KELLOGG SCHOOL OF MANAGEMENT

Game Theory and Strategic Decision-Making

DECS-452 Professor Bob Weber
Week #2

"Oh, what a tangled web we weave,
When first we practice to deceive."

Marmion, Sir Walter Scott

Readings: Please read the attached material.

Prepare solutions to the following problems (1a, and all of 2 and 3), to be turned in next Tuesday (day section) or Thursday (evening section). If you wish, you may work in groups of up to three. (If you've already formed a larger group in anticipation of the final project, just split into two groups for this and next week's exercises.) Note that the class webpage contains a spreadsheet for "solving" zero-sum games.

1. (Ware Medical Corporation) Ignore (for now) the final sentence of the case, and assume that both firms have access only to the $17.5 million estimate. To make all of our computations comparable, don't discount this year's expenditures, discount next year's once, and so on. (This will yield $6.031 million for the discounted profit stream, as indicated near the middle of the second page.)

(a) Clearly, Ware and National each have two available strategies: "in" (attempt development) and "out" (don't take any action). Give the strategic representation of the competitive problem (i.e., for each pairing of strategies, indicate the expected payoff to each of the two firms). To standardize our representations, take Ware as the "row" player, and use (0, 0) for the ("Ware: out", "National: out") payoffs.

Thought only (no need to write an answer):

(b) If you were Piper, what would you do?

(c) If you were making National's decision, what would you do?

2. (Another poker example) Consider the following simple two-person game. Each player puts $1 into the pot. Player I (the only player to receive a card!) is dealt a card which is equally likely to be “high” or “low”. He looks at this card, and then chooses to either “bet” (he adds an additional dollar to the pot) or “check” (no additional money is put in the pot). If I bets, II chooses to either “call” (adding a dollar to the pot) or “fold” (conceding the pot to I); if I checks, II chooses to either “call” (no additional money is placed in the pot) or “bet” (one additional dollar). Finally, if I checks and II bets, I chooses to either “call” (adding a dollar) or “fold”. If a player folds, the other wins all the money in the pot. If a player calls, I then reveals his card: If it is high, he takes the pot, and if it is low, II does.

(a) Represent this game in extensive form.

(b) How many pure strategies does I have? List those (hint: four) which are essentially distinct, and undominated.

(c) Argue (convincingly) that II should never bet when he hears a check.

(d) Give the four-by-two strategic representation that results from the analysis in (b) and (c).

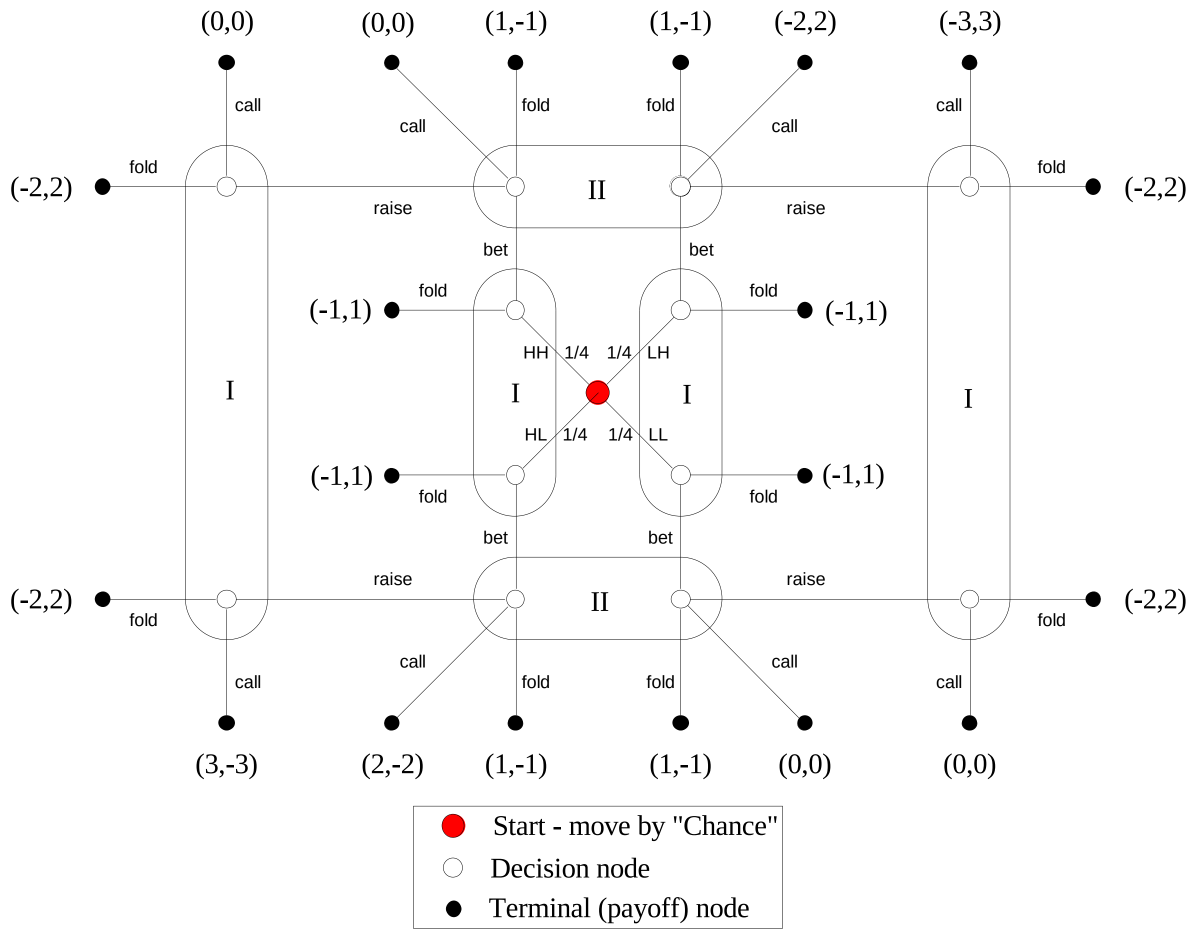
(e) Find an optimal strategy for each player, and determine the “value” of the game.

