Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion

By ANNETTE VISSING-JØRGENSEN AND ORAZIO P. ATTANASIO*

Many of the empirical rejections of the consumption CAPM can be explained by the fact that the marginal rate of substitution between present and future consumption, which in the standard model functions as the pricing kernel for all assets for which a consumer is not at a corner, seems to vary too little to be consistent with sensible values of the parameters. Moreover, when one considers more than one asset at a time, one typically gets strong rejections of the overidentifying restrictions implied by the model. The failure seems to be both in terms of unconditional and conditional moments.

Two recent papers Attanasio et al., 2002 [henceforth ABT]; Vissing-Jørgensen, 2002 [henceforth VJ] have shown that, if one focuses on the consumption of individuals participating in the stock market, one does not reject some implications of the model. In particular, both VJ and ABT find that, using the consumption of stockholders, conditional Euler equations lead to sensible preference parameters and, in the case of ABT, fail to reject the overidentifying restrictions even when considering two assets (stocks and bonds) at the same time.

While these results constitute a first empirical success, they do not necessarily constitute a solution to the equity premium puzzle. As argued by Robert E. Hall (1988) and Attanasio and Guglielmo Weber (1989), the estimates of the coefficient on the interest rate in a log-linearized Euler equation should be interpreted as the elasticity of intertemporal substitution (EIS) and can only be informative about the degree of risk-aversion under more restrictive assumptions (CRRA utility).

In this paper we argue that considering the consumption growth of stockholders and two asset returns not only yields values of the EIS that are plausible, but also helps explain the equity premium puzzle. The evidence we have on the consumption, income, and portfolios of stockholders is consistent with fairly plausible values of the coefficient of relative risk-aversion when using the preference specification of Larry G. Epstein and Stanley E. Zin (1991). We use three Euler equations, one for each of the two assets considered, and one for the household's total wealth portfolio, whose return will be denoted as R_m. R_m is, in principle, an unobservable variable in that it depends on the returns on all assets an individual consumer holds, including human capital. In this paper, we specify the conditional expected return to human capital as a linear function of the conditional expected returns to stocks and bonds. We suggest a novel approach to estimating the coefficients in this function based on conditional Euler equations for stockholders with two asset returns on the right-hand side. Then, considering unconditional moments, we estimate the risk-aversion of stockholders using the log-linearized equation for the equity premium from Epstein-Zin utility also explored by John Y. Campbell (1996). Unlike Campbell, our preferred approach does not substitute out consumption. Furthermore, we emphasize the importance of allowing for bonds in households' portfolios as well as not restricting the conditional expected human-capital return to equal that of stocks.

I. Epstein-Zin Preferences and Implied Euler Equations

Following Epstein and Zin (1991), we assume that consumers choose consumption and asset holdings to maximize lifetime utility, given, at time t, by the following:

* Vissing-Jørgensen: Finance Department, Kellogg School of Management, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208-2001, NBER, and CEPR; Attanasio: Department of Economics, University College London, IFS, and NBER.

(1)

$$U_t = \{C_t^{1-(1/\sigma)} + \beta[E_t(U_{t+1}^{1-\gamma})]^{[1-(1/\sigma)]/(1-\gamma)}\}^{1/[1-(1/\sigma)]}$$

where C_t is consumption, E_t denotes the expectation conditional on information available at time t, γ is the coefficient of relative risk-aversion, σ is the elasticity of intertemporal substitution, and β is the time discount factor. When $\gamma = 1/\sigma$, one obtains the standard expected-utility maximization problem (for easier comparability of subsequent formulas our notation follows that of Campbell [1996]). The period-t budget constraint to this problem is $W_{t+1} = R_{m,t+1}(W_t - C_t)$, where wealth W includes both financial and (unobserved) human capital and $R_{m,t+1}$ is the gross return on total wealth (and thus captures labor income).

Epstein and Zin (1991) derive two types of Euler equations from this approach:

$$(2) \quad E_t\left[\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-(1/\sigma)}\right)^{\theta} R_{m,t+1}^{\theta-1} R_{i,t+1}\right] = 1$$

and

$$(3) \quad E_t\left[\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-(1/\sigma)}\right)^{\theta} R_{m,t+1}^{\theta}\right] = 1$$

where $\theta = (1-\gamma)/(1-1/\sigma)$ and $R_{i,t+1}$ is the gross return on asset i. Equation (2) should hold for any asset for which the consumer is not at a corner and is a generalization of the standard Euler equation under expected utility preferences. The novelty of the Epstein-Zin utility specification is the fact that the pricing kernel for each individual asset depends, in this model, not only on present and future consumption, but also on the household's total portfolio return.

