

# The Aggregate Demand for Treasury Debt

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## Abstract

Investors value the liquidity and safety of U.S. Treasury bonds. We document this by showing that changes in Treasury supply have large effects on a variety of yield spreads. As a result, Treasury yields are reduced by 73 basis points, on average over the period from 1926-2008. Both the liquidity and safety attributes of Treasuries are driving this phenomenon since changes in Treasury supply change both the equilibrium price of liquidity and the equilibrium price of safety. We document this by analyzing the spread between assets with different liquidity (but similarly safety) or with different safety (but similar liquidity). The low yield on Treasuries due to their extreme safety and liquidity suggests that Treasuries in important respects are similar to money.

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# 1 Introduction

Money, such as currency or checking accounts, offers a low rate of return relative to other assets. The reasons behind this phenomenon are well understood. Money is a medium of exchange for buying goods and services, of high liquidity, and of extremely high safety in the sense of offering absolute security of nominal repayment. Investors value these attributes of money and drive down the yield on money relative to other assets.

We argue that a similar phenomenon affects the prices of Treasury bonds. The high liquidity and safety of Treasuries drive down the yield on Treasuries relative to assets that do not to the same extent share these attributes. Figure 1 provides vivid evidence for this assertion. The figure graphs the yield spread between Aaa rated corporate bonds and Treasury securities against the US government Debt-to-GDP ratio (i.e. the ratio of the market value of publicly held US government debt to US GDP). The figure reflects a Treasury demand function, akin to a money demand function. When the supply of Treasuries is low, the value that investors assign to the liquidity and safety attributes offered by Treasuries (referred to below as the Treasury convenience yield) is high. As a result the yield on Treasuries is low relative to the yield on the Aaa corporate bonds which offer less liquidity and safety. The opposite applies when the supply of Treasuries is high. We present detailed econometric evidence of the relation reflected in Figure 1 using several alternative yield spread measures and controlling for corporate bond default risk.

We further show that it is the liquidity and safety attributes of Treasuries that drive investors' high valuation of Treasuries. We examine the yield spread between a pair of assets which are different only in terms of their liquidity, as well as the yield spread between a pair of assets which are different only in terms of their safety. Under the hypothesis that liquidity/safety are priced attributes, the yield spread between these pairs of assets should reflect the equilibrium price of liquidity/safety. We show that changes in Treasury supply affect each of these yield spreads. The results indicate that Treasuries offer liquidity and safety so that changes in the supply of Treasuries separately change the equilibrium prices of liquidity and safety.

We compute that the value investors have paid on average over our main sample from 1926 to 2008 for the liquidity and safety attributes of Treasuries is 73 basis points per year of which 46 basis points is for liquidity and 27 basis points for safety. Our findings imply that the government collects seignorage from the liquidity and safety attributes of Treasuries and we compute that the government saves interest costs of 0.25% of GDP per year because of investors' demand for Treasuries. This figure is comparable in magnitude to the traditional notion of seignorage, which stems from the public's willingness to hold fiat money at zero interest. We compute that the latter seignorage is also around a quarter of a percent of GDP per year. Our results also indicate that Treasury interest rates are not an appropriate benchmark for "riskless" rates. Cost of capital computations using the CAPM should use a higher riskless rate than the Treasury rate; a company with a beta of zero cannot raise funds at the Treasury rate. In addition, the equity premium measured relative to Treasury rates will partly be driven by the liquidity and safety of Treasuries.

### *Relation to Literature*

Longstaff, Mithal, and Neis (2005) use default risk as estimated from the price of credit default swaps to measure the component of the spread between corporate and Treasury yields that is due to default considerations. They find a large unexplained non-default component. This finding is in keeping with many papers in the corporate bond pricing literature. Compared to the prior literature, the novelty of our work is to offer more direct evidence of the existence of a non-default component by documenting that the amount of Treasuries outstanding is a key driver of the non-default component of the corporate bond spread. Furthermore, our paper shows that the non-default component is driven by the liquidity and safety attributes of Treasury bonds.

We are aware of only a few papers in the literature that have noted a relation between the supply of government debt and interest rate spreads. Cortes (2003) documents a relation between the US Debt/GDP ratio and swap spreads over a period from 1994 to 2003. Longstaff (2004) documents a relation between the supply of Treasury debt and the spread between Refcorp bonds and Treasury bonds over a period from 1991 to 2001. Relative to these papers we study a much longer sample, provide a theoretical basis to study the relation, use several approaches to rule out that the relation could be driven by time-varying default risk, and decompose the Treasury convenience yield into a liquidity and safety component.

There is also a literature that seeks to examine whether the relative supplies of long and short-term Treasury debt has an effect on the term structure of Treasury yields. Early work in this literature was motivated by the 1962-64 “operation twist,” where the government tried to flatten the term structure by shortening the average maturity of government debt (see for example Modigliani and Sutch, 1966). More recently, Reinhart and Sack (2000) show that the projected government deficit is positively related to the slope of the Treasury yield curve, suggesting that this is evidence of a supply effect. More systematic evidence of a relative supply effect is provided in Greenwood and Vayanos (2010), who examine data from 1952 to 2005 and show that relative supply of long and short Treasuries is related to the slope of the yield curve. Krishnamurthy and Vissing-Jorgensen (forthcoming) study the Federal Reserve quantitative easing policies in 2008 to 2011 whereby the supply of long-term Treasuries was reduced. Using an event-study methodology, they show that long-term Treasury yields fell relative to short-term yields and attribute this to demand for extremely safe assets of specific maturities. These papers suggest that the demand for Treasury attributes varies by maturity, and are complementary to our study.

In macroeconomics, there is a large literature exploring the Ricardian equivalence proposition (Barro, 1974), that the financing choices of the government used to fund a given stream of government expenditures is irrelevant for equilibrium quantities and prices. One implication of the Ricardian equivalence proposition is that the size of government debt has no causal effect on interest rates. Despite a large amount of research devoted to studying this topic, there is yet no clear consensus on the effects of debt on interest rates (see, for

example, the survey by Elmendorf and Mankiw (1999)). It is important to note that the effect we identify is on the spread between government interest rates and corporate interest rates. It is possible that Ricardian equivalence fails in a way that government debt has an effect on the general level of interest rates, both corporate and government. Since we focus on spreads, we are unable to isolate such an effect. On the other hand, from an empirical standpoint, the advantage of focusing on spreads rather than the level of interest rates is that the spread measure is unaffected by other shocks (such as changes in expected inflation) that affect the level of interest rates and complicate inference. We also bypass endogeneity issues stemming from government behavior, since it is unlikely that the government chooses debt levels based on the corporate bond spread.

This paper is laid out as follows. The next section lays out a theoretical framework to relate the demand for the attributes offered by Treasuries to the price of Treasuries relative to other assets. The section develops a series of predictions of the theory. We test each of the theoretical predictions in Section 3. The paper also includes an appendix providing details on the data construction as well as mathematical derivations.

## 2 Theoretical Framework

We articulate our theory by modifying a standard representative agent asset-pricing model to include a term whereby agents derive utility directly from holdings of a “convenience” asset. The modification is along the lines of Sidrauski (1967) who considers a model where agents derive utility from their holdings of money. The representative agent maximizes,

$$E \sum_{t=1}^{\infty} \beta^t u(C_t). \quad (1)$$

Suppose that  $C_t$  is the sum of an endowment of  $c_t$  plus “convenience” benefits:

$$C_t = c_t + \nu(\theta_t^A, GDP_t; \xi_t). \quad (2)$$

The benefits are a function of the holdings of convenience assets,  $\theta_t^A$ . One example which we elaborate on below is that the function  $\nu(\cdot)$  captures the notion that holding more Treasury securities reduce costs that would otherwise be incurred by transacting in a less liquid security such as corporate bonds.<sup>1</sup> The argument  $\theta_t^A$  is the market value of the agent’s real holdings of convenience assets and include both Treasuries,  $\theta_t^T$ , and any other private sector assets,  $\theta_t^P$ , that provide services similar to Treasuries:

$$\theta_t^A = \theta_t^T + k^P \theta_t^P. \quad (3)$$

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<sup>1</sup>To be more precise, we can define  $C_t = c_t - cost(\theta_t^A, GDP_t; \xi_t)$ , where the function  $cost(\cdot)$  reflects costs that will be incurred by holding less liquid securities. By holding more Treasuries, these costs are reduced. This is just a renormalization relative to our defining a benefit function  $\nu(\cdot)$  that is increasing in  $\theta_t^A$ . The important aspect of the modeling is that  $dC_t/d\theta_t^A > 0$ .

The constant  $k^P$  measures the convenience services provided by the private sector assets relative to Treasuries. The term  $\xi_t$  in the convenience function is a preference shock that affects how much utility is derived from convenience assets. An example of such a shock is a “flight-to-quality” as during a financial crisis, where investors may temporarily increase their valuation of convenient assets such as Treasuries.

We assume that the convenience function is homogeneous of degree one in  $GDP_t$  and  $\theta_t^A$ . This captures the idea that liquidity benefits double if both income and convenience assets double. Thus define,

$$v\left(\frac{\theta_t^A}{GDP_t}; \xi_t\right) GDP_t \equiv \nu(\theta_t^A, GDP_t; \xi_t). \quad (4)$$

We assume that the convenience function is increasing in  $\frac{\theta_t^A}{GDP_t}$ , but the marginal convenience benefit is decreasing in  $\frac{\theta_t^A}{GDP_t}$  and has the property  $\lim_{\frac{\theta_t^A}{GDP_t} \rightarrow \infty} v'(\frac{\theta_t^A}{GDP_t}; \xi_t) = 0$ . That is, holding more convenience assets reduces the marginal value of an extra unit of convenient assets. Furthermore, this marginal value approaches zero if the agent is holding a large amount of convenient assets.

Let us next consider what underlies our reduced form convenience function  $v(\cdot)$ . We argue that Treasuries are valued for their liquidity and safety. Papers such as Vayanos and Vila (1999) and Rocheteau (2009) show how the superior liquidity of an asset will lead investors to pay a higher price for that asset. Under these theories, an increase in the holding of liquid assets will lower the marginal liquidity service provided by any liquid asset. That is, our earlier assumption that the marginal convenience,  $v'(\cdot)$ , is decreasing in  $\frac{\theta_t^A}{GDP_t}$  is a natural outcome of these models. We refer to these theories as describing a liquidity attribute.

A second benefit of Treasuries is that they are widely believed to provide absolute certainty of nominal repayment. Under some theories, this safety attribute can drive a convenience yield that is declining in the supply of safe assets.

