Statistics

… is all about making inferences concerning a population through the use of sample data.

… must therefore worry about exposure to sampling error, i.e., bad luck in the sampling process which leads to a somewhat-misrepresentative sample.

… deals with this problem by looking at the end results of statistical procedures as random variables, and using the tools of probability to study these random variables.

Estimation

Estimating a population mean, using simple random sampling:

estimate		margin of error (at 95%-confidence)
$\bar{x}$	$\pm$	$(\sim 2)\cdot\frac{s}{\sqrt{n}}$

Estimating a proportion:

	estimate		margin of error (at 95%-confidence)
precisely	$\hat{p}$	$\pm$	$(\sim 2)\cdot\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$
conservatively	$\hat{p}$	$\pm$	(less than) $\frac{1}{\sqrt{n}}$

The estimate is typically used for decision analysis, and the margin of error for risk/sensitivity analysis.

Setting the Sample Size

When estimating a **mean** or **proportion** using simple random sampling, *with a preliminary study at hand:*

$$needed\ sample\ size = \left(\frac{current\ margin\ of\ error}{target\ margin\ of\ error}\right)^2 \cdot (current\ sample\ size)$$

When estimating a **proportion** using simple random sampling:

The actual margin of error (at the 95%-confidence level) will always be somewhat less than $1/\sqrt{n}$, so

$$sample\ size = \frac{1}{(target\ margin\ of\ error)^2}$$

will always suffice.

Hypothesis Testing

We begin with a statement – the null hypothesis – “on trial.” At the end of the trial, we will find that the evidence at hand either contradicts the statement to some extent (i.e., the evidence supports a finding of “guilty”), or doesn’t really contradict the statement (i.e., the evidence doesn’t support a finding of “guilty,” so we find it “not (shown to be) guilty”).

Think of a hypothetical world in which the statement on trial is true. (If there’s more than one such world, choose the one which most closely fits the observed data.)

The *significance level* of the data (with respect to the statement on trial) is

Prob(we’d see data at least as contradictory to the statement as is the data at hand | the study that yielded the data at hand were to be conducted in the hypothetical world where the statement is true) .

We interpret the significance level of the data using this “translation” table:

If the numeric significance level of the data is	**then the data, all by itself, makes us**	**and the data supports the alternative**
above 20%	not at all suspicious	not at all
between 10% and 20%	a little bit suspicious	a little bit
between 5% and 10%	moderately suspicious	moderately
between 2% and 5%	very suspicious	strongly
between 1% and 2%	extremely suspicious	very strongly
below 1%	overwhelmingly suspicious	overwhelmingly

We never conclude that the evidence *supports* the statement on trial (i.e., the statement is never found “innocent”). Therefore, if our ultimate goal is to see if evidence supports a statement, we must put the opposite statement on trial, and see if the evidence contradicts that opposite statement.