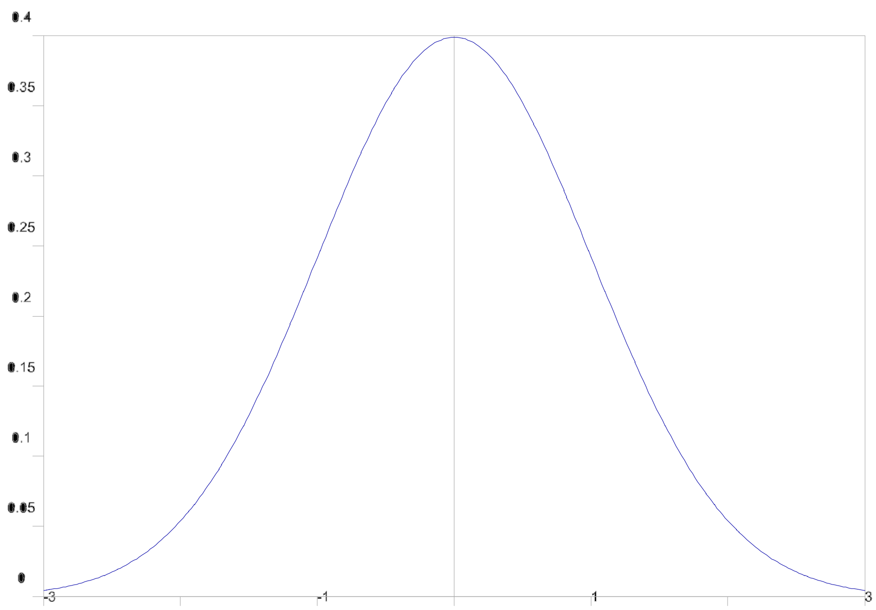
Continuous Random Variables

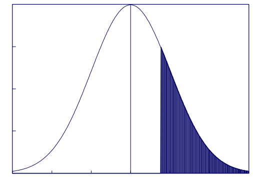
A *continuous random variable* is a random variable which can take any value in some interval. A continuous random variable is characterized by its *probability density function*, a graph which has a total area of 1 beneath it: The probability of the random variable taking values in any interval is simply the area under the curve over that interval.

The ***normal distribution***: This most-familiar of continuous probability distributions has the classic "bell" shape (see the graph below). The peak occurs at the mean of the distribution, i.e., at the expected value of the normally-distributed random variable with this distribution, and the standard deviation (the square root of the variance) indicates the spread of the bell, with roughly 68% of the area within 1 standard deviation of the peak.



The Central Limit Theorem: The normal distribution arises so frequently in applications due to an amazing fact: If you take a bunch of independent random variables (with comparable variances) and add (or average) them, the result will be roughly normally distributed, *no matter what the distributions of the separate variables might be*. Many interesting quantities, ranging from IQ scores (across a demographically-homogeneous group of individuals), to changes in share prices over time, to demand for a retail product (as it varies from day to day), to lengths of toy tractor axles (being cut in an automated process), are actually a composite of many separate random variables, and hence are roughly normally distributed.

If X is normal, and $Y = aX+b$, then Y is also normal, with $E[Y] = a \cdot E[X] + b$ and $StdDev[Y] = |a| \cdot StdDev[X]$. If X and Y are normal (independent or not), then $X+Y$ and $X-Y = X+(-Y)$ are also normal (intuition: the sum of two bunches is a bunch). Any normally-distributed random variable can be transformed into a "standard" normal random variable (with mean 0 and standard deviation 1) by subtracting off its mean and dividing by its standard deviation. Hence, a single tabulation of the cumulative distribution for a standard normal random variable (attached) can be used to do probabilistic calculations for *any* normally-distributed random variable.



Right-Tail Probabilities of the Normal Distribution

		+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09	+0.10
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	0.4602
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247	0.4207
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859	0.3821
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483	0.3446
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121	0.3085
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776	0.2743
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451	0.2420
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148	0.2119
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867	0.1841
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611	0.1587
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	0.1357
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	0.1151
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985	0.0968
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	0.0808
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	0.0668
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	0.0548
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	0.0446
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	0.0359
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	0.0287
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	0.0228
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	0.0179
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	0.0139
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	0.0082
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	0.0062
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	0.0047
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013

Simulating Random Variables

Sometimes one wishes to simulate a random process (so as to compare alternative policies on a computer before trying them out in real life). Simulations require the generation of random variables fitting particular distributions. Most computer languages and spreadsheets can provide (pseudo-) random numbers, uniformly distributed between 0 and 1. Fortunately, random variables with *any* probability distribution can be generated from these uniform random variables:

Consider any random variable X. Simply take a uniformly-distributed observation u, solve the equation $\Pr(X \le x) = u$ for x, and let the resulting x be your observation of X.

For example, in Excel the function RAND() returns a random value uniformly distributed between 0 and 1. In addition, the function NORMINV(.,.,.) returns the inverse of the cumulative normal probability distribution. Therefore, to generate a random observation from a normal distribution with a particular mean and standard deviation, one can simply enter the formula =NORMINV(RAND(),mean,stddev) in a cell of the spreadsheet.