Consider short-term Treasuries, such as three or six month maturity bills, which carry negligible price risk. Suppose that some investors face costs of understanding investment in risky assets, as in the literature on limited participation of investors in the stock market (Vissing-Jorgensen, 2003). These investors will have a unique demand for riskless assets, driving up the price of riskless assets. In addition, in many limited participation models expanding the stock of riskless assets reduces risk premia and raises riskless rates (see Gomes and Michaelides, 2008). Another explanation for safety demand stems from the use of Treasuries as collateral in many financial transactions. Gorton (2010) notes that there is a substantial demand for collateral for purposes of mitigating counterparty risk in derivatives and settlement systems. The collateral in these transactions is required to be extremely safe, thus also driving the demand for a safety attribute. Bansal and Coleman (1996) argue that commercial banks and money market funds use Treasuries to back checkable deposits. Treasuries thus inherit some of the medium-of-exchange convenience of money, lowering the yield on Treasuries. In this explanation, it is again the safety of Treasuries that makes them good backing for checking accounts. We will offer empirical evidence that the safety attribute of Treasuries is one of the

drivers of the convenience yield. However, we will not distinguish further on whether the underlying driver of safety demand is due to limited participation, collateral, or the check-backing explanations.

The safety explanation for low Treasury yields is distinct from that suggested by any of the standard representative agent model explanations of high risk premia in asset markets. This literature has demonstrated how altering the preferences of a representative agent to feature extreme forms of risk aversion can produce low riskless interest rates and high risk premia. Thus, in the representative agent model there will be negative relation between the price of a bond and its default risk. However, the quantity of convenient assets is unrelated to asset prices in the representative agent model. A way to think about how safety demand works is that the relation between price and default risk is very steep near zero-default-risk, over and above the negative relation implied by the representative agent model. Furthermore, the slope of this curve near zero-default-risk decreases in Treasury supply. This latter prediction generates a negative relation between the corporate-Treasury bond spread and Treasury supply (at a given level of corporate bond default risk) and is how to distinguish the safety explanation from a standard risk-based explanation (Figure 1 in Krishnamurthy and Vissing-Jorgensen (forthcoming) illustrates this relation).

The safety attribute may also apply to long-term Treasuries, such as 30-year bonds, which carry significant price risk because of interest rate volatility. Here, the limited participation, collateral usage, or check-backing explanations are unlikely to be relevant. Instead, Greenwood and Vayanos (2010) suggest that investors such as defined benefit pension funds have a special demand for certain long-term payoffs to back long-term nominal obligations. The same motive may apply to insurance companies that write long-term policies. Chalmers (1998) describes how long-term Treasury bonds are posted as collateral by municipalities to secure their own long-term borrowings. Broadly, this explanation is similar to the preferred habitat hypothesis of the term structure of interest rates (Modigliani and Sutch, 1966), under which investors are hypothesized to prefer certain maturities of bonds, but applied only (or to a larger extent) to extremely safe bonds. We refer to these theories as a describing a long-term safety attribute.

We can represent these different theoretical rationales for convenience in our specification of  $v(\cdot)$ . Denote  $\theta_t^{T,long}$  as the stock of long-term Treasury bonds, and  $\theta_t^{T,short}$  as the stock of short-term Treasuries ( $\theta_t^T = \theta_t^{T,long} + \theta_t^{T,short}$ ). Also define  $\theta_t^{P,liq}$  as the stock of non-Treasury liquid assets,  $\theta_t^{P,short-safe}$  as the stock of non-Treasury short-term safe assets, and  $\theta_t^{P,long-safe}$  as the stock of non-Treasury long-term safe assets. Suppose that total convenience on short-term Treasuries can be written as the sum of two convenience components:

$$v_{T,short}(\cdot) = v_{liq} \left( \frac{\theta_t^T + k^{liq} \theta_t^{P,liq}}{GDP_t}; \zeta_t^{liq} \right) + v_{short-safe} \left( \frac{\theta_t^{T,short} + k^{short-safe} \theta_t^{P,short-safe}}{GDP_t}; \zeta_t^{short-safe} \right). \quad (5)$$

Similarly, we can specify the convenience on long-term Treasuries as

$$v_{T, long}(\cdot) = v_{liq} \left( \frac{\theta_t^T + k^{liq} \theta_t^{P, liq}}{GDP_t}; \xi_t^{liq} \right) + v_{long-safe} \left( \theta_t^{T, long} + k^{long-safe} \theta_t^{P, long-safe} GDP_t; \xi_t^{long-safe} \right). \quad (6)$$

The constants,  $k^{liq}$ ,  $k^{short-safe}$  and  $k^{long-safe}$ , measure the convenience that the private sector assets offer relative to Treasuries.

Our specification emphasizes that the safety attributes may differ across short and long-term assets and thus lead to a difference in convenience value in long-term assets relative to short-term assets. In contrast, our specification assumes that both long and short-term Treasuries offer equal liquidity services. The empirical literature has documented the existence of significant liquidity premia on both long-term Treasuries (Krishnamurthy, 2002, Longstaff, 2004) and short-term Treasuries (Amihud and Mendelsen, 1991). We are making the somewhat stronger assumption that long and short-term Treasuries are equally liquid.

## 2.1 Spreads and Supply

We derive pricing expressions for short and long-term bonds based on these different specifications of convenience. As we describe below, decomposing the convenience in the manner above also yields empirical tests of the existence of priced safety and liquidity attributes. Before describing these tests, let us turn to asset pricing. We initially derive predictions of the convenience yield theory which do not distinguish between the liquidity and safety motives. We then turn to predictions implied by each of these separate motives. In terms of the framework above, our initial set of predictions implicitly assume that both  $v_{T, long}$  and  $v_{T, short}$  are only functions of  $\theta_t^T$  (as opposed to functions of both total Treasury supply and short or long-term Treasury supply). This will be the case if long and short Treasury supply moves in parallel (and if the demand shocks are perfectly correlated), or if only a liquidity motive is present. We relax this assumption later.

Denote the price level at date  $t$  as  $Q_t$ . If the agent buys a zero coupon nominal Treasury bond for a nominal price  $P_t^T$ , his real holdings  $\theta_t^A$  rises by  $\frac{P_t^T}{Q_t}$ .<sup>2</sup> The first order condition for Treasury bond holdings is then,

$$-\frac{P_t^T}{Q_t} u'(C_t) + \beta E_t \left[ \frac{P_{t+1}^T}{Q_{t+1}} u'(C_{t+1}) \right] + \frac{P_t^T}{Q_t} v'(\theta_t^A / GDP_t; \xi_t) u'(C_t) = 0 \quad (7)$$

Define the pricing kernel for nominal payoffs as,

$$M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_t}{Q_{t+1}}, \quad (8)$$

so that,

$$P_t^T = E_t[M_{t+1} P_{t+1}^T] + P_t^T v'(\theta_t^A / GDP_t; \xi_t) \Rightarrow P_t^T = \frac{E_t[M_{t+1} P_{t+1}^T]}{1 - v'(\theta_t^A / GDP_t; \xi_t)}. \quad (9)$$

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<sup>2</sup>We deriving pricing expressions for zero-coupon Treasury and corporate bonds. In our empirical work, we examine coupon bonds and assume that the impact of Treasury supply on coupon bond spreads is qualitatively similar.

This expression indicates that a positive marginal value of convenience,  $v'(\cdot)$ , raises the price of Treasuries,  $P_t^T$ .

Let us next derive pricing expressions for a zero-coupon corporate bond that offers no convenience services. Suppose that the corporate bond may default next period with probability  $\lambda_t$  and in default pays  $1 - L_{t+1}$ , where  $L_{t+1}$  measures the amount of losses suffered in default. If the bond does not default, it is worth  $P_{t+1}^C$ . Then, since the bond offers no convenience, its price satisfies

$$P_t^C = \lambda_t E_t[M_{t+1}(1 - L_{t+1})|\text{Default}] + (1 - \lambda_t) E_t[M_{t+1}P_{t+1}^C|\text{No Default}] \quad (10)$$

In our empirical work we estimate the convenience demand  $v'(\cdot)$  by relating  $\theta_t^T$  to two different measures of the price difference between  $P_t^C$  and  $P_t^T$ , short-maturity yield spreads between corporate and Treasury bonds and long-maturity yield spreads. We now derive expressions for each of these price measures and compare them. For simplicity, we focus our derivations on zero-coupon bonds and continuously compounded yields.

Consider first the case of one period bonds. For such bonds,  $P_{t+1}^C = P_{t+1}^T = 1$ . Then,

$$e^{-i_t^T} = P_t^T = \frac{E_t[M_{t+1}P_{t+1}^T]}{1 - v'(\theta_t^A/GDP_t; \xi_t)} \approx e^{v'(\theta_t^A/GDP_t; \xi_t)} E_t[M_{t+1}] \quad (11)$$

For the corporate bond, define  $\tilde{L}_{t+1}$  as a random variable that is equal to zero if there is no default, and equal to  $L_{t+1}$  if there is default. Then,

$$e^{-i_t^C} = P_t^C = E_t[M_{t+1}] - E_t[M_{t+1}] E_t[\tilde{L}_{t+1}] - cov_t[M_{t+1}, \tilde{L}_{t+1}] \approx e^{-\lambda_t E_t[L_{t+1}] - cov_t[M_{t+1}, \tilde{L}_{t+1}]/E_t[M_{t+1}]} E_t[M_{t+1}]$$

We thus have the following prediction:

**Prediction 1** (*Impact of Treasury supply on short-term spreads*)

*The one-period yield spread between corporate and Treasury bonds is related to the stock of Treasuries as follows:*

$$S_{t,1} \equiv i_t^C - i_t^T = v' \left( \frac{\theta_t^T + k^P \theta_t^P}{GDP_t}; \xi_t \right) + \lambda_t E_t[L_{t+1}] + cov_t[M_{t+1}, \tilde{L}_{t+1}]/E_t[M_{t+1}]. \quad (12)$$

*The yield spread reflects the sum of three terms: the convenience yield on Treasuries, the expected default rate on the corporate bond, and a risk premium associated with the covariance between default and the pricing kernel. Assuming that  $v''(\cdot) < 0$ ,  $S_{t,1}$  is declining in  $\theta_t^T + k^P \theta_t^P$ . Consider next the relationship between  $S_{t,1}$  and  $\theta_t^T$ . Project  $\theta_t^P$  linearly on  $\theta_t^T$ , so that  $\theta_t^P = a_0 + a_1 \theta_t^T + \text{error}$ , where the error is uncorrelated with  $\theta_t^T$ . Then  $\theta_t^T + k^P \theta_t^P = k^P a_0 + (1 + k^P a_1) \theta_t^T + k^P \text{error}$ . If  $1 + k^P a_1 > 0$ , then  $S_{t,1}$  is declining in  $\theta_t^T$ . The latter condition will be satisfied if  $a_1 > -1/k^P$ , i.e. unless the private sector reduces its supply of substitutes by more (in effective terms,  $k^P \theta_t^P$ ) than the increase in the Treasury supply.*

We verify the prediction of the convenience model that an increase in  $\theta_t^T$  causes the yield spread to fall. Our regressions of the yield spread on  $\theta_t^T$  recover  $v'(\cdot) \left(1 + k^P \frac{\partial \theta_t^P}{\partial \theta_t^T}\right)$  rather than  $v'(\cdot)$  because of private sector reaction to changes in Treasury supply. In order to recover  $v'(\cdot)$ , we further need knowledge of  $k^P$  and  $\frac{\partial \theta_t^P}{\partial \theta_t^T}$ . We do not explore that in this paper because for most questions of interest, it is more important to know  $v'(\cdot) \left(1 + k^P \frac{\partial \theta_t^P}{\partial \theta_t^T}\right)$  rather than  $v'(\cdot)$ .