3. You are trying to decide whether to acquire the license to a particular new technology. You have one competitor who might also wish to acquire the license. The licensing rules have been announced: Any interested party may submit a letter of interest together with a $750,000 payment. If only one letter is received, the payment is kept and an exclusive license is granted. If two letters are received, each party will receive a $250,000 rebate and a non-exclusive license. If no letters are received, the license will be awarded to an overseas firm.

To complicate matters (and to make money), a large data-collection service has just announced the creation of a new database containing information of relevance in estimating the size of the market for products made using this technology. For $75,000, anyone can purchase access to this database. The information in the database can be briefly summarized: It will be either “good news” or “bad news.”

(a) Assume that neither you nor your competitor can learn whether the other chooses to purchase access to the database. List the essentially-distinct pure strategies that are available to you (where you must decide whether to purchase access, and then whether to apply for a license). Use whatever shorthand you wish to create.

(b) Assume that the data-collection service is required to publish a list of subscribers to the database *after* both firms have made their purchase-of-access decisions. List the essentially-distinct pure strategies available to you.



Player I's Strategies

Pure	essentially distinct	undominated
HbcLbc	HbcLbc	HbcLbc
HbcLbf	HbcLbf	HbcLbf
HbcLfc	HbcLf (2)	HbcLf
HbcLff		
HbfLbc	HbfLbc	
HbfLbf	HbfLbf	
HbfLfc	HbfLf (2)	
HbfLff		
HfcLbc	HfLbc (2)	
HfcLbf	HfLbf (2)	
HfcLfc	HfLf (4)	
HfcLff		
HffLbc		
HffLbf		
HffLfc		
HffLff		

Player II's Strategies

pure	undominated
HrLr	HrLr
HrLc	HrLc
HrLf	HrLf
HcLr	
HcLc	
HcLf	
HfLr	
HfLc	
HfLf	

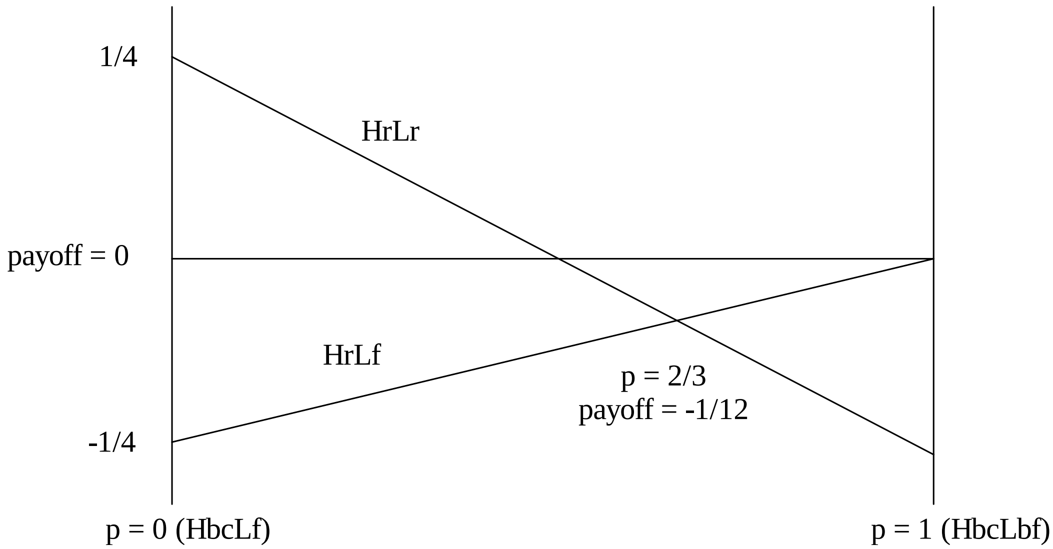
Payoffs to Player I, if deal is {HH, HL, LH, LL}

