Limited Stock Market Participation and the Equity Premium Puzzle

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Contents

Abstract

The paper analyzes whether accounting for limited stock market participation improves the performance of the consumption capital asset pricing model and thus helps resolve the equity premium puzzle. A simple condition is given under which the equity premium predicted by the standard CCAPM is only a fraction λ of the true equity premium generated by the process for consumption, where λ is the fraction of stockholders in the population. Correspondingly, estimating Euler equations involving stock returns without excluding nonstockholders will result in an estimate of relative risk aversion which is too high by factor $1/\lambda$. With an average value of λ of about 20 percent in postwar US data, this suggests limited stock market participation as a plausible explanation of the equity premium puzzle. I test this hypothesis using micro consumption and asset holding data from the Consumer Expenditure Survey. The empirical results show that accounting for differences in consumption patterns of stockholders and nonstockholders should be considered a major part of the solution to the puzzle.

1 Introduction

Empirical tests of the consumption capital asset pricing model with constant relative risk aversion have rejected the model in several ways. In a general equilibrium model Mehra and Prescott (1985) showed that the covariance of US per capita consumption growth and stock returns is too low to explain the observed equity premium unless risk aversion is assumed to be implausibly high. Hansen and Jagannathan (1991) derived a lower bound for the standard deviation of any valid stochastic discount factor and found that the stochastic discount factor implied by the CCAPM only satisfies the bound if risk aversion is very large. Hansen and Singleton (1983) found that when the loglinearized conditional Euler equation for an assumed representative agent was estimated for the nominally riskfree rate or jointly for an aggregate stock return and a nominally riskfree rate, the overidentifying restrictions strongly rejected the model. This indicates predictable deviations from the Euler equations. They furthermore found that estimates based on unconditional Euler equations lead to large risk aversion estimates, consistent with Mehra and Prescott's results. Hansen and Singleton (1982), (1984) found that risk aversion estimates from GMM estimation of nonlinear conditional Euler equations were negative in many of the cases considered.

Mehra and Prescott's finding is the well known equity premium puzzle. As Weil (1989) emphasized it implies a risk free rate puzzle. If consumers with CRRA utility are very averse to differences in consumption across states, they are also very averse to differences in consumption across time. Since the low observed riskfree rate offers little incentive to save, the observed average growth rate of per capita consumption is inconsistent with the large estimate of risk aversion unless the representative agent has a negative discount rate. In Hansen and Jagannathan's test this puzzle takes the form of the CCAPM stochastic discount factor entering the bound at a mean implying a very large riskfree rate.

This paper proposes limited stock market participation as a unified framework for explaining these rejections of the model. If the consumption growth of nonstockholders covaries less with stock returns than that of stockholders, including the consumption of nonstockholders in the consumption measure used to test the CCAPM will lead to an upward biased estimate of risk aversion. Furthermore, since Euler equations involving stock returns will not hold for nonstockholders, limited stock market participation has the potential of explaining the rejections of the model based on tests of overidentifying restrictions. Finally, the fact that a fraction of households hold all stock market risk could endogenously cause their consumption growth and thus their stochastic discount factor to be more volatile than that of nonstockholders.

The first section of the paper contains a theoretical analysis of the effect of limited stock market participation on Euler equation estimation. If nonstockholder consumption growth is independent of stock returns, the equity premium predicted by the standard CCAPM is only a fraction λ of the true equity premium generated by the process for consumption. λ is the fraction of stockholders in the population. Correspondingly, estimating Euler equations involving the stock return without excluding nonstockholders will result in an estimate of relative risk aversion which is too high by factor $1/\lambda$. The derivations of these results are based on aggregation of Euler equations without imposing any general equilibrium structure. This is why the condition for them to hold takes the form of a condition directly on the consumption of nonstockholders.

Mankiw and Zeldes (1991) first emphasized that nonstockholders should be excluded in tests of the CCAPM, and estimated Euler equations for stockholders and nonstockholders separately using data from the Panel Study of Income Dynamics. Based on unconditional Euler equations, they found large differences between the two groups but the estimate of relative risk aversion remained as high as 35.2 for the richest group of stockholders. However, the PSID only contains data on food consumption. If preferences are not separable in food and non-food components this is problematic. I therefore use data on consumption of nondurables and services from the Consumer Expenditure Survey to test the limited stock market participation theory. Brav and Geczy (1996) (and the updated and extended version by Brav, Constantinides and Geczy (1999) confirm Mankiw and Zeldes' findings for unconditional Euler equations using CEX data. They show that risk aversion estimates are much larger for assetholders than for nonassetholders, and that risk aversion estimates decline as they look at still wealthier layers of assetholders. They estimate risk aversion to be 12 for the richest assetholders.[1] Mankiw and Zeldes (1991) and Brav, Constantinides and Geczy (1999) do not provide standard errors for their estimates. I use bootstrap methods to show that the standard errors of risk aversion estimates based on unconditional Euler equations are extremely large. This suggests that adding information about

[1] The asset measure used by Brav, Constantinides and Geczy (1999) is assets in savings accounts plus assets in stocks, bonds and mutual funds. They refer to households with positive holdings of either as assetholders. In my study I focus on whether households have assets in stocks, bonds or mutual funds to (partially) be able to separate out stockholders. This will be discussed in detail in the data section.

predictable movements in expected consumption growth rates and expected asset returns is valuable. I therefore estimate risk aversion based on conditional Euler equations as well. My estimates of $1/\gamma$ for stockholders are significantly larger than zero. The point estimates imply values of γ in the range 1 to 7. This is driven by low risk aversion estimates for richer layers of stockholders. For nonstockholders on the other hand the estimates of $1/\gamma$ are typically close to zero. While the results are somewhat sensitive to choice of data frequency and instrumental variables, they are overall supportive of the limited stock market participation theory. The finding of significant differences in risk aversion estimations between stockholders and nonstockholders based on conditional Euler equations contrast with the findings of Jacobs (1997) based on food consumption data from the PSID. Interestingly, a contemporaneous paper on UK data by Attanasio, Banks, and Tanner (1998) also finds support for the limited stock market participation theory. They use a consumption definition similar to mine. An important negative finding in my results is that there is no clear tendency for the tests of overidentifying restrictions to reject for nonstockholders. Attanasio, Banks and Tanner (1998) do find a such pattern in the UK data.

In the Hansen-Jagannathan bound analysis, I again find the distinction between stockholders and nonstockholders to be important. For nonstockholders, the stochastic discount factor does not enter the bounds for any value of risk aversion. For stockholders, the stochastic discount factor enters the bound for values of risk aversion around 10.[2] The HJ bound analysis furthermore shows that it is important to account for cohort effects in aggregate consumption growth in order to reconcile the findings based on the CEX and the findings on US per capita consumption data.

Measurement error is a serious problem for any study using microdata for consumption. Under restrictive (but not unrealistic) conditions on the nature of the measurement errors, I show theoretically how measurement error affects the empirical findings. The measurement error results are valid as the number of households in the cross sections goes to infinity. To analyze the effect of having a finite and fairly small number of households in the cross section I conduct a Monte Carlo study. The main conclusions are that log-linearizing Euler equations is preferable to estimating nonlinear Euler equations when the standard deviation of the measurement error is large. Method of moments estimation based on nonlinear Euler equations may not have a

[2] Several adjustments must be made to the standard HJ analysis due to the use of micro data. The number 10 does not account for idiosyncratic risk.

unique solution for risk aversion. Furthermore, when it does lead to a unique estimate, this has poorer properties than that based on log-linearized Euler equations. This motivates the focus on log-linearized Euler equations in the empirical part of the paper. The Monte-Carlo analysis also considers the effect of measurement error on the Hansen-Jagannathan bound analysis.

With limited stock market participation, markets are incomplete in the extreme sense of one group of people not trading in the stock market at all. In general equilibrium the fact that a fraction of the population must hold all stock market risk endogenously causes their consumption to have a higher correlation with dividends than it would have if all agents shared the risk. As a result, the equilibrium market price of risk is increased. In a recent paper, Basak and Cuoco (1997) building on work by Saito (1992), elegantly derive this effect in a continuous time model. A drawback of their paper is that the separation between stockholders and nonstockholders is exogenous. It is clear that the reason for nonparticipation matters for the general equilibrium effects of limited stock market participation. Suppose for example that stockholders have much lower risk aversion than nonstockholders. Then as more agents enter, the 'average' risk aversion of stockholders increase and it is not a priori clear whether increased participation should decrease the market price of risk and the equity premium. In general, the equilibrium analysis of limited stock market participation is still at an early stage. It is too early to say if adding this feature to standard general equilibrium models can generate an equity premium close to that observed.[3] Understanding the general equilibrium effects of limited stock market participation is important not only for confirming that this type of model can generate a higher equity premium. There has been a dramatic increase in participation and risk sharing in the US since the 1950s. General equilibrium models of limited participation are needed for evaluating how this may have contributed to the path of asset prices and returns which has been observed, and to consider the potential effects of continued entry in the future.

2 Euler equations and limited stock market participation

Suppose the observed consumption and asset return data are generated by an economy characterized by limited stock market participation. Given the consumption data, how much smaller is the equity premium predicted by the model which ignores limited participation compared to

[3] For a simplified example of general equilibrium analysis with limited stock market participation, see Chapter 3 of Vissing-Jørgensen (1998).

the equity premium predicted using only the consumption of stockholders? In addition, how different is the estimate of relative risk aversion which is obtained based on each of the models? These are the questions in focus in the analysis which follows. The analysis is based on aggregation of Euler equations with no general equilibrium structure explicitly imposed. This is the most relevant setup if the purpose is to derive the bias in risk aversion estimates from estimations of Euler equations which do not account for limited stock market participation. It does, however, imply that the condition needed for the results must be stated directly as a condition on the consumption of nonstockholders and not as conditions on the underlying fundamentals. Such more fundamental conditions can only be derived by imposing more structure on the problem. It is clear that differences in consumption patterns between stockholders and nonstockholders depend in the reason why nonparticipants have chosen to stay out of the stock market. I will return to this issue after deriving the condition on nonstockholder consumption which is needed for limited stock market participation to be the explanation of implausible risk aversion estimates based on aggregate data.

2.1 Upward biased risk aversion estimates

To separate the effects of limited participation from those of incomplete markets discussed by e.g. Constantinides and Duffie (1996), this section will focus on Euler equations which take cross-sectional heterogeneity in consumption into account. The results are similar if one assumes a representative agent within each of the groups of agents in focus, as long as the cross-sectional variance of consumption growth is uncorrelated with asset returns.

An analysis which ignores limited stock market participation will assume that Euler equations for stocks and bonds hold for all households[4]. With CRRA preferences this leads to an aggregated Euler equation of the form

$$E_t\left[M_{t+1}^T R_{i,t+1}\right] = 1, \qquad M_{t+1}^T \equiv E_h\left[\delta\left(\frac{C_{t+1}^h}{C_t^h}\right)^{-\gamma}\right]. \tag{1}$$

$R_{i,t+1}$ denotes the gross return to holding asset i from date t to t+1. i=s will be used to refer to stocks, and i=f to refer to one period riskless bonds. E_h denotes the cross-sectional mean across

[4] The Euler equations are the first order conditions for intertemporal optimization. I assume absence of market frictions throughout the paper. Therefore the Euler equations hold with equality for households who are at an interior solution with respect to the asset in focus.

households at time t. E_t refers to the conditional expectation given all information known at time t. The parameter δ is the discount factor equal to $\frac{1}{1+\beta}$ where β is the discount rate. γ is the coefficient of relative risk aversion. Preferences are assumed identical for all agents. T denotes that the total set of households are used. I will use 's' to refer to stockholders and 'ns' to refer to nonstockholders. For stocks (1) is not valid under limited stock market participation, since nonstockholders are included. In other words, $M_{t+1}^T$is not a valid stochastic discount factor for pricing stocks, since nonstockholders are at a corner. Thus their consumption does not satisfy the Euler equation for $R_{s,t+1}$. M_{t+1}^T is a valid stochastic discount factor for pricing riskless assets under the maintained assumption that all households are at an interior solution with respect to riskless assets (borrowing is allowed).

The true aggregated Euler equation for the stock return only includes stockholders in the cross-sectional aggregation

$$E_t\left[M_{t+1}^s R_{s,t+1}\right] = 1, \qquad M_{t+1}^s \equiv E_{h,s}\left[\delta\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)^{-\gamma}\right]. \tag{2}$$

The stochastic discount factor M_{t+1}^T can be rewritten as

$$M_{t+1}^T \equiv E_h\left[\delta\left(\frac{C_{t+1}^{h}}{C_t^{h}}\right)^{-\gamma}\right] \tag{3}$$

$$= \left(\lambda E_{h,s}\left[\delta\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)^{-\gamma}\right] + (1-\lambda)E_{h,ns}\left[\delta\left(\frac{C_{t+1}^{h,ns}}{C_t^{h,ns}}\right)^{-\gamma}\right]\right) = \left[\lambda M_{t+1}^s + (1-\lambda)M_{t+1}^{ns}\right]. \tag{4}$$

λ is the proportion of stockholders in the population. As M_{t+1}^T, M_{t+1}^{ns} is not a valid stochastic discount factor for pricing stocks. Using (1) for stocks and bonds we obtain the equity premium predicted by the model which ignores limited participation

$$\left(E_t R_{s,t+1} - R_{f,t+1}\right)^T = -\frac{cov_t\left(R_{s,t+1}, \lambda M_{t+1}^s + (1-\lambda)M_{t+1}^{ns}\right)}{E_t\left(\lambda M_{t+1}^s + (1-\lambda)M_{t+1}^{ns}\right)} \tag{5}$$

$$= -R_{f,t+1} cov_t\left(R_{s,t+1}, \lambda M_{t+1}^s + (1-\lambda)M_{t+1}^{ns}\right). \tag{6}$$

Using (2) we get the equity premium which would in fact be consistent with the consumption processes observed [5]. If the CCAPM holds for the set of stockholders this will equal the observed

[5] We could equivalently use the stochastic discount factor based on the consumption of any of the stockholders in this relation. With the Euler equation for the stock return holding for each stockholder, the covariance of

equity premium

$$E_t R_{s,t+1} - R_{f,t+1} = -\frac{cov_t\left(R_{s,t+1}, M_{t+1}^s\right)}{E_t\left(M_{t+1}^s\right)} = -R_{f,t+1} cov_t\left(R_{s,t+1}, M_{t+1}^s\right). \qquad (7)$$

For limited stock market participation to explain the equity premium puzzle it must be the case that $\left(E_t R_{s,t+1} - R_{f,t+1}\right)^T < E_t R_{s,t+1} - R_{f,t+1}$. Faced with actual asset returns, estimations based on the total set of households will then lead to an upward biased estimate of risk aversion[6]. Comparing (6) and (7) we immediately get the following result.

RESULT 1:

a) $cov_t\left(R_{s,t+1}, M_{t+1}^{ns} - M_{t+1}^s\right) = 0 \Rightarrow \left(E_t R_{s,t+1} - R_{f,t+1}\right)^T = E_t R_{s,t+1} - R_{f,t+1}$

b) $cov_t\left(R_{s,t+1}, M_{t+1}^{ns}\right) = 0 \Rightarrow \left(E_t R_{s,t+1} - R_{f,t+1}\right)^T = \lambda\left(E_t R_{s,t+1} - R_{f,t+1}\right)$.

Proof: The result follows directly from comparing (6) and (7).

In words, if the assumed stochastic discount factor differs from the true one only by a quantity which is uncorrelated with the stock return, the two models will predict the same equity premium. The intuition is that under this condition the consumption growth of nonstockholders covaries with the stock return in the same way as the consumption growth of stockholders. Therefore, including nonstockholder consumption in the stochastic discount factor will not lead to a different prediction for the equity premium. However, as discussed in more detail below, we would expect consumption growth of nonstockholders to be less correlated with stock returns than that of stockholders. Result 1b) shows that if the stochastic discount factor based on nonstockholder consumption does not covary with the stock return then including nonstockholders in the CCAPM will lead to a predicted equity premium which is too low by factor λ.[7] **Thus if only 20 percent of the population are stockholders and the true equity premium is 6 percent, the CCAPM with nonstockholders included in the consumption measure will predict an equity premium of only 1.2 percent**[8].

stock returns with the consumption growth of each stockholder is identical (for identical stockholder preferences as assumed here). Note that this, along with the restriction on consumption growth for each agent implied by the Euler equation for the riskless asset, does not imply a degenerate cross-sectional distribution of consumption growth for stockholders as long as markets are incomplete. Only with complete markets will the consumption growth rates of any two stockholders be equalized state by state.

[6] This will be derived explicitly below.

[7] The condition in result 1b) will be satisfied if nonstockholder consumption growth is independent of the stock return.

[8] The fraction of stockholders in the adult US population was around 6-8 percent in the 1950s. It has been

Result 1 is quite general. All that has been assumed is that the Euler equations for stocks and bonds hold for stockholders and the Euler equation for bonds hold for nonstockholders. No specific distributional assumptions have been imposed, neither cross-sectionally nor in the time series dimension. Note also that while the average (across nonstockholders) of the covariances of $\left(\frac{C_{t+1}^{h,ns}}{C_t^{h,ns}}\right)^{-\gamma}$ and $R_{s,t+1}$ must equal zero, the covariance for each non-stockholder need not be zero.