Note that it is possible that Treasury supply or private sector supply reacts accommodating to demand shocks ( $\xi_t$ ) or to increases in corporate default risk. This will bias the relation between spreads and Treasury supply towards finding a positive relation, the opposite of the causal negative relation from Treasury supply to spreads. However, we view it as unlikely that overall Treasury supply reacts substantially to demand shocks or changes in the risk of corporate bonds. The more plausible reaction involves the private sector supply or the government's supply of particular maturities.

Let us next consider multi-period bonds. Define the  $\tau$  period yields on corporate and Treasury bonds as,

$$i_{t,\tau}^T = -\frac{1}{\tau} \ln P_t^T \quad \text{and} \quad i_{t,\tau}^C = -\frac{1}{\tau} \ln P_t^C. \quad (13)$$

The spread between these bonds is  $S_{t,\tau} = i_{t,\tau}^C - i_{t,\tau}^T$ .

Consider again the derivation for corporate bonds. Our derivation for multi-period bonds closely follows Duffie and Singleton (1999), reflecting the standard practice in the corporate bond pricing literature. Suppose that the event of default or no-default is non-systematic (i.e. uncorrelated with  $M_{t+1}$ ). Then, we can drop the conditioning on default/no-default and rewrite (10) as,

$$P_t^C = E_t[M_{t+1} (\lambda_t(1 - L_{t+1}) + (1 - \lambda_t)P_{t+1}^C)] \quad (14)$$

Assume that we can write the expected present value of the payment in default as,

$$E_t[M_{t+1}(1 - L_{t+1})] = E_t[M_{t+1}P_{t+1}^C](1 - D_t) \quad (15)$$

for a suitable process  $D_t$ .<sup>3</sup> This is Duffie and Singleton's "recovery of market value" assumption (RMV). Then,

$$P_t^C = (\lambda_t(1 - D_t) + (1 - \lambda_t))E_t[M_{t+1}P_{t+1}^C] \approx e^{-\lambda_t D_t} E_t[M_{t+1}P_{t+1}^C]. \quad (16)$$

Note that the term  $P_{t+1}^C$  is a function of  $D_{t+1}$  and  $\lambda_{t+1}$ , which embody changes in future default expectations such as downgrades. For high-grade corporate bonds, which are the focus our study, almost all of the default-related risk is of this form rather than in terms of the bonds defaulting between  $t$  and  $t + 1$ . In our setup, the latter default-related risk may be correlated with  $M_{t+1}$  and carry a risk premium. Thus, our restriction

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<sup>3</sup>Note that in expression (15), the left-hand side expectation is conditioning on default, while the right-hand side expectation is conditioning on no-default. However, given the assumption that the default-event is non-systematic, we can drop the conditioning.

that the default event in the next period is non-systematic is not a substantively important restriction, but does help to simplify our pricing expressions.

**Prediction 2** (*Impact of Treasury supply on long-term spreads*)

The yield spread between  $\tau$ -period corporate and Treasury bonds is related to the stock of Treasuries as follows,

$$S_{t,\tau} = \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t[v'(\theta_j^A/GDP_j; \xi_j)] + \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t[\lambda_j D_j] - \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} cov_t(m_{j+1}, \tilde{R}_{j+1}) \quad (17)$$

where,  $m_{j+1} = \log M_{j+1}$  ( $= \log \beta \frac{u'(C_{j+1})}{u'(C_j)} \frac{Q_j}{Q_{j+1}}$ ), is the log pricing kernel, and  $\tilde{R}_{j+1}$  is the one-period excess return of corporate bonds over Treasury bonds. As long as  $\theta_j^A$  increases with  $\theta_j^T$ , increases in Treasury supply lower the spread,  $S_{t,\tau}$ .

The derivation of this spread expression is in the Appendix. The derivation assumes that all relevant variables, including  $m_t$  and changes in the corporate and Treasury bond yields, are normally distributed.

The spread reflects three terms: (1) the expected average convenience benefit over the next  $\tau$  periods; (2) the expected average amount of default; and (3) a risk premium that depends on the covariance between the pricing kernel and the excess return on corporate over Treasury bonds.

Let us compare the short-term and long-term spread expressions. Note that shocks to both  $\theta_t^A/GDP_t$  and  $\xi_t$  have an impact on the short-term spread. The impact of these shocks on the long-term spread depends on the persistence of the shocks. In the data, a flight to quality (liquidity and safety) shock ( $\xi_t$ ) is likely to be short-lived, and should primarily affect short-term spreads. The debt-to-GDP ratio is quite persistent so that shocks to  $\theta_t^A/GDP_t$  will have a significant impact on both short and long-term spreads. This logic tells us that the convenience yield as embodied in the long-term spread is primarily driven by  $\theta_t^A/GDP_t$ , while variability in the short-term spread will partly be driven by  $\xi_t$ .<sup>4</sup> This is an advantage of using the long-term spread and data on  $\theta_t^A/GDP_t$  to estimate convenience yields. On the other hand, the corporate bonds we use to construct the short-term spread are close to default free. The corporate bonds used in the long-term

<sup>4</sup>Here is a simple case to formalize these points. Suppose that the convenience yield function is

$$v'(\theta_t^A/GDP_t; \xi_t) = b_0 + b_1 \log(\theta_t^A/GDP_t) + \log \xi_t.$$

Here, we have written the demand shock,  $\xi_t$ , to enter additively in the convenience yield, and assumed a log convenience yield function, as we do in most of our empirical tests. The short-term spread equally reflects a supply term  $b_1 \log(\theta_t^A/GDP_t)$  and a demand term  $b_1 \log \xi_t$ . Suppose that  $\log(\theta_t^A/GDP_t)$  is AR(1) with coefficient  $\rho$  and that  $\log \xi_t$  is i.i.d. with mean zero. Then it is easy to verify that the convenience yield component of the long-term spread is,

$$\sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t[v'(\theta_j^A/GDP_j; \xi_j)] = b_0 + b_1 \log(\theta_t^A/GDP_t) (1 + \rho + \rho^2 + \dots + \rho^{\tau-1}) \frac{1}{\tau} + \frac{\log \xi_t}{\tau}.$$

If we take  $\tau = 20$  years (the maturity for the long spread in our study), and  $\rho = 0.95$  (consistent with data on the Debt-to-GDP ratio), then the supply coefficient  $(1 + \rho + \rho^2 + \dots + \rho^{\tau-1}) \frac{1}{\tau}$  is 0.64, while the demand coefficient  $1/\tau$  is 0.05.

spread carry greater default risk. Thus, the results based on the long-term spread are more sensitive to precise controls for default risk.

## 2.2 Liquidity and Safety Attributes

We now reintroduce the different liquidity and safety attributes of Treasuries and consider how one can test if these attributes are priced. Following equations (5) and (6), long and short-term assets should be expected to have different convenience yields. To be precise, let us reconsider the short and long-term spread expression. The short-term spread reflects liquidity and short-term safety:

$$S_{t,1} = v'_{liq} \left( \frac{\theta_t^T + k^{liq} \theta_t^{P,liq}}{GDP_t}; \xi_t^{liq} \right) + v'_{short-safe} \left( \frac{\theta_t^{T,short} + k^{short-safe} \theta_t^{P,short-safe}}{GDP_t}; \xi_t^{short-safe} \right) + \lambda_t E_t[L_{t+1}] + cov_t[M_{t+1}, \tilde{L}_{t+1}] / E_t[M_{t+1}]. \quad (18)$$

The long-term spread (the spread for large  $\tau$ ) reflects the expected liquidity and long-term safety attributes over the term of the bond:

$$S_{t,\tau} = \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t \left[ v'_{liq} \left( \frac{\theta_j^T + k^{liq} \theta_j^{P,liq}}{GDP_j}; \xi_j^{P,liq} \right) + v'_{long-safe} \left( \frac{\theta_j^{T,long} + k^{long-safe} \theta_j^{P,long-safe}}{GDP_j}; \xi_j^{long-safe} \right) \right] + \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t[\lambda_j D_j] - \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} cov_t(m_{j+1}, \tilde{R}_{j+1}) \quad (19)$$

We consider pairs of assets which have either similar liquidity and different safety or similar safety and different liquidity. The yield spread between these assets only reflects the price of liquidity or the price of safety. We can then test whether the price of the attribute captured by the yield spread changes with the relevant supply of Treasuries.

Consider first the spread between  $P2$  and  $P1$  rated commercial paper. The former has a higher default risk than the latter. The assets are short-term but similarly illiquid as we document in the next section. Thus the  $P2 - P1$  spread purely reflects the value of short-term safety convenience.

### **Prediction 3** (*Impact of Treasury supply on price of short-term safety*)

Consider that  $P2$  and  $P1$  rated commercial paper are equally liquid (i.e.  $k_{P2}^{liq} = k_{P1}^{liq}$ ), but that  $k_{P1}^{short-safe} > k_{P2}^{short-safe}$ . Then, the spread between these bond yields is related to the stock of short-term Treasuries as follows:

$$S_{t,1}^{P2-P1} = (k_{P1}^{short-safe} - k_{P2}^{short-safe}) v'_{short-safe} \left( \frac{\theta_t^{T,short} + k^{short-safe} \theta_t^{P,short-safe}}{GDP_t}; \xi_t^{short-safe} \right) + \lambda_{t,P2} E_t[L_{t+1,P2}] - \lambda_{t,P1} E_t[L_{t+1,P1}] + cov_t[M_{t+1}, \tilde{L}_{t+1,P2} - \tilde{L}_{t+1,P1}] / E_t[M_{t+1}] \quad (20)$$

If short-term safety is a priced attribute, then increases in the supply of short-term Treasuries will lower  $S_{t,1}^{P2-P1}$  (as long as  $\theta_t^{T,short} + k^{short-safe} \theta_t^{P,short-safe}$  increases in  $\theta_t^{T,short}$ ).

In terms of the estimation, the P2-P1 spread is directly a function of the supply of short-term convenient assets. There is extensive evidence that both the private sector and the Treasury actively manages the maturity structure of debt. To get around any endogeneity issues stemming from this behavior, we use instrumental-variables regressions, using  $\theta_t^T$  as an instrument for  $\theta_t^{T,short}$  (a similar comment applies for testing Prediction 4 below).