	HrLr	HrLc	HrLf
HbcLbc	0,3,-3,0	0,2,-3,0	0,1,-3,1
HbcLbf	0,3,-2,-2	0,2,-2,0	0,1,-2,1
HbcLf	0,3,-1,-1	0,2,-1,-1	0,1,-1,-1

The Strategic Representation

	HrLr	HrLc	HrLf
HbcLbc	0	-1/4	-1/4
HbcLbf	-1/4	0	0
HbcLf	1/4	0	-1/4

Player I's Strategic Problem



Games in Extensive Form

The typical competitive problem has numerous informational features: private information may be held by some parties, the moves of some parties may be observable by others prior to their moves, there might be multiple stages of moves with new information gained between stages, and the like. One way to represent such features is to write the original problem in extensive form.

A **game in extensive form** is, in short, a graphical representation of the various features of an actual competitive problem. The basic "move" structure is represented by a "rooted tree" (which looks very much like a classical decision tree). There is a single **starting position**, from which play commences. Each (non-terminal) node in the tree "belongs" to a particular player (in which case, we call it a "decision node"), or to "chance." If it belongs to a player, the branches leaving that node correspond to the different actions available to the player if the play of the game reaches that node; if it belongs to chance, the branches have probabilities assigned to them.

The terminal nodes of the tree (i.e., those with no emanating branches) correspond to the potential **outcomes** of the game, and assign a payoff to each player. While we will often view the payoffs in monetary terms, it is more appropriate to view them as the players' "utilities" for the various outcomes. (An individual's "utility function" translates money into "happiness" in such a way that the individual – even if risk averse – will seek to maximize expected utility.)

An essential feature of competitive problems is that players often encounter situations wherein they must choose their actions while remaining uncertain about the state of the world, or about actions taken by other players. Such situations correspond to sets of nodes of the tree, all of which belong to the same player and confront him with the same set of alternative actions; when he chooses his action, he does not know at which particular node the game stands. A player's "information partition" is a clustering of his decision nodes into **decision situations (information sets)**, where each situation corresponds to a collection of indistinguishable (to him, at the time he must act) decision nodes.

We can view the play of an extensive-form game in the following manner: Each player is locked in a closet, and has in his possession a copy of the tree, indicating the information sets and payoffs of the players together with the probabilities associated with chance moves. A referee holds a master copy of the tree, with a marker on the root node.

If the marker sits on a node belonging to chance, the referee does the appropriate randomization, and moves his marker along the selected branch. If the marker sits on a player's decision node, the referee goes to the closet of that player, and tells the player which information set the marked node lies in (i.e., which situation has arisen which requires his choice of an action). The player chooses an action that is valid for that information set. The referee hits the player on the head (causing total amnesia), and then returns to his master copy and moves the marker along the selected branch. This procedure continues until some terminal node is reached, and then (finally) the players are called out from their closets, and receive the payoffs listed at that node.

(Obviously, no real-life referee will go around hitting people on their heads. I describe the play this way only to emphasize that knowledge of the currently-relevant situation summarizes everything a player actually knows when he must choose a move.)

With this definition, we can formally define a **pure strategy** for a player to be a function which assigns a particular move to each of the player's decision situations. Think of a strategy as a book with many chapters, each one page long. Each chapter title describes a situation where that player is called upon to act, and the text of the chapter simply indicates what move the player will make if that situation arises during the play of the game.

If the strategies of the players are all specified (i.e., once each player chooses a particular "book" from his shelf of strategies), the referee can play through the entire game and determine the outcome without visiting any closets. Equivalently, a player called out of town on business can simply hand his selected book to an agent, with full confidence that the agent will never face a situation for which he does not have complete instructions about how to act.

