which card I holds. Each player whose guess is correct receives one dollar from the other.

(a) Before you do *any* analysis or simplification, how many pure strategies does each player have?

(b) Solve this game, i.e., determine the value of the game and optimal strategies for the players.

(Hint: Feel free to exploit any apparent symmetries in simplifying the strategy spaces, by describing strategies in a way that doesn't use the specific names of the cards. In doing so, you'll find that the large numbers of strategies available to the players (from (a)) really include only a few qualitatively different strategies.

Once you've determined your shortened lists of strategies and written down the strategic form of the game, you can use the "Zero-Sum Games" workbook from Week 2 to solve the game. If you get to this point, and have any trouble with the workbook, contact me.)

Comments on “Common Knowledge”

A very extensive discussion of the meaning of “common knowledge” is found in Peter Vanderschraaf’s "Common Knowledge", *The Stanford Encyclopedia of Philosophy (Summer 2002 Edition)*, Edward N. Zalta (ed.), URL = http://plato.stanford.edu/archives/sum2002/entries/common-knowledge/.

Several additional “puzzle”-type problems can be found there.

As discussed in class, the primary practical “takeaway” is that there is NOTHING in human existence which is common knowledge, even amongst two people who know one another well. This is what breaks the backward “reasoning” that seems to argue in favor of never cooperating in the finitely-repeated Prisoners’ Dilemma, and makes mutually-advantageous cooperation possible (at least for *most* of the stages of the repeated game), even between rational actors.

The Number-Guessing Game

There are two individuals, A and B, facing one another in a room. Two consecutive positive integers are selected (by a referee; the choice probabilities are not relevant to this problem, except that *any* two such numbers might have been chosen). One of the numbers is written (in charcoal) on A’s forehead, and the other on B’s: Each sees the other’s number, but not his own. Alternately, starting with A, each is asked the question, “Do you know what your number is?” If he answers, “No,” the question is then asked of the other. (That is, A is asked first, then B, then A again, and so on.) For example, if A sees a 1, he answers, “Yes,” when first asked (and the game ends). If B sees a 2, and hears A say, “No,” then he can say, “Yes,” when first asked. (He knows he has a 3.)

(a) Assume the numbers are 3 (on A) and 4 (on B). What, if anything, is common knowledge before A answers the first question?

The only thing that is common knowledge, i.e., each knows, and knows the other knows, and knows the other knows they know, and so on …, is that A has an odd number on his forehead, and B has an even number. Nothing else is commonly known.

For example, A knows he doesn’t have a “1.” He also knows that B knows A doesn’t have a “1”. But A doesn’t know whether B knows that A knows he doesn’t have a “1”. (A, at the start, knows he has either a “3” or a “5” on his forehead. If B is looking at a “3,” then B doesn’t know whether or not he has a “2” or a “4”; if he had a “2,” then A wouldn’t know whether he had a “1” or not. But if A had a “5,” then B would know he had at least a “4,” and that A would then know he didn’t have a “1.”)

(b) Both know (on the basis of what they see in the case mentioned in (a)) that the first two answers will be “No.” Given this, what information is conveyed by A’s first answer? By B’s?

No information is conveyed by A's first statement: It was already common knowledge that A had an odd number, and the person with an odd number can never say "Yes" when the first speaker.

When B says "No," it becomes common knowledge that A does NOT have a "1" on his forehead. (Each knew this before B spoke, but it wasn't common knowledge, as shown by the discussion of part (a).).

(c) In what cases will the game eventually end in a "Yes"? You might first consider the case in (a).

If A says "No" in the second round, it becomes common knowledge that B does not have a "2." (If B had a "2," he could only have said "No" the first time if he was looking at a "3." So, if B had a "2," A would say, "Yes, I have a '3'!" in the second round.)

B, after hearing A's second "No," knows he doesn't have a "2." But he's looking at a "3." So, in the second round, he says, "Yes, I have a '4'!"

The game ALWAYS ends. In general, if the one with an odd number speaks first, and if the numbers are actually "2k" and "2k+1" (i.e., the even number is the smaller one), A will say "Yes" in round k+1. If the numbers are "2k" and "2k-1," B will say "Yes" in round k.

The Missionary Problem

Forty couples live on a remote island. In their society, three rules are rigidly adhered to:

1. Whenever a woman is unfaithful to her husband, the other thirty-nine men meet and commonly share this information.

2. No one ever tells a man that his wife has been unfaithful to him.

3. If a man ever goes to bed with his wife in the evening, *knowing* that she has been unfaithful to him at some time in the past, he kills her before morning (and his action becomes public knowledge in the morning).

The actual state of "affairs" is that all forty wives have at some time in the past been unfaithful (although each has — to herself — foresworn further affairs). Each man therefore knows that there are at least thirty-nine unfaithful wives on the island (the wives of the other men), but is unsure about his wife's fidelity. For years, everyone has been living happily (if a bit uneasily) in this setting.

One day, a missionary comes to the island for a visit. He stays for several weeks, during which he visits privately with each resident. On the day of his departure, as the eighty residents gather to watch him row away, he calls out, "I enjoyed my visit, but was disappointed to learn that there is at least one unfaithful wife on the island." He then passes beyond earshot, before any questions can be asked of him. Of course, none of the husbands are surprised to hear his statement. And indeed, the next morning no wives are found to have been killed. Yet on the fortieth night after his departure, just when the wives are beginning to relax, all forty men kill their spouses.

Explain what happened.

The simplest explanation is inductive.

If there were only one unfaithful wife on the island, her husband would have been surprised by the missionary's announcement, and would have killed her on night 1.

If there were exactly two unfaithful wives, each of their husbands would initially know only about the other's unfaithful wife, and would be expecting – if his own wife were faithful – for the other man to act on night 1. When the other man doesn't act (i.e., on the morning of day 2), he knows his own wife must not have been faithful (and he acts on night 2).

Climbing up the logic ladder, assume there were three unfaithful women. Then each man with an unfaithful wife knew, even before the missionary's statement, that there were either two or three unfaithful wives on the island. If there were only two, the previous argument shows that, after the missionary's statement, both husbands with unfaithful wives would kill their wives on night 2. On the morning of day 3, when it becomes clear that nothing violent happened the previous night, each of these men knows there must be THREE unfaithful women on the island, one of which is his wife.

And, step by step, the argument eventually reaches the 40-unfaithful-wives case.

What did the missionary tell the men that they didn't know before? It's not what he said – It's the fact that he said it in a public forum. He made the existence of at least one unfaithful wife ***common knowledge***.

In the two-unfaithful-wives case, each of their husbands knew, before the missionary spoke, that there was at least one unfaithful woman on the island. But the husband of each of the two unfaithful women didn't know if the husband of the other unfaithful wife knew or not. In the three-unfaithful-wives case, everyone knew there were at least two unfaithful women, and everyone knew that everyone else knew there was at least one. But the husband of unfaithful woman A DIDN'T know whether (or not) the husband of unfaithful woman B knew that the husband of unfaithful woman C knew there was at least one.

In the full case, every man knew there were at least 39 unfaithful wives on the island, and knew that everyone else knew there were at least 38, and knew that everyone else knew that the others knew there were at least 37, and so on. But man 1 DIDN'T know whether man 2 knew that man 3 knew that … man 40 knew there was at least one, until after the missionary spoke.