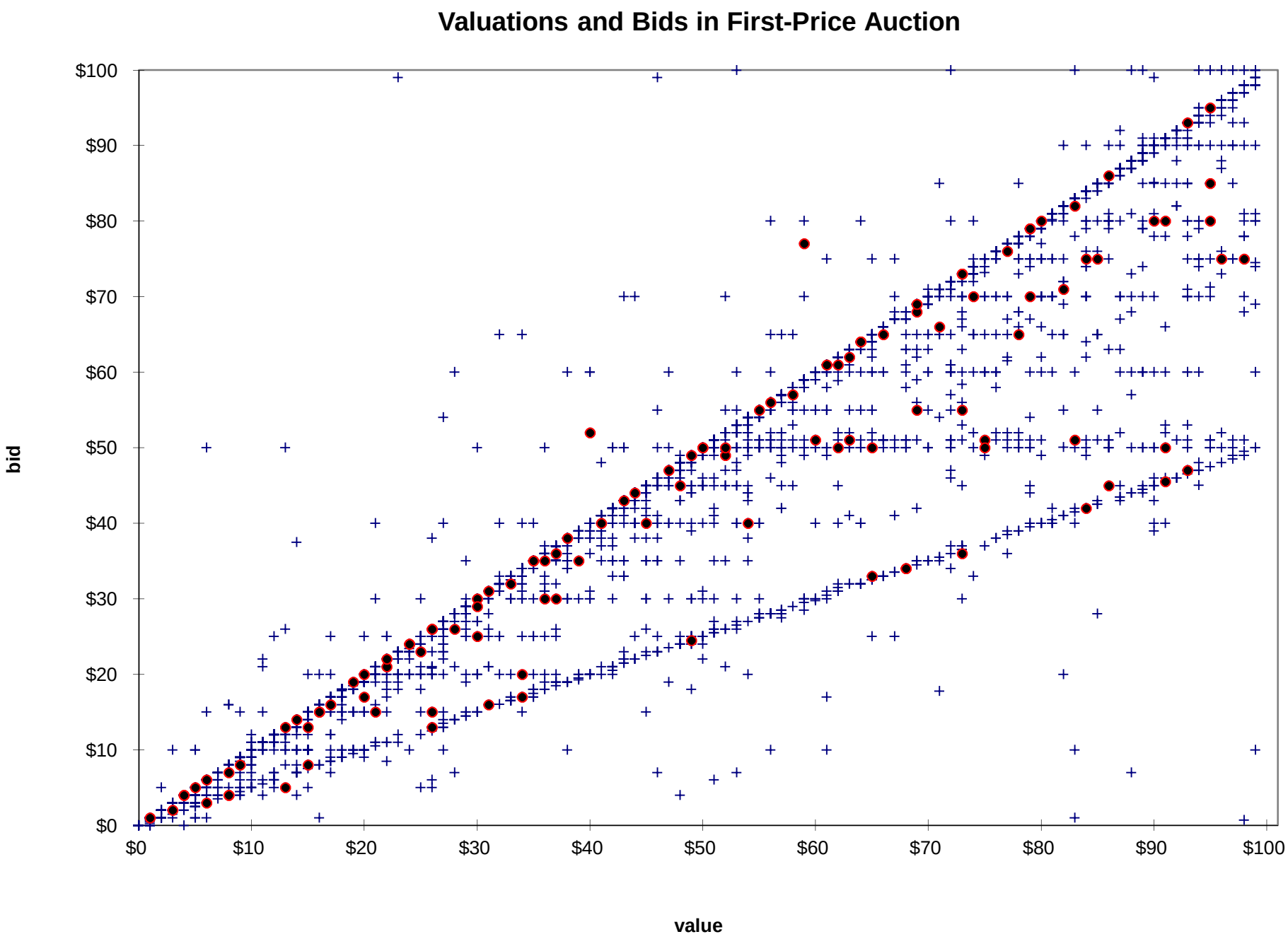
Valuations and Bids in First-Price Auction
bid
value
$0
$10
$20
$30
$40
$50
$60
$70
$80
$90
$100