Clearly, some books on the shelf may be identical except for pages that will never be read. (This is sort of like having several "Choose-Your-Own-Adventure" books that differ only on pages never referenced by any other page.) In addition, some books may be inferior to others when matched against any collection of opposing books (chosen by the other players). These correspond to "essentially equivalent" and "dominated" strategies, respectively.

Language Summary

position

> A view of the current status of the game from the perspective of an omniscient observer. In the graphical (extensive-form) representation of a game, each node corresponds to a different position. There are three types of positions: positions where a chance move occurs, positions where a specific player must act, and ending positions.

• position where a chance move occurs

> A single branching node in the graphical representation of the game, with a probability distribution over the outgoing branches.

• position where a player must act

> A single branching node in the graphical representation of the game, with labels on the outgoing branches which indicate the various actions available to the player.

• ending position

> An outcome of the game, where each player finally receives a payoff. Each ending position corresponds to a terminal (non-branching) node in the graphical representation of the game. Each ending position is connected to the starting point of the game by a unique path: This path is sometimes called a "play" of the game.

A (decision-making) situation, or "information set"

> A view of the game-in-progress from the perspective of a player who must now choose an action. The same situation can encompass several different positions: Two positions correspond to the same situation if the information available to the player, and the actions from which he must now choose, are precisely the same at both positions. In the formal language of "game theory," a *situation* is called an *information set*.

action

One of the choices available to a player as he stands at a position where he must act. The same set of actions is available at each of the positions corresponding to a single situation.

pure strategy (for a player)

A listing of all the situations which that player might encounter, and a selection of a specific action to be taken in each of those situations.

Example

Consider the following simple two-person game. Each of two players is dealt a high or low card, with equal probability. Each player antes $1, then looks (privately) at his or her own card. Player I (the dealer) may either fold, or bet $1. If I bets, then II may fold, call (matching the bet), or raise $1 (putting two more dollars on the table). Finally, if II raises, I may either fold or call (matching the $1 raise). A player wins the pot if his opponent folds, or if a call occurs and he holds a higher card than his opponent. If they hold equal cards, a call results in the pot being split evenly.

There are 33 positions: one belonging to chance (which also happens to be the starting position of the game), eight where Player I must act, four where Player II must act, and twenty ending positions. The eight positions where Player I must act correspond to four different situations which Player I could encounter; the four positions where Player II must act correspond to the two situations which Player II could encounter.

Therefore, a pure strategy for Player I must specify an action in each of the four situations Player I might face. A pure strategy for Player II, on the other hand, need only specify the action to be taken in each of the two situations Player II might face.

For Player I, a pure strategy specifies, for each of the situations described in quotes, one of the actions appearing within the following braces:

"was dealt a high card"	… {bet, fold}
"was dealt a high card, bet, and heard Player II raise"	… {call, fold}
"was dealt a low card"	… {bet, fold}
"was dealt a low card, bet, and heard Player II raise"	… {call, fold}

There are 2x2x2x2 = 16 ways to specify a particular action to be taken in each of the four situations, so Player I has 16 pure strategies.

For Player II, the quoted phrases below describe the only two situations he might encounter, since he doesn't know, at the time he must act in either situation, what card Player I was dealt. The first situation consists of two positions, as does the second. A pure strategy for Player II specifies one of the actions in braces for each of the two situations.

"was dealt a high card, and heard Player I bet"	… {fold, call, raise}
"was dealt a low card, and heard Player I bet"	… {fold, call, raise}

Randomized Strategies

A player, facing some decision situation, could certainly elect to leave the choice of an action to chance. For example, Player I in the poker example could, when holding a low card and needing to decide whether to bet or fold, could roll a die and then bet only if the die comes up 5 or 6. Continuing to think of a strategy as a book, with each chapter prescribing the decision to make when a particular situation arises, we could have some chapters which – rather than specifying a particular action, as in a pure strategy – specify instead a probability distribution over the available actions. Such a strategy is called a **behavioral strategy**.

Alternatively, instead of selecting a particular pure strategy to be employed in the game, a player could leave the choice of pure strategy to chance, assigning probabilities to each of the pure strategies. Such a “meta“-strategy is called a **mixed strategy**.

Conveniently, as long as each player is permitted to remember everything he previously learned or did during the play of a game (i.e., as long as the game has "perfect recall," which is typically the case in managerial settings), these two notions of randomization coincide.

Continuing with the poker example, Player I could specify the following behavioral strategy to an agent: “If dealt a low card, fold with probability 1/3, and bet with probability 2/3; if, holding that low card, you bet and get raised, fold – if dealt a high card, bet; if, holding that high card, you bet and get raised, call.”

