The Promise of Strategic Customer Behavior: On the Value of Click Tracking

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Click tracking is gaining in popularity and the practice of web analytics is growing fast. Whether strategic customers are willing to visit a website when they know their clicks may be tracked is an important yet complex problem which depends on various factors. Using a parsimonious newsvendor framework, we examine this problem by focusing on four key factors that may affect customer incentives to click: inconvenience cost, valuation uncertainty, preference learning, and price. We first demonstrate how the magnitude of the inconvenience cost plays a crucial role in determining the equilibrium outcomes. For low costs, a strong Nash equilibrium always exists where all customers are willing to click. We further study whether customers with uncertain valuation are willing to click. Clicking allows such customers and the firm to learn their preferences but yields *noisy* advance demand information. We find that customer incentives to click are fairly robust to noise. While firm preference-learning always strengthens incentives, customer preference-learning does so only under certain conditions. Price-sensitive demand and markdown pricing may reduce incentives, but price commitment and product personalization can mitigate negative effects for the firm. Compared with "traditional" operations and marketing strategies such as quantity commitment, availability guarantees, quick response and advance selling, click tracking is typically advantageous to both the firm and its customers. Contrary to the conventional wisdom that strategic customer behavior is typically a *peril* for the firm, we demonstrate its *promise* in the context of click tracking.

Key words: Click tracking, clickstream, advance demand information, customer behavior, information technology, Internet, game theory, newsvendor, operations-marketing interface, advance selling

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1. Introduction

Recent Internet click tracking technology has generated the fast growing practice of web analytics[1] and stimulated ongoing research in academia. This research paper, together with an accompanying empirical study (Huang and Van Mieghem 2011), is motivated by our interaction with a U.S. manufacturer of industrial products, hereafter referred to as "the company." The company makes high-end roll-up doors that are customized for industrial and commercial buildings with regards to size, type of material, type of environment, etc. The doors can go into new buildings or can

[1] Forrester forecasts that US businesses will spend $953 million dollars on web analytics software in 2014 with an average compound annual growth rate of 17% (http://www.forrester.com/Research/Document/Excerpt/0,7211,53629,00.html, Retrieved on Oct 18, 2011).

replace older doors. Prices for a door range from the thousands to tens of thousand dollars. Like many others, the company provides current and potential customers with company, product, and contact information on its website. In contrast to e-commerce firms, however, the website is non-transactional and the company sells its products *offline*, either direct or through dealers. The company hires the services of a web analytics firm that specializes in click tracking to help demand forecasting, procurement and inventory planning.

This paper studies the value of click tracking as a mechanism of advance demand (and preference) information. Our companion paper reports on an empirical study of the company's clickstream and sales data to demonstrate the effectiveness of click tracking: The empirical study shows that clickstream data provides the company with advance demand information (ADI) in terms of not only purchasing probabilities and amount, but also purchasing timing. Our data suggests that the company can reduce the inventory holding and backordering cost by about 5% by using clickstream data. The improvement is significant given the noise in the clickstream data and demonstrates how click tracking yields ADI.

While this empirical finding is of great value from the company's perspective, it may sound striking to "average" consumers (or customers): At present, would a significant number of consumers give any thought to how their *individual* clicking decision to visit a website would influence the company's production quantity decisions? Typically, customers visit a website for a variety of reasons: search for products/services needed, learn more information and so on. Given that click tracking is still fairly novel, many consumers are *not* aware of its usage. Hence, a customer does not necessarily consider how the firm will use her online clicking behavior. However, we *expect* that in the near future more customers will realize that their clicks may be tracked. Therefore, this paper studies the value of click tracking technology assuming *strategic* customers are aware that the firm is tracking their clicks and anticipate the firm's optimal actions.

Our "forward-looking" research questions are: Will such strategic customers still be willing to click? What is the resulting value of click tracking technology from an operations management perspective; i.e., how can it help production, inventory, and pricing decisions? And how does it compare with other "traditional" strategies? These questions are important for at least the following reasons: First, whether strategic customers are willing to share their information determines the value of click tracking in the future. Second, answering these questions helps firms decide on which strategy should be adopted. For example, well-known strategies such as pre-orders (i.e., advance selling) have been used to collect demand information. Click tracking is essentially an information sharing channel for the firm and its customers. How do they differ, and which one should be adopted? Our study in §6.2 offers interesting insights.