Auction Results, 1989-2016

Probability that a Bid Wins

Bid	Pr(win)	Bid	Pr(win)	Bid	Pr(win)	Bid	Pr(win)
$0.00	0.52%	$25.00	30.93%	$50.00	60.54%	$75.00	85.10%
$0.50	1.09%	$25.50	31.82%	$50.50	62.50%	$75.50	85.79%
$1.00	1.78%	$26.00	32.26%	$51.00	63.62%	$76.00	86.16%
$1.50	2.37%	$26.50	32.64%	$51.50	64.74%	$76.50	86.50%
$2.00	2.72%	$27.00	33.00%	$52.00	65.33%	$77.00	86.77%
$2.50	3.12%	$27.50	33.40%	$52.50	65.81%	$77.50	87.00%
$3.00	3.76%	$28.00	34.03%	$53.00	66.20%	$78.00	87.32%
$3.50	4.29%	$28.50	34.70%	$53.50	66.56%	$78.50	87.64%
$4.00	5.07%	$29.00	35.15%	$54.00	67.04%	$79.00	87.98%
$4.50	5.89%	$29.50	35.52%	$54.50	67.49%	$79.50	88.32%
$5.00	6.62%	$30.00	36.52%	$55.00	68.27%	$80.00	88.94%
$5.50	7.32%	$30.50	37.48%	$55.50	69.05%	$80.50	89.64%
$6.00	7.91%	$31.00	37.84%	$56.00	69.46%	$81.00	89.89%
$6.50	8.48%	$31.50	38.19%	$56.50	69.84%	$81.50	90.14%
$7.00	9.05%	$32.00	38.73%	$57.00	70.35%	$82.00	90.44%
$7.50	9.63%	$32.50	39.28%	$57.50	70.80%	$82.50	90.71%
$8.00	10.26%	$33.00	39.81%	$58.00	71.08%	$83.00	90.88%
$8.50	10.92%	$33.50	40.31%	$58.50	71.37%	$83.50	91.06%
$9.00	11.56%	$34.00	40.88%	$59.00	71.85%	$84.00	91.27%
$9.50	12.09%	$34.50	41.40%	$59.50	72.26%	$84.50	91.49%
$10.00	13.00%	$35.00	42.15%	$60.00	73.24%	$85.00	91.99%
$10.50	13.87%	$35.50	42.95%	$60.50	74.22%	$85.50	92.52%
$11.00	14.37%	$36.00	43.34%	$61.00	74.55%	$86.00	92.79%
$11.50	14.85%	$36.50	43.75%	$61.50	74.91%	$86.50	93.06%
$12.00	15.35%	$37.00	44.25%	$62.00	75.28%	$87.00	93.36%
$12.50	15.83%	$37.50	44.68%	$62.50	75.61%	$87.50	93.63%
$13.00	16.42%	$38.00	45.10%	$63.00	75.91%	$88.00	93.93%
$13.50	17.04%	$38.50	45.53%	$63.50	76.21%	$88.50	94.23%
$14.00	17.47%	$39.00	45.85%	$64.00	76.46%	$89.00	94.50%
$14.50	17.91%	$39.50	46.21%	$64.50	76.71%	$89.50	94.73%
$15.00	18.84%	$40.00	47.60%	$65.00	77.58%	$90.00	95.26%
$15.50	19.69%	$40.50	48.99%	$65.50	78.42%	$90.50	95.80%
$16.00	20.05%	$41.00	49.39%	$66.00	78.61%	$91.00	96.06%
$16.50	20.57%	$41.50	49.79%	$66.50	78.81%	$91.50	96.33%
$17.00	21.19%	$42.00	50.46%	$67.00	79.02%	$92.00	96.46%
$17.50	21.67%	$42.50	51.07%	$67.50	79.24%	$92.50	96.58%
$18.00	22.19%	$43.00	51.64%	$68.00	79.45%	$93.00	96.99%
$18.50	22.65%	$43.50	52.17%	$68.50	79.67%	$93.50	97.40%
$19.00	23.29%	$44.00	52.69%	$69.00	79.95%	$94.00	97.56%
$19.50	23.93%	$44.50	53.21%	$69.50	80.24%	$94.50	97.72%
$20.00	25.09%	$45.00	54.17%	$70.00	81.23%	$95.00	97.99%
$20.50	26.26%	$45.50	55.11%	$70.50	82.23%	$95.50	98.25%
$21.00	26.80%	$46.00	55.57%	$71.00	82.51%	$96.00	98.42%
$21.50	27.31%	$46.50	56.02%	$71.50	82.83%	$96.50	98.58%
$22.00	27.81%	$47.00	56.57%	$72.00	83.07%	$97.00	98.72%
$22.50	28.26%	$47.50	57.00%	$72.50	83.30%	$97.50	98.86%
$23.00	28.72%	$48.00	57.35%	$73.00	83.55%	$98.00	99.11%
$23.50	29.18%	$48.50	57.69%	$73.50	83.80%	$98.50	99.29%
$24.00	29.61%	$49.00	58.17%	$74.00	84.08%	$99.00	99.39%
$24.50	30.06%	$49.50	58.65%	$74.50	84.38%	$99.50	99.50%
$25.00	30.93%	$50.00	60.54%	$75.00	85.10%	$100.00	99.72%