A. *Intertemporal Substitution and Conditional Expected Portfolio Returns*

Attanasio and Weber (1989) showed that assuming joint log-normality of consumption growth and asset returns the Epstein-Zin Euler equations can be used to obtain an approximate equation for individual asset returns of the following form:

$$(4) \quad E_t \Delta c_{t+1} = \beta_{0,i} + \sigma E_t r_{i,t+1}$$

where $\Delta c_{t+1} = \ln(C_{t+1}/C_t)$ and $r_{i,t+1} = \ln(R_{i,t+1})$, and where the constant term $\beta_{0,i}$ is a function of the preference parameters and the variances and covariances of Δc_{t+1}, $r_{i,t+1}$, and $r_{m,t+1}$. This is the type of equation estimated by ABT and VJ. Equation (4) does not require information on the total portfolio return $R_{m,t+1}$. However, the coefficient of relative risk-aversion γ is buried into the intercepts of the log-linearized Euler equations and cannot be identified. Attanasio and Weber (1989) showed how one can use information on the variances of the interest-rate processes and the estimates of $h_{0,i}$ for two different assets to derive bounds on γ. Here, instead, we generalize the approach of Campbell (1996) in that we make specific assumptions about R_m to be able to identify the risk-aversion coefficient. In particular, we assume that, given the return on bonds $R_{1,t+1}$, on stocks $R_{2,t+1}$, and on human capital $R_{y,t+1}$, the return on the household's total portfolio $R_{m,t+1}$ is given by

$$(5) \quad R_{m,t+1} = (1-v)[\phi_1 R_{1,t+1} + \phi_2 R_{2,t+1}] + vR_{y,t+1}$$

where ϕ_1 is the share of bonds in the financial portfolio, $\phi_2 = 1 - \phi_1$ is the share of stocks in the financial portfolio, and v is the share of human capital in total household wealth. In logs, this equation leads to the approximation $r_{m,t+1} \simeq (1-v)[\phi_1 r_{1,t+1} + \phi_2 r_{2,t+1}] + vr_{y,t+1}$.

The return on human capital is difficult to measure. We assume that the conditional expected return on human capital is a linear combination of those on bonds and stocks

$$(6) \quad E_t r_{y,t+1} = \omega_0 + \omega_1 E_t r_{1,t+1} + \omega_2 E_t r_{2,t+1}.$$

Notice that the assumption is made on the conditional expectations of asset returns and does not require that $r_{y,t+1} = \omega_0 + \omega_1 r_{1,t+1} + \omega r_{2,t+1}$. Our assumptions are more general than the ones made in the literature. Campbell (1996), for instance, assumes that $\omega_1 = 0$, $\omega_2 = 1$. Ravi Jagannathan and Zhenyu Wang (1996), in a different asset-pricing context involving labor income, instead assume $\omega_1 = \omega_2 = 0$ (i.e., that human-capital returns are unpredictable).

Importantly, our assumptions furthermore differ from Campbell's in that we allow for bonds (and other relatively riskless assets) in the financial portfolio, $\phi_1 > 0$. This is important since even stockholders hold substantial amounts of less risky financial assets (bonds, cash, etc.), about 60 percent according to our calculations using the 1989, 1992, 1995, and 1998 Survey of Consumer Finances (SCF) for all stockholders, and around 43 percent for those in the top third in terms of stockholdings.

Assuming joint log-normality of consumption growth and the return on the household's total wealth portfolio, equation (3) can be log-linearized to give

(7)

$$\begin{aligned} E_t\Delta c_{t+1} &= \beta_0 + \sigma E_t r_{\mathrm{m},t+1} \\ &= \beta_0^* + \sigma[(1-v)\phi_1 + v\omega_1]E_t r_{1,t+1} \\ &\quad + \sigma[(1-v)(1-\phi_1) \\ &\quad + v\omega_2]E_t r_{2,t+1} \end{aligned}$$