How does the difference in the predicted equity premium translate into bias in estimates of the coefficient of relative risk aversion when the model ignoring limited participation is estimated on a set of consumption and asset return data? To get a closed form answer to this question more structure must be imposed. Let I^s denote the set of stockholders and I^{ns} the set of nonstockholders. I make the following distributional assumptions:

ASSUMPTION:

a) $\frac{C_{t+1}^{h,s}}{C_t^{h,s}}$, $R_{s,t+1} \sim$ joint log-normal, $\forall$h$\in I^s$, conditional on information known at t

b) $\frac{C_{t+1}^{h,ns}}{C_t^{h,ns}}$, $R_{s,t+1} \sim$ joint log-normal, $\forall$h$\in I^{ns}$, conditional on information known at t.

For stockholders the Euler equation for stocks imposes that the covariance of $\ln\frac{C_{t+1}^{h,s}}{C_t^{h,s}}$ and $\ln R_{s,t+1}$ must be the same for all stockholders, but this need not be the case for nonstockholders.

Using this assumption the Euler equation for each $h \in I^s$ can be log-linearized as shown by Hansen and Singleton (1983)

$$0 = \ln\delta + E_t \ln R_{s,t+1} - \gamma E_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right) + \frac{1}{2}V_t \ln R_{s,t+1} \tag{8}$$

$$+\frac{1}{2}\gamma^2 V_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right) - \gamma\, cov_t\left(\ln R_{s,t+1}, \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right). \tag{9}$$

Summing (8) over stockholders implies

$$0 = \ln\delta + E_t \ln R_{s,t+1} - \gamma E_{h,s}\left[E_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right] + \frac{1}{2}V_t \ln R_{s,t+1} \tag{10}$$

$$+\frac{1}{2}\gamma^2 E_{h,s}\left(V_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right) - \gamma\, E_{h,s}\left[cov_t\left(\ln R_{s,t+1}, \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right)\right].$$

Similarly for the riskless rate

$$0 = \ln\delta + \ln R_{f,t+1} - \gamma E_{h,s}\left[E_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right] + \frac{1}{2}\gamma^2 E_{h,s}\left(V_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right). \tag{11}$$

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gradually increasing since then. The latest available estimate is 41 percent in the 1995 Survey of Consumer Finances. See Vissing-Jørgensen (1998) for a description of the trends.

Under appropriate conditions for changing the order of integration, the cross-sectional expectation can be moved inside $E_t(\cdot)$ and $cov_t(\cdot)$. One method of estimating δ and γ is then to replace expectations by actual values plus an expectational error, isolate either the consumption measure or the interest rate as the dependent variable and an instrumental variables estimation method[9][10]. (10) and (11) can be estimated separately or jointly. Alternatively, one can subtract (11) from (10), assume constant conditional variances and covariances and use the law of iterated expectations to do a simple method of moments estimation like that of Mankiw and Zeldes (1991)

$$\gamma = \frac{E\left(\ln R_{s,t+1} - \ln R_{f,t+1}\right) + \frac{1}{2}V \ln R_{s,t+1}}{cov\left(\ln R_{s,t+1}, E_{h,s} \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right)}. \tag{12}$$

The corresponding approach which ignores limited stock market participation will include all households in the summations. Letting γ^* denote the estimate of relative risk aversion when nonstockholders are included this would result in

$$\gamma^* = \frac{E\left(\ln R_{s,t+1} - \ln R_{f,t+1}\right) + \frac{1}{2}V \ln R_{s,t+1}}{cov\left(\ln R_{s,t+1}, E_{h} \ln\left(\frac{C_{t+1}^{h}}{C_t^{h}}\right)\right)}. \tag{13}$$

When nonstockholder consumption growth have a lower covariance with stock returns than stockholder consumption growth, γ^* is an upward biased estimate of γ. The contribution of this section is to quantify the bias.

RESULT 2: $\text{cov}\left(lnR_{s,t+1}, E_{h,ns} \ln\left(\frac{C_{t+1}^{h,ns}}{C_t^{h,ns}}\right)\right) = 0 \Rightarrow \gamma^* = \frac{1}{\lambda}\gamma.$

Proof: Rewrite the denominator of (13)

$$\begin{aligned} & cov\left(\ln R_{s,t+1}, E_h \ln\left(\frac{C_{t+1}^{h}}{C_t^{h}}\right)\right) \\ = \; & \lambda cov\left(\ln R_{s,t+1}, E_{h,s} \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right) + (1-\lambda) cov\left(\ln R_{s,t+1}, E_{h,ns} \ln\left(\frac{C_{t+1}^{h,ns}}{C_t^{h,ns}}\right)\right). \end{aligned}$$

Under the condition assumed, the second term is zero. Comparing (12) and (13) then implies

$$\gamma\, cov\left(\ln R_{s,t+1}, E_{h,s} \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right) = \gamma^* \lambda cov\left(\ln R_{s,t+1}, E_{h,s} \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right).$$

[9] Ordinary least squares would lead to inconsistent estimates. The expectational errors enter the error term causing the regressor to be correlated with the error term.

[10] It must be assumed that conditional variances and covariances are constant over time or uncorrelated with left hand side variables (and instruments).

The result follows by rearrangement.

The estimates of γ based on instrumental variables estimation will be related by the same formula under the additional assumption that $E_{h,ns} \ln \left(\frac{C_{t+1}^{h,ns}}{C_t^{h,ns}} \right)$ be uncorrelated with all instruments used.

Result 2 says that if the consumption growth of nonstockholders is uncorrelated with stock returns then an estimation based on the CCAPM but including the consumption of nonstockholders in the consumption measure, will lead to an upward biased estimate of risk aversion. The factor determining the bias is again λ, the proportion of stockholders in the population. **Thus if only 20 percent of the population are stockholders and the true coefficient of relative risk aversion is 4, the CCAPM with nonstockholders included in the consumption measure will result in an estimate of 20.** For λ as low as in the 1950s, i.e. around 7 percent, γ^* will be biased upward by a factor of 14. It is important to note that it is the fraction of the population who are stockholders which matter for the bias and not the fraction of consumption accounted for by stockholders. This is due to the fact that I have aggregated taking the cross-sectional heterogeneity in consumption growth rates into account. If some households have higher levels of consumption than others, they do not get a higher 'weight' in the calculations since what matters is consumption growth rates. Had I assumed a representative agent and thus used the growth rate of average consumption, the bias in the estimate of γ would be determined by the fraction of consumption accounted for by nonstockholders. This fraction is smaller than $1 - \lambda$ but still large.

The condition in Result 2 is the same as the one in Result 1b but with the log-normality assumptions imposed. It is written here as an unconditional covariance, but the derivation of (12) relies on constant conditional variances and covariances (conditional on information known at time t). Thus the condition for Result 2 is in fact the similar to that for Result 1b.[11] Alternatively one could allow time-variation in variances and covariances and impose unconditional joint lognormality of stock returns, bond returns and consumption growth rates (for each household h). The numerator of (12) and (13) would then be modified by a term involving the unconditional variance of the bond rate, and the covariance of consumption growth and the bond rate would enter the denominator. Both of these extra terms are small for aggregate consumption

[11] The condition in Result 1b) requires nonstockholder consumption growth to be independent of the stock return. The condition in Result 2 requires the log consumption growth rate of nonstockholders to be uncorrelated with the stock return.

data and for all groups of households in the CEX which we consider. Therefore, empirically, the assumptions of conditional joint log-normality and unconditional joint log-normality lead to similar results.

Summing up, this section has shown that the condition that nonstockholder consumption growth be independent of stock returns implies two results. Firstly, for given consumption processes the equity premium predicted by a relation which ignores limited stock market participation is only a fraction λ of the true equity premium generated by those consumption processes, where λ is the fraction of stockholders in the population. Secondly, estimating Euler equations involving the stock return without taking limited stock market participation into account will result in an upward biased estimate of the coefficient of relative risk aversion γ. The bias can be large as the examples showed.

2.2 Plausibility of the condition on nonstockholder consumption growth

Whether limited participation based on these results should be considered a promising explanation of the equity premium puzzle obviously depends on the plausibility of the condition on nonstockholder consumption growth. Suppose, as an extreme case of the literature on uninsurable idiosyncratic income shocks, that nonparticipants have chosen to stay out of the stock market because their idiosyncratic labor income is strongly positively correlated with stock returns and they face a short sales constraint on stocks[12]. Then consumption growth of nonstockholders could have a higher covariance with stock returns than consumption growth of stockholders. Since I find that the covariance of consumption growth with stock returns is much lower for nonstockholders than for stockholders in the CEX, labor income shocks of the above type are unlikely to be the main reason for nonparticipation.

Several empirical papers directly analyze reasons for nonparticipation in the stock market. In Vissing-Jørgensen (1999) I use income and portfolio data from the PSID to provide a direct test of whether nonfinancial income affects portfolio choice as predicted by theory. I find only weak evidence of a negative effect of the covariance of nonfinancial income with stock returns on the probability of being a stockholder and the optimal proportion of financial wealth invested

[12] By definition, the idiosyncratic risk of all agents cannot be correlated with stock returns. It is possible, however, that the correlation is positive for one set of agents, and correspondingly negative for another set of agents.

in stocks conditional on participation. Strong effects of the mean and variance of nonfinancial income are found, but these effects do not invalidate the hypothesis of a low covariance of nonstockholder consumption growth with stock returns.[13] Souleles (1999) does find that the covariance of consumption risk with the excess return on stocks enters significantly in a model of stock purchases.

Most empirical papers on the determinants of stockownership find that the probability of stockownership is increasing in wealth and education[14]. The dependence on wealth is consistent with a fixed entry cost or a per period participation cost which is distributed identically across agents. A higher probability of stockownership for more educated individuals is consistent with education lowering the cost. Vissing-Jørgensen (1999) contains a more detailed empirical analysis of the structure and dollar amounts of stock market participation costs.

The results of Blume and Zeldes (1994) and Bertaut and Haliassos (1995) show that people who indicate a willingness to take average or above average risk are more likely to be stockholders. This points toward heterogeneous risk aversion as another dimension of heterogeneity which causes some but not others to enter the stock market (a fixed entry cost still needs to be present to explain zero and not just small stockholdings). Furthermore, the significance of wealth may be a reflection of different discount rates to the extent that more patient individuals will tend to have higher wealth at a given age.

Neither of these additional factors (heterogeneity in wealth, risk aversion or discount rates, each combined with the presence of a fixed cost of entry) invalidate the claim that nonstockholder consumption growth is likely to have a low conditional covariance with stock returns. In any economy in which stock market risk is the only type of uncertainty and in which this risk is not shared by labor, consumption growth of households holding only riskless bonds is conditionally riskless. It therefore has a conditional covariance with stock returns of zero, as needed for Result 1b and 2. For nonstockholder consumption growth to have a positive correlation with stock returns, it must be the case that labor income (or business income) of nonstockholders is positively correlated with the stock market and that nonstockholders face a short-sales constraint for stocks, as discussed above. Alternatively, additional sources of uncertainty must be present.

Before turning to the empirical section, I should address the question of whether the rea-

[13] My findings are consistent with those of Heaton and Lucas (1999) and Guiso et al. (1996).

[14] Additional references include Mankiw and Zeldes (1991), Blume and Zeldes (1994) and Bertaut and Haliassos (1995).

son for nonparticipation matters for the validity of Euler equation estimations for stockholders and nonstockholders. For nonstockholders the Euler equation involving the stock return is by assumption not valid. The issue of validity concerns, for stockholders, the Euler equations for stocks and for the riskless asset, and for nonstockholders the Euler equation for the riskless asset.

Firstly, conditional on participation, the Euler equations hold for any process for nonfinancial income. One of the attractive features of estimating Euler equations is precisely that one does not need information about income or wealth. Secondly, in the presence of fixed entry or per period participation costs, the Euler equation for stocks (as well as all Euler equations involving the riskless rate) still holds with equality for agents who are stockholders. Trading costs could cause violation of the Euler equations in periods where the household does not change its asset holdings. Thirdly, if heterogeneity in discount rates is important, estimates of relative risk aversion based on the log-linearized model are still valid. Estimates of risk aversion based on the simple calibrations in (12) and (13) do not involve estimation of discount rates and are thus unaffected by heterogeneity in discount rates. Instrumental variables estimation of γ based on equations (10) and (11) is also essentially unaffected by heterogeneity in discount rates. This is because the discount factor is isolated in the constant term. When individual log-linearized Euler equations are summed across agents, as shown in (10) and (11) for the set of stockholders, the term $\ln \delta$ is replaced by an average of agents' log discount factors. This does not affect estimates of γ.

If risk aversion is heterogeneous, implying that it will be households with low risk aversion who pay the fixed cost and enter the stock market, then complications arise. If long time series of data were available for each agent, Euler equations could be estimated for each stockholder and each nonstockholder to get consistent estimates of each agent's coefficient of relative risk aversion. However, in the Consumer Expenditure Survey, each household is not observed for a sufficiently long period for this to be feasible. As will be explained below, a simple cohort technique must be applied. This involves using consumption growth observations for similar agents from earlier and later periods to obtain an estimate of risk aversion for a given type of agents. If risk aversion differs within the set of agents, the resulting estimate will be a (complicated) function of the risk aversion coefficients of the agents involved. This implies that to get a precise estimate of risk aversion, it is desirable to be able to identify groups of agents with similar values of risk aversion. Splitting agents into stockholders and nonstockholders by itself goes some way towards solving this problem. In addition, I split the set of stockholders into three layers by size of stockholdings. Aside from providing more precise estimates of risk aversion in the presence of risk aversion heterogeneity, this is of interest with respect to the issue of diversification. Suppose risk aversion is homogeneous but that an Euler equation involving the return on an aggregate stock market index is estimated for stockholders whose stock portfolio is not as diversified as the index. The result is likely to be an upward biased estimate of risk aversion. To the extent that wealthier stockholders are more diversified than less wealthy stockholders, risk aversion estimates will be less biased for richer layers of stockholders[15].

3 Empirical results based on the Consumer Expenditure Survey

3.1 Empirical strategy

The empirical analysis contains three parts. First, a simple estimation corresponding to equation (12). Second, estimation of the loglinearized conditional Euler equations in (10) and (11), using a linear GMM estimation approach (i.e. an instrumental variables approach). Third, a Hansen-Jagannathan bound analysis. Each of these parts emphasizes different features of the data. The estimation of equation (12) does not use conditioning information. Instrumental variables estimation uses information about time-variation in expected consumption growth and asset returns. The Hansen-Jagannathan analysis focuses on the volatility of the stochastic discount factor proposed by the CCAPM. The null hypothesis is that the CCAPM is a satisfactory description of the equilibrium relation between consumption and asset return data once we focus on the consumption of stockholders. Under this hypothesis, the risk aversion estimates for stockholders obtained from each part should be identical. For each of the three parts, the model is estimated/tested first for the set of all households in the sample, then for stockholders and nonstockholders separately and finally for the bottom, middle and top layer of stockholders ranked by size of stockholdings. I will return to the issue of aggregation within groups below.

3.2 Data

The CEX data available cover the period 1980:1-1996:1. In each quarter approximately 5000 households are interviewed. Each household is interviewed five times, the first time is practice

[15] The number of different stocks held is strongly related to wealth in the Survey of Consumer Finances.

and the results are not in the data files. The interviews are three months apart and when interviewed households are asked to report consumption for the previous three months separately. Information about other variables is reported on a quarterly basis. Financial information is gathered in the fifth quarter only. Aside from attrition, the sample is representative of the US population. Attrition is quite substantial with only about half the households making it through all five quarters.

The CEX does not allow a perfect separation of households into stockholders and nonstockholders. Households are asked for holdings of "stocks, bonds, mutual funds and other such securities".[16] Since many households who do not hold stocks may hold positive amounts of bonds and non-equity mutual funds, some nonstockholders will unavoidably be classified as stockholders. In general, inability to perfectly identify stockholders and nonstockholders biases against finding differences in risk aversion estimates for the two groups. Below I will refer to the answer reported for the above asset category as stockholdings.

To avoid potential sample selection problems, stockholding status must be defined based on stockholdings at the beginning of period t (when considering $\Delta \ln C_{t+1}$). Two additional CEX variables are used for this purpose. The first one reports whether the household holds the same amount or more or less of the above asset category. The second one reports the "dollar difference in the estimated market value of all stocks, bonds, mutual funds and other such securities held by CU last month compared with the value of all securities held a year ago last month". A household was defined as a stockholder if it either reported having decreased its stockholdings or kept them constant, or had increased its stockholdings but by a dollar amount less than the current reported stock.[17]

It is known from e.g. the Survey of Consumer Finances that many households hold stocks only in their pension plan. Whether these should be considered stockholders or not depends on the type of pension plan. In a defined contribution pension plan households can adjust their contributions and thus ensure that the Euler equation for the stock return is satisfied. In a defined benefit plan the bearers of the stock market risk is not the employees but the owners

[16] The other component of financial assets in the CEX is amounts in "savings accounts in banks, savings and loans, credit unions, etc.".

[17] A small number of households reported increasing their stockholdings by more than the value of the reported end of period stock. Since it cannot be determined if they were stockholders initially, these households were categorized as nonstockholders.

of the company which provides these benefits. Thus households with defined benefit plans in effect hold a riskless asset. Unfortunately, it is not possible to determine whether households with defined contribution plans report their stockholdings in these plans when answering the CEX questions. The percent of stockholders in the CEX is smaller than in other sources (it is about 22 percent, see below).[18] This may indicate that many households with stockholdings in defined contribution plans do not report these. This will lead them to be misspecified as nonstockholders. Again, this should bias against finding differences between the two groups.