Analysis of the Bidding Exercise

Assume that your own valuation is \$70, and that you make a bid of \$b.
Then your expected payoff will be

$$(70 - b) \cdot \text{Prob(your bid of \$b wins)} .$$

A plausible model of opponent behavior is that, whatever value \$V he has, he will bid a fixed fraction f of that value (i.e., he will bid \$fV). Values of f near 1 correspond to "aggressive" bidding behavior; smaller values to less aggressive behavior.

Then

$$\begin{aligned} &\text{Prob(your bid of \$b wins)} \\ &= \text{Prob(} fV < b \text{)} \\ &= \text{Prob(} V < b/f \text{)} \\ &= (b/f)/100 . \end{aligned}$$

Therefore, your own expected payoff is

$$(70 - b) \cdot b / (100f) ;$$

this is maximized by taking \$b = \$35, **no matter what value f takes**. (Of course, if $f < ½$, don't bid more than \$100f.) More generally, against any opponent who fits the model, your optimal bid is one-half of your valuation.

Note:

Bidders typically bid far more than can be justified (either theoretically *or* empirically.)

Possible explanations:

They worry too much about getting *something*, and not enough about how much they actually get (i.e., they are "conflict averse")

They "project" their own valuations onto their opponents.

Auction Results, 1989-2016

Optimal Bids

Value	Optimal Bid	Expected Payoff	"Half" Strategy	Expected Payoff
$0.00	$0.00	$0.00	$0.00	$0.00
$1.00	$0.50	$0.01	$0.50	$0.01
$2.00	$1.00	$0.02	$1.00	$0.02
$3.00	$1.00	$0.04	$1.50	$0.04
$4.00	$2.00	$0.05	$2.00	$0.05
$5.00	$1.50	$0.08	$2.50	$0.08
$6.00	$3.00	$0.11	$3.00	$0.11
$7.00	$4.00	$0.15	$3.50	$0.15
$8.00	$4.50	$0.21	$4.00	$0.20
$9.00	$4.50	$0.27	$4.50	$0.27
$10.00	$5.00	$0.33	$5.00	$0.33
$11.00	$5.50	$0.40	$5.50	$0.40
$12.00	$5.50	$0.48	$6.00	$0.47
$13.00	$6.00	$0.55	$6.50	$0.55
$14.00	$6.50	$0.64	$7.00	$0.63
$15.00	$7.00	$0.72	$7.50	$0.72
$16.00	$8.00	$0.82	$8.00	$0.82
$17.00	$8.50	$0.93	$8.50	$0.93
$18.00	$10.50	$1.04	$9.00	$1.04
$19.00	$10.50	$1.18	$9.50	$1.15
$20.00	$10.50	$1.32	$10.00	$1.30
$21.00	$10.50	$1.46	$10.50	$1.46
$22.00	$10.50	$1.60	$11.00	$1.58
$23.00	$10.50	$1.73	$11.50	$1.71
$24.00	$10.50	$1.87	$12.00	$1.84
$25.00	$11.00	$2.01	$12.50	$1.98
$26.00	$11.00	$2.16	$13.00	$2.13
$27.00	$12.00	$2.30	$13.50	$2.30
$28.00	$13.50	$2.47	$14.00	$2.45
$29.00	$15.50	$2.66	$14.50	$2.60
$30.00	$15.50	$2.86	$15.00	$2.83
$31.00	$15.50	$3.05	$15.50	$3.05
$32.00	$15.50	$3.25	$16.00	$3.21
$33.00	$15.50	$3.45	$16.50	$3.39
$34.00	$15.50	$3.64	$17.00	$3.60
$35.00	$15.50	$3.84	$17.50	$3.79
$36.00	$20.50	$4.07	$18.00	$3.99
$37.00	$20.50	$4.33	$18.50	$4.19
$38.00	$20.50	$4.60	$19.00	$4.43
$39.00	$20.50	$4.86	$19.50	$4.67
$40.00	$20.50	$5.12	$20.00	$5.02
$41.00	$20.50	$5.38	$20.50	$5.38
$42.00	$20.50	$5.65	$21.00	$5.63
$43.00	$20.50	$5.91	$21.50	$5.87
$44.00	$20.50	$6.17	$22.00	$6.12
$45.00	$20.50	$6.43	$22.50	$6.36
$46.00	$21.00	$6.70	$23.00	$6.61
$47.00	$21.00	$6.97	$23.50	$6.86
$48.00	$21.50	$7.24	$24.00	$7.11
$49.00	$21.50	$7.51	$24.50	$7.36
averages		$10.32		$10.12