To answer our research questions, we first analyze the impact of click tracking using a standard newsvendor model where customer valuation is certain and known to the price-taking firm. We demonstrate how the role of the magnitude of the inconvenience cost impacts the existence of Nash equilibria both in pure and mixed strategies. Interestingly, for low costs, a *strong* Nash equilibrium always exists where *all* strategic customers are willing to "click"[2] regardless of the small *individual* impact on the *aggregate* demand. In contrast to other settings, the presence of strategic customer behavior makes also the firm better off: In theory, it could gather *perfect* demand information from strategic customers. Second, we study noisy clicks and preference learning in a more realistic setting where strategic customers have uncertain valuations. Interestingly, customer incentives to click are fairly robust to noise. Then, we investigate settings where pricing may impact customer incentives to click: price-sensitive demand and markdown pricing. We propose *price commitment* and *product personalization* to induce customers to click.

Finally, in a unified framework, we compare click tracking to traditional operations and marketing strategies such as quantity commitment, availability guarantees, quick response and advance selling. Different researchers (a literature review follows in §2) have studied how these strategies improve a newsvendor firm's profit when selling to strategic customers. Our model allows a unified comparison of the strategies that can be used by traditional retailers with the new click tracking technology available to firms with Internet access. This comparison provides insight into the key drivers of these different strategies and provides a recommendation as to which strategy is more valuable in certain stylized settings. We show that click tracking can bring *more* value to the firm than strategic instruments such as quantity commitment and availability guarantees. The reason is that the latter strategies provide incentives to affect strategic customer behavior while click tracking also reduces demand-supply mismatches. We also compare click tracking with advance selling, and we provide a novel threshold-type condition on when one strategy outperforms the other. For example, our finding suggests that click tracking can be better than advance selling for the firm, especially when selling popular products. Compared with all these traditional strategies, notably, click tracking is typically advantageous to both the firm and its customers.

There are three main contributions of the paper. First, driven by both practice and empirical evidence, this appears to be the first study to explore the value of the important click tracking

[2] One can interpret "click" at a higher abstract level as "provide information" throughout the paper.

practice in operations management. Second, our comparisons of clicking tracking with other operations and marketing strategies provide novel managerial insights on which strategy should be adopted. Third, our study also contributes to the recent literature on strategic customer behavior. While strategic customer behavior is typically a *peril* for firms, we demonstrate its *promise* in the context of click tracking.

The outline of this paper is as follows. After reviewing related literature in §2, we present a simple model in §3. In §4, we model noisy clicks and explore the implications of both firm and customer preference-learning. In §5, we investigate how price risk may impact customer incentives to click by extending the simple model to price-sensitive demand and markdown pricing. In §6, we compare click tracking with other operations and marketing strategies. Finally, we provide discussion and point out limitations. All proofs are relegated to the Online Supplement.

2. Related Literature

Our paper is related to several branches of research in operations management, economics, marketing, and information systems literature.

Advance Demand Information and Inventory Management: There is a vast body of literature modeling perfect and imperfect ADI for production planning and inventory control; see, for example, Hariharan and Zipkin (1995), Gallego and Özer (2001, 2003), Özer and Wei (2004), Wang and Toktay (2008) and Gayon et al. (2009). All these papers assume that the firm has ADI and study how to use ADI in inventory management and thus quantify the value of ADI. In this paper, we conduct a complementary study to this literature by focusing on how ADI is obtained and how the interaction between strategic customers and the firm affects the quality of ADI. We study whether strategic customers are willing to click given our empirical validation that click tracking technology does provide ADI (Huang and Van Mieghem 2011).

Strategic Consumer Behavior in Operations: There is a significant literature that explicitly considers strategic consumer behavior; see, for example, Aviv and Pazgal (2008), Besanko and Winston (1990), Cachon and Swinney (2009), Prasad et al. (2010), Swinney (2011) and references therein. We study the willingness of strategic consumer to click and thereby provide ADI, which is a timely addition to this literature. While strategic consumer behavior typically hurts the firm in the literature, in our setting it is a benefit.