Because the financial information is reported in interview five, and because I wish to calculate consumption growth value by households, households must be matched across quarters. Therefore, I drop households for which any of interview two to five are missing. Matching households across interviews creates problems around 1985-86 since sample design and household identification numbers were changed, with no records being kept of which new household identification numbers correspond to which old ones. I initially attempted to use other variables than household ID to match households for the quarters affected by this problem. However, not enough households could be uniquely matched for this to be useful. Rather, I chose to exclude households who did not finish their interviews before the ID change. This implies that no observations are available for some month around the ID change (10 months when considering quarterly consumption growth rates, 16 months when considering semiannual consumption growth rates). This is taken into account when programming corrections for autocorrelation and when doing bootstrapping.

In addition to the split between stockholders and nonstockholders, the set of stockholders is split into three layers of approximately equal size based on dollar amounts reported. Consistent with the definition of stockholding status based on stockholdings at the beginning of the household's participation in the study, this was done based on initial stockholdings. These were calculated as current holdings minus the increase in amount in the current period (which can be negative). The bottom layer consists of those reporting initial stockholdings of $2-$3500 in real 1982-1984 dollars, using the CPI to deflate the nominal values. The middle and top layers are those with real initial stockholdings of $3500-$20000 and above $20000, respectively. For interviews conducted from 1991 onwards, about 5 percent of households report stockholdings of $1. I contacted the BLS to ask if this was a coding error, but they were not sure how to interpret

[18]This is the case whether using the CEX weights or not.

the \$1 answers. Since these households therefore cannot be classified by layer of stockholding, I chose to classify them as nonstockholders for the purpose of defining layers of stockholders. Thus the total number of stockholders used when defining layers is smaller than the total amount of stockholders when not doing this split.

The consumption measure used is nondurables and some services aggregated as carefully as possible from the disaggregate CEX consumption categories to match the definitions of non-durables and services in the NIPA. The service categories excluded are housing expenses (but not costs of household operations), medical care costs, and education costs. This was done since many of these three types of costs have a substantial durable component. Attanasio and Weber (1995) used a similar definition of consumption. In leaving out durables, it is implicitly assumed that utility is separable in durables and nondurables/services. Nominal consumption values are deflated by the BLS deflator for nondurables.

Following Dynarski and Gruber (1997) and Zeldes (1989) extreme outliers are dropped under the assumption that these reflect reporting or coding errors. Specifically, I drop observations for which C_{t+1}/C_t <0.2 or C_{t+1}/C_t >5. In addition, nonurban households (missing for part of the sample) and households residing in student housing are dropped as are households with incomplete income responses. Furthermore, I drop households who report a change in age of household head between any two quarters different from 0 or 1 year. These exclusions are standard. More drastically, I drop all consumption observations for households interviewed in 1980 and 1981, since several measures indicated low data quality in this first part of the survey. For example, the ratio of total annual family consumption to family income after tax is 4.4 percent higher for households interviewed in 1980 or 1981 than the subsequent years.[19] Attanasio and Weber (1995) show that the share of food in nondurable consumption was much higher in 1980 and 1981 than subsequently.

The final sample consists of 30535 semiannual consumption growth observations, and 95162 quarterly consumption growth observations (more on the choice of data frequencies below). The proportion of stockholders is 22.7 for the semiannual data, and 22.4 for the quarterly data. Consistent with data from other sources, the proportion of stockholders is upward trending.

Table 1 gives the average number of observations per month for the various household groups,

[19] Based on median values of this variable across households and excluding rural households for comparability across years.

along with the mean and standard deviation of their log consumption growth (not annualized). For the semiannual case, the average number of observations for stockholders per month is 50, while the average number of observations for nonstockholders is 168. The restricted sample consisting of single individual households only, has substantially fewer observations. Therefore it is not possible to consider layers of stockholders in this case for the semiannual data. For the quarterly data the average number of observations per stockholder layer is very small for this case, but none of the results reported below for this case are driven by one or a few outliers.[20] The summary statistics on consumption in Table 1 show that log consumption growth is higher for stockholders than for nonstockholders. Since the measure of consumption growth is the cross sectional sum of $\Delta \ln C_{t+1}^{h}$, its standard deviations depends on the number of households in the given category. In the Monte Carlo simulations discussed below I analyze the effect of the number of households for the various estimations.

Monthly NYSE value weighted returns are used as the stock return measure and monthly T-bill returns as the measure of nominally riskless returns. The CPI for total urban consumption is used to calculate real returns. Quarterly and semiannual returns are aggregated up from the real monthly returns. As instruments for the log stock return and the log T-bill return I use the dividend price ratio, the lagged real value weighted NYSE return, the lagged real T-bill return, the government bond horizon premium, and the bond default premium (in addition to seasonal dummies and family size controls). The dividend price ratio, the bond horizon premium and the bond default premium are based on data from Ibbotson (1997). The dividend price ratio used is the ratio of dividends over the previous 12 months to the current price (the S&P500 Index). The bond horizon premium is defined as $\frac{1+R_{f,t}^{\text{long term govt. bonds}}}{1+R_{f,t}^{\text{short term govt. bonds}}}$, where 'long term' means 20 years to maturity and 'short term' means approximately 1 month to maturity. The bond default premium is defined as $\frac{1+R_{f,t}^{\text{long term corp. bonds}}}{1+R_{f,t}^{\text{long term govt. bonds}}}$ where long term again means 20 years to maturity. The monthly values are aggregated multiplicatively to quarterly and semiannual values.

For reference, data for real US per capita consumption of nondurables and services will be needed. I use nominal consumption and price deflators for nondurables and services from the Bureau of Economic Analysis, and population data from the U.S. Census Bureau.

[20] For two of the stockholder layers there was one month with no observations for the single individual households.

3.3 Econometric issues

3.3.1 Measurement error

As in all studies based on micro data the issue of measurement error arises. Although, to my knowledge, no validation study has been done for the Consumer Expenditure Survey we know from the PSID validation study that measurement errors in micro data can be large. Duncan and Hill (1985) find that 15 to 30 percent of the cross-sectional variation in earnings is measurement error. It is likely that people remember their earnings more accurately than their consumption resulting in much larger measurement error for consumption. Again for the PSID, Runkle (1991) estimates that 75 percent of the part of consumption growth variation which is unexplained by family specific interest rates is noise.

The conditions on measurement error under which consistent estimates of relative risk aversion can be obtained based on estimation of Euler equations are strict. The measurement error in individual consumption must be multiplicative and independent of the true consumption level and asset returns[21,22]. This is the case whether the Euler equations are log-linearized or estimated in the original nonlinear form. To be specific, suppose we had a long time series of consumption observations for an agent h and wished to test the CCAPM. Let $C_t^{h,*}$ be the true consumption of agent h at t and assume observed consumption is given by $C_t^h = C_t^{h,*}\varepsilon_t^h$, where ε_t^h is the measurement error. The true Euler equation is

$$E_t\left[\delta\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma}R_{i,t+1}\right]=1. \tag{14}$$

However, our estimates $\hat{\delta}$ and $\hat{\gamma}$ are based on the sample equivalent of

$$E_t\left[\hat{\delta}\left(\frac{C_{t+1}^{h}}{C_t^{h}}\right)^{-\hat{\gamma}}R_{i,t+1}\right]=1 \Leftrightarrow E_t\left[\hat{\delta}\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^h}{C_t^{h,*}\varepsilon_t^h}\right)^{-\hat{\gamma}}R_{i,t+1}\right]=1. \tag{15}$$

As an example, in the case of GMM estimation with two interest rates and two instruments, one of which is a column of ones, our estimates $\hat{\delta}$ and $\hat{\gamma}$ satisfy the sample equivalents of the above

[21] Fortunately, introspection suggest that multiplicative measurement errors are more plausible than additive, since people are more likely to misreport their consumption by some (stochastic) fraction than to misreport it by the same dollar amount no matter how large the true level is.

[22] For log-linearized Euler equations measurement errors must be uncorrelated with the true level of consumption, not necessarily independent of the level.

equation with equality. If ε_{t+1}^{h} and ε_{t}^{h} are conditionally independent of C_{t+1}^{h}, C_{t}^{h} and $R_{i,t+1}$, (15) implies

$$E_t\left[\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_{t}^{h}}\right)^{-\hat{\gamma}}\right]E_t\left[\hat{\delta}\left(\frac{C_{t+1}^{h,*}}{C_{t}^{h,*}}\right)^{-\hat{\gamma}}R_{i,t+1}\right]=1. \tag{16}$$

ε_{t}^{h} and ε_{t+1}^{h} are unobservable and thus not included in the time t information set.
If $E_t\left[\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_{t}^{h}}\right)^{-\hat{\gamma}}\right]$ is constant over time as would be the case with i.i.d. measurement errors. It follows that the estimator of δ will be inconsistent by this factor, whereas γ will be consistently estimated. ε_{t+1}^{h} and ε_{t}^{h} are bounded from below by zero under the reasonable assumption that no households reports negative consumption. Thus $E_t\left[\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_{t}^{h}}\right)^{-\hat{\gamma}}\right]$ is positive. If measurement errors are lognormal, $\ln\varepsilon_{t}^{h}\sim N\left(\mu_{\varepsilon},\sigma_{\varepsilon}^{2}\right)\forall t$, the estimate of δ will be inconsistent by the factor

$$\frac{1}{E_t\left[\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_{t}^{h}}\right)^{-\gamma}\right]}=\frac{1}{\exp\left(\gamma^{2}\left(\sigma_{\varepsilon}^{2}-\sigma_{\varepsilon,\varepsilon}\right)\right)} \tag{17}$$

where $\sigma_{\varepsilon,\varepsilon}$ is the covariance of $\ln\varepsilon_{t+1}^{h}$ and $\ln\varepsilon_{t}^{h}$. Similarly, $\ln\delta$ in the log-linearized model will be inconsistent by the quantity $-\gamma^{2}(\sigma_{\varepsilon}^{2}-\sigma_{\varepsilon,\varepsilon})$. If we do not have a long time series of consumption for each agent and instead aggregate over consumers within each period as in (2) the inconsistency in δ would remain the same if we had infinitely many households in the cross section. To see this, suppose that at least two observations are available for each household such that C_{t+1}^{h}/C_{t}^{h} can be calculated. When estimating δ and γ using a time series of cross sections of C_{t+1}^{h}/C_{t}^{h} observations, $\widehat{\delta}$ and $\widehat{\gamma}$ will be based on the sample equivalent of

$$E_t\left[\hat{\delta}\frac{1}{H}\Sigma_h\left[\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\hat{\gamma}}\right]R_{i,t+1}\right]=1\Leftrightarrow E_t\left[\hat{\delta}\frac{1}{H}\Sigma_h\left[\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^{h}}{C_{t}^{h,*}\varepsilon_{t}^{h}}\right)^{-\hat{\gamma}}\right]R_{i,t+1}\right]=1.$$

or as $H\rightarrow\infty$

$$E_t\left[\hat{\delta}E_h\left[\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\hat{\gamma}}\right]R_{i,t+1}\right]=1\Leftrightarrow E_t\left[\hat{\delta}E_h\left[\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^{h}}{C_{t}^{h,*}\varepsilon_{t}^{h}}\right)^{-\hat{\gamma}}\right]R_{i,t+1}\right]=1.$$

Under the independence assumptions stated above and assuming the same distribution of measurement errors in the cross section at each date as for one individual over time, this implies

$$
\begin{aligned}
E_t\left[\hat{\delta} E_h\left[\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\hat{\gamma}}\right] E_h\left[\left(\frac{\varepsilon_{t+1}^h}{\varepsilon_t^h}\right)^{-\hat{\gamma}}\right] R_{i,t+1}\right] &= 1 \Leftrightarrow \\
\exp\left(\widehat{\gamma}^2\left(\sigma_\varepsilon^2-\sigma_{\varepsilon,\varepsilon}\right)\right) E_t\left[\hat{\delta} E_h\left[\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\hat{\gamma}}\right] R_{i,t+1}\right] &= 1.
\end{aligned}
$$

Thus if we had infinitely many households in the cross section, the estimate $\widehat{\delta}$ would be inconsistent by the same factor as above and $\widehat{\gamma}$ would again be consistent. Similarly, if the Euler equation involving the excess return on stocks $R_{s,t+1} - R_{f,t+1}$ is estimated, the estimate of γ would be consistent under the independence assumptions stated (δ cancels in this case).[23]

Similar comments apply for the analysis of stochastic discount factors for the Hansen-Jagannathan bound analysis. The true stochastic discount factor is $\delta \frac{1}{H}\Sigma_h\left[\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma}\right]$ or as $H \to \infty$, $\delta E_h\left[\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma}\right]$. With measurement error, it is only possible to observe $\delta \frac{1}{H}\Sigma_h\left[\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^h}{C_t^{h,*}\varepsilon_t^h}\right)^{-\gamma}\right]$ or as $H \to \infty$,

$$
\delta E_h\left[\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^h}{C_t^{h,*}\varepsilon_t^h}\right)^{-\gamma}\right] = \exp\left(\gamma^2\left(\sigma_\varepsilon^2-\sigma_{\varepsilon,\varepsilon}\right)\right) \delta E_h\left[\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma}\right].
$$

Thus both the mean and the standard deviation of the stochastic discount factor will be biased by the factor $\exp\left(\gamma^2\left(\sigma_\varepsilon^2-\sigma_{\varepsilon,\varepsilon}\right)\right)$. However, if one assume a representative agent (within groups), measurement error has no effect as $H \to \infty$, since

$$
\delta\left[\frac{E_h\left[\left(C_{t+1}^h\right)^{-\gamma}\right]}{E_h\left[\left(C_t^h\right)^{-\gamma}\right]}\right] = \delta\left[\frac{E_h\left[\left(C_{t+1}^{h,*}\right)^{-\gamma}\right] E_h\left[\left(\varepsilon_{t+1}^h\right)^{-\gamma}\right]}{E_h\left[\left(C_t^{h,*}\right)^{-\gamma}\right] E_h\left[\left(\varepsilon_t^h\right)^{-\gamma}\right]}\right] = \delta\left[\frac{E_h\left[\left(C_{t+1}^{h,*}\right)^{-\gamma}\right]}{E_h\left[\left(C_t^{h,*}\right)^{-\gamma}\right]}\right]
$$

assuming that the distribution of measurement errors is stationary over time. Brav, Constantinides and Geczy (1999) conclude that tests of the representative agent (complete insurance) model against the nonrepresentative agent (incomplete insurance) model have very low power. Their results are based on a Monte Carlo simulation designed to capture the main features of

[23] Note that consistency here concerns the probability limit of the estimator as $T \to \infty$.

the CEX data set. Given this and given the robustness of the stochastic discount factor to measurement error when a representative agent is assumed, I chose to make the representative agent assumption in the empirical Hansen-Jagannathan bound analysis.

The above derivations assumed $H \to \infty$. It remains to determine what happens for empirically relevant values of H. How large a value of H is needed to ensure that measurement error has only negligible effects on the properties of the risk aversion estimate? Intuitively, this should depend on how large the standard deviation of the measurement error is relative to the standard deviation of the true value of consumption. It may also depend on whether the Euler equations are log-linearized or not (log-linearization is not an issue for the Hansen-Jagannathan bound analysis) and on whether a representative agent is assumed within the group of stockholders and within the group of nonstockholders.

To address this issue, Appendix A contains a small scale Monte Carlo study of the properties of risk aversion estimators based on the original nonlinear Euler equations and risk aversion estimators based on the log-linearized Euler equations. The simulation also addresses the importance of measurement error for the properties of stochastic discount factors, relevant for the Hansen-Jagannathan bound analysis. The results are presented in Table 2 and Figures 1 and 2. I draw three conclusions based on the findings of the Monte Carlo study.

Firstly, for values of H and σ_ε^2 in the empirically relevant range, the properties of risk aversion estimators based on log-linearized Euler equations are reasonably close to the properties of estimators in the case without measurement error. This conclusion does not depend on whether a representative agent is assumed.

Secondly, this is not the case for risk aversion estimators based on method of moments estimation of nonlinear Euler equations. The nonlinear Euler equation analyzed represents one moment in one unknown (when using the equity premium $R_{s,t+1} - R_{f,t+1}$ such that δ drops out). However, depending on the particular draw of the data, there may be zero, one or several values of γ which sets the moment equal to zero. The proportion of cases in which there is not a unique solution is substantial for empirically relevant values of H and σ_ε^2. Therefore, this estimator may not be applicable in a particular sample, and even if applicable is shown to generally have poorer properties than estimators based on log-linearized Euler equations. This motivates the focus on log-linearized Euler equations in the empirical part of the paper.

Thirdly, when plotting $Std(M)$ against $E(M)$ for various values of γ, the shape of the curve is largely unaffected by measurement error if a representative agent is assumed. The value of risk aversion corresponding to a given point on the curve is lower than the true value due to measurement error. As expected this effect is stronger the lower H is.