Next consider a similar prediction but based on the spread between two long-term corporate bonds:

**Prediction 4** (*Impact of Treasury supply on price of long-term safety*)

Take two long-term corporate bonds, an Aaa-rated bond and a Baa-rated bond. Consider that these bonds are equally liquid (i.e.  $k_{Aaa}^{liq} = k_{Baa}^{liq}$ ), but that  $k_{Aaa}^{long-safe} > k_{Baa}^{long-safe}$ . Then, the spread between these bond yields is related to the stock of long-term Treasuries as follows:

$$S_{t,\tau}^{Baa-Aaa} = (k_{Aaa}^{long-safe} - k_{Baa}^{long-safe}) \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t \left[ v'_{long-safe} \left( \frac{\theta_j^{T,long} + k^{long-safe} \theta_j^{P,long-safe}}{GDP_j}; \xi_j^{long-safe} \right) \right] + \sum_{j=t}^{t+\tau-1} E_t [\lambda_j^{Baa} D_j^{Baa} - \lambda_j^{Aaa} D_j^{Aaa}] - \sum_{j=t}^{t+\tau-1} \frac{1}{\tau} cov_t(m_{j+1}, \tilde{R}_{j+1}^{Baa-Aaa}). \quad (21)$$

If long-term safety is a priced attribute, then increases in the supply of long-term Treasuries will lower  $S_{t,\tau}^{Baa-Aaa}$  (as long as  $\theta_t^{T,long} + k^{long-safe} \theta_t^{P,long-safe}$  increases in  $\theta_t^{T,long}$ ).

A similar comparison, but now getting at the liquidity attribute is through the following.

**Prediction 5** (*Impact of Treasury supply on price of liquidity*)

Consider a one-period Treasury bond which offers one unit of liquidity and is default free. Consider also an FDIC insured bank deposit which is default free but only offers  $k^{liq} < 1$  units of liquidity. Then, the one-period spread between these bonds is related to the stock of Treasuries as follows:

$$S_{t,1}^{FDIC} = i_t^{FDIC} - i_t^T = (1 - k^{liq}) v'_{liq} \left( \frac{\theta_t^T + k^{liq} \theta_t^{P,liq}}{GDP_t}; \xi_t^{liq} \right). \quad (22)$$

If liquidity is a priced attribute, then increases in the supply of Treasuries will lower  $S_{t,1}^{FDIC}$  (as long as  $\theta_t^T + k^{liq} \theta_t^{P,liq}$  increases in  $\theta_t^T$ ).

### 3 Evidence

Details on the data construction for each table as well as the sources for all variables used in our regressions are in the data appendix. The regressions all use data at an annual frequency and for as long a period as is feasible given the variables included in the regression.

### 3.1 Impact of Treasury Supply on Spreads

Predictions (1) and (2) state that under the convenience yield hypothesis increases in Treasury supply should reduce short spreads and long spreads. Table I presents regressions confirming these predictions.

The key explanatory variable in the regressions reported in the tables is the log of  $Debt/GDP$ , where  $Debt/GDP$  is the market value of the outstanding stock of US Treasuries divided by US GDP. The measure of government debt corresponds to what is referred to as publicly held debt. It includes debt held by the Federal Reserve, but excludes debt held by other parts of the government such as the Social Security Trust Fund. Our results do not change appreciably if we exclude the holdings of the Federal Reserve. The debt measure is as of the end of the government's fiscal year, i.e. end of June up to and including 1976 and end of September from 1977 on. Statistics on government debt are typically reported in face value terms, while we are interested in the market value of debt.<sup>5</sup> The appendix details how we adjust face values to come up with the market value of debt.

The theoretical measure of convenience yield (convenience benefit) is  $v'(\theta_t^A/GDP_t)$ , where  $\theta_t^A$  includes both Treasury debt and private sector convenient assets. Predictions (1) and (2) are that the convenience yield is declining in  $\theta_t^A$ . As we have noted, as long as private sector convenience asset supplies do not change more than one-for-one (in the opposite direction) in response to changes in Treasury debt, we can verify predictions (1) and (2) by examining the impact of changes in Treasury debt on price measures. The regression coefficient on Treasury debt should be interpreted as reflecting the effect, net of private sector supply response, of a change in Treasury debt on the Treasury convenience yield.

#### 3.1.1 Long-term spreads

In Table I the Treasury debt variable is the logarithm of the Debt-to-GDP ratio, while the dependent variable in each regression is a bond yield spread measured in percentage terms. Panel A and B present results for long-term spreads, namely the spread between long-term Aaa-rated corporate bonds and long-term Treasuries (Panel A) and the spread between long-term Baa-rated corporate bonds and long-term Treasuries (Panel B). We use the log functional form because it provides a good fit and requires estimation of only one parameter. A drawback with the log function is that it does not asymptote to zero as Debt-to-GDP rises. We estimate an alternative functional form with the asymptote property in Section 3.3.

The regressions in Table I are estimated using ordinary least squares (OLS). We report  $t$ -statistics adjusting the standard errors assuming an AR(1) error structure. The AR(1) structure is motivated by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The first-order autocorrelations are included in the table. Serial correlation is pronounced

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<sup>5</sup>Doepke and Schneider (2006) encounter the same issue in studying how inflation affects the market value of debt. They offer a procedure to compute market values of debt.

only in the long-term spread regressions of Panel A. For consistency across columns, we use the AR(1) adjustment in all columns though it makes little difference except in Panel A where t-statistics based on “standard” OLS standard errors (assuming an i.i.d. error term) would be substantially larger than those presented. An alternative to OLS estimation with an AR(1) standard error adjustment would be to use GLS estimation. However, we suspect that  $\log(\text{Debt}/\text{GDP})$  is not a perfect measure of the convenience state variable in the long-term spread regressions,  $\sum_{j=t}^{t+\tau-1} \frac{1}{\tau} E_t[v'(\theta_j^A/\text{GDP}_j; \xi_j)]$ . It is likely that the private sector expectations of this sum involve variables other than the current Debt-to-GDP ratio (notably information about the likely development of the government budget). If so, then Debt-to-GDP is measured with error implying downward bias in the impact of Treasury supply on spreads (under standard assumptions about the measurement error). GLS estimation would be more affected by the measurement error issue. To see this, suppose the error autocorrelation was close to one. Then GLS would effectively transform the data to run a first difference regression. While in levels the variance of  $\log(\text{Debt}/\text{GDP})$  is likely to be large relative to that of the measurement error, in first differences this will not be the case since the high persistence of  $\log(\text{Debt}/\text{GDP})$  means that the volatility of its first difference is low (a formal Monte-Carlo study of this issue is available upon request). These considerations lead us to present OLS regressions with standard errors adjusted for the serial correlation. We emphasize that our OLS regression coefficients are likely to be conservative, understating the impact of Treasury supply on convenience yields.<sup>6</sup>

The coefficient of  $-0.746$  in column (1) of Table I implies that a decrease of one standard deviation in Debt-to-GDP from its mean value of 0.439 to 0.243 increases the convenience yield component of the Aaa-Treasury spread by as much as 0.44% (44 basis points).

Default risk is an important component of the bond spreads. In column (2) and (3) we show that the impact of Treasury supply on the Aaa-Treasury spread is robust to including default controls. Moody’s Investors Service (2005) estimates, based on data from 1920 to 2004, that the default rate on Aaa rated bonds over a 10 year period is around 1%, while for Baa bonds this default rate is 8%. In column (2), we control for default risk using a default measure computed by Moody’s Analytics, who are the current industry standard in calculating default probabilities for corporate bond pricing. Their computation is based on Merton (1974) which treats the debt of a firm as a riskless asset minus a put option on the firm’s assets. Using capital structure information and Merton’s option decomposition of capital structure, they infer the firm’s asset value and asset value volatility. This information allows them to compute the distance to default

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<sup>6</sup>A standard solution to the bias problem would be an IV approach. If the error term was serially uncorrelated one could use lagged values of  $\log(\text{Debt}/\text{GDP})$  as instrument for  $\log(\text{Debt}/\text{GDP})$ . With serially correlated errors one should lag the instrument far enough to avoid correlation between the instrument and the error term. We find that the coefficient on  $\log(\text{Debt}/\text{GDP})$  is larger in IV estimations and keeps increasing as the instrument is lagged further. This suggesting that the measurement error concern is relevant, but that it is unclear how far to lag the instrument and therefore more conservative to report only OLS results.

on debt (i.e. moneyness of the put option). Using historical default information in a non-linear regression, they estimate how distance-to-default translates into default probabilities. We use the median EDF reported by Moody’s Analytics for large firms (defined as firms with book value of assets > \$300 million in current dollars). The EDF measure is available from 1969-2008. The results in column (2) shows that the EDF default measure is informative. Crucially, the coefficient on  $\log(Debt/GDP)$  remains highly significant and of roughly the same magnitude as in other specifications.<sup>7,8</sup> The EDF measure is only available back to 1969, while we would like a default measure that goes back to 1926. Because the EDF measure is option-pricing based, the key input into the measure is stock return volatility. Thus for the longer sample we use stock return volatility. We measure the weekly returns on the value-weighted S&P index based on daily returns. As the volatility measure for a given year, we compute the standard deviation of the weekly log returns over the year leading up to the end of the government fiscal year (the date of the Debt-to-GDP observation). We annualize this standard deviation by multiplying by the square root of 52. Over the 39 years for which we have both EDF data and stock market volatility estimates, the correlation of these two default measures is 0.76. This provides strong support for the use of stock market volatility as a default control over the full sample. Column (3) presents results using the stock return volatility measure. Volatility is significantly related to the spread, but the coefficient on  $\log(Debt/GDP)$  is roughly the same as in previous specifications. Furthermore, the economic effects of changes in the default risk measures on the Aaa-Treasury spread are modest compared to the effect of Treasury supply. A one standard deviation increase in the EDF increases the Aaa-Treasury spread by 23 basis points while a one standard deviation increase in stock return volatility increases it by 10 basis points.

The regressions in column (2) and (3) include the slope of the yield curve as a further control. We measure the slope as the spread between the 10 year Treasury yield and the 3 month Treasury yield (*slope*). The slope of the yield curve is a measure of the state of the business cycle. It is known to predict the excess returns on stocks and may also pick up time-varying risk premia on corporate bonds. For example, if investors are more risk averse in a recession, when the slope is high, they will demand a higher risk premium to hold corporate bonds. Thus, the slope of the yield curve serves as a measure of variation in the risk premium component of the bond spread, i.e. the term involving  $cov_t(\cdot)$  in Prediction 2. We also note that

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<sup>7</sup>Another issue that arises in interpreting the long-term spread regressions is callability. Duffee (1988) points out that the Moody’s Aaa index includes callable corporate bonds. Thus, the Aaa-Treasury spread may also reflect an interest rate option. Duffee proxies for the moneyness of the call option using the level of interest rates and shows that yield spreads vary significantly with the level of interest rates. Following this approach, we have investigated adding levels of short and long-term interest rates as explanatory variables in our long-term spread regressions and have found that it has no appreciable effects on the coefficient on  $\log(Debt/GDP)$ .