where β_0^* is a function of ω_0, the preference parameters and the variances and covariances of Δc_{t+1}, $r_{1,t+1}$, and $r_{2,t+1}$. We will denote $\sigma[(1-v)\phi_1 + v\omega_1]$ by β_1 and $\sigma[(1-v)(1-\phi_1) + v\omega_2]$ by β_2. This equation can be estimated by instrumental-variables estimation (replacing expected by realized values), assuming that conditional expected returns on stocks and bonds, as measured by the projection of the actual returns on the instruments, are not perfectly correlated. An important caveat is that independent movements in expected stock and bond returns suggest that the higher-order moments are not constant and may be correlated with the instruments. This could lead to bias in estimates of β_1 and β_2. Tests of overidentifying restrictions did not reject the hypothesis of no correlation between instruments and the error term in (7), although one of our estimations based on (4) did lead to rejection of the test of overidentifying restriction (see Table 1). Furthermore, we constructed measures of each of the higher-order moments based on the residuals for Δc_{t+1}, $r_{1,t+1}$, and $r_{2,t+1}$ from the VAR discussed below. Including such measures in the estimation of (7) had little effect on estimates of β_1 and β_2.

Since β_1 and β_2 are functions of the five structural parameters σ, v, ϕ_1, ω_1, and ω_2, one cannot identify all of these parameters from estimates of β_1 and β_2. We consider four alternative assumptions (the first two for reference with the literature and the second two that we consider more realistic). For all cases we consider several possible values for v.

Case 1 (Jagannathan and Wang): $\omega_1 = \omega_2 = 0$. Then, β_1 and β_2 identify σ and ϕ_1.

Case 2 (Campbell): $\omega_1 = 0$, $\omega_2 = 1$. Then, β_1 and β_2 again identify σ and ϕ_1. Campbell also assumes $\phi_1 = 0$, but in general, both assumptions cannot be satisfied in this context.

Case 3 (our first alternative): $\omega_1 + \omega_2 = 1$, $\phi_1 = ½$. Then, β_1 and β_2 identify ω_1, $\omega_2 = 1 - \omega_1$ and σ.

Case 4 (our second alternative): σ estimated based on (4), $\phi_1 = ½$. Then, β_1 and β_2 identify ω_1 and ω_2.

B. *Risk-Aversion*

The log-linearized Euler equations for the stock and bond returns can be used to derive the following equation for the equity premium (similar to equation 6 in Campbell [1996]):

$$(8) \qquad E_t r_{2,t+1} - E_t r_{1,t+1} + \frac{V_{22} - V_{11}}{2} = \theta\,\frac{V_{2-1,c}}{\sigma} + (1-\theta)V_{2-1,\mathrm{m}}$$

where $V_{ii} = V(r_{i,t+1} - E_t r_{i,t+1})$, $V_{2-1,c} = \mathrm{Cov}[(r_{2,t+1} - E_t r_{2,t+1}) - (r_{1,t+1} - E_t r_{1,t+1}), c_{t+1} - E_t c_{t+1}]$, and $V_{2-1,\mathrm{m}} = \mathrm{Cov}[(r_{2,t+1} - E_t r_{2,t+1}) - (r_{1,t+1} - E_t r_{1,t+1}), r_{\mathrm{m},t+1} - E_t r_{\mathrm{m},t+1}]$.

Considering the unconditional expectation of (8), one obtains the following:

(9)

$$\theta = \frac{Er_{2,t+1} - Er_{1,t+1} + \dfrac{V_{22} - V_{11}}{2} - V_{2-1,\mathrm{m}}}{\dfrac{V_{2-1,c}}{\sigma} - V_{2-1,\mathrm{m}}}.$$

One can then estimate γ as $\gamma = 1 - \theta(1 - 1/\sigma)$. Unlike Campbell (1996), this approach does not substitute out consumption using the budget constraint. This allows our estimation to fit the covariance between observed consumption and asset returns and therefore avoids an implausibly high covariance between implied consumption innovations and asset returns.

To make (9) operational requires estimates of the moments on the right-hand side of the expression. Following Campbell, we estimate a first order VAR, in our case including average stockholder consumption growth in the VAR, and use the residuals to estimate V_{11}, V_{22}, $V_{2-1,c}$, and $V_{2-1,\mathrm{m}}$. Our VAR furthermore includes the two asset returns and the log dividend price ratio, the cay variable from Martin Lettau and Sydney C. Ludvigson (2001), the bond horizon premium, and the bond default premium (all at the semiannual frequency, and for the 1982–1996 period of Consumer Expenditure Survey data availability; details available from the authors upon request). Under our more general assumptions about the conditional expected return on human capital and allowing bonds in the financial portfolio we get