Value	Optimal Bid	Expected Payoff	"Half" Strategy	Expected Payoff
$50.00	$25.50	$7.80	$25.00	$7.73
$51.00	$25.50	$8.11	$25.50	$8.11
$52.00	$25.50	$8.43	$26.00	$8.39
$53.00	$25.50	$8.75	$26.50	$8.65
$54.00	$25.50	$9.07	$27.00	$8.91
$55.00	$25.50	$9.39	$27.50	$9.19
$56.00	$25.50	$9.71	$28.00	$9.53
$57.00	$25.50	$10.02	$28.50	$9.89
$58.00	$25.50	$10.34	$29.00	$10.19
$59.00	$30.50	$10.68	$29.50	$10.48
$60.00	$30.50	$11.06	$30.00	$10.96
$61.00	$30.50	$11.43	$30.50	$11.43
$62.00	$30.50	$11.81	$31.00	$11.73
$63.00	$30.50	$12.18	$31.50	$12.03
$64.00	$30.50	$12.56	$32.00	$12.39
$65.00	$30.50	$12.93	$32.50	$12.77
$66.00	$30.50	$13.31	$33.00	$13.14
$67.00	$30.50	$13.68	$33.50	$13.50
$68.00	$30.50	$14.06	$34.00	$13.90
$69.00	$30.50	$14.43	$34.50	$14.28
$70.00	$35.50	$14.82	$35.00	$14.75
$71.00	$35.50	$15.25	$35.50	$15.25
$72.00	$35.50	$15.68	$36.00	$15.60
$73.00	$35.50	$16.11	$36.50	$15.97
$74.00	$35.50	$16.54	$37.00	$16.37
$75.00	$35.50	$16.96	$37.50	$16.75
$76.00	$35.50	$17.39	$38.00	$17.14
$77.00	$40.50	$17.88	$38.50	$17.53
$78.00	$40.50	$18.37	$39.00	$17.88
$79.00	$40.50	$18.86	$39.50	$18.25
$80.00	$40.50	$19.35	$40.00	$19.04
$81.00	$40.50	$19.84	$40.50	$19.84
$82.00	$40.50	$20.33	$41.00	$20.25
$83.00	$40.50	$20.82	$41.50	$20.66
$84.00	$40.50	$21.31	$42.00	$21.19
$85.00	$40.50	$21.80	$42.50	$21.70
$86.00	$51.50	$22.34	$43.00	$22.20
$87.00	$51.50	$22.98	$43.50	$22.69
$88.00	$51.50	$23.63	$44.00	$23.18
$89.00	$51.50	$24.28	$44.50	$23.68
$90.00	$51.50	$24.93	$45.00	$24.38
$91.00	$51.50	$25.57	$45.50	$25.08
$92.00	$51.50	$26.22	$46.00	$25.56
$93.00	$51.50	$26.87	$46.50	$26.05
$94.00	$51.50	$27.52	$47.00	$26.59
$95.00	$51.50	$28.16	$47.50	$27.07
$96.00	$51.50	$28.81	$48.00	$27.53
$97.00	$51.50	$29.46	$48.50	$27.98
$98.00	$51.50	$30.11	$49.00	$28.50
$99.00	$51.50	$30.75	$49.50	$29.03
$100.00	$51.50	$31.40	$50.00	$30.27