3.3.2 Matching

With multiple consumption observations per household, household identification numbers can be used to match households across interviews thus exploiting the panel dimension of the CEX. This is necessary for estimating nonlinear Euler equations without a representative agent assumption as discussed above. For estimating log-linearized Euler equations one could consider not matching, and using as consumption measure $\left[\frac{1}{H_{t+1}}\sum_{h=1}^{H_{t+1}}\ln C_{t+1}^h\right]-\left[\frac{1}{H_t}\sum_{h=1}^{H_t}\ln C_t^h\right]$. However, with 25 percent of the sample being replaced each quarter, matching is important. Without matching, the extent to which the consumption level of the new households differs from that of those who are no longer in the sample enters the estimation as an additional element of noise.[24]

For the Hansen-Jagannathan bound analysis, matching is important whether a representative agent is assumed or not. Consider an overlapping generations setting. New generations are born with higher lifetime resources than existing generations due to productivity growth. In principle each individual can have a flat consumption profile during her lifetime (aside from shocks) at the same time as per capita consumption growth is positive. The reason is that the consumption of the oldest households at t is replaced by the consumption of young richer households at $t+1$. Then with matching the growth rate of average consumption

$$\frac{\frac{1}{H_t}\sum_{h=1}^{H_t}C_{t+1}^h}{\frac{1}{H_t}\sum_{h=1}^{H_t}C_t^h}-1$$

will be close to zero on average. However, the growth rate of average consumption without matching

$$\frac{C_{t+1}^T}{C_t^T}-1=\frac{\frac{1}{H_{t+1}}\sum_{h=1}^{H_{t+1}}C_{t+1}^h}{\frac{1}{H_t}\sum_{h=1}^{H_t}C_t^h}-1$$

will be positive on average. This point will be central when reconciling the Hansen-Jagannathan bound analysis based on the (matched) CEX data with the findings in the literature based on aggregate (and thus not matched) data.

[24] In any case, in the CEX matching across interviews is necessary for defining stockholder status as discussed in the data section.

3.3.3 Timing

The fact that households are interviewed every three months for a year, and in each interview report consumption for the previous three months separately leaves open a choice of data frequency for defining consumption growth rates. Let subscript t refer to month t. I consider two alternatives.

a) Semiannual consumption growth rates: $\frac{C_{t+6}+C_{t+7}+C_{t+8}+C_{t+9}+C_{t+10}+C_{t+11}}{C_t+C_{t+1}+C_{t+2}+C_{t+3}+C_{t+4}+C_{t+5}}$

b) Quarterly consumption growth rates: $\frac{C_{t+3}+C_{t+4}+C_{t+5}}{C_t+C_{t+1}+C_{t+2}}$.

Using semiannual consumption growth rates results in one consumption growth observation per household, compared to three growth rate observations per household when using quarterly growth rates. While it would be possible to calculate month to month consumption growth rates, this is likely to be less reliable. A substantial number of households report identical expenditures across the three reference months for many consumption categories. While some of these reports may be correct, many are likely to be due to households simply averaging their quarterly expenditures across months and reporting the average for each month. See Souleles (1995) for details about this problem.

While each household is interviewed three months apart, the interviews are spread out over the quarter implying that there will be households interviewed in each month of the sample. Thus the data frequency for both a) and b) is monthly. This implies that consumption growth observations for adjacent months will involve partially overlapping time periods and thus partially overlapping expectational errors. As a consequence the error term in the log-linearized model will have an MA(5) component when using semiannual consumption growth observations, and an MA(2) component when using quarterly consumption growth observations. In addition, measurement error in consumption generates an MA(3) component for case b), since some of the households interviewed in month t will also have been interviewed in month $t-3$.

The second issue regarding timing is the timing of interest rates. For the semiannual data, is the relevant interest rate $(1+R_t)(1+R_{t+1})...(1+R_{t+5})$ or $(1+R_{t+6})(1+R_{t+7})...(1+R_{t+11})$ or something else? Suppose consumers sell their assets (stocks or bonds) at the beginning of the month in which they would like to consume. Then shifting consumption from period t to $t+6$ would imply a gross return of $(1+R_t)(1+R_{t+1}).....(1+R_{t+5})$, whereas shifting consumption from period $t+5$ to $t+11$ would yield a return of $(1+R_{t+5})(1+R_{t+6})...(1+R_{t+10})$. Thus the relevant interest rate would be a weighted average of all $(1+R_t)$, $(1+R_{t+1})$, ...,

$(1+R_{t+10})$. For simplicity I chose to use the middle six months of relevant interest rates $(1+R_{t+2})(1+R_{t+3})...(1+R_{t+7})$.[25] Similar considerations lead to the choice of $(1+R_{t+1})(1+R_{t+2})(1+R_{t+3})$ when using quarterly consumption growth rates.

Autocorrelation raises the question of which lags of interest rates and other variables are valid instruments. The error term has an expectational error component and a measurement error component. Since interest rates are likely to be uncorrelated with the measurement error component of the error term, autocorrelation due to measurement error does not invalidate lags of interest rates as instruments. It does have an effect on which lags of consumption would be valid instruments but since lagged real consumption growth rates had low correlation with the real stock return and the real T-bill return they were not used. Autocorrelation in the error term due to overlapping expectational errors imply that interest rate lags six and further back are valid instruments for the semiannual data and that interest rates lag three and beyond are valid instruments for the quarterly data. Due to the fact that R_t and R_{t+1} for the semiannual case, and R_t in the quarterly case may be partly relevant as right hand side variables as discussed above, the interest rate instruments used are lagged further so that there is no overlap. Thus $(1+R_{t-6})(1+R_{t-5}).....(1+R_{t-1})$ is used as instrument for semiannual data and $(1+R_{t-3})(1+R_{t-2})(1+R_{t-1})$ for quarterly data. As for the specific interest rates used, I will return to when stock returns and T-bills returns are used when discussing the results. Similar considerations lead to using the dividend price ratio at the beginning of period t as an instrument, and the bond horizon premium and bond default premium over the period $t-6$ to $t-1$ for semiannual data and the period $t-3$ to $t-1$ for quarterly data.

3.3.4 Family size controls

Following a series of papers in the consumption literature I assume that family size enters the utility function multiplicatively, and thus include $\Delta \ln(\text{familiy size})$ in the log-linearized Euler equations. It is however not clear that such a simple correction accurately captures family size effects. The literature on equivalence scales considers this issue in detail. Here, I choose to repeat the estimations using households consisting of only a single individual at both t and $t+1$ to see if this affects the results.

[25] Results are similar when using $(1+R_{t+3})(1+R_{t+4})...(1+R_{t+8})$.

3.3.5 Seasonality

I assume that seasonality enters as a multiplicative factor in the utility function, such that, $U\left(C_t^h\right)=\frac{1}{1-\alpha}\left(C_{t+1}^h S_{m(t+1)}\right)^{1-\alpha}$ where $S_{m(t+1)}$ is the seasonal factor and $m\left(t+1\right)$ is the month at t+1 is. Under this assumption seasonal adjustment by dummies is valid in the log-linearized model. I therefore include 12 monthly dummies in the instrumental variables estimations. For the Hansen-Jagannathan bound analysis (which is not based on log-linearization), I use as the adjusted consumption ratio the exponential of the average of $\Delta\ln C_{t+1}$ plus the residual from a regression of $\Delta\ln C_{t+1}$ on 12 monthly dummies. In the estimations based on the unconditional Euler equation for the equity premium, the seasonal factor drops out if the equation is derived based on conditional joint log normality (but not if the equation is derived from unconditional joint log normality). For comparison with earlier results, I seasonally adjust $\Delta\ln C_{t+1}$ for this case as well.[26]

3.3.6 Summary: Estimated relations

The risk aversion estimate based on the equity premium and unconditional Euler equations is, with the relation for stockholders as the example

$$\widehat{\gamma}=\frac{E\,(\ln R_{s,t+1}\widehat{-}\ln R_{f,t+1})+\frac{1}{2}V\,(\ln\widehat{R}_{s,t+1})}{\widehat{cov}\left(\ln R_{s,t+1},\frac{1}{H^s}\sum_{h=1}^{H^s}\ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right)}.$$

The log-linearized conditional Euler equations are, with the stockholders' Euler equation for stock returns as an example

$$\frac{1}{H^s}\sum_{h=1}^{H^s}\ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)=\beta_1 D_1+\beta_2 D_2+...+\beta_{12}D_{12}+\alpha\Delta\ln\left(\text{family size}_{t+1}\right)+\frac{1}{\gamma}\ln R_{s,t+1}+u_t \tag{18}$$

[26] Ferson and Harvey (1992) considers the seasonal adjustment issue in detail. They emphasize that most of the empirical papers in the equity premium puzzle literature use aggregate consumption data which are seasonally adjusted using the X-11 seasonal adjustment program or a similar method. Since data adjusted by the X-11 method are weighted averages of past and, in revised data, future expenditures, this type of seasonal adjustment can induce spurious correlation between the error terms of a model and lagged values of the variables (e.g. spurious rejection of a model based on tests of overidentifying restrictions when the instruments include lagged consumption values). In addition the X-11 method changes the mean of the growth rate of a series which in the present context would cause biases in means of stochastic discount factors.

where

$$\beta_i \equiv \frac{1}{\gamma}\ln\left(\delta\left(\frac{S_{i+1}}{S_i}\right)^{1-\gamma}\right) + \frac{1}{2\gamma}V_t \ln R_{s,t+1} + \frac{1}{2}\gamma\frac{1}{H^s}\sum_{h=1}^{H^s}\left(V_t \ln\left(\frac{\varepsilon_{t+1}^{h,s}}{\varepsilon_t^{h,s}}\right)\right)$$
$$+\frac{1}{2}\gamma\frac{1}{H^s}\sum_{h=1}^{H^s}\left(V_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right) - cov_t\left(\ln R_{s,t+1}, \frac{1}{H^s}\sum_{h=1}^{H^s}\ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right),\ i = 1, ..., 12.$$

$$u_t \equiv \frac{1}{\gamma}\left(E_t \ln R_{s,t+1} - \ln R_{s,t+1}\right) - \frac{1}{H^s}\sum_{h=1}^{H^s}\left[\left(E_t \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right) - \ln\left(\frac{C_{t+1}^{h,s}}{C_t^{h,s}}\right)\right)\right]. \qquad (19)$$

If the conditional variances and covariances in the expression for the (seasonal) constant term are not constant, the stochastic components enter the error term. This does not cause problems for the estimation as long as these components are uncorrelated with the asset returns and the instruments used.

For each group of households, the Euler equation for the stock return and the Euler equation for the bond return are first estimated separately. The estimation method used is linear GMM estimation, or in other words 2SLS modified to account for autocorrelated error terms of the MA(3) form for quarterly data and the MA(5) form for semiannual data (commonly referred to as optimal IV estimation). Furthermore, I correct for heteroscedasticity of arbitrary form. Heteroscedasticity is likely to be present because of a varying number of observations per quarter. Instrumental variables estimation is used rather than OLS because of endogeneity of asset returns due to the expectational error being included in the error term.

The Euler equations for stock and bond returns are then estimated jointly, again using linear GMM estimation. Joint estimation is used to gain efficiency from exploiting cross-equation correlation in error terms caused by correlated expectational errors. Furthermore, it makes it possible to impose identical values for $1/\gamma$ and the coefficient on $\Delta \ln(\text{family size})$ and determine if this leads to rejection of the model according to overidentification tests. In the joint estimation, the coefficients on the seasonal dummies are allowed to differ for the stock and the bond equation since this is implied by the model when the two returns have different variances or different covariances with log consumption growth.

For the Hansen-Jagannathan bound analysis the stochastic discount factor based on a representative agent assumption within groups is

$$\delta\left(\frac{\frac{1}{H^s}\sum_{h=1}^{H^s} C_{t+1}^{h,s}}{\frac{1}{H^s}\sum_{h=1}^{H^s} C_t^{h,s}}\right)^{-\gamma}$$

again with stockholders as an example.

3.4 Results

3.4.1 Unconditional Euler equations

Tables 4 and 5 show the risk aversion estimates based on unconditional log-linearized Euler equations, corresponding to equations (12) and (13). For reference the results for US per capita consumption data are given in Table 3 for quarterly and annual data.

The results show large differences in relative risk aversion estimates between stockholders and nonstockholders, but the confidence intervals are very wide. For quarterly CEX data, with no representative agent assumption, the risk aversion estimate for stockholders is 55.4, compared to -300.7 for nonstockholders. The lower estimate for stockholders is driven by a value of 12.7 for the top layer of stockholders. When focusing on single individual households, the risk aversion estimate for stockholders decreases to 19.7, driven by a value of 4.5 for the top layer of stockholders. The results for semiannual data also generally show large differences in risk aversion estimates for stockholders and nonstockholders, but with more mixed results for single individual households. The finding of lower risk aversion estimates for richer stockholders are in line with the findings of Brav, Geczy, and Constantinides (1999) also using quarterly CEX data. As for the negative risk aversion estimate for nonstockholders, this is consistent with results of Poterba and Samwick (1995) based on the PSID, using the Skinner consumption index instead of only food consumption.

Since these estimates are based on only one moment restriction, large standard errors are to be expected (as was seen in the Monte Carlo analysis). Mankiw and Zeldes (1991) and Brav, Geczy and Constantinides (1999) do not report standard errors for their estimates. The last two columns of Tables 4 and 5 give 95 percent bootstrap confidence intervals for the risk aversion estimates. Even if consumption growth, the stock return and the T-bill return are joint lognormal, the finite sample distribution of the risk aversion estimate considered here is difficult to derive analytically. The asymptotic distribution can be found using the delta method. However, the asymptotic distribution which is a normal distribution does not seem to be a close approximation to the finite sample distribution which has a longer tail to the right (this was confirmed both from the estimates in the Monte Carlo simulation and by bootstrapping from the actual CEX data). This is the reason for using bootstrap simulation. The results in column 4 assume that the data are independent over time, whereas the results in column 5 allow for third order autocorrelation for the quarterly data and sixth order autocorrelation for the semiannual data. Accounting for autocorrelation is done by resampling blocks of data rather than individual data points. For a description of the block bootstrap see Horowitz (1998). Based on the presumption of third and sixth order autocorrelation, the bootstrap uses blocks of length three months for the quarterly data and length six months for the semiannual data. Nonoverlapping blocks are used. For each risk aversion estimate two bootstrap confidence intervals are given in each of column 4 and 5. The first is the 95 percent percentile interval $[\gamma_{0.025}, \gamma_{0.975}]$, where γ_α denotes the α^{th} percentile of the bootstrap distribution. The second is the 95 percent bias-corrected percentile interval as described in Efron and Tibshirani (1993). This approach corrects for possible bias in the estimator by using different percentiles of the bootstrap distribution. The results shown are based on 20000 bootstrap draws.

The bootstrap confidence intervals are extremely wide, more so for nonstockholders than for stockholders. Only for the top layer of stockholders is risk aversion estimated somewhat more precisely. The confidence interval for the top layer, when using quarterly data and not assuming a representative agent within the group is [1.2, 56.6] when autocorrelation is accounted for. For single individual households the corresponding interval is [0.4, 14.2]. The wide confidence intervals are consistent with the large standard errors for the correlation of stock returns and consumption growth rates reported by Poterba and Samwick (1995). The results are little affected by assuming a representative agent. This is consistent with the Monte-Carlo evidence of Brav, Constantinides and Geczy (1999) mentioned above.

Overall the results from the calibration show some evidence of differences in risk aversion estimates between stockholders and nonstockholders. The bootstrap confidence intervals indicate that one moment is too little to obtain precise estimates, although the bootstrap confidence intervals for the top layer of stockholders are somewhat narrower.

3.4.2 Conditional Euler equations

The results of the instrumental variables estimations of the log-linearized model in equation (18) are shown in Tables 6–11. Each table shows three sets of estimations corresponding to three different sets of instruments. All instrument sets include 12 seasonal dummies and $\Delta \ln(\text{family size})_{t+1}$. In addition instrument set 1 includes the log dividend price ratio. Instrument set 2 includes the log dividend price ratio, the lagged log real stock return and the lagged log real T-bill returns. Instrument set 3 includes the log dividend price ratio, the bond horizon premium and the bond default premium. See section 3.3.3 for the precise timing of these variables. Many previous studies have used lagged stock and bond returns as instruments. The dividend price ratio is well known to be able to forecast real stock returns. The use of the bond horizon premium and the bond default premium is motivated by the findings of Fama and French (1989) that these have predictive power for stock returns. In my sample, the log dividend price ratio was the strongest predictor of both the real stock return and the real T-bill return. Leaving out the family size variable, the R^2 from regressing the log real stock return on the variables in the three instrument sets was 0.138, 0.155, 0.200 for semiannual data, and 0.112, 0.206, 0.146 for quarterly data. The corresponding R^2 values for the log real T-bill return are 0.707, 0.707, 0.762 for semiannual data, and 0.578, 0.606, 0.585 for quarterly data.

While other variables in the information set should be uncorrelated with the error terms in the log-linearized Euler equations according to economic theory, the properties of the estimators may deteriorate if weak instruments are included. It is known that the 2SLS estimator tends to be biased towards the biased and inconsistent OLS estimator. If the explanatory power of the first stage is fixed while more instruments are added, the bias becomes progressively worse.[27] This motivates the use of a small set of instruments (adding in more instruments tended to push the estimates of $1/\gamma$ closer to 0 for all groups).

The results inTables 6-11 are favorable to the limited participation theory. Table 6 shows the joint estimation for semiannual data using both the Euler equation for the stock return and the one for the bond return. For instrument set 1, $1/\gamma$ is estimated to be 0.335 for stockholders implying an estimate of γ around 3. The estimate of $1/\gamma$ is significant at the 5 percent level. For nonstockholders, however, the estimate of $1/\gamma$ is close to zero. This implies a very large estimate of γ, but should rather be interpreted as evidence that the Euler equation for one or both assets does not hold for nonstockholders. The difference between stockholders and nonstockholders is even larger for single individual households. When adding more instruments the qualitative patterns remain the same, but the estimates of $1/\gamma$ for stockholders tend to decrease. This could be a result of adding weak instruments.