This behavioral strategy is equivalent to the following mixed strategy: “With probability 1/3, use the pure strategy that says ‘fold with a low card, and bet (and, if raised, call) with a high card’ with probability 2/3, use the pure strategy that says ‘bet (and, if raised, fold) with a low card, and bet (and, if raised, call) with a high card.’

This equivalence permits us to represent and analyze strategic problems in terms of mixed strategies, even if those strategies would ultimately be deployed as behavioral strategies.

Some Zero-Sum Games

1. Paper, stone, scissors

In this classic children's game, two players simultaneously display either an open hand (paper), a clenched fist (stone), or a pair of fingers (scissors). Paper beats (covers) stone, stone beats (breaks) scissors, and scissors beats (cuts) paper. Physical violence (inflicted by the winner upon the loser) follows.

Stone does the greatest damage, and scissors the least. We'll model this by assigning values of 3, 2, and 1 to victories by stone, paper, and scissors, respectively. (Played for cash, this makes an interesting bar game.)

Which of the three would you play most frequently, and which the least?

2. Duels

Early military-sponsored research considered optimal strategies in duels between airplanes engaged in aerial combat. The simplest (described more fancifully below as a duel-of-honor) had the following form:

Two protagonists, each armed with a single-shot pistol, face each other at a distance of 50 meters. They simultaneously have the right to fire. If neither does, or if one fires and misses, they walk forward, reducing the distance to 40 meters, and each who has a remaining bullet again has the right to fire. Again, if no-one is hit and both have not yet fired, they close to 30, then 20, then 10 meters, and finally to point-blank range. Both are equally-skilled: Each has a probability of 1 - (distance/50) of hitting the other at any particular distance.

If both are hit simultaneously, or both miss, the duel is a draw. If one hits first, he wins.

(a) If both have noisy pistols, what are optimal strategies?
(b) If both have silencers (a missed shot by one leaves the other uncertain as to whether a shot has yet been fired), what are optimal strategies?
(c) If only one has (and is known to have) a silencer, what advantage does he possess?

3. Chomp (This is more a brain-teaser than it is an illustration of anything important.)

This pencil-and-paper game begins with a rectangular grid of dots. The lower, left-hand (southwest) dot is “poisoned”: To “eat” it is to lose. The two players alternate moves. A move consists of “eating” a dot, and all other dots above and to the right of it (i.e., all northeasterly dots). Tom Ferguson has a website where you can play the 4-by-7 version against the computer: http://www.math.ucla.edu/~tom/Games/chomp.html.

If the starting position is a square (bigger than 1-by-1!), the first player to move has an easy win. (Eat the dot immediately to the northeast of the poisoned one, leaving a single row and single column of equal length. Then “match” each of the opponent's moves, always keeping the row and column of equal length.)

No one has yet discovered a general optimal strategy, or even an optimal first move, for all non-square rectangles. Yet it is easy to prove that the first player to move can always force a win. Can you show this?

Zero-Sum Games

In a strictly competitive situation (for example, in many two-player board and card games, as well as in numerous military and inspection problems) the payoffs of the two players sum to zero at every possible outcome. Obviously, in such situations no incentives exist for the players to cooperate.

A very conservative approach to the play of such a game is to seek a strategy that guarantees you the maximum possible gain, no matter what the other player does. (In particular, this strategy should maximize your payoff, given that the other player can predict perfectly the strategy you will follow.)

In the example below (where only the payoffs to Player I — the "row" selector — are shown; the payoffs to Player II are the "negatives" of the displayed numbers), strategy "D" guarantees Player I a payoff of at least 16 (and Player II a loss of at least 16). At the same time, a conservative Player II has a strategy, "b", which guarantees that he loses (and Player I wins) no more than 16. Therefore, in some sense "D" and "b" are *optimal* strategies, and the *value* of the game is 16.

	a	b	c	d
A	20	15	13	6
B	7	12	14	28
C	21	15	12	23
D	18	16	17	18

Contrast this example with the poker game we analyzed in class. Restricting himself to the "pure" selection of a single strategy, Player I can guarantee himself a loss of no more than 1/4. But Player II can guarantee himself a gain of only at most 0.