Auction Results, 1989-2016

Evaluating the "Half" Strategy

Valuation	Maximum Possible Expected Payoff	Expected Payoff from "Half"	Ratio	
$0	$0.00	$0.00	100.0%	
$5	$0.08	$0.08	94.0%	
$10	$0.33	$0.33	100.0%	
$15	$0.72	$0.72	99.8%	
$20	$1.32	$1.30	98.6%	98.1%
$25	$2.01	$1.98	98.4%	
$30	$2.86	$2.83	99.0%	Average
$35	$3.84	$3.79	98.8%	Yield from
$40	$5.12	$5.02	98.0%	"Half"
$45	$6.43	$6.36	98.8%	
$50	$7.80	$7.73	99.2%	
$55	$9.39	$9.19	97.9%	
$60	$11.06	$10.96	99.1%	
$65	$12.93	$12.77	98.7%	
$70	$14.82	$14.75	99.6%	
$75	$16.96	$16.75	98.8%	
$80	$19.35	$19.04	98.4%	
$85	$21.80	$21.70	99.6%	
$90	$24.93	$24.38	97.8%	
$95	$28.16	$27.07	96.1%	
$100	$31.40	$30.27	96.4%	

Further Comments on the Auction Experiment

The bid-half-of-your-valuation result should not be misinterpreted as "general" advice for how to act in an auction. Our derivation relied on the rules of the auction (a so-called "first-price auction," in which the high bidder pays his bid), and also on the facts that that there were only two bidders, that each knew his valuation (i.e., that there was no mutual uncertainty concerning the quality of the item being sold), that the bidders were risk-neutral, that the valuations were independently determined, that the valuations came from the same uniform distribution, and that this uniform distribution was anchored at 0. Changing *any* of these features would change our analysis.

For example, the second version of the auction (a so-called "second-price" auction), in which the winning (high) bidder pays the other's bid (i.e., the highest losing bid determines the price), leads to a very different result: Bidding your own valuation is a *dominant* strategy. (If, instead of bidding your own valuation, you bid a bit more, you'll still win in the cases when the other bidder bids less than your valuation, and you'll pay the same price. When the other bidder bids above your elevated bid, you'll still lose. But if the other bidder bids anywhere between your elevated bid and your actual valuation, you'll win and pay more than the item being sold is worth to you. An analogous argument shows that bidding less than your valuation can only hurt you.)

In the first-price auction we simulated, the pairing of strategies in which each bidder bids half of his valuation forms the only "equilibrium point" of the bidding game. (If our competitor is bidding half of his valuation, he fits our proposed model of his bidding behavior, and therefore we can't do any better than to bid half of our own valuation.

In the second-price auction, the pairing of strategies in which each bids his own valuation forms the only *symmetric* "equilibrium point" of the bidding game. However, there are other equilibria, such as the one in which one bidder bids $1,000,000 for the item, and the other bids $0. Clearly, neither gains personally by deviating from the indicated behavior. However, the million-dollar bidder can suffer if, for example, the other bidder follows his dominant "bid your own valuation" strategy instead of just bidding $0.

DECS-452 Course Outline

4. Extensive form
    a) Global thermonuclear war (and MAD)
    b) **Perfect equilibria**: A perfect equilibrium point specifies optimal behavior for all players at all locations in the "game tree." At the unique perfect equilibrium point of the GTW game, the Russians launch a first strike, expecting the U.S. to (optimally) not retaliate.
    c) One-sided revision of the game: By precommitting to retaliation, the U.S. creates a new game in which the unique perfect equilibrium point has the Russians refraining from attack. (cf. Dr. Strangelove)

5. The auction experiment
    a) You should set Pr(win) = value/100, unless prepared to argue why you believe your competitor would bid differently than you when in an identical situation.
    b) Bid = value is dominant in second-price auction.
    c) Bid = 1/2 * value is optimal against any multiplicative (bid = fixed fraction of private valuation) strategy in first-price auction. Note: This result holds for this **specific** game, with two bidders and independent, identically uniformly-distributed private valuations which range down to zero.