[27] See e.g. Bound, Jaeger, and Baker (1995) and Donald and Newey (1997) for useful discussions of the literature on this.

The larger estimates of $1/\gamma$ for stockholders are driven by larger estimates for the richest layer or the two richest layers of stockholders. As mentioned earlier, this could be interpreted either as lower risk aversion for richer stockholders or as a result of richer stockholders having better diversified portfolios than less wealthy stockholders. The estimates in Table 8 based only on the Euler equation for the T-bill return show the same pattern of higher estimates of $1/\gamma$ for the richest stockholders thus favoring the heterogeneous risk aversion interpretation.

The estimations in Table 7 and 8 using the stock return and the T-bill return separately both show larger estimates of $1/\gamma$ for stockholders than for nonstockholders. For nonstockholders the estimate of $1/\gamma$ is positive but insignificantly different from zero even when estimating the Euler equation for the T-bill return. One interpretation of this finding is that nonstockholders have a low elasticity of intertemporal substitution (and thus a high risk aversion given the CRRA preference assumption). Another possibility is that they face frequently binding borrowing constraints such that not even the Euler equation for the T-bill return hold in all periods.

Differences between stockholders and nonstockholders are also present for quarterly data, but with lower levels of significance and with the exception of instrument set 2. In this case differences between stockholders and nonstockholders remain when focusing on singles and the pattern of higher estimates of $1/\gamma$ for richer stockholders remains.

The coefficients on $\Delta \ln(\text{family size})_{t+1}$ for the estimations using all household sizes indicate that the semiannual estimations may be more reliable than the quarterly estimation. For semiannual data the coefficient on $\Delta \ln(\text{family size})_{t+1}$ (not shown in the tables) is generally around 0.4 for stockholders, and significant for the set of all stockholders or for the two top layers of stockholders. For nonstockholders the coefficient on $\Delta \ln(\text{family size})_{t+1}$ is smaller and typically insignificant. For quarterly data, the family size variable most often enters with a negative coefficient for both types of households.

An important negative finding based on the instrumental variables estimations is that there is no clear tendency for the tests of overidentifying restrictions to reject for nonstockholders but not for stockholders.

All estimations in Tables 5-10 were repeated under the representative agent assumption with results fairly similar to the results taking cross-sectional consumption growth heterogeneity into account. Furthermore, limited information maximum likelihood estimations (for simplicity without corrections for heteroscedasticity and autocorrelation) gave similar results. Unlike the linear GMM estimation, the LIML estimates are not affected by whether $\Delta \ln C_{t+1}$ is regressed on $\ln (1 + r_{i,t+1})$ or the other way around (assuming the same set of instruments are used). The estimate of γ from the latter regression will be the inverse of the estimate of $1/\gamma$ from the first regression.

Overall the instrumental variables results support the hypothesis that focusing on stockholders lead to much more reasonable risk aversion estimates and thus that limited stock market participation is an important part of the solution to the equity premium puzzle. The improvement in results compared to the unconditional Euler equation estimations is likely to be due to exploiting the time-variation in expected consumption growth and expected asset returns to estimate the parameters.

3.4.3 Hansen-Jagannathan bound analysis

For reference, the well known HJ bounds and stochastic discount factor points for US per capita data are reproduced in Figures 3 and 4 for annual and quarterly data respectively. Based on the estimates in Gourinchas and Parker (1999), the value of the discount rate is set to 4 percent per year. It is seen that the mean-standard deviation points for the stochastic discount factor only enter the bound for large and implausible values of risk aversion.[28]

However, as discussed in section 3.3.2 on the importance of matching, the growth rate of per capita consumption is too large since it includes the effect of replacing the oldest cohort no longer alive at $t+1$ with a younger richer cohort. Campbell (1992) pointed this out as a potential problem for Hansen-Jagannathan bound analysis but did not derive the precise effect. If aggregate consumption is lognormal, $\Delta \ln C_{t+1}^T \sim N\left(\mu, \sigma^2\right)$, then

$$E\left(M_t\right) = E\left(\delta \left(\frac{C_{t+1}^T}{C_t^T}\right)^{-\gamma}\right) = e^{-\gamma\mu}\left(\delta e^{\frac{1}{2}\gamma^2\sigma^2}\right), \qquad \sqrt{V\left(M_t\right)} = e^{-\gamma\mu}\left(\delta e^{\frac{1}{2}\gamma^2\sigma^2}\sqrt{e^{\gamma^2\sigma^2}-1}\right).$$

Thus reducing μ amounts to rescaling $E\left(M_t\right)$ and $\sqrt{V\left(M_t\right)}$ by the same factor. It therefore moves the point in the HJ diagram corresponding to a given value of γ along a line from the

[28] An interesting recent paper on Hansen-Jagannathan bound analysis is Luttmer (1996). He shows that there is little evidence against power utility specifications with a low risk-aversion parameter when proportional transactions costs are allowed for. It remains to be determined if actual transactions costs are mostly proportional and sufficiently large.

original point to $(0,0)$.

The consumption literature documents hump shaped consumption profiles over the lifetime, with little difference between consumption when old and when young, see e.g. Gourinchas and Parker (1999). Therefore, the cohort adjustment amounts (approximately) to adjusting μ down by the rate of technological progress g.[29] Estimates of the rate of technological progress are about 2 percent per year for the period in focus, see for example Blanchard and Fischer (1989), Chapter 1. Based on this, the second curve of points in Figures 3 and 4 shows the values of $E(M_t)$ and $\sqrt{V(M_t)}$ when adjusting the US per capita consumption growth rate by the factor $\frac{1}{1.02}$ for the annual data and $\frac{1}{1.005}$ for the quarterly data. Now, the stochastic discount factors pass close under the bound for much smaller values of risk aversion, around 12 for annual data and around 30 for quarterly data. Thus, the cohort adjustment fundamentally changes the results.

There is still a central role for the stockholder-nonstockholder distinction. This is clear from the CEX results shown in Figures 5 to 8. When using all households, the stochastic discount factor for stockholders enters the bound at $\gamma = 10$ for semiannual data, and $\gamma = 11$ for quarterly data.[30] These estimates should be adjusted up a bit due to the downward bias documented in the Monte Carlo study. Thus the risk aversion estimate for stockholders based on HJ analysis is a bit above 10 or 11 when using a representative agent assumption. The stochastic discount factors for nonstockholders do not enter the bound at any value of γ. Notice furthermore how the points corresponding to the set of all households (but matched across periods to avoid the cohort problem) look similar to the points based on the cohort adjusted aggregate data.

For single individual households the stockholder-nonstockholder distinction remains important. None of the points enter the bound due to lower consumption growth for single family households. However, the points for stockholders come very close to the bound for $\gamma = 2$ for both the semiannual and the quarterly data. Again, these values should be seen as lower bounds

[29] Suppose individuals live economically independent of their parents from age 20 to 80. Denote the consumption level of a 20 year old in period t by $\overline{C}$. Then with flat consumption profiles over the lifetime of each individual (and aside from shocks) $\frac{C_{t+1}^T}{C_t^T} = \frac{\overline{C}\left((1+g)+\ldots+(1+g)^{61}\right)}{\overline{C}\left(1+(1+g)+\ldots+(1+g)^{60}\right)} = 1+g$. Thus $\Delta \ln C_{t+1}^T = \ln(1+g) \simeq g$, whereas $E\left(\Delta \ln C_{t+1}^h\right) = 0$ for each individual h. The adjustment is of similar magnitude with hump shaped lifetime consumption profiles.

[30] Including $\Delta \ln$(family size) along with the seasonal dummies in the construction of the seasonally adjusted consumption data for the HJ bound analysis made little difference.

for risk aversion based on the results of the Monte Carlo study.

It remains to consider the role of idiosyncratic risk. As discussed earlier, measurement error makes it difficult to use the micro data for this purpose. However, under lognormality assumptions one case at least theoretically derive the role of idiosyncratic risk by generalizing the results of Constantinides and Duffie (1996). Appendix B gives the derivation. It is a slight generalization of teaching material from John Campbell's asset pricing class at Harvard (all errors are mine). The only change is that I allow for consumption growth rates which are not independent over time. The derivation shows that

$$M_{t+1}^{T} = M_{t+1}^{RA} \exp\left(\frac{1}{2}\gamma(\gamma+1) V_{h,t+1}\left[\Delta \log C_{t+1}^{h}\right] + \gamma cov_{h,t+1}\left[\log C_{t}^{h}, \Delta \log C_{t+1}^{h}\right]\right)$$

$M_{t+1}^{T} \equiv E_{h,t+1}\left[\delta\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\gamma}\right]$ is the stochastic discount factor based on cross-sectional summation of the stochastic discount factors for a group of households. If all households in the group participate in the asset market in focus. $M_{t+1}^{RA} \equiv \delta\left(\frac{E_{h,t+1}\left[C_{t+1}^{h}\right]}{E_{h,t}\left[C_{t}^{h}\right]}\right)^{-\gamma}$ is the stochastic discount factor for this group of households when a representative agent within the group is assumed. It is assumed that the cross-sectional distribution of consumption is lognormal at each date. The formula shows that if $V_{h,t+1}\left[\Delta \log C_{t+1}^{h}\right]$ and $cov_{h,t+1}\left[\log C_{t}^{h}, \Delta \log C_{t+1}^{h}\right]$ are (approximately) constant over time, accounting for idiosyncratic risk amounts to adjusting $E(M^{RA})$ and $\sqrt{V\left(M^{RA}\right)}$ by the factor $\exp\left(\frac{1}{2}\gamma(\gamma+1) V_{h}\left[\Delta \log C_{t+1}^{h}\right] + \gamma cov_{h}\left[\log C_{t}^{h}, \Delta \log C_{t+1}^{h}\right]\right)$. Constantinides and Duffie (1996) assume i.i.d. consumption growth rates and their results therefore do not have the covariance component. As for the cohort adjustment derived earlier, the adjustment to account for idiosyncratic risk amounts to moving the point for a given value of γ along a line from the original point to $(0,0)$. I drew in one of these lines in each of Figures 3-8. The line shown is the one such that adjustment along the line for this value of γ would make the point enter the bottom of the HJ bound (for the right amount of adjustment). In Figures 5-8 the adjustment line drawn is for the stochastic discount factor points for stockholders.[31]

It is not clear whether the points should be moved left or right along these lines. $V_{h,t+1}\left[\Delta \log C_{t+1}^{h}\right]$ is positive, but $cov_{h,t+1}\left[\log C_{t}^{h}, \Delta \log C_{t+1}^{h}\right]$ is negative if consumption growth rates are mean reverting. Measurement error makes it very difficult to estimate either of $V_{h,t+1}\left[\Delta \log C_{t+1}^{h}\right]$ and $cov_{h,t+1}\left[\log C_{t}^{h}, \Delta \log C_{t+1}^{h}\right]$ based on micro data and I did not attempt

[31] Lettau (1998) also consider the effect of idiosyncratic risk on the stochastic discount factors. However, he focuses on the ratio $\frac{\sqrt{V(M_t)}}{E(M_t)}$ which under the above assumptions is unaffected by idiosyncratic risk.

such an estimation.

In sum, the HJ analysis shows that when assuming a representative agent within groups, the stochastic discount factor for stockholders enters the HJ bound or come close to the HJ bound for much smaller values of risk aversion than was the case for the (unadjusted) aggregate US. This is not the case for nonstockholders. Unfortunately, measurement error makes it difficult to adjust for idiosyncratic risk within groups, but it was shown theoretically that such an adjustment could move the points either left or right. Given the difficulties with implementing this adjustment I did not attempt to calculate standard errors for the stochastic discount factor points. Luttmer (1996) gives standard errors for the bound.

4 Conclusion

I conclude that limited stock market participation should be considered an important part of the solution to the equity premium puzzle. The theoretical section showed that under the condition that the conditional correlation of nonstockholder consumption growth with stock returns is zero, estimation of Euler equations involving stock returns without excluding nonstockholders will result in an upward biased estimate of relative risk aversion. The bias is given by the factor $1/\lambda$, where λ is the fraction of stockholders in the population.

This hypothesis was tested using micro consumption data from the Consumer Expenditure Survey. Differences in risk aversion estimates for stockholders and nonstockholders are large, although the results differ somewhat across data frequencies and instrument sets. The empirical section furthermore showed that the distinction between stockholders and nonstockholders is important for Hansen-Jagannathan bound analysis. A negative finding in the empirical analysis was that tests of overidentifying restrictions on the Euler equations do not consistently reject for nonstockholders.

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5 Appendix A. Monte Carlo study of the effects of measurement error

5.1 Part 1. The benefits of log-linearization for Euler equation estimation

Table 2 shows the results of a small scale Monte Carlo simulation of the properties of four different risk aversion estimators. T is kept fixed and the simulation shows the effect of increasing H on the small sample properties of the four estimators. This is done for several different values of the standard deviation of the measurement error in consumption.

All four risk aversion estimators are based on the Euler equation for the excess return on stocks over T-bills. Thus the Euler equation contains only one preference parameter γ. For simplicity I do not consider situations in which instruments are available (the basic points should carry over to this case). The four estimators differ in whether they are based on log-linearized Euler equations or not, and in whether the Euler equation assumes a representative agent or not.

1. Nonlinear Euler equation, no representative agent assumption. $\widehat{\gamma}$ solves
$$\frac{1}{T}\Sigma_t\left[\frac{1}{H}\Sigma_h\left[\left(\frac{C_{t+1}^h}{C_t^h}\right)^{-\hat{\gamma}}\right](R_{s,t+1}-R_{f,t+1})\right]=0.$$

2. Log-linearized Euler equation, no representative agent assumption. $\widehat{\gamma}$ is given by
$$\widehat{\gamma}=\frac{\frac{1}{T}\Sigma_t\left(\ln R_{s,t+1}-\ln R_{f,t+1}\right)+\frac{1}{2}\widehat{V}\left(\ln R_{s,t+1}\right)}{\widehat{cov}\left(\ln R_{s,t+1},\frac{1}{H}\Sigma_h\ln\left(\frac{C_{t+1}^h}{C_t^h}\right)\right)}$$
with the standard estimators of variance and covariance.

3. Nonlinear Euler equation, representative agent assumption. $\widehat{\gamma}$ solves
$$\frac{1}{T}\Sigma_t\left[\left[\left(\frac{\frac{1}{H}\Sigma_h C_{t+1}^h}{\frac{1}{H}\Sigma_h C_t^h}\right)^{-\hat{\gamma}}\right](R_{s,t+1}-R_{f,t+1})\right]=0.$$

4. Log-linearized Euler equation, representative agent assumption: $\widehat{\gamma}$ is given by
$$\widehat{\gamma}=\frac{\frac{1}{T}\Sigma_t\left(\ln R_{s,t+1}-\ln R_{f,t+1}\right)+\frac{1}{2}\widehat{V}\left(\ln R_{s,t+1}\right)}{\widehat{cov}\left(\ln R_{s,t+1},\ln\left(\frac{\frac{1}{H}\Sigma_h C_{t+1}^h}{\frac{1}{H}\Sigma_h C_t^h}\right)\right)}.$$

The Monte Carlo simulation consists of repeating the following steps a large number of times in order to be able to characterize the distribution of the each estimator. Each of the four estimators are method of moments estimators.

Step 1. Draw a time series for each of the three aggregate variables, $\Delta \ln C_{t+1}$, $\ln(1+R_{s,t+1})$, and $\ln(1+R_{f,t+1})$. These are assumed joint lognormal with means and variance-covariance matrix as in the annual US data for the period 1930-1996. The specific data series are as described in the data section.

Step 2. For each t, H idiosyncratic shocks ε_{t+1}^h are then drawn. These are assumed lognormal, $\ln \varepsilon_t^h \sim N\left(0, \sigma_{\ln \varepsilon}^2\right)$ and independent over time. These shocks represent both genuine cross sectional variation in consumption growth rates, as well as measurement error. The cross sectional variation in consumption growth rates is assumed independent of the aggregate variables. Idiosyncratic consumption risk therefore enters the equations in the same way as measurement error.[32] The value of $\sigma_{\ln \varepsilon}^2$ is chosen based on the CEX data. For quarterly data the cross-sectional standard deviation of consumption growth $\sqrt{V_h\left(\Delta \ln C_t^h\right)}$ equals 0.34 for quarterly data seasonally adjusted using monthly dummies.[33] The standard deviation of US quarterly nondurables and services per capita consumption growth for the same period (1982-1995) is $\sqrt{V\left(\Delta \ln C_t^*\right)} = 0.04$.[34] This implies $\sqrt{V\left(\Delta \ln \varepsilon_{t+1}\right)} = \sqrt{0.34^2 - 0.04^2} = 0.338$. Thus the standard deviation of $\Delta \ln \varepsilon_{t+1}^h$ is 8 to 9 times larger than that of $\Delta \ln C_{t+1}$ in the quarterly data. Assuming the same ratio for the annual data implies a value of $\sigma_{\Delta \ln \varepsilon} = 9 * 0.023 = 0.207$ for the annual data.[35] With independent measurement errors over time $\sigma_{\Delta \ln \varepsilon}^2 = 2\sigma_{\ln \varepsilon}^2$, so $2\sigma_{\ln \varepsilon}^2 = 0.207^2$ or $\sigma_{\ln \varepsilon}^2 = 0.021$. Since this is only a rough calibration, the sensitivity of the results to various values of this parameter is considered. Table 2 gives the parameter choices in

[32] By definition idiosyncratic consumption shocks are not correlated with the stock or the bond return. Thus for the present purposes, idiosyncratic consumption shocks have the same effect on the properties of risk aversion estimators as measurement errror.

