The Analytical Basics

The basic rules of probability:

Pr(A) + Pr(not-A) = 1; Pr(A and B) + Pr(A and not-B) = Pr(A)

Pr(A or B) = 1 – Pr(not-A and not-B)

Pr(A or B) = Pr(A) + Pr(B) – Pr(A and B)

The basic rules of conditional probability:

Definition: Pr(A|B) = Pr(A and B) / Pr(B)

Pr(A and B) = Pr(A)·Pr(B|A); when A and B are independent, Pr(A and B) = Pr(A)·Pr(B)

Pr(A and B and C) = Pr(A)·Pr(B|A)·Pr(C|A and B), and so on

Pr(A) = Pr(A|B_1) · Pr(B_1) + … + Pr(A|B_k) · Pr(B_k), when B_1, …, B_k are disjoint and exhaustive

Bayes' Rule, and how it works using probability trees

The basic rules of expectation:

E[aX+b] = a·E[X] + b

E[X+Y] = E[X] + E[Y]

E[X] = E[X|B_1]·Pr(B_1) + … + E[X|B_k]·Pr(B_k), when B_1,…, B_k are disjoint and exhaustive

E[XY] = E[X]·E[Y], if X and Y are independent

The basic rules of variability:

Definitions: Var(X) = E[X^2] - $(E[X])^2$ = E[$(X-E[X])^2$]; StDev(X) = √Var(X)

Var(aX+b) = a^2 Var(X);   StDev(aX+b) = |a| StDev(X)

Var(X+Y) = Var(X) + Var(Y) + 2 Cov(X,Y)

Var(X+Y+Z) = Var(X) + Var(Y) + Var(Z) + 2 Cov(X,Y) + 2 Cov(X,Z) + 2 Cov(Y,Z), and so on.

Definition: Cov(X,Y) = E[XY] - E[X]·E[Y] = E[(X-E[X]) · (Y-E[Y])]

Cov(aX+b,cY+d) = ac·Cov(X,Y)

Definition: Corr(X,Y) = Cov(X,Y) / (StDev(X) · StDev(Y))

If X, X_1, …, X_n are independent and identically distributed:

E[X_1+…+X_n] = n·E[X]

Var(X_1+…+X_n) = n·Var(X); StDev(X_1+…+X_n) = √n · StDev(X),

Var((X_1+…+X_n) / n) = Var(X) / n; StDev((X_1+…+X_n) / n) = StDev(X) / √n