Clickstream Research in Marketing: Empirical research on clickstream data is an ongoing active research area in marketing. Moe and Fader (2004), Van den Poel and Buckinx (2005), and Hui et al. (2009) provide a comprehensive literature review. This stream of research focuses mainly on how to model online consumer behavior to best "fit" the observed click behavior with purchase probabilities in e-commerce settings. Different from this empirical literature, our study is theoretical. We are interested in offline-transaction firms with informational websites as well e-commerce using click tracking to collect ADI. It is reported that e-commerce sales only account for 1.2% of all retail sales.[3] Hence, the vast majority of commerce still is executed offline and thus our research setting addresses a larger part of the economy beyond e-commerce. In addition, given the offline ordering lag relative to clicking, we investigate how clicking can be used as ADI for better operations management. Clearly, in an e-commerce setting like Amazon, the time lag between clicks and orders could be on the order of minutes, too short to adjust operational plans. Even in that setting, as we will see in Figure 4, e-commerce such as Amazon can enjoy ADI, the operational benefit, from click tracking by *proactively* asking customers to "click" *before* its procurement or production decisions. In contrast, the company we study observes lead times on the order of weeks and even months. Using newsvendor models that incorporate customers who realize their clicks are tracked for collecting advance information, we provide complementary theory to analyze how strategic customer behavior and click tracking technology affect firms' production, inventory and pricing decisions.

Information Systems: Our work is also related to the information systems literature. Aron et al. (2006) study the impact of intelligent agents on electronic markets with the features of customization, preference revelation and pricing. Murthi and Sarkar (2003) present a literature review of personalization and Yang and Padmanabhan (2005) survey the evaluation of online personalization systems. Trust is often an issue in e-commerce. McKnight et al. (2002) propose and validate measures for a multidisciplinary, multidimensional model of trust in e-commerce. In our theoretic model, we assume information revelation is verifiable and trusted. While the focus of our work is quite different from theirs, we propose personalization as a strategy to mitigate the negative effect of consumers' strategic behavior purely based on operations models.

3. Simple Model

3.1. Model Description

Consider a firm that features a product on its website, uses Internet click tracking technology, and sells a product with a per-unit production cost c at a *fixed* price p to a random number D of discrete customers. Following Deneckere and Peck (1995) and Dana (2001), these customers are randomly drawn by nature from a large population (which we call "potential customers") into the market. We

[3] http://www.ecommercetimes.com/story/19145.html?wlc=1292379670, Retrieved on Oct 22, 2011.

assume that all potential customers are homogeneous: This implies that each potential customer faces the same probability of being selected by nature. After being selected and having entered the market, each customer is only informed of her own presence, but not of the demand realization D. The demand D is a non-negative discrete random variable with cumulative distribution function F, probability mass function f, and expectation $\mu = \mathbb{E}(D) < \infty$.

Customers and the firm are rational decision makers that maximize expected utility and expected profit, respectively. Customer homogeneity implies that each customer has the same utility function U. Specifically, each customer derives deterministic utility v (mnemonic for "valuation") from buying the product, and zero when not buying (the outside option). We assume that one customer buys at most one unit. To avoid trivialities, we assume $v > p$. Before purchasing the product, each customer has the option to visit the firm's informational website. The firm tracks the number of visits (or "clicks") X to predict the number of customers. Each customer incurs an inconvenience cost t (mnemonic for travel or time cost) per visit or click. (Ellison and Ellison 2005 and Fay et al. 2009 assume that t is arbitrarily small. However, for now, we allow any finite cost t because we want to understand how its magnitude affects the equilibrium results.[4])

The **information structure** of the game is as follows: The price p, production cost c, cost t, the demand distribution F and the valuation v are common knowledge. Only X and production quantity q are private information to the firm. Every customer's presence in the market and her click decision are privately known by herself. Notice that the firm has perfect preference information, and thus advance preference information (API) is irrelevant in this simple model.