[33] This is the average value of the standard deviation across periods, the values for each period vary around this value. The calculations are based on my CEX sample which excludes observations for which C_{t+1}^h / C_t^h is above 5 or less than 2.

[34] The US data are seasonally adjusted by the BEA using a variety of the X11 method. Unadjusted US data were not available.

The consumption measure used is US per capita log consumption growth of nondurables and services, 1930-96. I use the '*' notation here since I assume that aggregate consumption is not measured with error.

[35] 0.023 is the standard deviation of annual US per capita log consumption growth of nondurables and services, 1930-96.

terms of $\sigma_{\Delta \ln \varepsilon}$ since this is easier to interpret.

Step 3. Calculate the value of each of the four estimators. This step is time consuming since the values of the two risk aversion estimators based on the nonlinear Euler equations must be calculated using grid search. This is necessary because there are no, one or several values of γ which sets the moment equal to zero, depending on the draw of the data. Without measurement error, the moment is a monotonic downward sloping function of γ for both estimator 1 and 3 with a unique value of γ settting it equal to zero. With measurement error this not always the case. These features of the solution were determined by plotting the moments against γ for many different draws of the data. I focus on the cases where the moment is positive at very low values of γ, negative at very positive values, and has a unique value of γ that sets it equal to zero. These are referred to as cases for which the estimator has a 'valid' solution. The grid search was done over the interval -150 and 150 with 301 grid points. For the cases with a valid solution the estimate found by grid search is chosen to be the point half way between the two relevant grid points, and will thus by at most 0.5 away from the exact value of γ which sets the moment equal to zero.

The current results are based on 1500 Monte Carlo iterations. This is sufficient to illustrate the main features of the results. More iterations are needed to calculate the precise means, medians, and percentiles of the estimators and will be provided for the next draft of the paper. Figure 1 illustrates the results for $\sigma_{\Delta \ln \varepsilon} = 0.15$. Each graph has 1500 points corresponding to the 1500 Monte Carlo iterations. On the horizontal axis, the estimate based on the log-linearized model for the case with no measurement error is plotted (without measurement error, and with the data being joint lognormal, it has very little effect whether the Euler equation is log-linearized or not). On the vertical axis in the top three pictures, the values of estimators 1 and 2 are plotted. In the bottom three pictures the values of estimators 3 and 4 are shown. If a given estimation procedure was little affected by measurement error in consumption, then all the points should lie close to the 45 degree line. For large H, $H = 1000$, this is what is found for estimator 2 and 4 based on log-linearized Euler equations. The lower H, the more scattered the points are around the 45 degree line. For estimators 1 and 3 based on the nonlinear Euler equations, the cases for which a valid solution for the estimator is not available are plotted as zeros. The percent of cases for which a valid solution is available is around 40 percent for estimator 1 and 50 percent for estimator 2. The graph clearly shows that even when a valid solution is available it may be far from the solution for the case with no measurement error (and the same draw of the aggregate variables). Table 2 provides detailed statistics from the simulation for various values of H and $\sigma_{\Delta \ln \varepsilon}$. Notice that even for $\sigma_{\Delta \ln \varepsilon} = 0.05$, substantially smaller than what the CEX data suggest, the problem of no valid solution for the nonlinear Euler equations still occurs quite frequently. It seems fair to conclude that with measurement error, log-linearization is preferable. Furthermore, except for $H = 25$, estimators 2 and 4 based on the log-linearized Euler equation are typically close to the estimators for the case with no measurement errors. They are close to median unbiased and have 95 percent confidence intervals close to those when no measurement errrors are present, cf. Table 2. The confidence intervals are wide (as is the case in the corresponding estimation based on actual data), suggesting that more moments should be used. The main point that log-linearization is benefical should carry over to this case.

The following simple calculation provides some intution for the poorer small sample properties of estimates based on the nonlinear Euler equations. In the log-linearized model, consumption enters as

$$-\gamma\left(\Delta \ln C_{t+1}^{h}\right) = -\gamma\left(\Delta \ln C_{t+1}^{h,*}\right) - \gamma\left(\Delta \ln \varepsilon_{t+1}\right).$$

whereas in the original nonlinear Euler equation consumption enters as

$$\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\gamma} = \left(\frac{C_{t+1}^{h,*}}{C_{t}^{h,*}}\right)^{-\gamma}\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_{t}^{h}}\right)^{-\gamma}.$$

Suppose $\Delta \ln C_{t+1}^{h,*} \sim N\left(\mu, \sigma^2\right)$, $\Delta \ln \varepsilon_{t+1} \sim N\left(0, \sigma_{\Delta \ln \varepsilon}^2\right)$, and $\Delta \ln C_{t+1}^{h,*}$ and $\Delta \ln \varepsilon_{t+1}$ independent. Define $k = \frac{\sigma_{\Delta \ln \varepsilon}^2}{\sigma^2}$. Then

$$\begin{aligned} V_1 &= \frac{V\left(-\gamma\left(\Delta \ln \varepsilon_{t+1}\right)\right)}{V\left(-\gamma\left(\Delta \ln C_{t+1}^{h,*}\right)\right)} = \frac{k\gamma^2\sigma^2}{\gamma^2\sigma^2} = k \\ V_2 &= \frac{V\left(\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_{t}^{h}}\right)^{-\gamma}\right)}{V\left(\left(\frac{C_{t+1}^{h,*}}{C_{t}^{h,*}}\right)^{-\gamma}\right)} = \frac{e^{k\gamma^2\sigma^2}\left(e^{k\gamma^2\sigma^2} - 1\right)}{e^{-2\gamma\mu}e^{\gamma^2\sigma^2}\left(e^{\gamma^2\sigma^2} - 1\right)}. \end{aligned}$$

Suppose that $\sigma_{\varepsilon} = 9\sigma$ as suggested above. Then $k = 81$. In addition, suppose $\mu = 0$ and $\sigma^2 = 0.023^2 = 0.000529$. Then for $\gamma = 1$, $V_2 = 86$, but for $\gamma = 5$, $V_2 = 415$ and for higher γ, V_2 is very large. Thus, unless risk aversion is very low, the variation in the consumption term which enters the Euler equation will be substantially more dominated by variation in the measurement error part in the nonlinear Euler equation than in the log-linearized Euler equation. Thus one would expect poorer properties of risk aversion estimators based on the nonlinear Euler equations.

5.2 Part 2. The effect of measurement error on stochastic discount factors

Figure 2 shows the Monte Carlo simulation for the stochastic discount factors under a representative agent assumption. The data generating process is as for the above Monte Carlo exercise. For a given γ, the values plotted are the medians of $Std(M)$ and $E(M)$ across the 1500 Monte Carlo draws. The top left picture shows the benchmark case of no measurement error or idiosyncratic shocks. The other three pictures correspond to different values of H. The shape of the curves are seen to be largely unaffected by the presence of measurement error (due to the representative agent assumption). However, for $H = 25$, the value of γ corresponding to a given point on the curve is only about half the value at the corresponding graph based on no measurement error. This problem diminishes with larger H.

6 Appendix B. Derivation of aggregation correction for stochastic discount factor

Let $M_{t+1}^{T} \equiv E_{h,t+1}\left[\delta\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\gamma}\right]$ be the stochastic discount factor based on cross-sectional summation of the stochastic discount factors for a set of households. Furthermore, let $M_{t+1}^{RA} \equiv \delta\left(\frac{E_{h,t+1}\left[C_{t+1}^{h}\right]}{E_{h,t}\left[C_{t}^{h}\right]}\right)^{-\gamma}$ be the (invalid) stochastic discount factor based on a representative agent assumption. Assume that the cross-sectional distribution of consumption is lognormal at each date: $\log C_{t}^{h} \sim N\left(\mu_{t}, \sigma_{t}^{2}\right) \quad \forall t \quad (A1)$. Then

$$
\begin{aligned}
\log M_{t+1}^{T} &= \log\delta + \log E_{h,t+1}\left[\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\gamma}\right] \\
&= \log\delta + E_{h.t+1}\left[\log\left(\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\gamma}\right)\right] + \frac{1}{2}V_{h,t+1}\left[\log\left(\left(\frac{C_{t+1}^{h}}{C_{t}^{h}}\right)^{-\gamma}\right)\right] \text{ by ass. } A1. \\
&= \log\delta - \gamma E_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] + \frac{1}{2}\gamma^{2}V_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] \\
\log M_{t+1}^{RA} &= \log\delta + \log\left(\left(\frac{E_{h,t+1}\left[C_{t+1}^{h}\right]}{E_{h,t}\left[C_{t}^{h}\right]}\right)^{-\gamma}\right) \\
&= \log\delta - \gamma\log E_{h,t+1}\left[C_{t+1}^{h}\right] + \gamma\log E_{h,t}\left[C_{t}^{h}\right] \\
&= \log\delta - \gamma\left(E_{h,t+1}\left[\log C_{t+1}^{h}\right] + \frac{1}{2}V_{h,t+1}\left[\log C_{t+1}^{h}\right]\right) \\
&\quad +\gamma\left(E_{h,t}\left[\log C_{t}^{h}\right] + \frac{1}{2}V_{h,t}\left[\log C_{t}^{h}\right]\right) \\
&= \log\delta - \gamma E_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] - \frac{1}{2}\gamma\left(V_{h,t+1}\left[\log C_{t+1}^{h}\right] - V_{h,t}\left[\log C_{t}^{h}\right]\right) \\
&= \log\delta - \gamma E_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] - \frac{1}{2}\gamma\left(\begin{array}{c} V_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] \\ +2cov_{h,t+1}\left[\log C_{t}^{h}, \Delta\log C_{t+1}^{h}\right]\end{array}\right).
\end{aligned}
$$

The last line follows from the identity $\log C_{t+1}^{h} = \log C_{t}^{h} + \Delta\log C_{t+1}^{h}$. Thus

$$
\begin{aligned}
\log M_{t+1}^{T} &= \log M_{t+1}^{RA} + \frac{1}{2}\gamma^{2}V_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] \\
&\quad +\frac{1}{2}\gamma\left(V_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] + 2cov_{h,t+1}\left[\log C_{t}^{h}, \Delta\log C_{t+1}^{h}\right]\right) \\
&= \log M_{t+1}^{RA} + \frac{1}{2}\gamma\left(\gamma+1\right)V_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] + \gamma cov_{h,t+1}\left[\log C_{t}^{h}, \Delta\log C_{t+1}^{h}\right]
\end{aligned}
$$

or

$$
M_{t+1}^{T} = M_{t+1}^{RA}\exp\left(\frac{1}{2}\gamma\left(\gamma+1\right)V_{h,t+1}\left[\Delta\log C_{t+1}^{h}\right] + \gamma cov_{h,t+1}\left[\log C_{t}^{h}, \Delta\log C_{t+1}^{h}\right]\right).
$$

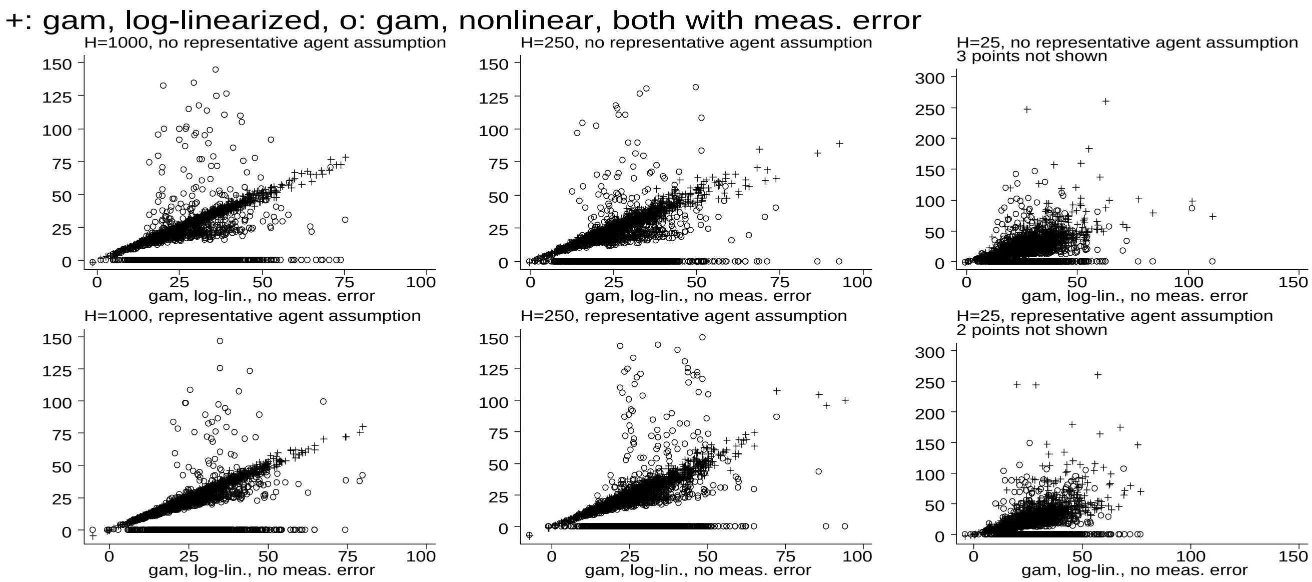

Figure 1. Monte Carlo simulation of properties of risk aversion estimates based on log-linearized Euler equations versus nonlinear Euler equations, just identified case, 1500 Monte Carlo iterations, $\sigma_{\Delta \ln \varepsilon} = 0.15$. Points along the horizontal axis represent cases where the nonlinear Euler equation did not have a 'valid' solution (see text for definition of 'valid')

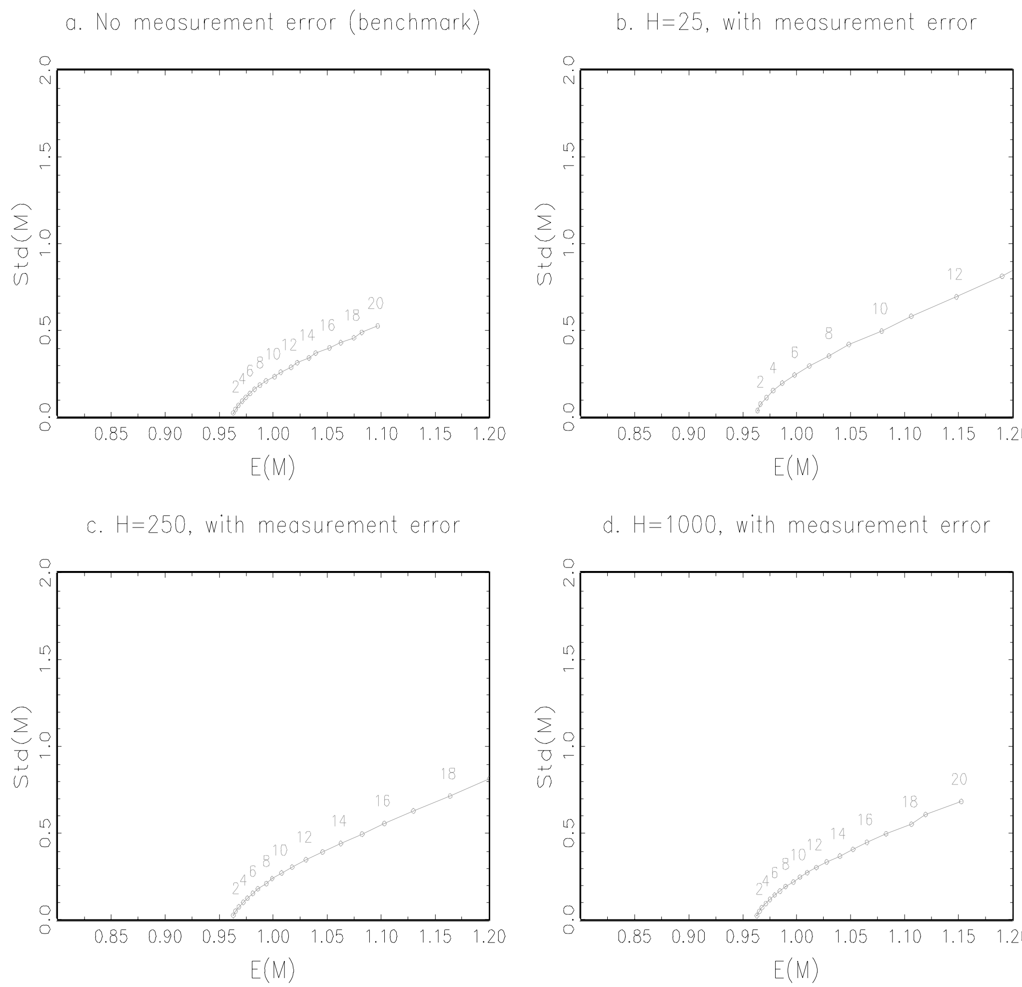

Figure 2. Monte Carlo simulation of properties of stochastic discount factors when consumption is measured with error. 1500 Monte Carlo iterations, $\sigma_{\Delta \ln \varepsilon} = 0.15$, median of standard deviation of stochastic discount factor plotted against median of mean of stochastic discount factor, representative agent assumption, discount rate 4 percent p.a., $\gamma = 1, ..., 20$.