	HrLr	HrLf
HbcLbf	-1/4	0
HbcLf	1/4	-1/4

However, by being somewhat unpredictable, Player I can improve his guaranteed payoff. If he plays his "upper" strategy with probability p, and his "lower" strategy with probability 1-p, then Player I guarantees himself at least the smaller of $-0.25p + 0.25(1-p) = 0.25 - 0.5p$ (if Player II plays "left") and $-0.25(1-p) = -0.25 + 0.25p$ (if Player II plays "right"). For any p strictly between 0 and 1, Player I guarantees himself an expected loss of less than 1/4. And for $p = 2/3$, he guarantees himself a loss of only 1/12, no matter what Player II does. (This is indicated graphically in the notes on the poker game. Or, by noting that his expected payoff against "left" decreases as p is increased, and his expected payoff against "right" increases, we see that the lower of the two expected payoffs is maximized when the two are equal, i.e., when $0.25 - 0.5p = -0.25 + 0.25p$.) Similarly, Player II can guarantee himself an expected gain of at least 1/12 by choosing "left" with probability 1/3, and "right" with probability 2/3. Again, it makes sense to call these strategies "optimal" for the two players, and to call -1/12 (to Player I) the "value" of the game.

[Note that the phrase “optimal strategy” means that we are guaranteeing ourselves a particular payoff, and our opponent not only can in fact hold us to this payoff, but also actually *wants* to hold us to it. Also note that, against a dumb opponent, we might do better with some other strategy. But, in order to do better, we must risk doing worse (against an opponent who is “playing dumb” in order to “sucker” us.]

The two most striking facts in the strategic analysis of zero-sum games are:

1. In a two-player zero-sum game of “perfect information” (in which all information sets are single nodes; equivalently, there is no private information, the players never move simultaneously, and all moves are publicly observable — chess and checkers are examples of such games), both players have optimal pure (nonrandomized) strategies.

2. (The “Minimax Theorem”) In *any* two-player zero-sum game, both players have optimal (possibly randomized) strategies.

An optimal strategy can be found using linear programming. For instance, in the poker example, Player I solves the problem

$$
\begin{aligned}
&\text{maximize} && u - v \\
&\text{subject to} && \\
& && -1/4\, p_1 + 1/4\, p_2 \geq u - v \\
& && 0\, p_1 - 1/4\, p_2 \geq u - v \\
& && 1\, p_1 + 1\, p_2 = 1 \\
&\text{with} && p_1, p_2, u, v \geq 0
\end{aligned}
$$

(We use u-v to represent the value of the game, in order to permit that value to be either positive or negative.) The tables below indicate optimal strategies (and, in the upper-lefthand corner, the value of the game to the row-chooser), for some examples.

Rock-Paper-Scissors:

0.000		0.167	0.500	0.333
		rock	**paper**	**scissors**
0.167	**rock**	0.000	-2.000	3.000
0.500	**paper**	2.000	0.000	-1.000
0.333	**scissors**	-3.000	1.000	0.000

Rock-paper-scissors in Wikipedia (https://en.wikipedia.org/wiki/Rock-paper-scissors).

The noisy-noisy duel (a strategy for either player is of the form "Fire at a distance of __ unless the other player has already fired and missed; in the latter case, walk up to point-blank range before firing"):

0.000		0.000	0.000	0.000	1.000	0.000	0.000
		50	**40**	**30**	**20**	**10**	**0**
0.000	**50**	0.000	-1.000	-1.000	-1.000	-1.000	-1.000
0.000	**40**	1.000	0.000	-0.600	-0.600	-0.600	-0.600
0.000	**30**	1.000	0.600	0.000	-0.200	-0.200	-0.200
1.000	**20**	1.000	0.600	0.200	0.000	0.200	0.200
0.000	**10**	1.000	0.600	0.200	-0.200	0.000	0.600
0.000	**0**	1.000	0.600	0.200	-0.200	-0.600	0.000

Each should shoot at a distance of 20 meters.

The silent-silent duel (a strategy for either player is of the form "Fire at a distance of __"):

0.000		0.000	0.000	0.455	0.455	0.000	0.091
		50	**40**	**30**	**20**	**10**	**0**
0.000	**50**	0.000	-0.200	-0.400	-0.600	-0.800	-1.000
0.000	**40**	0.200	0.000	-0.120	-0.280	-0.440	-0.600
0.455	**30**	0.400	0.120	0.000	0.040	-0.080	-0.200
0.455	**20**	0.600	0.280	-0.040	0.000	0.280	0.200
0.000	**10**	0.800	0.440	0.080	-0.280	0.000	0.600
0.091	**0**	1.000	0.600	0.200	-0.200	-0.600	0.000

Each is most likely to shoot at distances of 30 or 20 meters, but occasionally crosses his fingers and walks up to point-blank range (hoping the other has already fired and missed).