The **timing** of the game is as follows: At the beginning of the sales season, all customers decide whether or not to visit the website (and "click") simultaneously but *independently*. Upon observing the number of clicks X, the firm updates its demand distribution and then decides its production quantity q. After the firm's production decision has been made, each customer decides whether or not to purchase the product. If $D \leq q$, then all customers are served. Otherwise, the product is rationed anonymously and uniformly, i.e., all customers at the firm receive one unit with probability $\frac{q}{D} < 1$. Ex ante, after a customer enters the market but before clicking, she faces the availability probability $s(q) = \frac{\mathbb{E}\min\{D,q\}}{\mathbb{E}(D)}$, which is also called fill rate or service level, given that the firm produces quantity q (cf. Deneckere and Peck 1995 for how Bayesian updating yields this expression and Dana 2001 for more discussion[5]). Throughout the paper, we assume that the firm can *distinguish*

[4] Assuming an opportunity cost of time of roughly \$20/hr, the inconvenience cost t can be on the order of (1 sec to 1 min)×\$20/hr= \$0.005 to \$0.33.

[5] The only difference is that we assume demand distributions are discrete, which does not change the result.

clicks coming from different customers from IP addresses (e.g., the company in our accompanying empirical study) or email accounts (Figure 4 illustrates an example). Hence, multiple clicks (or clickstreams) from a single customer will be treated as one unit of demand by the rational firm. [6]

Throughout the paper, we assume that the production lead time, i.e., the length of time required to produce the product, does not exceed the click lead time, i.e., the length of time between clicking and purchasing. In other words, the firm has sufficient time to produce to satisfy demand after observing clicks. Notice that it is perfectly admissible for clicks to occur sequentially. Given that click decisions are private information, all we need is that the firm observes the cumulative number of clicks at least before the production lead time.

We use Bayesian Nash equilibrium as our solution concept. Since Bayesian updating is commonly used as default, we can simply refer to Bayesian Nash as Nash. In our model, this is defined as follows: Let $a_i = (\xi_i, \eta_i)$ be customer i's clicking and purchasing strategy profile, where $\xi_i \in [0,1]$ denotes the clicking probability and $\eta_i \in [0,1]$ denotes the purchasing probability. Let $\mathbf{a} = \prod_{i=1}^{D} a_i$ be the vector of all the customers' strategy profile. We denote $\mathbf{a}_{-i}$ as the customers' strategy profile other than customer i. Let $X_i \in \{0,1\}$ be the *realized* clicking decision of customer i. Note that we allow customers to use mixed strategies, while the firm is restricted to pure strategies in choosing its production quantity decision. We also denote $\Pi(q, \mathbf{a})$ as the firm's *expected* profit function, and $U_i(q, a_i, \mathbf{a}_{-i})$ as customer i's *expected* utility function.

A **Nash Equilibrium** $(q^*, \mathbf{a}^*)$ of the game between the firm and customers satisfies:

1. The firm plays a best response given customer behavior: $q^* \in \arg\max_q \Pi(q, \mathbf{a}^*)$. The expected profit involves Bayesian updating of the demand distribution upon observing the number of clicks $X = \Sigma_{i=1}^{D} X_i$.
2. Each customer i plays a best response given firm behavior and other customers' behavior: $a_i^* \in \arg\max_{a_i} U_i(q^*, a_i, \mathbf{a}_{-i}^*)$.

Apparently, our game is a dynamic game with incomplete information, where various refinements of Nash equilibria such as subgame perfection may be desirable and are commonly used in the economics literature. However, for the ease of more fairly comparing the click tracking strategy with other strategies studied in the operations management literature, we decide to simply adopt the Nash equilibrium concept without using refinements: Su and Zhang (2009) use rational expectations equilibrium (REE) to study the value of quantity commitment and availability guarantees,

[6] Moreover, the firm may even *perfectly* observe the identity of each customer from her click (or clickstream), then the firm can first satisfy customers who have clicked which we will call "priority rationing," which just strengthens our findings.

and Cachon and Swinney (2009) use REE to study quick response in dynamic games with incomplete information. Given that "rational expectations" is already *implicitly imbedded* in the Nash equilibrium concept (Kailai 2011), the addition of the term "rational expectations" is not strictly necessary. Notably, Cachon (2010) and Cachon and Swinney (2011) have made it clear that the REE used in the prior literature is essentially Nash.[7] Simply adopting the Nash solution concept is thus consistent with the recent literature on strategic consumer behavior in operations management; see also, Su and Zhang (2008), Tereyağoğlu and Veeraraghavan (2010), Swinney (2011) and references therein.