<sup>8</sup>A further issue that affects spreads between corporate bonds and Treasuries is that Treasuries are exempt from state taxes, while corporate bonds typically are not. We have run regressions that include a control for the state tax effect on the spread (the product of the average state tax rate and the Aaa yield) and find that our results are largely unchanged.

to the extent that corporate default risk is likely to vary with the business cycle, the slope variable can furthermore help control for the default risk in the yield spread.

Turning to the results for the Baa-Treasury spread in Panel B, the coefficient on  $\log(Debt/GDP)$  in column (5) is considerably higher than the coefficient in column (3) for the Aaa-Treasury spread, and imply that a decrease of one standard deviation in Debt-to-GDP from its mean value of 0.439 to 0.243 increases the convenience yield component of the Baa-Treasury spread by 0.77% (77 basis points). The difference in results between Panel A and B suggests that Aaa bonds offer some convenience services of Treasuries and thus that the Baa-Treasury spread is more appropriate for capturing the full effect of Treasury supply on the Treasury convenience yield. We present further evidence for this interpretation in Table II below.

### 3.1.2 Short-term spreads

Panel C and D of Table I are based on short-term bond spreads, with Panel C showing results for the spread between the highest rated commercial paper and Treasury bills, both of 3-month maturity, and Panel D focusing on the spread between lower-grade commercial paper (A2/P2 rated paper) and Treasury bills. The shorter time-period in column (9) is due to yields for lower-grade commercial paper being available only starting in 1974.<sup>9</sup>

Short and long-term spreads may contain different convenience attributes, so we should not expect the coefficients on  $\log(Debt/GDP)$  to be the same across the different maturities. Nonetheless, the regressions for short-term spreads show that the effect of changes in Treasury supply on short-term spreads are of fairly similar magnitudes as the effect on long-term spreads. Consistent with the results for long-term spreads, the higher coefficients in Panel D than in Panel C is also indicative that the high grade commercial paper in Panel C (like the Aaa-rated long-term bonds in Panel A) has some convenience attributes.

It is important to note that our evidence on the spread between commercial paper and T-Bills is less likely to be affected by issues of omitted expected default or default risk premium controls than the evidence on the Aaa-Treasury spread. This is the case because historical default rates on the highest rated commercial paper (A1/P1) are very low, with literally zero defaults during the period 1972-2000 for which Moodys provide data on commercial paper defaults. This latter observation can also explain why the default controls are statistically not different from zero in the commercial paper regressions in columns (7) and (9). Over a three month period, a highly rated firm may run into financial difficulties and be downgraded, but is unlikely to enter bankruptcy.

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<sup>9</sup>When analyzing lower-grade commercial paper yields we drop 2008. This year is a dramatic outlier in terms of the lower-grade commercial paper yield, likely due to increased default risk. Since our EDF default control which pertains to corporate bonds may not be as good a control for default risk for commercial paper as for corporate bonds, we drop 2008 in our analysis of lower-grade commercial paper.

### 3.2 Evidence for the Existence of Priced Liquidity and Safety Attributes

We next test Predictions (3)-(5) that the convenience yield on Treasuries is a reflection of two priced attributes: liquidity and safety. We consider pairs of assets with different liquidity but similar safety or with different safety but similar liquidity. Table II presents the results.

The first two columns of Table II present evidence for the existence of a unique demand for long-term safe assets leading to a positive convenience price of the long-term safety attribute. The dependent variable is the spread between Baa and Aaa corporate bonds. Baa bonds carry more default risk than Aaa bonds but are similarly illiquid. Chen, Lesmond and Wei (2008) study corporate bond liquidity in a large sample of corporate bonds over the period 1995 to 2003 and report that the spread between bid and ask prices (as a percentage of the price) on Aaa corporate bonds averaged 52 *bps* while they averaged 59 *bps* on the Baa corporate bonds. These spreads compare to about 10 to 20 *bps* for Treasury bonds (calculated based on typical dollar spreads of two to three 32nds per \$100 face value according to Longstaff, 2004). Thus any convenience yield as measured in the Baa-Aaa spread can only reflect the price of the long-term safety attribute, as opposed to a liquidity effect. Column (1) shows that an increase in Treasury supply leads to a decrease in the Baa-Aaa spread, confirming the existence of a priced long-term safety attribute driven by Treasury supply. Theory suggests that the supply of long-term safe assets rather than the total supply of Treasuries should drive the price of long-term safety. In column (2), we presents results using the supply of long-term Treasury debt, measured as the supply of Treasuries with greater than 10 years maturity divided by GDP. Since the maturity structure of government debt is likely to be endogenous, we instrument the long-term supply by powers of ( $Debt/GDP$ ). The first-stage of the instrumental variables regression is not reported for brevity, but shows a highly significant relation between the total debt and the measure of long-term debt. The coefficient of  $-0.306$  means that a one standard deviation decrease in the supply of long term Treasuries (as a ratio to GDP) from its mean value of 0.079 to 0.021 raises the price of long-term safety by 41 basis points.

Column (3) and (4) present evidence for the existence of a priced short-term safety attribute driven by Treasury supply. The dependent variable is the spread between three month A2/P2 rated and A1/P1 rated commercial paper. Over a three month period, based on data from 1972 to 2000, Moody's Investors Service (2000) estimates that the default probability on A2/P2 paper is 0.02%, while it is 0% for A1/P1 paper. Thus there is a (small) difference in the safety of these short-term assets. As for liquidity, there is little secondary market trading activity in commercial paper of any rating. Covitz and Downing (2007) based on data from 1998 to 2003 report that most of the commercial paper activity is in terms of new issuance. Secondary market transactions account for less than 2% of the new issuance activity on any day. Using trading volume as a proxy for liquidity and EDF, credit ratings, and stock volatility as proxies for credit risk, Covitz and Downing find that, in the cross-section of commercial paper, liquidity differences explain very little of the

variation in rates. Thus, any convenience yield as measured in the P2-P1 spread reflects the price of the short-term safety attribute. In column (3) we use total Treasury supply while column (4) presents an IV regression, where the supply measure is of Treasuries with maturity less than 1 year divided by GDP. The results in both column (3) and (4) confirm the existence of a significant price of short-term safety. The coefficient of  $-1.462$  in column (4) means that a one standard deviation decrease in the supply of short-term Treasuries (as a ratio to GDP) from its mean value of 0.124 to 0.104 raises the price of short-term safety by 26 basis points.

The last two columns present evidence for the existence of a priced liquidity attribute of Treasuries, comparing assets with similar safety but different liquidity. The dependent variable in column (5) is the spread between the interest rate customers receive on 6 month FDIC-insured CDs and 6 month Treasury bills. We start the CD series in 1984, corresponding to the phasing out of Regulation Q (see Gilbert, 1986). Given FDIC insurance, any convenience yield documented via Treasury supply affecting this spread can only reflect a liquidity attribute. The supply variable in the regression is the total supply of Treasuries, since all Treasuries carry the liquidity attribute. The impact of Treasury supply on the CD-Treasury spread confirms the existence of a significant price of liquidity over the 1984-2008 period. To obtain evidence going back further, column (6) uses data on the spread between the average interest rate paid on time and savings deposits and Treasury bill rates. Since interest rate ceilings set by Regulation Q were binding at various points during the period from 1966 until their phase-out in the mid 1980s (see again Gilbert, 1986), we focus on data from 1934 – the first year of FDIC insurance – to 1965 (we do not have CD rate data from this earlier period). The interest rate on time and savings deposits is less ideal than the CD rate because FDIC insurance does not apply to large deposits, so the interest rate on time and savings deposits reflect a mix of insured accounts and non-insured accounts. The FDIC reports that insurance applied to approximately half of all deposits at commercial banks over the period from 1934 to 1965 (Federal Deposit Insurance Corporation: *The First Fifty Years, a History of the FDIC, 1933-1983* (Washington: FDIC, 1984)). As a result, there is a slight difference in the safety of the average time and savings deposit and of Treasuries.<sup>10</sup> In addition, the time and savings deposits are an amalgam of deposits with different maturities, making it more difficult to maturity match the series to Treasuries. We match it to 6-month Treasuries and include a slope control to deal with the maturity mismatch. Subject to these qualifiers, the evidence in column (6) of an impact of Treasury supply on the interest rate spread between time and savings deposits and Treasuries provides further support that investors value liquidity with Treasury supply driving the price of liquidity.<sup>11</sup>

We can gauge the magnitude of the liquidity effect in two different ways. First, note that the deposits in

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<sup>10</sup>Including the fraction of deposits that are insured as a control does not affect the results and this variable is not significant of the spread.

<sup>11</sup>Note that we focus on time and savings account deposit rates. Checking deposits, and not time and savings deposits, are conventionally thought to provide a medium-of-exchange attribute.

column (6) include very short-term savings accounts which are quite liquid. That is the spread in column (6) reflects the difference between a very liquid Treasury and a liquid bank account, so that the spread reflects an underestimate of how much Treasury yields are reduced by the liquidity attribute of Treasuries. The spread in column (5) is a more pure read on the liquidity impact on Treasury yields, although the sample is quite short. The coefficient of  $-1.884$  means that a one standard deviation decrease in the supply of Treasuries (as a ratio to GDP) from its mean value of 0.439 to 0.243 raises the price of liquidity by 111 basis points. A second estimate of the impact of Treasury supply on the price of liquidity comes from the regression in Table I, column (7), which is the high-grade CP spread regression from 1969 to 2008. Considering the spread between CP and Treasury bills also deals with concerns that the spread between insured CD rates and Treasury bills partly reflects intermediation costs or retail deposit effects. We have noted that data from 1972 on indicates that there has never been a default on high grade CP. The coefficient on the default controls in that regression are also small and statistically not different from zero. Using this spread as a measure of liquidity, we find that a one standard deviation decrease in the supply of Treasuries (as ratios to GDP) from its mean value of 0.439 to 0.243 raises the price of liquidity by 57 basis points.

### 3.3 Quantifying the convenience yield

We next turn to quantifying the level of the convenience yield. Results from the log-specification pin down the derivative of the convenience yield with respect to Treasury supply, but do not pin down the level of the convenience yield as a function of Treasury supply. Theory suggests that the value of convenience should go to zero given sufficient convenience assets. Imposing this asymptote property allows us quantify the level of the convenience yield, since the convenience yield at some  $\theta_t^T$  given an estimated relation between spreads and Treasury supply of  $f(\theta^T/GDP)$  is simply  $f(\theta_t^T/GDP_t) - f(\infty)$  (i.e. the distance between the predicted spread and the estimated asymptote).