(10)

$$V_{2-1,\mathrm{m}} = (1 - v)\phi_1 V_{2-1,1} + (1 - v)\phi_2 V_{2-1,2} + vV_{2-1,y} - v\omega_1 V_{2-1,h_1} - v\omega_2 V_{2-1,h_2}$$

where

$$V_{2-1,y} = \mathrm{Cov}[(r_{2,t+1} - E_t r_{2,t+1}) - (r_{1,t+1} - E_t r_{1,t+1}), (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta y_{t+1+j}]$$

$$V_{2-1,h_i} = \mathrm{Cov}[(r_{2,t+1} - E_t r_{2,t+1}) - (r_{1,t+1} - E_t r_{1,t+1}), (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{i,t+1+j}]$$

can be calculated based on the VAR.

The parameter ρ equals $1 - \exp(c - w)$, where $c - w$ is the mean log consumption wealth ratio around which the budget constraint is linearized. In our estimations we set $c - w$ to log(0.05) when using annual data and log(0.025) when using semi-annual data.[1] Given assumptions and estimates for v, ϕ_1, $\phi_2\,(=1 - \phi_1)$, ω_1, ω_2, and σ, we can in turn estimate γ for each of the four cases outlined above.

Alternatively one can follow Campbell's (1996) approach and substitute out consumption. Under our more general assumptions,

(11)

$$V_{2-1,c} = (1 - v)[\phi_1 V_{2-1,1} + \phi_2 V_{2-1,2}] + vV_{2-1,y} + \{(1 - \sigma)[(1 - v)\phi_1 + v\omega_1] - v\omega_1\}V_{2-1,h_1} + \{(1 - \sigma) \times [(1-v)\phi_2 + v\omega_2] - v\omega_2\}V_{2-1,h_2}$$

which leads to $\gamma = A/B$ with

(12)

$$A = E_t r_{2,t+1} - E_t r_{1,t+1} + \frac{V_{22} - V_{11}}{2} + [(1 - v)\phi_1 + v\omega_1]V_{2-1,h_1} + [(1 - v)\phi_2 + v\omega_2]V_{2-1,h_2}$$

[1] The correct value for the consumption wealth ratio is hard to pin down since it will vary with age, preferences, etc. The chosen value is approximately consistent with a flat expected consumption profile for a person facing a 3-percent real interest rate on his portfolio and a remaining lifetime of 30 years.

(13)

$$B = (1 - v)(\phi_1 V_{2-1,1} + \phi_2 V_{2-1,2}) + (1 - v) \times (\phi_1 V_{2-1,h_1} + \phi_2 V_{2-1,h_2}) + v V_{2-1,y}.$$

Note that this does not require an estimate of σ.

II. Data

The data on stockholder consumption growth are from the 1982–1996 Consumer Expenditure Survey (CEX) and are identical to the data used in VJ. The CEX contains a year of consumption data for each household, and we calculate semiannual consumption growth at the household level (last six months' consumption divided by first six months' consumption). Interviews are staggered across months of the year, so semiannual consumption growth rates are available at the monthly frequency. For each month, log consumption growth rates are then averaged across stockholders. A household is classified (somewhat noisily) as a stockholder if it has positive holdings of "stocks, bonds, mutual funds and other such securities" at the beginning of the year for which consumption data are available. We also consider the wealthiest one-third of households based on holdings of this wealth category. (See VJ for details on these definitions.)

The asset returns used are monthly T-bill returns and monthly NYSE value-weighted returns, calculated in real terms using the CPI for total consumption of urban households, and aggregated to the semiannual frequency. Data on the dividend price ratio, bond horizon premium, and bond default premium are from Ibbotson Associates (2000). (See VJ for details on lags when using these variables as instruments.) Finally, we supplement the data from VJ with data on the log consumption wealth ratio measure cay of Lettau and Ludvigson (2001). We linearly interpolate the quarterly values of cay to have data available at the monthly frequency.

Semiannual income data are not available in the CEX. We therefore estimate $V_{2-1,y}$ based on a VAR using semiannual aggregate U.S. data for 1952–1996. This VAR includes the same variables as described above except consumption growth and adds in as a measure of Δy the log growth rate of real compensation of employees based on NIPA table 1.14.

Finally, when substituting out consumption for comparison with Campbell (1996) we use data for 1890–1995 obtained from Campbell. This differs slightly from the sample period in Campbell (1996) which is 1871–1990.