Only for the sake of demonstrating the robustness of our equilibrium results, we will also *occasionally* use the concept of strong Nash equilibrium (Aumann 1959), which is a Nash equilibrium under which no coalition of players has any profitable deviation. We refer readers to Nessah and Tian (2009) and references therein for the recent literature about the theory and applications of the strong Nash equilibrium. In our setting, such coalition can be formed as follows: *Before* potential customers are drawn into the market, they can freely discuss their strategies without making any binding commitments with each other. Furthermore, we even allow customers to discuss their strategies with the firm. The strong Nash concept is criticized as too "strong" in that it allows for unlimited private communication. For example, a strong Nash equilibrium has to be Pareto efficient. As a result of these stringent requirements, a strong Nash equilibrium rarely exists in general games. However, in our game played by the newsvendor firm and its customers, interestingly, we will specify Nash equilibria that turn out to be also strong.

3.2. Pure Strategy Equilibria

We focus on pure strategy equilibria in this section. There are two counterbalancing forces that drive customer clicking decisions: Clicking induces increased availability but incurs an inconvenience cost t. Proposition 1 below shows that that for any positive cost $t \le \bar{t} \equiv \frac{v-p}{\mu}$, it is in the best interest of every strategic customer to click.

Proposition 1. *(i) If and only if $t \le \bar{t}$, there exists a strong Nash equilibrium where $X^* = q^* = D$: All customers click with probability one and the firm produces the quantity that is equal to the number of observed clicks. In equilibrium, the firm's expected profit is $\Pi^* = (p-c)\mu$, and each customer's expected utility is $U^* = v - p - t$.*

[7] The proof that REE in our game is Nash is available from the authors. We can also show that all the Nash equilibria we are interested in survive the subgame perfection when assigning corresponding beliefs for each node in the information set (see Fudenberg and Tirole 1991 for a general discussion on Nash and subgame perfection in extensive-form games).

(ii) There always exists a Nash equilibrium where $X^* = 0$ *and* $q^* = \min\left\{q \geq 0 : F(q) \geq \frac{p-c}{p}\right\}$*: No customers click and the firm produces the newsvendor quantity. Furthermore, if* $t \geq \bar{t}\mathbb{E}\max\{0, D - q^*\}$*, this equilibrium is a strong Nash equilibrium.*

The key insight of Proposition 1 is that in the pure-strategy equilibrium in this simple model, strategic customer behavior and clicks yield perfect information. The interesting finding is that individual customer's incentive to click *never* disappears as long as $t \leq \bar{t} = \frac{v-p}{\mu}$. It is surprising that each customer is always willing to click even if his own *individual* impact to the *aggregate* market appears small, for example, even if there may be infinitely many customers in the market. This is driven by the fact that $\bar{t} > 0$ no matter whether demand D is unbounded or not. In particular, there is no free-rider problem for small inconvenience costs.

The threshold $\bar{t}$ captures the expected marginal benefit of clicking, which is greatly influenced by the information structure and rationing rule. Under our default uniform rationing, one more click brings one more unit of product into the market given the firm's strategy, but each customer in the market gets the additional unit with equal probabilities. In contrast, if each click also reveals customer identity, one can prioritize allocation and the threshold becomes simply $v - p$, which is independent of the population size. (Note that such setting becomes almost equivalent to advance sales, but without the purchasing commitment, as we will discuss their differences in §6.2.) This demonstrates the significant value of customer identity recognition from click tracking, especially when the expectation of demand μ is large.