The silent-noisy duel (a strategy for Player I is of the form “Fire at a distance of __ unless the other player has already fired and missed; in the latter case, walk up to point-blank range before firing”; a strategy for Player II is of the form “Fire at a distance of __”):

0.018		0.000	0.000	0.000	0.909	0.000	0.091
		50	**40**	**30**	**20**	**10**	**0**
0.000	**50**	0.000	-0.200	-0.400	-0.600	-0.800	-1.000
0.000	**40**	1.000	0.000	-0.120	-0.280	-0.440	-0.600
0.455	**30**	1.000	0.600	0.000	0.040	-0.080	-0.200
0.545	**20**	1.000	0.600	0.200	0.000	0.280	0.200
0.000	**10**	1.000	0.600	0.200	-0.200	0.000	0.600
0.000	**0**	1.000	0.600	0.200	-0.200	-0.600	0.000

The duelist with the silencer fires at a distance of 30 or 20 meters (unless he has already heard the other fire and miss). The duelist with the noisy gun usually fires at 20 meters, but occasionally gambles that the other has already fired and missed.

The game of “Chomp”:

As a game of perfect information that cannot end in a draw, either Player I has a winning strategy, or Player II does. For any rectangular starting configuration, consider an opening move by Player I of taking only the northeastern-most point. Either this is the start of a winning strategy (in which case, Player I can force a win by starting with this move), or it's not. If not, then Player II has a winning response. **But Player I could have started with that move instead**, so again, Player I has a winning strategy.

Note that this analysis is totally non-constructive: It doesn't even tell us how Player I should start. And in fact, for large starting rectangles, the “right” first move is unknown. Wikipedia has a long article on the game: http://en.wikipedia.org/wiki/Chomp.

Kellogg School of Management
Northwestern University

WARE MEDICAL CORPORATION[1]

Louis Richardson, a polymer chemist at Ware Medical Corporation routinely scanned the contents of the new *Federal Bulletin of Materials* every month, but the June issue this year brought him quite a shock when he saw the announcement of FR 8420. FR 8420 was a new acrylic polymer with silicate side-chains that had just been developed by NASA scientists, as a by-product of their research on materials for use in the space shuttle. Richardson immediately recognized that this new discovery could threaten Ware's position in the market for dental materials.

Ten years ago, after six years of research, Ware Medical Corporation had received a patent for a new method of making translucent composite materials that combined glass particles in a matrix of plastic. Before that time, prosthetic dental restoratives had been predominantly made of plastic and porcelain. Porcelain had the advantage of being more resistant to abrasion, but it was also more brittle than plastic, so that neither material was clearly superior for all purposes. Ware's composite material, which was marketed under the name of Dentosite, combined the best properties of both materials. Dental laboratories and manufacturers of artificial teeth were quick to recognize the advantages of Dentosite. According to a study that was published four years ago by Data Research Corporation, Dentosite had captured a 60 percent share in the market for materials used in dental prosthetics.

National Dental Corporation was the largest supplier of materials for dental prosthetics before Dentosite. Five years ago, National had entered into lengthy negotiations for the right to manufacture and sell composite materials using Ware's patent process. At times, it had seemed that an agreement was imminent; but negotiations broke down two years ago and National initiated a lawsuit to contest Ware's patent. Although the suit was still pending, Ware's lawyers were confident of winning.

Charles Piper was the vice president of Ware responsible for the dental products group. On June 21, three days after Richardson read the announcement of FR 8420, Piper held a meeting with Louis Richardson and Benjamin Gretter, who had general responsibility for the Dentosite product.

Richardson began the discussion. “Our Dentosite material has held a unique position in the market essentially because of our patented process for preparing the glass particles to bond to plastic. However, such preparations could be entirely omitted if the usual plastic materials were replaced by this new material FR 8420, because it bonds directly to glass. Of course, it would take some time to develop a new composite with FR 8420 that could serve as a dental material. The main problem is that the glass-plastic bond that one could get with FR 8420 would not be as strong as what we get with our process. The only way to overcome that problem would be to try to use a fibrous glass component. I figure that there is a 50% chance that an acceptable translucent composite is feasible using fibers with FR 8420. So if we are lucky, it might not be feasible, but we cannot count on such luck. It seems to me that our best bet is to work on developing a translucent fibrous composite ourselves. If the technique is feasible, then we would have just as good a chance as National of being the first to prove it. Then, if we developed it first, we could extend our patent protection to this technique and prevent any competitor from making fibrous composites with FR 8420.”

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[1]Prepared for classroom discussion by Roger B. Myerson. Any resemblance to actual companies or individuals is completely coincidental.

