Homework Assignment 2

PROBLEM 1: The WarmUp Company

Your firm, the WarmUp Company, sells only two products: winter hats and coffee mugs. (This is quite a limited portfolio of products!) In terms of revenue, you charge prices of $20 per hat and $10 per mug. However, you are uncertain how many hats and mugs you will sell this year. Let \( H \) denote the number of hats you will sell this year, and let \( M \) denote the number of mugs you will sell this year. Based on forecasts from your sales team, you believe that...

\[
E(H) = 22,000, \quad \text{Var}(H) = 9,000,000, \quad E(M) = 8,000, \quad \text{Var}(M) = 1,000,000
\]

A. What is your expected total revenue for the year from these two products?
B. Assuming \( H \) and \( M \) are independent, what is the variance of your total revenue for the year?

Problem 2: Pooled Testing

Read the Pooled Testing case on the class website. Answer the following two questions (ignore the wording of questions in the case).

A. We found that by pooling \( n=20 \) blood samples, the expected testing cost is 35.5 baht per person. Would it be cheaper to pool \( n=10 \) samples at a time? Or \( n=50 \) samples at a time? Create a spreadsheet showing the per-person cost of testing when pooling \( n \) blood samples, for \( n = 2,3,4,\ldots,100 \).
B. Which \( n \) minimizes the expected per-person cost of testing?
C. Redo the above problem (b) when there is a 0.1% (0.001) probability that any particular person has the disease for which we are testing.

Problem 3: Bad Debt encore

Reconsider the Bad Debt problem from Homework 1, and its accompanying dataset. The second column in the dataset provides the variable “Invoice amount”. Suppose that the firm separates invoices into two categories: an invoice is “Small” if the invoice amount is less than $4,000, and it is “Large” if it is $4,000 or more. How would this firm estimate its bad debt? We answer this by performing the following steps.

(A) Create two copies of the probability tree structure you used in Homework 1 (reproduced a couple of pages below); label one copy as “Small” and one as “Large”. Using the data from Homework 1 (BadDebt.xls), fill in probability information for the “Small” tree using data on small invoices. Do the same for the “Large” tree using data on large invoices. (Hint: you are merely dividing the data into two sets, and doing two new versions of the same exercise you did in Homework 1!) Your answer should be two complete trees.
(B) Using your trees, compute the following probabilities. (Use your “Small” tree for the first four, and your “Large” tree for the other four.)

i. \( \text{Pr}(\text{bad debt} \mid \text{small and not yet overdue}) \)

ii. \( \text{Pr}(\text{bad debt} \mid \text{small, overdue no more than 60 days}) \)

iii. \( \text{Pr}(\text{bad debt} \mid \text{small, overdue between 61 and 90 days}) \)

iv. \( \text{Pr}(\text{bad debt} \mid \text{small, overdue more than 90 days}) \)

v. \( \text{Pr}(\text{bad debt} \mid \text{large and not yet overdue}) \)

vi. \( \text{Pr}(\text{bad debt} \mid \text{large, overdue no more than 60 days}) \)

vii. \( \text{Pr}(\text{bad debt} \mid \text{large, overdue between 61 and 90 days}) \)

viii. \( \text{Pr}(\text{bad debt} \mid \text{large, overdue more than 90 days}) \)

(C) The firm has the following outstanding invoice totals:

<table>
<thead>
<tr>
<th>Age of Accounts Receivable</th>
<th>Receivable Balance “Small” Invoices</th>
<th>Receivable Balance “Large” Invoices</th>
<th>Receivable Balance Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>$30,000</td>
<td>$20,000</td>
<td>$50,000</td>
</tr>
<tr>
<td>1-60 days past due</td>
<td>$18,000</td>
<td>$12,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>61-90 days past due</td>
<td>$9,000</td>
<td>$6,000</td>
<td>$15,000</td>
</tr>
<tr>
<td>Over 90 days past due</td>
<td>$1,000</td>
<td>$4,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>Total</td>
<td>$58,000</td>
<td>$42,000</td>
<td>$100,000</td>
</tr>
</tbody>
</table>

Using the outstanding invoice totals from above (and your estimated probabilities from (B)), what is the expected bad debt that will accrue from small invoices? What is it for large invoices? (Provide two separate numbers. This exercise mirrors what we discussed in class.)

(D) *(Food-for-thought question: worth zero points.*) In class, we estimated this firm’s bad debt without breaking invoices into two categories, and got an answer of $2900. How does this compare to the sum of your two answers in (B)? Should these two approaches give the same answers?