III. Results

Panel A of Table 1 shows the results of estimating (4) for T-bills and then for the NYSE value-weighted stock index using semiannual consumption data for stockholders from the Consumer Expenditure Survey. After-tax returns are calculated assuming a constant 30-percent tax rate. The first column only uses the log dividend–price ratio as an instrument and is comparable to VJ (table 2, instrument set 1), with the exception that those tables used pretax returns. The second column uses six instruments: the log dividend–price ratio, the lagged six-month after-tax T-bill return, the lagged six-month after-tax stock return, Lettau and Ludvigson's consumption–wealth ratio measure cay, the lagged bond horizon premium, and the lagged bond default premium. All estimations of conditional Euler equations are performed using the normalization-free, continuous-updating GMM estimator of Lars P. Hansen et al. (1996). We furthermore include the log family-size growth rate as well as seasonal dummies as in VJ. For the group of all stockholders, the estimate of σ using after-tax T-bill returns is 1.44 and 1.03 (results for the six-instrument case are similar if the log dividend–price ratio is dropped as an instrument). Estimates using the returns on Vanguard's short-term municipal bond fund, which is nontaxable, gave similar results (1.42 and 1.23), confirming the importance of using after-tax returns when estimating Euler equations for assets whose returns are taxed. When using the stock index, σ estimates are in all cases lower. This may reflect the much lower predictive power of the instruments for stock returns which could lead to poorer small-sample properties of the estimator. It may be specific to the U.S. data since σ estimates remain high in the U.K. data in the estimations in ABT. When using a σ estimate to proceed with our case-4 estimations below, we rely on the average of the

TABLE 1—CONDITIONAL EULER EQUATION ESTIMATION

A. *One Return on Right-Hand Side (T-bill/Stock Index)*

Sample	One instrument σ	Six instruments σ	Six instruments OID-test
T-bill			
All stockholders	1.44	1.03	12.40
	(0.54)	(0.44)	[p = 0.03]
Top third	2.34	2.19	7.81
	(1.00)	(0.98)	[p = 0.17]
Stock index			
All stockholders	0.42	0.30	6.68
	(0.20)	(0.08)	[p = 0.26]
Top third	0.67	0.35	6.47
	(0.45)	(0.18)	[p = 0.26]

B. *Two Returns on Right-Hand Side (T-bill and Stock Index)*

Sample	β_1	β_2	OID-test
All stockholders	0.99	0.18	5.63
	(0.52)	(0.09)	[p = 0.34]
Top third	1.62	0.14	7.11
	(1.19)	(0.19)	[p = 0.21]

Case 1: $\omega_1 = \omega_2 = 0$ (Jagannathan and Wang); σ and ϕ_1 estimated

v	All stockholders σ	All stockholders ϕ_1	Top third σ	Top third ϕ_1
⅓	1.75	0.85	2.64	0.92
	(0.70)	(0.12)	(1.64)	(0.14)
⅔	3.50	0.85	5.28	0.92
	(1.40)	(0.12)	(3.28)	(0.14)
0.9	11.68	0.85	17.60	0.92
	(4.66)	(0.12)	(10.92)	(0.14)

Case 2: $\omega_1 = 0$, $\omega_2 = 1$ (Campbell); σ and ϕ_1 estimated

v	All stockholders σ	All stockholders ϕ_1	Top third σ	Top third ϕ_1
⅓	1.17	1.28	1.76	1.38
	(0.47)	(0.18)	(1.09)	(0.21)
⅔	1.17	2.55	1.76	2.77
	(0.47)	(0.36)	(1.09)	(0.41)
0.9	1.17	8.50	1.76	9.22
	(0.47)	(1.21)	(1.09)	(1.38)

Case 3: $\omega_1 + \omega_2 = 1$, $\phi_1 = ½$ (our first alternative); σ and ω_1 estimated

v	All stockholders σ	All stockholders ω_1	Top third σ	Top third ω_1
⅓	1.17	3.25	1.76	3.61
	(0.47)	(0.36)	(1.09)	(0.41)
⅔	1.17	1.45	1.76	1.59
	(0.47)	(0.18)	(1.09)	(0.21)
0.9	1.17	0.98	1.76	1.07
	(0.47)	(0.13)	(1.09)	(0.15)

TABLE 1—*Continued.*

Case 4: ω_1 and ω_2 estimated (our second alternative); $\phi_1 = ½$ and σ from panel A T-bill

v	All stockholders ω_1	All stockholders ω_2	Top third ω_1	Top third ω_2
⅓	1.41	−0.57	1.15	−0.82
	(1.26)	(0.23)	(1.58)	(0.25)
⅔	0.96	−0.04	0.83	−0.16
	(0.63)	(0.11)	(0.79)	(0.13)
0.9	0.84	0.10	0.74	0.01
	(0.47)	(0.08)	(0.59)	(0.09)

Note: Standard errors are in parentheses and account for heteroscedasticity and autocorrelation up to order five.