The Nash equilibrium in part (ii) is not a strong Nash equilibrium when $t < \bar{t}\mathbb{E}\max\{0, D - q^*\}$, since the grand coalition of all customers and the firm have incentives to deviate to the equilibrium in part (i). This fact demonstrates the role of preplay communication among the players. If t is sufficiently large, namely, $t \geq \bar{t}\mathbb{E}\max\{0, D - q^*\}$, then the inconvenience cost outweighs the upper bound value of clicking $\bar{t}\mathbb{E}\max\{0, D - q^*\}$, which occurs when one more click guarantees availability. To avoid trivialities, we assume $\mathbb{E}\max\{0, D - q^*\} \geq 1$.

To gain some intuition for the $\bar{t}\mathbb{E}\max\{0, D - q^*\}$ threshold, suppose the discrete demand distribution can be approximated by a normal distribution with mean μ and standard deviation σ, then $\mathbb{E}\max\{0, D - q^*\} = \sigma[\phi(z) - (1 - \beta)z]$, where $\beta = \frac{p-c}{p}$ is the critical fractile, $z = z_\beta = \Phi^{-1}(\beta)$, and ϕ and Φ are the density and cumulative distribution function of the standard normal random variable. Hence, this threshold is increasing in the demand volatility, which implies that the region where the strong Nash equilibrium yields no ADI shrinks as σ increases. In other words, when ADI becomes more valuable, customers are more likely to click.

3.3. Mixed Strategy Equilibria

Proposition 1 characterizes the existence of equilibria in which the firm collects either *perfect* information or *no* information at all. It is natural to ask whether there are equilibria in which the number of clicks provides *imperfect* (or partial) information about the demand realization. This could result from customers using mixed strategies of clicking or not. (Later in this paper we will consider valuation uncertainty as another mechanism that yields imperfect ADI.) To be specific, consider mixed strategies where the *coalition* of all customers chooses the same probability of clicking $\xi \in (0,1)$.[8] One can think of such a mixed strategy as a distribution of pure strategies chosen by *different* customers. For instance, one can treat the mixed-strategy ξ as a fraction of customers who click with probability one.

We analyze these mixed strategies as follows: Given that the demand realization is $D = d$ and each customer clicks with probability ξ, the number of clicks X follows a binomial distribution with parameters d and ξ: $\mathbb{P}(X = n|D = d) = \binom{d}{n}\xi^n(1-\xi)^{d-n}$, where $n \leq d$. Upon observing $X = n$, the firm derives the posterior demand distribution which we denote as $F_{D|X}$ using Bayesian updating: $\mathbb{P}(D = d|X = n) = \frac{\mathbb{P}(X=n|D=d)f(d)}{\sum_{d' \geq n}\mathbb{P}(X=n|D=d')f(d')}$. We denote $q^*(n) = \min\{q \geq 0 : F_{D|n}(q) \geq \frac{p-c}{p}\}$ as the optimal newsvendor quantity when demand distribution is $F_{D|n}$. Given the firm's strategy $q^*(n)$, customers choose the clicking probability ξ^* that maximizes their expected utility $U(\xi, q^*(X)) = (v-p)\frac{\mathbb{E}_D\mathbb{E}_{X|D}\min\{D,q^*(X)\}}{\mathbb{E}(D)} - \xi t = \bar{t}\mathbb{E}_D\mathbb{E}_{X|D}\min\{D, q^*(X)\} - \xi t$. Let $\xi^* \in \arg\max_{\xi \in (0,1)} U(\xi, q^*(X))$. There exists a mixed strategy equilibrium where each customer randomizes to click with probability ξ^* and the firm produces quantity $q^*(n)$ when n clicks are observed, if and only if $\xi^* \in (0,1)$ exists.[9]

In conclusion, the existence of mixed strategy equilibria depends on verification of the existence of $\xi^* \in (0,1)$. Offering a closed-form condition in terms of the model parameters for the existence of a mixed-strategy equilibrium is challenging, since the dependence of the utility function $U(\xi, q^*(X))$ on the clicking probability ξ is intricate through the newsvendor fractile.[10] However, it is straightforward to numerically investigate this problem. To illustrate the analysis and the surprising results, we start with a simple numerical example.

Example: Suppose demand D follows a two-point distribution: $\mathbb{P}(D = 0) = f(0) = 1-\theta$, and $\mathbb{P}(D = d) = f(d) = \theta \in (0,1)$, where d is a strictly positive integer.