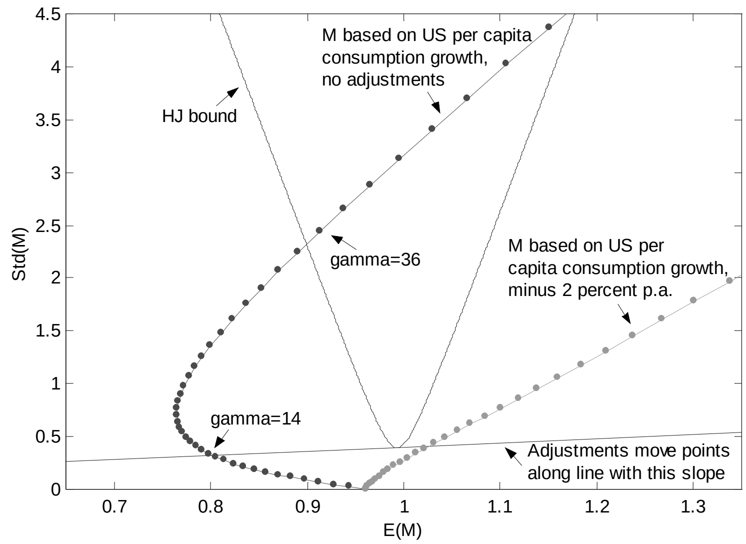

Figure 3. Hansen-Jagannathan bound. Annual US per capita data, nondurables and services consumption, NYSE value weighted and T-bill returns, 1929-1996, discount rate 4 percent p.a., $\gamma = 0, 1, ..., 50$

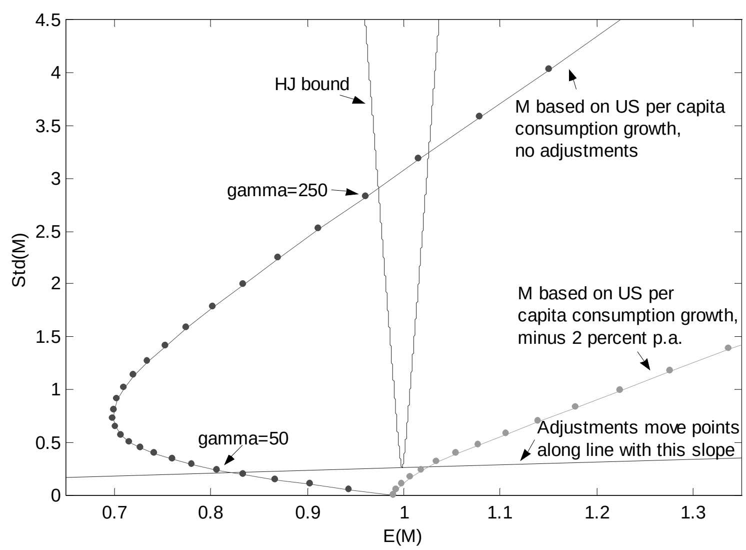

Figure 4. Hansen-Jagannathan bound. Quarterly US per capital data, nondurables and services consumption (seasonally adjusted by the BEA), NYSE value weighted and T-bill returns, 1946:1-1996:4, discount rate 4 percent p.a., $\gamma = 0, 10, 20, ..., 300$

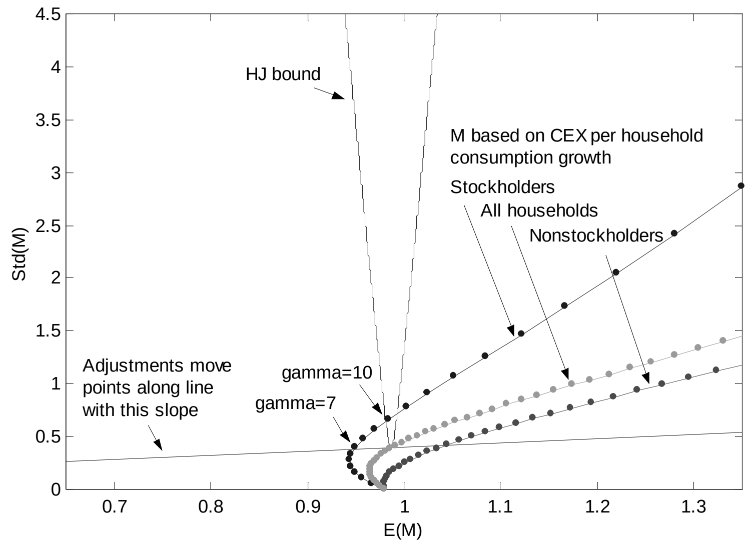

Figure 5. Hansen-Jagannathan bound. Semiannual CEX data, all household sizes, nondurables and services consumption (seasonally adjusted using seasonal dummies), NYSE value weighted and T-bill returns, 1982 (first half)-1995 (first half), discount rate 4 percent p.a., $\gamma = 0, 1, 2, ..., 30$

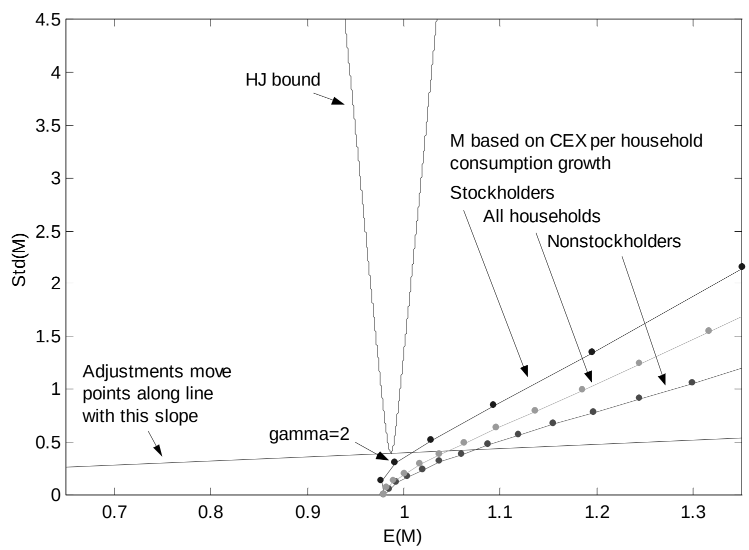

Figure 6. Hansen-Jagannathan bound. Semiannual CEX data, single individuals, nondurables and services consumption (seasonally adjusted using seasonal dummies), NYSE value weighted and T-bill returns, 1982 (first half)-1995 (first half), discount rate 4 percent p.a., $\gamma = 0, 1, 2, ..., 30$

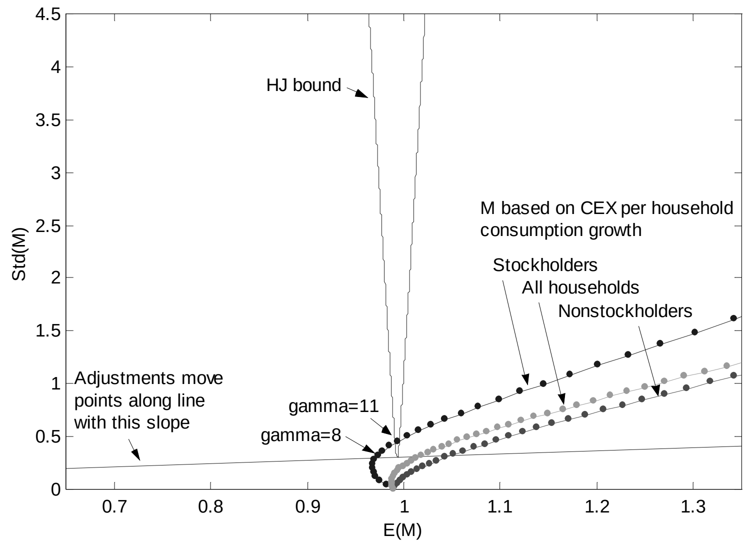

Figure 7. Hansen-Jagannathan bound. Quarterly CEX data, all household sizes, nondurables and services consumption (seasonally adjusted using seasonal dummies), NYSE value weighted and T-bill returns, 1982:1-1995:2, discount rate 4 percent p.a., $\gamma = 0, 1, 2, ..., 30$

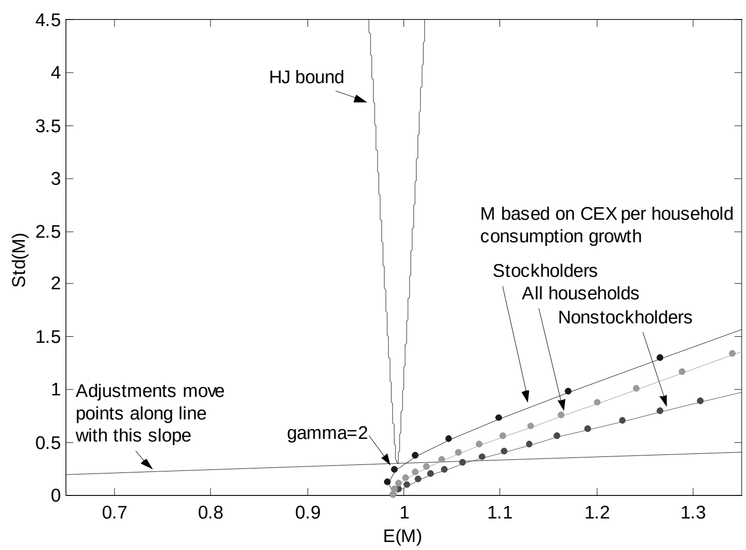

Figure 8. Hansen-Jagannathan bound. Quarterly CEX data, single individuals, nondurables and services consumption (seasonally adjusted using seasonal dummies), NYSE value weighted and T-bill returns, 1982:1-1995:2, discount rate 4 percent p.a., $\gamma = 0, 1, 2, ..., 30$

Data	Group	Mean number of observations per month	Mean of $\Sigma_{h=1}^{H_t} \Delta \ln C_{t+1}^h$ over the sample	Standard deviation of $\Sigma_{h=1}^{H_t} \Delta \ln C_{t+1}^h$ over the sample
Semiannual, all household sizes	All	218	0.003	0.025
	Stockholders	50	0.013	0.043
	Nonstockholders	168	0.0005	0.028
	Bottom layer	11	0.022	0.085
	Middle layer	11	0.002	0.098
	Top layer	11	0.011	0.102
Semiannual, single individual households	All	55	0.002	0.046
	Stockholders	11	0.015	0.099
	Nonstockholders	44	-0.0002	0.053
Quarterly, all household sizes	All	639	0.0003	0.022
	Stockholders	143	0.007	0.032
	Nonstockholders	496	-0.002	0.024
	Bottom layer	31	0.008	0.061
	Middle layer	32	0.005	0.056
	Top layer	32	0.010	0.080
Quarterly, single individual households	All	150	-0.001	0.038
	Stockholders	30	0.009	0.075
	Nonstockholders	120	-0.004	0.042
	Bottom layer	6	-0.005	0.172
	Middle layer	7	0.013	0.165
	Top layer	7	0.007	0.215

Note: $\Sigma_{h=1}^{H_t} \Delta \ln C_{t+1}$ is seasonally adjusted using seasonal dummies. The seasonally adjusted value is the mean of the series plus the residual from a regression on 12 dummies.

Table 1: Summary statistics, CEX data, 1982-1995

		Pct of cases with unique 'valid' solution	Median($\widehat{\gamma}$)	E($\widehat{\gamma}$)	[$\widehat{\gamma}_{0.025}$, $\widehat{\gamma}_{0.975}$]	Std.($\widehat{\gamma}$)	Corr($\widehat{\gamma}$, $\widehat{\gamma}_{\text{log-lin., no meas. error}}$)
No idiosyncratic risk/measurement error							
Log-linearized		100.0	25.9	27.3	[9.1, 51.2]	10.9	
Nonlinear		100.0	23.7	25.3	[8.2, 50.8]	11.4	
Idiosyncratic risk/measurement error, no representative agent assumption							
$\sigma_{\Delta \ln \varepsilon} = 0.05$							
Log-linearized	H=25	100.0	25.7	27.1	[8.8, 53.7]	11.1	0.97
	H=250	100.0	25.5	27.1	[9.7, 52.9]	11.1	1.00
	H=1000	100.0	25.7	26.8	[8.1, 51.1]	11.0	1.00
Nonlinear	H=25	75.5	22.5	24.3	[7.5, 55.5]	11.6	0.38
	H=250	84.7	22.5	24.3	[8.5, 49.5]	11.3	0.52
	H=1000	86.7	22.5	24.2	[6.5, 47.5]	11.7	0.55
$\sigma_{\Delta \ln \varepsilon} = 0.15$							
Log-linearized	H=25	100.0	26.1	26.5	[10.7, 57.9]	94.8	-0.02
	H=250	100.0	25.4	26.5	[10.2, 46.5]	11.3	0.97
	H=1000	100.0	26.1	27.3	[11.5, 33.7]	11.0	0.99
Nonlinear	H=25	40.7	16.5	23.3	[5.5, 85.5]	20.3	0.10
	H=250	42.1	18.5	23.7	[7.5, 76.5]	17.9	0.13
	H=1000	43.1	20.5	26.3	[6.5, 96.5]	20.6	0.14
$\sigma_{\Delta \ln \varepsilon} = 0.30$							
Log-linearized	H=25	100.0	24.2	41.8	[-67.8, 183.0]	458.7	0.09
	H=250	100.0	25.6	27.5	[8.8, 59.8]	13.0	0.87
	H=1000	100.0	25.9	27.4	[8.8, 52.4]	11.3	0.97
Nonlinear	H=25	30.6	10.5	16.8	[5.5, 49.5]	17.2	0.03
	H=250	29.9	12.5	17.7	[6.5, 45.5]	17.3	0.05
	H=1000	33.3	13.5	19.4	[7.5, 52.5]	18.8	0.07
Idiosyncratic risk/measurement error, representative agent assumption							
$\sigma_{\Delta \ln \varepsilon} = 0.05$							
Log-linearized	H=25	100.0	25.8	27.1	[8.6, 53.0]	11.5	0.97
	H=250	100.0	26.0	27.3	[8.1, 51.5]	11.0	1.00
	H=1000	100.0	25.5	27.0	[8.4, 53.1]	11.4	1.00
Nonlinear	H=25	83.7	22.5	24.4	[7.5, 52.5]	12.0	0.48
	H=250	91.2	23.5	24.3	[7.5, 47.5]	10.8	0.58
	H=1000	93.0	22.5	24.2	[7.5, 50.5]	10.9	0.66
$\sigma_{\Delta \ln \varepsilon} = 0.15$							
Log-linearized	H=25	100.0	25.9	29.0	[10.5, 61.7]	32.8	0.34
	H=250	100.0	25.5	27.1	[11.2, 48.3]	12.0	0.97
	H=1000	100.0	25.3	26.9	[10.9, 46.2]	10.9	0.99
Nonlinear	H=25	43.9	18.5	23.9	[7.5, 80.5]	18.1	0.12
	H=250	50.5	20.5	27.5	[7.5, 106.5]	22.6	0.24
	H=1000	55.1	20.5	24.2	[7.5, 75.5]	15.6	0.21
$\sigma_{\Delta \ln \varepsilon} = 0.30$							
Log-linearized	H=25	100.0	24.7	54.0	[-82.8, 159.1]	488.8	0.03
	H=250	100.0	26.2	28.3	[8.2, 59.2]	15.1	0.83
	H=1000	100.0	25.6	27.1	[8.6, 55.7]	11.6	0.97
Nonlinear	H=25	33.9	13.5	19.8	[5.5, 83.5]	19.8	0.00
	H=250	35.5	15.5	21.6	[7.5, 82.5]	17.8	0.06
	H=1000	37.4	16.5	23.9	[7.5, 87.5]	23.9	0.11

Note: The statistics shown in the last 6 columns are for the cases where a valid solution for the estimator is found. The true value of risk aversion in the simulation is 25.3. See text for calibration of parameters.

Table 2: Monte Carlo simulation of properties of risk aversion estimates based on log-linearized Euler equations versus nonlinear Euler equations, just identified case, 1500 Monte Carlo iterations.