σ estimates obtained for the two instrument sets when using T-bills (1.23 for the group of all stockholders).

Panel B of Table 1 presents estimates of (7). We present results for several values of v. Using the 1989, 1992, 1995, and 1998 SCF (in which stockholding information is of higher quality than in the CEX), and estimating v based on income flows (as the share of nonfinancial income in total income), v is around 0.90 for all stockholders. It is around 0.83 even for the top third of households in terms of stockholdings. Case 3 (where σ is estimated) again leads to σ estimates above 1. Equally importantly, both case 3 and case 4 estimates of ω_1 are quite large and positive, implying that the conditional expected return to labor income co-moves positively with the expected return to T-bills. When allowing both ω_1 and ω_2 to be estimated in case 4, it is furthermore seen that ω_2 is negative or close to zero suggesting little co-movement of excepted labor-income returns and expected stock returns. This is important for the subsequent estimation of risk-aversion because the labor-income coefficients ω_1 and ω_2 determine the part of the human-capital return due to innovations in expected future returns. Notice also that in the most realistic case of high human-capital shares in total wealth, the estimate of ω_1 is significantly different from zero, counter to previous assumptions made in the literature. Consistent with this, restricting ω_1 to be zero in case 1 and especially case 2 leads to implausible estimates of the share of bonds in the financial portfolio ϕ_1. The estimates of β_1

TABLE 2—ESTIMATION OF RELATIVE RISK-AVERSION USING T-BILL AND STOCK-INDEX RETURN

A. *Consumption Not Substituted Out*

v	γ: All stockholders ($V_{2-1,c} = 0.000191$)	γ: Top third ($V_{2-1,c} = 0.000218$)
Case 1: $\omega_1 = \omega_2 = 0$ (Jagannathan and Wang); σ and ϕ_1 estimated		
⅓	27.0	50.6
	[4.5, 55.8]	[8.5, 185.0]
⅔	32.2	39.7
	[6.6, 56.5]	[8.0, 75.5]
0.9	34.3	36.0
	[7.7, 59.4]	[7.6, 62.7]
Case 2: $\omega_1 = 0$, $\omega_2 = 1$ (Campbell); σ and $\phi_1$estimated		
⅓	10.6	47.6
	[−1.7, 49.8]	[−24.5, 4,391.1]
⅔	8.3	31.1
	[−8.0, 56.6]	[−5.9, 1,031.1]
0.9	6.3	25.1
	[−0.9, 33.5]	[−11.3, 619.1]
Case 3: $\omega_1 + \omega_2 = 1$, $\phi_1 = ½$ (our first alternative); σ and ω_1 estimated		
⅓	−9.7	−21.2
	[−202.4, 94.8]	[−269.4, 172.4]
⅔	10.2	43.6
	[0.8, 56.0]	[6.7, 2,415.7]
0.9	4.9	14.5
	[−0.0, 10.4]	[0.8, 45.8]
Case 4: ω_1 and ω_2 estimated (our second alternative); $\phi_1 = ½$ and σ from Table 1 (panel A) T-bill		
⅓	8.1	28.4
	[−0.6, 18.5]	[0.4, 79.1]
⅔	6.3	19.9
	[0.3, 13.9]	[0.3, 48.2]
0.9	5.5	16.5
	[0.7, 11.8]	[0.7, 39.8]