The log function does not have the asymptote property. Table III reports results in which we model the convenience yield with a function that is piecewise linear in Treasury supply,  $b_1 \max[b_2 - Debt/GDP, 0]$ . Visually, the piecewise linear function appears to be a good fit of the relation as depicted in Figure 1. Indeed, the  $R^2$ s for the piecewise linear function rise somewhat relative to the log function. Regressions are estimated by non-linear least squares. As in Table I, the standard errors are adjusted assuming the error term is AR(1).

The term  $b_1 \max[b_2 - Debt/GDP, 0]$  is the convenience yield on long-term Treasury debt as a function of the Debt-to-GDP ratio.<sup>12</sup> Consider the estimates from column (4) for the Baa-Treasury spread. We can

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<sup>12</sup>There is an important caveat in interpreting the piecewise linear estimates. The estimates of  $b_0$  and  $b_2$  are heavily affected by observations in the right tail of Figure 1 (i.e. high Debt-to-GDP ratios). These high ratios only occur during the 1940s and 1950s, so the parameter estimates are influenced by the idiosyncrasies of the war period. In an effort to help finance the war debt at low interest rates, the Federal Reserve committed to purchase long-term Treasury bonds at a floor below prevailing market prices, thereby inducing long-term Treasury rates to be lower than market forces would have otherwise dictated. As

evaluate the convenience yield function at historical values of the Debt-to-GDP ratio from 1926 to 2008, and average these numbers to come up with the average value of the convenience yield over our sample. This computation gives 73 basis points. The Baa-Treasury spread reflects both long-term safety and liquidity. We have argued that the Baa-Aaa spread is only driven by long-term safety, while the Aaa-Treasury spread is primarily driven by liquidity given the low default rate on Aaa bonds. To provide a sense of the relative magnitudes of the safety and liquidity convenience components, we make the simplifying assumption that the Aaa-Treasury spread only reflects liquidity, while the Baa-Aaa spread only reflects safety. Based on the Aaa-Treasury estimates of column (2), the average liquidity convenience is 46 basis points. This number should be interpreted as an upper bound for liquidity convenience, since the Aaa-Treasury spread does contain some safety convenience as well (since Aaa bonds are not quite as safe as Treasuries). Further, subtracting 46 from 73, gives us that safety convenience is worth 27 basis points on average (a lower bound since Aaas do carry some default risk).

Are these estimates plausible based on the existing literature? Consider liquidity first. Figure 1 in Krishnamurthy (2002) documents that the yield spread between the on-the-run liquid 30 year Treasury bond and the off-the-run less-liquid 30 year Treasury bond was 12 basis points on February 9, 2001. In price terms, taking the duration of the bonds to be 12 years, the on-the-run bond was priced at 1.44% over the off-the-run bond. Krishnamurthy notes that this difference will converge to a price premium of 0.36% points over the next 9 months. Thus, the annualized liquidity convenience yield on the on-the-run bond was 144 basis points  $(=(1.44 - 0.36) \times \frac{12}{9} \times 100)$ .<sup>13</sup> This computation demonstrates that liquidity premia on Treasury bonds are indeed quite large. Our estimate, which is roughly one-third of that from the on-the-run bond, is likely due to the fact that we measure a liquidity premium on all Treasuries and not only the exceptionally

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events played out, the Federal Reserve did not in fact accumulate many long-term Treasuries, but even so, the commitment to a floor should be expected to affect prices. As a result, our estimates of  $b_0$  are biased upwards – implying that our above estimate of an average Treasury convenience yield of 73 basis points is conservative – and estimates of  $b_2$  biased downwards. The Federal Reserve intervened much more strongly in the Treasury bill market. Policy at the time effectively allowed commercial banks to carry Treasury bills as interest bearing reserves because the Federal Reserve allowed banks to freely exchange Treasury bills for reserves at a fixed price. In addition, beginning in 1943, the Federal Reserve directly purchased a large share of newly issued Treasury bills, while long-term bonds were held by a variety of investors. The Federal Reserve directly held 72% of total Treasury bills outstanding in 1944. See Wicker (1969) for further details on all of these points. We present piecewise linear regressions for the long-term spreads, subject to the caveat noted above, but we do not present results for the short-term spreads on grounds that they are likely largely uninformative about convenience yields. We also note, as can be seen from columns (2), (4), (7), and (9) in Table I, that our main results about the impact of Treasury supply on yields hold for subsamples that do not include the war period.

<sup>13</sup>It is a coincidence that the 1.44% price premium and the 144 basis point convenience yield are the same numbers in the example. For the sample studied in Krishnamurthy (2002) from 1990 to 1999, the paper finds that the on-the-run/off-the-run yield spread converges at the rate of 3.2 basis points per 90 days (Table 6, first column), which translates to an annualized liquidity convenience yield of 156 basis points  $(=12 \times 3.2 \times \frac{365}{90})$ .

liquid on-the-run bond.

Consider long-term safety next. Longstaff, Mithal and Neis (2006) study the pricing of corporate bonds relative to Treasury bonds, using information from the credit-default swap market to pin down the default component.<sup>14</sup> As with most papers in the literature, their paper finds that there is a substantial non-default component in the corporate bond spreads. Their estimates put the average non-default component at 65 basis points, which is in line with our overall convenience yield estimates. Most relevant for the safety premium, they find that the non-default component is about 50 basis points for Aaa and Aa rated bonds, and about 70 basis points for lower-rated bonds. Thus the premium due to safety for the Aaa bond is 20 basis points which is similar to our estimate of 27 basis points.

## 4 Conclusion

Investors value the liquidity and safety attributes of Treasuries. We document this by showing that changes in Treasury supply have large effects on a variety of yield spreads. Evaluated based on the Baa-Treasury spread (for long-term bonds), a one standard deviation reduction in Treasury supply (starting from mean supply) lowers Treasury yields by 77 basis points relative to corporate bond yields. The effects based on the CPP2-Bills spread (for short-term bonds) is of a roughly similar magnitude.

By studying pairs of assets with similar liquidity but different safety (higher and lower grade corporate bonds and commercial paper) or with similar safety but different liquidity (FDIC insured CDs and Treasury bills) we document that changes in Treasury supply drive both the equilibrium price of safety and the equilibrium price of liquidity. This implies that Treasuries carry both a safety and liquidity attribute. Using a piecewise linear specification, we estimate that the average convenience yield on long-term Treasuries over the period 1926-2008 is 73 basis points, of which 46 basis points is driven by the liquidity of Treasuries and 27 basis points by the safety of Treasuries.

One way of putting these basis point estimates in perspective is to compute the seignorage the government collects from being able to finance the US federal debt with securities that offer convenience benefits. The historical average Debt-to-GDP ratio is 0.439. At this Debt-to-GDP, the convenience yield from the Baa-Treasury piecewise linear specification in column (4) of Table III evaluates to 53 basis points. Multiplying these numbers together gives seignorage of 0.23% of GDP. Alternatively, if we evaluate the convenience yield at each of the Debt-to-GDP ratios in our sample, multiplying by that Debt-to-GDP ratio, and then averaging,

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<sup>14</sup>In the piecewise linear regressions, the constant term  $b_0$  is the average default component of the spread, since the EDF, volatility and slope controls in the regressions are demeaned. Consider column (4) for the Baa-Treasury spread. This regression indicates that  $b_0$  is 1.199, which is within the range of estimates from the corporate bond price literature of the default component of Baa bonds (see Longstaff, Mithal and Neis (2006)).

we find that average seignorage is also 0.23% of GDP.<sup>15</sup> For comparison, consider the seignorage that the government derives from households' willingness to hold fiat money at no interest. The monetary base at the end of 2007, prior to the Federal Reserve's quantitative easing experiment, was \$824 Billion, corresponding to 6% of GDP. Suppose the federal government had to repurchase the monetary base by issuing Treasury bills and that these Treasury bills had a 4% nominal yield. Then the annual interest expense of this additional debt would be  $4 \times 0.06 = 0.24\%$  of GDP per year. Together, these calculations suggest that the total benefit from the convenience yield on Treasuries is large and that the seignorage to the government from being able to finance the current level of debt with securities that have a convenience yield is of the same order of magnitude as the seignorage resulting from the public's willingness to hold fiat money at no interest.

Our results have many implications for asset pricing. Since Treasury debt carries a convenience yield, the common practice of identifying the Treasury interest rate with models' riskless interest rate is incorrect. We have argued that the observed price of a one-period Treasury is  $P_t^T = \frac{E_t[M_{t+1}]}{1 - v'(\theta_t^A/GDP_t; \xi_t)}$  (implying an interest rate of  $i_t^T \approx -\ln E_t[M_{t+1}] - v'(\cdot)$ ). The Treasury interest rate is lower than the "true" riskless interest rate. In order to recover the true riskless rate from the data (the rate that can meaningfully be compared to the riskless rate from a model that ignores the convenience benefits of Treasuries), one has to estimate the convenience yield and adjust Treasury rates by this convenience yield. Our estimated demand curves may be used to measure the convenience yield and make the adjustment. The literature has found that the high spread between corporate bond rates and Treasury rates, and the high equity premium, measured as the average excess stock market return over Treasury bill rates, are hard to reconcile based on standard factors. Accounting for a true riskless rate that is higher than the Treasury rate can go part of the way towards reconciling these findings.

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<sup>15</sup>These computations implicitly assume that the convenience yield for short Treasuries is similar to that for long Treasuries. Due to the wartime interventions noted above, we cannot estimate the level of the convenience yield for short Treasuries using the kinked specification. However, we found quite similar sensitivity of long and short spreads to changes in Debt/GDP in Table I suggesting that this may be a reasonable assumption.

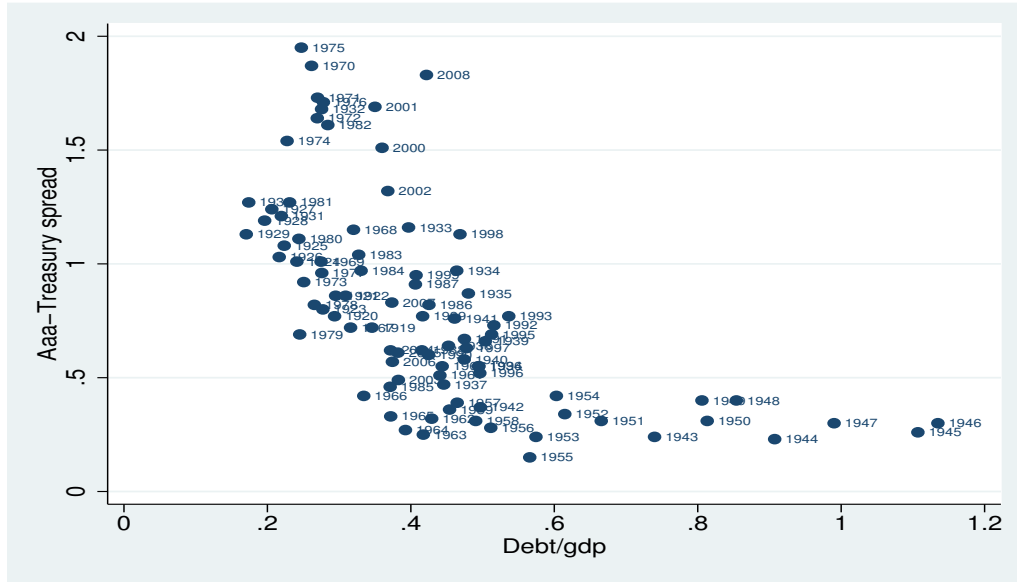
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Figure 1: Corporate Bond Spread and Government Debt



The figure plots the Aaa-Treasury corporate bond spread ( $y$ -axis) against the Debt-to-GDP ratio ( $x$ -axis) based on annual observations from 1919 to 2008. The corporate bond spread is the difference between the percentage yield on Moody's *Aaa* long maturity bond index and the percentage yield on long maturity Treasury bonds.

