Statistics

… is all about making inferences concerning a population through the use of sample data.

… must therefore worry about exposure to sampling error, i.e., bad luck in the sampling process which leads to a somewhat-misrepresentative sample.

… deals with this problem by looking at the end results of statistical procedures as random variables, and using the tools of probability to study these random variables.
Estimation

Estimating a population mean, using simple random sampling:

\[ \bar{x} \pm (\sim 2) \cdot \frac{s}{\sqrt{n}} \]

Estimating a proportion:

\[ \hat{p} \pm (\sim 2) \cdot \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \]

precisely

conservatively

(less than) \[ \frac{1}{\sqrt{n}} \]

The estimate is typically used for decision analysis, and the margin of error for risk/sensitivity analysis.

Setting the Sample Size

When estimating a mean or proportion using simple random sampling, with a preliminary study at hand:

\[ \text{needed sample size} = \left( \frac{\text{current margin of error}}{\text{target margin of error}} \right)^2 \cdot (\text{current sample size}) \]

When estimating a proportion using simple random sampling:

The actual margin of error (at the 95%-confidence level) will always be somewhat less than \( \frac{1}{\sqrt{n}} \), so

\[ \text{sample size} = \frac{1}{(\text{target margin of error})^2} \]

will always suffice.
Hypothesis Testing

We begin with a statement – the null hypothesis – “on trial.” At the end of the trial, we will find that the evidence at hand either contradicts the statement to some extent (i.e., the evidence supports a finding of “guilty”), or doesn’t really contradict the statement (i.e., the evidence doesn’t support a finding of “guilty,” so we find it “not (shown to be) guilty”).

Think of a hypothetical world in which the statement on trial is true. (If there’s more than one such world, choose the one which most closely fits the observed data.)

The significance level of the data (with respect to the statement on trial) is

\[
\text{Prob}(\text{we'd see data at least as contradictory to the statement as is the data at hand | the study that yielded the data at hand were to be conducted in the hypothetical world where the statement is true})
\]

We interpret the significance level of the data using this “translation” table:

<table>
<thead>
<tr>
<th>If the numeric significance level of the data is</th>
<th>then the data, all by itself, makes us</th>
<th>and the data supports the alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>above 20%</td>
<td>not at all suspicious</td>
<td>not at all</td>
</tr>
<tr>
<td>between 10% and 20%</td>
<td>a little bit suspicious</td>
<td>a little bit</td>
</tr>
<tr>
<td>between 5% and 10%</td>
<td>moderately suspicious</td>
<td>moderately</td>
</tr>
<tr>
<td>between 2% and 5%</td>
<td>very suspicious</td>
<td>strongly</td>
</tr>
<tr>
<td>between 1% and 2%</td>
<td>extremely suspicious</td>
<td>very strongly</td>
</tr>
<tr>
<td>below 1%</td>
<td>overwhelmingly suspicious</td>
<td>overwhelmingly</td>
</tr>
</tbody>
</table>

We never conclude that the evidence supports the statement on trial (i.e., the statement is never found “innocent”). Therefore, if our ultimate goal is to see if evidence supports a statement, we must put the opposite statement on trial, and see if the evidence contradicts that opposite statement.