Data	$\widehat{\gamma}$	95 pct bootstrap confidence interval, data assumed i.i.d. [percentile] [bias-corrected]
US annual, 1930-96		
Using R_t	195.0	[-1587.9, 1729.2]
		[37.7, 152697.2]
Using R_{t-1}	25.3	[7.3, 70.5]
		[6.2, 65.4]
US quarterly, 1947:2-1996:4		
Using R_t	358.2	[107.1, 1915.9]
		[109.8, 1959.3]
Using R_{t-1}	253.2	[111.9, 634.9]
		[107.3, 608.2]

Table 3: Calibration of γ based on log linearized Euler equations, US per capita data, nondurables and services

Data	Group	$\widehat{\gamma}$	95 pct bootstrap confidence interval, data assumed i.i.d. [percentile] [bias-corrected]	95 pct bootstrap confidence interval, nonoverlapping blocks [percentile] [bias-corrected]
CEX semiannual,	All	-2150.8	[-2192.9, 2230.3]	[-1400, 1571]
No representative			[-3225788, -2112.4]	[-384773, -3287]
agent assumption	Stockholders	51.4	[-225.8, 408.1]	[-350.5, 481.6]
			[-98.3, 621.4]	[-198.1, 672.4]
	Nonstockholders	-147.1	[-1574.3, 1412.3]	[-1207, 1200]
			[-28773.5, 103.3]	[-47613 -21.4]
	Bottom layer	2020.7	[-796.1, 809.2]	[-971.5, 1046.9]
			[5809.3, 250055.6]	[3791, 211073]
	Middle layer	107.1	[-612.9, 554.2]	[-525.6, 487.1]
			[32.8, 112487.3]	[33.0, 164295]
	Top layer	45.4	[-413.3, 449.4]	[-391.3, 431.6]
			[-39.4, 6111.4]	[-23.9, 6109]
CEX semiannual,	All	240.9	[-1761.1867.1]	[-1339, 1313]
Representative			[66.0, 380215.4]	[79.3, 489245]
agent assumption	Stockholders	35.6	[5.6, 220.5]	[-116.6, 255.6]
			[9.1, 253.2]	[-67.3, 286.3]
	Nonstockholders	-169.6	[-1525.4, 1482.1]	[-1062, 1027]
			[-153848.2, -23.4]	[-188299, -32.8]
	Bottom layer	56.3	[-498.8, 611.1]	[-614.4, 666.9]
			[-43.9, 11449.0]	[-94.9, 4698]
	Middle layer	108.1	[-608.5, 610.4]	[-560.5, 622.6]
			[31.8, 577093.2]	[29.2, 158756]
	Top layer	42.7	[-373.2, 405.9]	[-352.8, 362.9]
			[-16.3, 6880.1]	[0.5, 14195]
CEX quarterly,	All	814.8	[-1378.1, 1552.6]	[-1244.8, 1392.5]
No representative			[472.6, 8245345]	[501.6, 774952.3]
agent assumption	Stockholders	55.4	[-287.7, 458.5]	[-402.8, 650.8]
			[-189.8, 531.0]	[-294.9, 791.1]
	Nonstockholders	-300.7	[-1204.8, 1350.2]	[-1050.9, 1041.0]
			[-2993930, -93.0]	[-3734607, -105.6]
	Bottom layer	-29.3	[-187.6, -5.3]	[-161.0, 0.6]
			[-206.9, -6.6]	[-173.9, -0.8]
	Middle layer	22.3	[5.8, 109.9]	[-0.7, 163.7]
			[5.6, 106.2]	[-2.2, 146.8]
	Top layer	12.7	[4.0, 41.7]	[1.2, 52.6]
			[3.6, 37.4]	[0.1, 43.9]
CEX quarterly,	All	226.8	[-1435.3, 1461.1]	[-1427.4, 1437.6]
Representative			[66.0, 334013]	[56.6, 357406.6]
agent assumption	Stockholders	51.2	[-398.6, 540.9]	[-484.3, 585.6]
			[-209.7, 929.8]	[-205.6, 1210.6]
	Nonstockholders	-260.6	[-1490.1, 1488.0]	[-1365.6, 1255.9]
			[-1855291, -72.5]	[-500145.2, -71.1]
	Bottom layer	-37.0	[-328.8, 275.6]	[-302.8, 250.7]
			[-907.4, 82.5]	[-860.1, 58.2]
	Middle layer	24.0	[2.1, 154.4]	[-27.7, 186.7]
			[4.33, 175.3]	[-7.1, 199.5]
	Top layer	13.2	[3.1, 72.4]	[-35.5, 98.2]
			[2.0, 60.5]	[-39.8, 95.0]

Note: The stock return used is the real value weighted NYSE return. The bond return is the real return on T-bills. 20000 bootstrap iterations. See text for bootstrap methodology

Table 4: Calibration of γ based on log linearized Euler equations, CEX data, all household sizes, 1982-1995

Data	Group	$\widehat{\gamma}$	95 pct bootstrap confidence interval, data assumed i.i.d. [percentile] [bias-corrected]	95 pct bootstrap confidence interval, nonoverlapping blocks [percentile] [bias-corrected]
CEX semiannual,	All	-45.8	[-412.9, 277.3]	[-602.5, 494.1]
No representative			[-744.7, 121.9]	[-2289.4, 129.2]
agent assumption	Stockholders	64.2	[-495.8, 498.5]	[-442.9, 463.4]
			[16.5, 30107.1]	[15.5, 27868]
	Nonstockholders	-30.8	[-185.2, -8.5]	[-277.5, 177.6]
			[-194.2, -8.8]	[-356.8, 107.3]
CEX semiannual,	All	-57.4	[-618.1, 553.0]	[-620.9, 559.9]
Representative			[-4643.2, 553.0]	[-12387, 40.0]
agent assumption	Stockholders	-54.8	[-437.2, 420.0]	[-431.9, 420.3]
			[-128754.7, -13.5]	[-37460, -9.6]
	Nonstockholders	-58.4	[-685.4, 540.6]	[-774.4, 666.8]
			[-3138.1, 98.0]	[-8600, 68.5]
CEX quarterly,	All	-660.5	[-940.2, 1026.9]	[-840.4, 845.2]
No representative			[-530768.9, -469.2]	[-801918.0, -557.2]
agent assumption	Stockholders	19.7	[-101.7, 166.2]	[-70.9, 168.6]
			[-61.9, 204.5]	[-66.8, 168.6]
	Nonstockholders	-73.0	[-735.7, 740.8]	[-736.6, 679.8]
			[-25594.9, 740.8]	[-67121.2, -1.0]
	Bottom layer	-16.7	[-181.3, 160.4]	[99.7, 178.1]
			[-765.0, 33.4]	[-196.2, 182.7]
	Middle layer	15.6	[-139.5, 168.8]	[-820.3, 46.2]
			[-35.7, 168.8]	[-46.9, 298.3]
	Top layer	4.5	[1.4, 12.7]	[0.4, 14.2]
			[1.3, 11.9]	[0.2, 13.0]
CEX quarterly,	All	48.2	[-369.4, 472.3]	[-381.4, 485.0]
Representative			[-62.6, 2456.3]	[-75.9, 1662.3]
agent assumption	Stockholders	12.6	[2.4, 77.2]	[0.9, 62.8]
			[2.4, 77.2]	[0.3, 54.0]
	Nonstockholders	-102.1	[-835.9, 870.4]	[-834.9, 818.8]
			[-55918.8, -14.6]	[-366409.3, -13.7]
	Bottom layer	-17.7	[-180.6, 168.3]	[-206.5, 182.6]
			[-1434.1, 22.7]	[-1833.6, 21.2]
	Middle layer	13.1	[-111.7, 141.2]	[-70.3, 120.6]
			[36.3, 357.6]	[-35.4, 166.7]
	Top layer	3.8	[1.2, 11.0]	[0.4, 11.9]
			[1.1, 10.5]	[0.2, 11.2]

Note: The stock return used is the real value weighted NYSE return. The bond return is the real return on T-bills. 20000 bootstrap iterations. See text for bootstrap methodology

Table 5: Calibration of γ based on log linearized Euler equations, CEX data, single individuals households, 1982-1995

	Instrument set 1		Instrument set 2		Instrument set 3	
	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=1, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=5, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=5, p-value
All household sizes						
All	0.027		0.030		0.018	
	(0.038)	0.153	(0.030)	0.427	(0.033)	0.688
Stockholders	**0.335**		0.284		0.184	
	(0.112)	0.187	(0.094)	0.675	(0.064)	0.271
Nonstockholders	**0.003**		0.010		0.000	
	(0.023)	0.401	(0.012)	0.495	(0.022)	0.919
Bottom layer	0.140		0.054		0.045	
	(0.147)	0.119	(0.101)	0.565	(0.048)	0.713
Middle layer	0.016		0.107		0.033	
	(0.090)	0.661	(0.129)	0.764	(0.028)	0.530
Top layer	0.748		0.602		0.402	
	(0.303)	0.055	(0.254)	0.382	(0.167)	0.095
Single individual households						
All	0.141		0.041		0.135	
	(0.150)	0.250	(0.059)	0.300	(0.093)	0.414
Stockholders	**0.838**		0.445		0.414	
	(0.369)	0.136	(0.259)	0.161	(0.182)	0.529
Nonstockholders	**0.009**		-0.008		0.028	
	(0.067)	0.630	(0.017)	0.487	(0.068)	0.572

Note: 12 monthly dummies included as explanatory variables and instruments. The estimations for all household sizes furthermore include $\Delta \ln$(family size) as explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: dividend price ratio. Instrument set 2: dividend price ratio, lagged log real value weighted NYSE return, lagged log real T-bill return. Instrument set 3: dividend price ratio, default premium, bond horizon premium.

Table 6: GMM estimation of log linearized Euler equation. Joint estimation using real value weighted NYSE return and real T-bill return. No representative agent assumption. CEX, 1982-1995. Semiannual data.

	Instrument set 1	Instrument set 2		Instrument set 3	
	$\widehat{\frac{1}{\gamma}}$ (std. error)	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=2, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=2, p-value
All household sizes					
All	0.092	0.062		0.071	
	(0.069)	(0.061)	0.159	(0.059)	0.521
Stockholders	**0.253**	0.227		0.183	
	(0.104)	(0.085)	0.426	(0.067)	0.184
Nonstockholders	**0.059**	0.021		0.049	
	(0.077)	(0.068)	0.144	(0.067)	0.673
Bottom layer	0.214	0.117		0.129	
	(0.168)	(0.121)	0.302	(0.099)	0.514
Middle layer	0.117	0.260		0.159	
	(0.277)	(0.202)	0.673	(0.245)	0.195
Top layer	0.552	0.521		0.326	
	(0.346)	(0.268)	0.251	(0.193)	0.037
Single individual households					
All	0.173	0.011		0.239	
	(0.138)	(0.101)	0.049	(0.107)	0.583
Stockholders	**0.601**	0.366		0.517	
	(0.339)	(0.217)	0.027	(0.202)	0.931
Nonstockholders	**0.062**	-0.070		0.133	
	(0.128)	(0.114)	0.133	(0.114)	0.325

Note: 12 monthly dummies included as explanatory variables and instruments. The estimations for all household sizes furthermore include $\Delta \ln(\text{family size})$ as explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: dividend price ratio. Instrument set 2: dividend price ratio, lagged log real value weighted NYSE return, lagged log real T-bill return. Instrument set 3: dividend price ratio, default premium, bond horizon premium.

Table 7: GMM estimation of log linearized Euler equation. Real value weighted NYSE return. No representative agent assumption. CEX, 1982-1995. Semiannual data.

	Instrument set 1	Instrument set 2		Instrument set 3	
	$\widehat{\frac{1}{\gamma}}$ (std. error)	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=2, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=2, p-value
All household sizes					
All	0.382	0.368		0.254	
	(0.245)	(0.237)	0.226	(0.231)	0.348
Stockholders	**0.967**	0.976		0.932	
	(0.440)	(0.420)	0.667	(0.442)	0.308
Nonstockholders	**0.240**	0.210		0.087	
	(0.281)	(0.268)	0.159	(0.244)	0.511
Bottom layer	0.803	0.695		0.773	
	(0.456)	(0.448)	0.388	(0.450)	0.861
Middle layer	0.444	0.598		1.012	
	(1.152)	(1.131)	0.385	(0.962)	0.294
Top layer	2.103	1.980		1.889	
	(0.692)	(0.684)	0.500	(0.680)	0.060
Single individual households					
All	0.659	0.538		0.225	
	(0.460)	(0.458)	0.047	(0.422)	0.091
Stockholders	**2.288**	2.179		1.800	
	(0.913)	(0.868)	0.076	(0.878)	0.263
Nonstockholders	**0.237**	0.166		-0.147	
	(0.479)	(0.474)	0.128	(0.423)	0.166

Note: 12 monthly dummies included as explanatory variables and instruments. The estimations for all household sizes furthermore include $\Delta \ln$(family size) as explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: dividend price ratio. Instrument set 2: dividend price ratio, lagged log real value weighted NYSE return, lagged log real T-bill return. Instrument set 3: dividend price ratio, default premium, bond horizon premium.

Table 8: GMM estimation of log linearized Euler equation. Real T-bill return. No representative agent assumption. CEX, 1982-1995. Semiannual data.

	Instrument set 1		Instrument set 2		Instrument set 3	
	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=1, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=5, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=5, p-value
All household sizes						
All	0.025		0.125		0.006	
	(0.079)	0.274	(0.059)	0.084	(0.027)	0.896
Stockholders	**0.225**		0.131		0.060	
	(0.181)	0.154	(0.079)	0.284	(0.050)	0.177
Nonstockholders	**0.014**		0.120		0.003	
	(0.067)	0.417	(0.059)	0.078	(0.022)	0.972
Bottom layer	-0.000		-0.000		0.005	
	(0.055)	0.713	(0.009)	0.992	(0.010)	0.895
Middle layer	0.463		0.092		0.100	
	(0.296)	0.235	(0.094)	0.597	(0.115)	0.576
Top layer	0.481		0.553		0.274	
	(0.316)	0.219	(0.231)	0.274	(0.171)	0.407
Single individual households						
All	0.002		0.197		0.009	
	(0.050)	0.827	(0.101)	0.045	(0.029)	0.879
Stockholders	**0.817**		0.609		0.118	
	(0.483)	0.118	(0.180)	0.100	(0.137)	0.151
Nonstockholders	**-0.000**		0.089		-0.000	
	(0.078)	0.777	(0.078)	0.118	(0.003)	0.998

Note: 12 monthly dummies included as explanatory variables and instruments. The estimations for all household sizes furthermore include $\Delta \ln(\text{family size})$ as explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: dividend price ratio. Instrument set 2: dividend price ratio, lagged log real value weighted NYSE return, lagged log real T-bill return. Instrument set 3: dividend price ratio, default premium, bond horizon premium.

Table 9: GMM estimation of log linearized Euler equation. Joint estimation using real value weighted NYSE return and real T-bill return. No representative agent assumption. CEX, 1982-1995. Quarterly data.

	Instrument set 1	Instrument set 2		Instrument set 3	
	$\widehat{\frac{1}{\gamma}}$	$\widehat{\frac{1}{\gamma}}$	Test of overid.,	$\widehat{\frac{1}{\gamma}}$	Test of overid.,
	(std. error)	(std. error)	df=2, p-value	(std. error)	df=2, p-value
All household sizes					
All	0.147	0.230		0.072	
	(0.144)	(0.080)	0.440	(0.098)	0.641
Stockholders	**0.256**	0.203		0.143	
	(0.172)	(0.087)	0.908	(0.109)	0.074
Nonstockholders	**0.119**	0.231		0.049	
	(0.153)	(0.084)	0.332	(0.104)	0.720
Bottom layer	0.088	-0.037		0.018	
	(0.253)	(0.139)	0.814	(0.182)	0.507
Middle layer	0.442	0.242		0.247	
	(0.271)	(0.141)	0.513	(0.194)	0.265
Top layer	0.548	0.656		0.374	
	(0.320)	(0.258)	0.766	(0.226)	0.210
Single individual households					
All	0.043	0.330		0.092	
	(0.196)	(0.114)	0.291	(0.135)	0.540
Stockholders	**0.603**	0.545		0.300	
	(0.433)	(0.183)	0.682	(0.221)	0.046
Nonstockholders	**-0.066**	0.251		0.019	
	(0.240)	(0.106)	0.192	(0.148)	0.894

Note: 12 monthly dummies included as explanatory variables and instruments. The estimations for all household sizes furthermore include $\Delta \ln(\text{family size})$ as explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: dividend price ratio. Instrument set 2: dividend price ratio, lagged log real value weighted NYSE return, lagged log real T-bill return. Instrument set 3: dividend price ratio, default premium, bond horizon premium.

Table 10: GMM estimation of log linearized Euler equation. Real value weighted NYSE return. No representative agent assumption. CEX, 1982-1995. Quarterly data.

	Instrument set 1	Instrument set 2		Instrument set 3	
	$\widehat{\frac{1}{\gamma}}$ (std. error)	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=2, p-value	$\widehat{\frac{1}{\gamma}}$ (std. error)	Test of overid., df=2, p-value
All household sizes					
All	0.558	1.059		0.551	
	(0.508)	(0.384)	0.040	(0.503)	0.789
Stockholders	**0.995**	0.966		1.326	
	(0.590)	(0.507)	0.119	(0.566)	0.097
Nonstockholders	**0.436**	1.039		0.397	
	(0.528)	(0.407)	0.031	(0.519)	0.887
Bottom layer	0.347	0.307		0.251	
	(0.952)	(0.916)	0.833	(0.935)	0.518
Middle layer	1.689	1.548		1.747	
	(1.041)	(0.987)	0.712	(1.025)	0.939
Top layer	2.072	2.292		2.217	
	(1.372)	(1.327)	0.069	(1.284)	0.320
Single individual households					
All	0.165	1.111		0.332	
	(0.749)	(0.687)	0.007	(0.709)	0.495
Stockholders	**2.283**	2.447		1.960	
	(1.093)	(1.062)	0.026	(1.025)	0.100
Nonstockholders	**-0.249**	0.932		-0.097	
	(0.879)	(0.773)	0.022	(0.816)	0.892

Note: 12 monthly dummies included as explanatory variables and instruments. The estimations for all household sizes furthermore include $\Delta \ln(\text{family size})$ as explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: dividend price ratio. Instrument set 2: dividend price ratio, lagged log real value weighted NYSE return, lagged log real T-bill return. Instrument set 3: dividend price ratio, default premium, bond horizon premium.

Table 11: GMM estimation of log linearized Euler equation. Real T-bill return. No representative agent assumption. CEX, 1982-1995. Quarterly data.