Table I

## Impact of Treasury Supply on Bond Spreads: Log Specification

The dependent variables are long and short-term yield spreads between corporate and Treasury bonds, both measured in percentage units. Independent variables are the log of the ratio of the market value of Treasury debt outstanding to US GDP, and controls for the default risk and default risk premium on corporate bonds. EDF is the expected default frequency for corporate bonds. Volatility is the annualized standard deviation of weekly log stock returns on the S&P500 index. Slope is the slope of the Treasury yield curve measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield. The data appendix provides the precise definitions of all variables. EDF, volatility, and slope control are demeaned. Regressions are estimated by ordinary least squares. The standard errors are adjusted assuming errors are AR(1). We use the Box-Jenkins methodology for identifying the error structure.  $\rho$  denotes the first order autocorrelation of the error terms. t-statistics in brackets.

	Panel A: Aaa-Treasury			Panel B: Baa-Treasury		Panel C: CP-Bills			Panel D: CPP2-Bills
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Period	1919-2008	1969-2008	1926-2008	1969-2008	1926-2008	1920-2008	1969-2008	1926-2008	1974-2007
$\log(Debt/GDP)$	-0.746 [-4.36]	-0.871 [-2.78]	-0.800 [-5.12]	-1.662 [-3.96]	-1.309 [-7.55]	-0.730 [-4.42]	-0.961 [-2.08]	-0.554 [-3.56]	-1.958 [-3.97]
$EDF$		1.040 [3.51]		1.453 [3.21]			0.126 [0.24]		0.040 [0.08]
$Volatility$			1.313 [1.93]		6.344 [6.81]			1.902 [2.28]	
$Slope$		0.058 [1.26]	0.081 [1.89]	0.221 [2.88]	0.312 [4.69]		-0.102 [-1.09]	-0.083 [-1.39]	-0.10 [-1.10]
$Intercept$	0.109 [0.62]	0.118 [0.35]	0.077 [0.49]	0.377 [0.84]	0.737 [4.33]	0.095 [0.57]	-0.200 [-0.40]	0.227 [1.48]	-0.849 [-1.66]
$R^2$	0.449	0.599	0.573	0.577	0.690	0.226	0.184	0.258	0.287
$\rho$	0.604	0.460	0.552	0.171	0.013	0.175	-0.022	0.016	0.074
$N$	90	40	83	40	83	89	40	83	34



Table II

Impact of Treasury Supply on the Price of Safety and the Price of Liquidity

The dependent variables are: The spread between Baa-rated corporate bonds and Aaa-rated bonds (column (1) and (2)), the spread between A2/P2-rated and A1/P1-rated commercial paper (column (3) and (4)), the spread between the interest rate on FDIC insured 6 month CDs and 6-month Treasury bills (column (5)), and the spread between the average interest rate paid by banks on time and savings deposits and 6-month Treasury bills (column (6)). Independent variables are the log of the ratio of the market value of Treasury debt outstanding to US GDP, the log of the ratio of the market value of Treasury debt with remaining maturity greater than 10 years to US GDP, the log of the ratio of the market value of Treasury debt with remaining maturity less than 1 year to US GDP, the annualized standard deviation of weekly log stock returns on the S&P500 index (*Volatility*), the expected default frequency for corporate bonds, (*EDF*), and the slope of the Treasury yield curve measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield in column (1)-(4) and as the spread between the 10-year Treasury yield and the 6-month Treasury yield in column (5)-(6). The data appendix provides the precise definitions of all variables. EDF, volatility, and slope control are demeaned. Regressions are estimated by ordinary least squares or instrumental variables (2SLS), where we use instruments  $Debt/GDP$ ,  $(Debt/GDP)^2$ , and  $(Debt/GDP)^3$ , as indicated. The standard errors are adjusted assuming errors are AR(1) or i.i.d. as indicated. We use the Box-Jenkins methodology for identifying the error structure. t-statistics in brackets.

	Panel A: Price of Safety				Panel B: Price of Liquidity	
	$S^{Baa-Aaa}$		$S^{P2-P1}$		$S^{FDIC\ insured\ CDs-Bills}$	$S^{Time\ \&\ Savings\ Accounts-Bills}$
	(1)	(2)	(3)	(4)	(5)	(6)
Period	1926-2008	1926-2008	1974-2007	1974-2007	1984-2008	1935-1965
$log(Debt/GDP)$	-0.510		-0.888		-1.884	-0.690
	[-3.45]		[-4.34]		[-2.49]	[-4.08]
$log(Debt > 10\ year\ mat/GDP)$ , instr. by $(Debt/GDP)$ , $(Debt/GDP)^2$ , $(Debt/GDP)^3$		-0.306				
		[-2.44]				
$log(Debt \leq 1\ year\ mat/GDP)$ instr. by $(Debt/GDP)$ , $Debt/GDP)^2$ , $(Debt/GDP)^3$				-1.462		
				[-2.84]		
<i>Volatility</i>	5.031	6.237				
	[6.47]	[6.65]				
<i>EDF</i>			0.074	0.019		
			[0.35]	[0.07]		
<i>Slope</i>	0.232	0.211	0.014	0.055	0.101	1.181
	[4.20]	[3.29]	[0.42]	[1.13]	[1.02]	[15.67]
<i>Constant</i>	0.660	0.258	-0.508	-2.679	-1.482	-0.118
	[4.54]	[0.71]	[-2.39]	[-2.47]	[-2.23]	[-1.09]
<i>N</i>	83	83	34	34	25	31
<i>R</i> <sup>2</sup>	0.600		0.497		0.233	0.898
Estimation method	OLS	IV	OLS	IV	OLS	OLS
Error term	AR(1)	AR(1)	AR(1)	AR(1)	i.i.d.	i.i.d.

Table III

**Impact of Treasury Supply on Bond Spreads: Piecewise Linear Specification**

This table estimates a piecewise linear specification for the relation between Treasury supply and bond spread. The function estimated is  $b_0 + b_1 \times \max[b_2 - Debt/GDP, 0]$ . The dependent variables are long-term yield spreads between corporate and Treasury bonds, measured in percentage units. Independent variables are the ratio of the market value of Treasury debt outstanding to US GDP, and controls for the default risk and default risk premium on corporate bonds. Volatility is the annualized standard deviation of weekly log stock returns on the S&P500 index. Slope is the slope of the Treasury yield curve measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield. The data appendix provides the precise definitions of all variables. Volatility and slope are demeaned. Regressions are estimated by non-linear least squares. The standard errors are adjusted assuming errors are AR(1). We use the Box-Jenkins methodology for identifying the error structure. t-statistics in brackets.

	Panel A: Aaa-Treasury		Panel B: Baa-Treasury	
	(1)	(2)	(3)	(4)
Period	1919-2008	1926-2008	1919-2008	1926-2008
$b_0$	0.319 [ 1.80 ]	0.347 [ 2.55 ]	1.019 [ 1.94 ]	1.200 [ 7.68 ]
$b_1$	2.590 [ 4.06 ]	3.075 [ 5.17 ]	4.309 [ 2.64 ]	4.952 [ 6.76 ]
$b_2$	0.584 [ 7.02 ]	0.548 [ 9.74 ]	0.626 [ 4.22 ]	0.546 [ 12.92 ]
<i>Volatility</i>		1.206 [ 1.93 ]		6.215 [ 6.98 ]
<i>Slope</i>		0.096 [ 2.43 ]		0.334 [ 5.07 ]
$R^2$	0.479	0.618	0.290	0.704
$N$	90	83	90	83

## A Derivation for Prediction 2

We rewrite (9), using the approximation that  $1 - v'(\cdot) \approx e^{-v'(\cdot)}$ , so that the return from  $t$  to  $t+1$  on holding Treasury bonds satisfies,

$$1 = e^{v'(\theta_t^A/GDP_t; \xi_t)} E_t[M_{t+1} P_{t+1}^T / P_t^T] = E_t[M_{t+1} e^{-(\tau-1)i_{t+1, \tau-1}^T + \tau i_{t, \tau}^T + v'(\theta_t^A/GDP_t; \xi_t)}]. \quad (23)$$

Likewise, for corporate bonds, rewriting (16), we find,

$$1 = e^{-\lambda_t D_t} E_t[M_{t+1} P_{t+1}^C / P_t^C] = E_t[M_{t+1} e^{-(\tau-1)i_{t+1, \tau-1}^C + \tau i_{t, \tau}^C - \lambda_t D_t}]. \quad (24)$$

The excess return on corporate bonds over Treasury bonds is,

$$\tilde{R}_{t+1} = \frac{P_{t+1}^C}{P_t^C} - \frac{P_{t+1}^T}{P_t^T} \approx -(\tau-1)S_{t+1, \tau-1} + \tau S_{t, \tau}.$$

We log-linearize to make the approximation. We make the following computations to arrive at an expression for the spread  $S_{t, \tau}$ . We assume that  $M_{t+1}$  is lognormally distributed (where  $m \equiv \ln M$ ) and that all interest rates are normally distributed. Then, it is straightforward to rewrite (23) as,

$$1 = e^{v'(\theta_t^A/GDP_t; \xi_t) + E_t[m_{t+1}] + 1/2 \text{var}_t[m_{t+1}] - (\tau-1)E_t[i_{t+1, \tau-1}^T] + \tau i_{t, \tau}^T + \frac{1}{2}(\tau-1)^2 \text{var}_t(i_{t+1, \tau-1}^T) - \text{cov}_t(m_{t+1}, (\tau-1)i_{t+1, \tau-1}^T - \tau i_{t, \tau}^T)},$$

and rewrite (24) as,

$$1 = e^{-\lambda_t D_t + E[m_{t+1}] + 1/2 \text{var}_t[m_{t+1}] - (\tau-1)E_t[i_{t+1, \tau-1}^C] + \tau i_{t, \tau}^C + \frac{1}{2}(\tau-1)^2 \text{var}_t(i_{t+1, \tau-1}^C) - \text{cov}_t(m_{t+1}, (\tau-1)i_{t+1, \tau-1}^C - \tau i_{t, \tau}^C)}.$$

We take logs and subtract these last two equations. We assume that the terms involving  $\text{var}_t(i_{t+1, \tau-1}^T)$  and  $\text{var}_t(i_{t+1, \tau-1}^C)$  are approximately the same. That is, innovations in both corporate and Treasury rates have the same variance. Then,

$$0 = \frac{1}{\tau}(v'(\theta_t^A/GDP_t; \xi_t) + \lambda_t D_t) + \frac{\tau-1}{\tau} E_t[S_{t+1, \tau-1}] - S_{t, \tau} + \frac{1}{\tau} \text{cov}_t(m_{t+1}, (\tau-1)S_{t+1, \tau-1} - \tau S_{t, \tau}),$$

or,

$$S_{t, \tau} = \frac{1}{\tau}(v'(\theta_t^A/GDP_t; \xi_t) + \lambda_t D_t) + \frac{\tau-1}{\tau} E_t[S_{t+1, \tau-1}] - \frac{1}{\tau} \text{cov}_t(m_{t+1}, \tilde{R}_{t+1}).$$

Solving this equation recursively for  $S_{t, \tau}$  we find the expression in the text.