“Lou and I have gone over the numbers to justify this plan,” Gretter said. “If the technique is feasible at all, it should take two years of work, for us or for National, to develop an acceptable fibrous composite using FR 8420. We would have to budget about $500,000 per year to the project. National would probably need to spend more, about $1 million per year for two years, because their goal would be to develop a product ready for mass production, whereas we are just trying to prove feasibility to get the patent.”

“Our current patent expires seven years from now, after which anybody can make composites like Dentosite using our current techniques,” Gretter continued. “So this alternative fibrous technology would be only valuable to National during the next seven years. That means that they really would need to get this new composite developed and into production within two years or it is just not worthwhile for them. On the other hand, if they do develop an acceptable fibrous composite in two years (and if we do not stop them by getting a patent on the process first), then during the last five years of our patent we will probably lose about one-half our market for Dentosite to them. From their point of view, it must look pretty risky, and I cannot imagine that anybody else besides National Dental Corporation would be willing to even consider trying to develop this technology.”

“According to the projections of the Data Research study, demand for our composite should be between $15 million and $20 million per year over the next seven years. Our profit margin has been 20% of sales of Dentosite, and I expect that National would follow a similar pricing policy. They would not need to start a price war to take market share from us in this field, once they had a product to sell. So if we project sales at $17.5 million per year and use a 10% discount rate, the present discounted value of profits from one-half the market for Dentosite during the period between two and seven years from now is $6.0 million.”

After Gretter finished, Piper made a few notes and tried to summarize the situation. “Our whole problem seems to depend on what National does,” he said. “It is foolish to spend money to develop a technology that we do not want to use if National is not trying to develop it. On the other hand, if National is trying to develop this technology, then we cannot afford to drop out of the race. So it all depends on how the people at National see this situation. Do you think that they see it as you have just described? Is there anything that we know that they do not know?”

“Everything that we have discussed so far is commonly known in the industry,” Gretter replied. “Certainly National has people who check the *Federal Bulletin* every month, just as we do. If they have not noticed FR 8420 yet, they certainly will soon. Except for a few minor details, we probably have the same information about the technology and economics of the situation. I used Data Research's expected projections precisely because they are what National would be considering. Actually, our annual sales have been around $16.0 million per year, and that is probably a better estimate of future annual sales than $17.5 million. But that does not look like a significant distinction, in view of all the other uncertainties.”

DECS-452 Course Outline

B. Decision trees and the extensive form

1. One-person decision problems

   a) **Information set**: A collection of a player's decision nodes, at each of which he holds precisely the same information. No strategy can call for systematically different actions at different nodes in the same information set.
   b) Elimination of information sets (actually, reduction to single nodes) by incorporating moves of Nature only when they become observable; this can always be done in a one-person problem
   c) Backward induction to find optimal decision, once information sets are all reduced to single decision nodes

2. Two-person games

   a) Information sets
   b) Persistence of information sets: It may be that no redrawing of the “game tree” eliminates all information sets
   c) Inapplicability of backward induction, and consequent need for “circular” reasoning.

C. Strategies

1. Poker example

2. **Pure strategy**: A specification of an action for every one of a player's information sets, i.e., a complete plan of action. Can be viewed as a book, with one page for each of the player's information sets, listing on each page the action to be taken at that information set.

3. **Essential equivalence**: Throwing away all but one of any set of books that differ only on pages that will never be read.

4. Dominance: Throwing away a book that is inferior to another against any opposing book.

5. Reduction to strategic representation: Determining the outcome of any pairing of strategy books.

6. Iterated domination: If we believe our opponent to be clever enough to not follow a dominated strategy, then we may find ourselves able to eliminate other of our own strategies via dominance arguments.

7. Randomized strategies: The need, on occasion, to hide behind the veil of unpredictability.

   a) Example (2x2) without dominance
   b) Discussion of inconsistent theories of strategic choice: Any theory which prescribed the use of a particular pure strategy (in the poker example) would be a theory which (once our opponent also learned it) we would choose to violate.
   c) The need to be "unpredictable" at times
   d) Unpredictability in practice: Actions based on private information (the second version of the Ware case will provide an example)

D. Analysis

1. Computation of optimal strategies in the poker example

   a) Equalizing arguments: If our opponent knows the theory, the best we can do against him is to randomize in such a way as to make him indifferent between his best alternative actions. (This is generally true for zero-sum games only.)
   b) The minimax theorem: For zero-sum games, both players have optimal strategies, and the game has a "value."
   c) The notions of "optimal strategies" and "value of the game" don't extend to non-zero-sum games. The classical "Battle of the Sexes" provides an example, and the Ware case will provide another.