B. *Consumption Substituted Out*

1. *Risk-Aversion Estimates*

v	γ: $V_{2-1,h_1}, V_{2-1,h_2} = 0$	γ: V_{2-1,h_1}, V_{2-1,h_2}, estimated
Campbell's Assumptions: $\omega_1 = 0$, $\omega_2 = 1$, $\phi_1 = 0$, $\phi_2 = 1$		
0	2.0	2.7
	[0.6, 3.8]	[0.5, 6.5]
⅓	3.0	3.9
	[1.0, 5.7]	[0.8, 10.8]
⅔	5.8	7.6
	[−0.3, 13.3]	[−21.0, 33.2]
0.9	17.5	21.4
	[−20.8, 784.1]	[−8.2, 2,328.6]
Our Alternative Assumptions: $\omega_1 = 1$, $\omega_2 = 0$, $\phi_1 = ½$, $\phi_2 = ½$		
0	4.1	5.8
	[1.4, 7.8]	[−0.8, 16.6]
⅓	6.1	8.8
	[1.8, 11.8]	[−0.5, 24.7]
⅔	11.6	17.0
	[−16.0, 52.6]	[−39.9, 196.0]
0.9	31.7	43.9
	[−5.3, 4,293.2]	[9.8, 7,185.9]

2. *Correlation of Implied Consumption Innovation and Real Stock-Return Innovation* (V_{2-1,h_1}, V_{2-1,h_2} estimated)

v	Correlation coefficients: $\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$
Campbell's Assumptions: $\omega_1 = 0$, $\omega_2 = 1$, $\phi_1 = 0$, $\phi_2 = 1$			
0	1	0.982	0.948
⅓	0.986	0.942	0.897
⅔	0.909	0.852	0.810
0.9	0.775	0.745	0.723
Our Alternative Assumptions: $\omega_1 = 1$, $\omega_2 = 0$, $\phi_1 = ½$, $\phi_2 = ½$			
0	0.997	0.975	0.916
⅓	0.874	0.763	0.661
⅔	0.289	0.208	0.158
0.9	−0.043	−0.077	−0.097

Note: Numbers in square brackets give 95-percent confidence intervals, based on a block-bootstrap.

and β_2 show that consumption growth is quite sensitive to the expected T-bill return but less to the expected stock return, which assuming $\omega_1 = 0$ can only be the case if ϕ_1 is implausibly large.

Armed with estimates of ω_1, ω_2, and σ from our case-3 and case-4 estimations we can proceed to the risk-aversion estimation which is shown in Table 2. In panel A we do not substitute out consumption. Consistent with our conditional Euler equation estimations, we use the after-tax excess return on stocks, again assuming 30-percent tax rates for both stocks and bonds. A more detailed study would attempt to calculate the (time-varying) marginal tax rate for the typical stockholder. If our assumption of a 30-percent tax rate is too high, our risk-aversion estimates will be a bit too low, and conversely. The confidence intervals shown in Table 2 are calculated based on a block-bootstrap in which six-month clusters are used to account for the autocorrelation likely induced by overlapping expectational errors due to our use of semiannual data at the monthly frequency (the confidence intervals do not account for estimation uncertainty in σ, ω_1, ω_2, or $V_{2-1,y}$). The results for cases 3 and 4 show that quite reasonable risk-aversion estimates are obtained when focusing on all stockholders (with the exception of case 3 for low v). The lowest risk-aversion estimates are around 5. Risk-aversion estimates are lower for higher v because the term $V_{2-1,y}$, which measures the covariance between the excess stock-return innovation and current and future labor-income growth innovations is positive (0.0008), and because higher v implies less negative or slightly positive estimates of ω_2. A less negative ω_2 increases $V_{2-1,\mathrm{m}}$ by lowering the value of $\omega_2 V_{2-1,h_2}$ (the term capturing predictability of stock returns, V_{2-1,h_2} is estimated to be -0.0024). Risk-aversion estimates for case 2 are also reasonable, but we believe this is the result of the effect of unrealistic parameter values cancelling out. The unrealistic large ϕ_1 estimate for case 2 tends to increase γ, but the unrealistic large ω_2 assumption tends to lower γ because V_{2-1,h_2} is negative. As for the results for the richest third of stockholders, these show larger risk-aversion estimates than those for all stockholders even for the most realistic cases, 3 and 4. This is due to the larger estimates for σ from Table 1, since $\hat{\gamma} = 1 - \hat{\theta}(1 - 1/\hat{\sigma})$ and $\hat{\theta}$ estimates are negative given the low value of $V_{2-1,c}$ relative to $V_{2-1,\mathrm{m}}$. Note also that since using the consumption-growth data for all households, as opposed to stockholders only, leads to σ estimates much below 1, the risk-aversion estimates for the set of all households would be negative. This emphasizes the importance of excluding households for which we do not expect the Euler equations to be satisfied.