## B Data

## C Data

### Table I:

**Aaa-Treasury yield spread:** The percentage spread between Moody’s Aaa-rated long maturity corporate bond yield and the yield on long maturity Treasury bonds. The Moody’s Aaa index is constructed from a sample of long maturity ( $\geq 20$  years) industrial and utility bonds (industrial only from 2002 onward), available from 1919-2008. The yield on long maturity Treasury bonds is the average yield on long-term government bonds, available from 1919 - 1999. The Treasury bonds included are due or callable after 8 years for 1919-1925, 12 years for 1926-1941, 15 years for 1941-1951, 12 years for 1952, and 10 years for 1953-1999. We use the yield on 20-year maturity Treasury bonds for 2000 - 2008. All three data series are from the Federal Reserve’s FRED database (series AAA, LTGOVTBD, and GS20), with the exception that the long-term Treasury yield data for 1919-1924 are from Banking and Monetary Statistics, 1914-1941, Table 128. We use annual observations, sampled in July of the year up to (and including) 1976 and in October of each year after that.

**Baa-Treasury yield spread:** The percentage spread between Moody’s Baa-rated long maturity corporate bond yield and the yield on long maturity Treasury bonds. The Moody’s Baa index is constructed from a sample of long maturity ( $\geq 20$  years) industrial and utility bonds (industrial only from 2002 onward), available from 1919-2008. The Baa data series is also from the FRED database (series BAA). We use annual observations, sampled in July of the year up to (and including) 1976 and in October of the year after that.

**CP-Bills yield spread:** The percentage yield spread between commercial paper and Treasury bills. For 1971-2008 the commercial paper yield is from the FRED database. For 1971-1996 it is the series CP3M (the average of offering rates on 3-month commercial paper placed by several leading dealers for firms whose bond rating is AA or equivalent) and for 1997-2008 the series CPN3M (the 3-Month AA nonfinancial commercial paper rate). Prior to 1971 we use the commercial paper series for prime commercial paper, 4-6 month maturity, from Banking and Monetary Statistics (Table 12.5 for 1941-1970 and Table 120 for 1920-1940). The Treasury bill yield is for 3-month Treasury bills from 1971-2008 (from FRED, series TB3MS), 6-month Treasury bills for 1959-1970 (from FRED, series TB6MS), and 3-6 month Treasury bills for 1920-1958 from the NBER Macro History database (series M13029b for 1931-1958 and series m13029a for 1920-1930). We use annual observations, sampled in July of the year up to (and including) 1976 and in October of each year after that.

**CPP2-Bills yield spread:** The percentage yield spread between lower-grade commercial paper and Treasury bills. Calculated as the sum of the CP-bills yield spread described above (i.e. high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and

30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York and Bloomberg.<sup>16</sup>

**Debt/GDP:** The supply of Treasuries scaled by GDP, at market value. Calculated as (Debt/GDP with Debt at face value)\*(Total market value of Treasuries/Total face value of Treasuries).

The Debt/GDP series (with debt at face value) is from Henning Bohn's web page for years 1919-2008. The debt measure includes debt held by the Federal Reserve, but excludes debt held by other parts of the government such as the Social Security Trust Fund. Debt is for the end of the government's fiscal year, i.e. end of June up to and including 1976 and end of September from 1977 on. GDP is for the same fiscal year.

The CRSP Monthly US Treasury Database is used to calculate the factor for translating face values to market values (Total market value of Treasuries/Total face value of Treasuries). This data base starts in 1926. Prior to 1949 the amounts outstanding are missing for a lot of the Treasuries. From 1949 onward, at least 97% of Treasuries in the database have values for amounts outstanding. We multiply Bohn's Debt/GDP series by the CRSP ratio of Treasury market to face value to get a series for the market value of Treasury Debt/GDP. Over the period 1949-2008 the correlation between Debt/GDP at face value and Debt/GDP at market value is 0.996 and the means and standard deviations of the two series are very similar. Prior to 1949, one can still calculate the ratio of Treasury market to face value for the set bonds in the CRSP data base for which the data are available. This set is somewhat skewed towards better coverage of long maturities. We get similar results whether we use Debt/GDP at face value and Debt/GDP at market value in the pre-1949 period. We report the results which use the market Debt/GDP series.

Prior to 1926 the CRSP data base is not available and we therefore do not make the market value adjustment. In general, over the 1926-2008 period our results throughout the paper are very similar whether the market value adjustment is made or not.

**Volatility:** Annualized standard deviation of weekly log stock returns on the S&P500 index. We calculate weekly returns on the value-weighted S&P index based on daily returns obtain from CRSP. As the volatility measure for a given year, we compute the standard deviation of the weekly log returns over the year leading up to the end of the government's fiscal year. We annualize the standard deviation of weekly log returns by multiplying by the square root of 52.

**EDF:** Expected default frequency for corporate bonds. The data are obtained from Moody's-KMV. We use the median EDF reported by Moody's-KMV for large firms (defined as firms with book value of assets > \$300 million in current dollars). The EDF measure is available from 1969 to 2008. We use annual observations, sampled in July of the year up to (and including) 1976 and in October of each year after that.

**Slope:** Slope of the Treasury yield curve measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield. The interest rate on Treasuries with 10 year maturity is from FRED from

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<sup>16</sup>We thank Michael Fleming at the Federal Reserve for help in obtaining this data.

1953 to 2008 (series GS10). Prior to 1953 we use series m13033a (1926-1941) and m13033b (1942-1952) from the NBER Macro History Database, with both these series referring to the yield on long-term Treasuries. The interest rate on Treasuries with 3 month maturity is from FRED from 1934 to 2008 (series TB3MS) and from the NBER Macro History database prior to that (series m13029a for 1926-1933, referring to 3-6 month Treasuries). We use annual observations, sampled in July of the year up to (and including) 1976 and in October of each year after that.

In all tables, volatility, EDF and slope variables are demeaned.

Table II, Panel A:

**Baa-Aaa yield spread:** The percentage yield spread between the Moody's Baa-rated long maturity bond yield and Moody's Aaa-rated long maturity bond yield. Both series are described under Table 1 above. We use annual observations, sampled in July of the year up to (and including) 1976 and in October of each year after that.

**A2/P2-A1/P1 yield spread:** The yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York and Bloomberg (Tickers: H15S030D, CPDR3ANC). We use annual observations, sampled in July of the year up to (and including) 1976 and in October of each year after that.

**(Debt > 10 year maturity)/GDP:** The ratio of Treasury debt with more than 10 year remaining maturity to GDP. From 1949-2008 the amount of Treasury debt is at market value and calculated using the CRSP Monthly US Treasury Database. From 1926-1948 the amount of Treasury debt is at face value and obtained from Banking and Monetary Statistics 1914-1941 Table 147 and Banking and Monetary Statistics 1941-1970 Table 13.5 D.

**(Debt ≤ 1 year maturity)/GDP:** The ratio of Treasury debt with a year or less of remaining maturity to GDP. From 1949-2008 the amount of Treasury debt is at market value and calculated using the CRSP Monthly US Treasury Database (we only use this series in column (2) which is based on data from 1974 onward).

For the above two variables, the data for GDP used are from Henning Bohn's web page.

**Debt/GDP, volatility, slope:** As for Table I.

Table II, Panel B:

**(FDIC insured CDs)-Bills yield spread:** The percentage yield spread between the yield on 6-month certificates of deposit and Treasury bills. The CD yields are the national average rates obtainable by depositors on small-denomination (and thus FDIC insured) CDs of 6 month maturity, where small-denomination means the account-opening minimum. The rates are from Bank Rate Monitor and are obtained from the

New York Times for 1984-1997 and from Bloomberg for 1998-2008 and are as of the end of September. The Treasury bill yield used is the yield on 6-month bills obtained from FRED (series TB6MS), as of September.

**(Time & Savings Accounts)-Bills yield spread:** The percentage yield spread between time and savings accounts and Treasury bills. We estimate the yield on time and savings accounts using data from FDIC Historical Statistics on Banking, Table CB06 and CB15. Specifically, we divide the total annual interest on deposits in domestic offices by the sum of savings and time deposits in domestic offices (using averages of beginning and end of year values for the deposits). The FDIC series is available from 1935 onward. We compare this yield series to the yield on 6-month Treasuries, with data obtained from FRED (series TB6MS) from 1959 on and from the NBER Macro History database prior to that (series m13029b for 1931-1958 and series m13029a for 1926-1930, both referring to 3-6 month Treasuries). Since the yield on time and savings accounts is an annual average, we use the annual calendar year average of 6-month monthly Treasury bill rates for comparison.

**Slope:** Slope of the Treasury yield curve measured as the spread between the 10-year Treasury yield and the 6-month Treasury yield. The interest rate on Treasuries with 10 year maturity is from FRED from 1953 to 2005 (series GS10). Prior to 1953 we use series m13033a (1926-1941) and m13033b (1942-1952) from the NBER Macro History Database, with both these series referring to the yield on long-term Treasuries. The interest rate on Treasuries with 6 month maturity is from FRED from 1959 to 2008 (series TB6MS) and from the NBER Macro History database prior to that (series m13029b, referring to 3-6 month Treasuries). The slope variable in column (5) is based on September values to match the timing of the left-hand side variable. The slope variable in column (6) is (annual calendar year average of 10-year monthly yields)-(annual calendar year average of 6-month monthly yields), in order to match the timing of the dependent variable which is a calendar year average.

**Debt/GDP:** As for Table I.

Table III: Same data as Table I.