Finally, in panel B of Table 2, we provide the results of following Campbell's approach and substituting out consumption. Results can be compared to Campbell (1996 tables 6 and 9). As an alternative to Campbell's assumption, we assume $\omega_1 = 1$, $\omega_2 = 0$, $\phi_1 = 1/2$, and $\phi_2 = 1/2$. For $v = 2/3$ our assumptions result in a risk-aversion estimate of 17 (but with a very wide confidence interval). This is about twice what is obtained under Campbell's assumptions because we assume that half of the financial portfolio is not in stocks, thus lowering the covariance between the households' total portfolio returns and the stock returns. Notice that risk-aversion estimates now increase in v. This is partly due to the assumption that the ω_2 value does not depend on the assumed v value (and in our case is assumed to be zero). Thus increasing v now lowers $V_{2-1,\mathrm{m}}$. It also lowers $V_{2-1,c}$, which is now calculated based on the consumption innovations implied by the budget constraint. The total effect is an increase in risk-aversion estimates for higher v. As shown in the bottom part of panel B, the advantage of the more realistic assumptions on ϕ_1 and ω_1 is to dramatically reduce the correlation of stock returns and the consumption innovation implied by the budget constraint. Furthermore our assumptions lead to a reduction in the standard deviation of this consumption innovation (not shown), for the case of $v = 0.9$ and $\sigma = 1.5$ from 14.9 percent to 11.6 percent.

IV. Conclusion

Overall, we conclude that the elasticity of substitution for stockholders is likely to be above 1 and that risk-aversion estimates for stockholders as low as 5–10 can be obtained for realistic assumptions about the covariance of consumption growth and stock returns, the share of stocks in the financial wealth portfolio, the properties of the expected returns to human capital, and the share of human capital in overall wealth. Our parameter estimates imply a preference for early resolution of uncertainty ($\theta < 1$). Risk-aversion estimates obtained when not substituting out consumption are quite sensitive to the value of the elasticity of intertemporal substitution, and Campbell's approach of substituting out consumption provides a useful supplement to our main risk-aversion estimates because it does not rely on an estimate of σ. Interestingly, our findings based on Euler equation estimations are consistent with the general-equilibrium results of Ravi Bansal and Amir Yaron (2002). They show that, in an endowment economy with Epstein-Zin preferences and a persistent expected growth-rate component in dividends and consumption, their model is consistent with the observed riskless rate and equity premium for risk aversion of 9.5 and an EIS of 1.5. This further strengthens the argument for loosening the link between risk-aversion and intertemporal substitution.

REFERENCES

Attanasio, Orazio P.; Banks, James and Tanner, Sarah. "Assets Holding and Consumption Volatility." *Journal of Political Economy*, August 2002, *110*(4), pp. 771–92.

Attanasio, Orazio P. and Weber, Guglielmo. "Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption." *Economic Journal*, Supplement 1989, *99*(395), pp. 59–73.

Bansal, Ravi and Yaron, Amir. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." Working paper, The Wharton School, University of Pennsylvania, 2002.

Campbell, John Y. "Understanding Risk and Return." *Journal of Political Economy*, April 1996, *104*(2), pp. 298–345.

Epstein, Larry G. and Zin, Stanley E. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Return: An Empirical Analysis." *Journal of Political Economy*, April 1991, *99*(2), pp. 263–86.

Hall, Robert E. "Intertemporal Substitution in Consumption." *Journal of Political Economy*, April 1988, *96*(2), pp. 339–57.

Hansen, Lars P.; Heaton, John and Yaron, Amir. "Finite-Sample Properties of Some Alternative GMM Estimators." *Journal of Business and Economic Statistics*, July 1996, *14*(3), pp. 262–80.

Ibbotson Associates. *Stocks, bonds, bills, and inflation yearbook*. Chicago: Ibbotson Associates, 2000.

Jagannathan, Ravi and Wang, Zhenyu. "The Conditional CAPM and the Cross-Section of Expected Returns." *Journal of Finance*, March 1996, *51*(1), pp. 3–53.

Lettau, Martin and Ludvigson, Sydney C. "Resurrecting the (C)CAPM: A Cross-Sectional Test when Risk Premia are Time-Varying." *Journal of Political Economy*, December 2001, *109*(6), pp. 1238–86.

Vissing-Jørgensen, Annette. "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution." *Journal of Political Economy*, August 2002, *110*(4), pp. 825–53.