[8] We refer the readers to Deneckere and Peck (1995) and Dana (2001) for a similar approach. Deneckere and Peck (1995) argue that it is "without loss of generality" to restrict consideration to equilibria in which all customers choose the *same* mixed strategy. One can similarly define the equilibrium by treating customers individually maximizing their utilities in mixed-strategies. However, it is not even amenable to computational analysis.

[9] Note that the simple commonly-used rule that a player should be indifferent to pure strategies when a mixed strategy equilibrium exists for simple bi-matrix games does *not* apply here since $U(\xi, q^*(X))$ depends on ξ intricately.

[10] Even working with conjugate distributions does not yield analytical tractability.

First, Bayesian updating yields the conditional distribution of $D|X$ for the firm: $\mathbb{P}(D=0|X=0)=\frac{1-\theta}{(1-\xi)^d\theta+1-\theta}$, and $\mathbb{P}(D=d|X=0)=\frac{(1-\xi)^d\theta}{(1-\xi)^d\theta+1-\theta}$, while $\mathbb{P}(D=0|X>0)=0$ and $\mathbb{P}(D=d|X>0)=1$.

Second, the firm sets optimal production quantity $q^*(n)=\min\{q\geq 0: F_{D|X=n}(q)\geq\beta\}$ for $n=0,1,...,d$. Only observing no clicks is non-trivial, and indeed $F_{D|X=0}$ is case specific: If $\frac{1-\theta}{(1-\xi)^d\theta+1-\theta}\geq\beta$, i.e., $\xi\geq 1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, then $q^*(0)=0$; otherwise, $q^*(0)=d$. Clearly $q^*(n)=d$ for $n=1,...,d$.

The third step is to derive the customer utility function. If $\xi\geq 1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, then some calculation yields $U(\xi)=U(\xi,q^*(x))=(v-p)[1-(1-\xi)^d]-\xi t$; otherwise, $U(\xi)=v-p-\xi t$.

Next, we investigate whether a mixed-strategy equilibrium exists. If $\xi<1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, it is clear that there is no $\xi^*\in(0,1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}})$ that maximizes $U(\xi)$. For the first case, the first-order optimality condition (which is both necessary and sufficient) yields

$$\xi^*=1-\sqrt[d-1]{\frac{t}{(v-p)d}}.$$

Only if $\xi^*\geq 1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, i.e., $\beta\leq\overline{\beta}=\frac{1}{1+\frac{\theta}{1-\theta}\frac{t}{(v-p)d}\sqrt[d-1]{\frac{t}{(v-p)d}}}$, and $t<(v-p)d$, does there exist a mixed-strategy equilibrium in which each customer clicks with probability ξ^*. Denote $\bar{t}_m=(v-p)d$. Notice that $\bar{t}_m=\theta d^2\bar{t}$, so that $\bar{t}_m$ is quite different from $\bar{t}$ because they serve different purposes and have different interpretations: In the pure strategy equilibrium case, $\bar{t}$ can be interpreted as the marginal benefit of clicking versus not clicking. However, any mixed strategy equilibrium ξ^* balances the marginal benefit of availability improvement (which is intricate) and the marginal cost t, and $\bar{t}_m$ merely ensures that such a balance is sustained. If $\beta>\overline{\beta}$, then no mixed strategy equilibrium exists for any $t>0$. The reason is that, when β is large, the optimal newsvendor quantity already ensures full availability, and thus not clicking is each customer's optimal strategy.

Finally, we are interested in comparing the mixed-strategy equilibrium (which yields imperfect ADI) with the pure strategy equilibrium (which yields perfect ADI) in terms of consumer utility, firm profit and social welfare. Customers are always better off in the mixed strategy equilibrium than in the pure strategy equilibrium by definition of the mixed strategy equilibrium. Indeed, with the two-point distribution, direct calculation yields the consumer utility in the mixed-strategy equilibrium:

$$U(\xi^*)=U^*+\frac{d-1}{d}t\sqrt[d-1]{\frac{t}{(v-p)d}}>U^*,$$

for any $d>1$. Plugging in the equilibrium clicking probability and the firm's quantity decisions, we obtain the firm's expected profit:

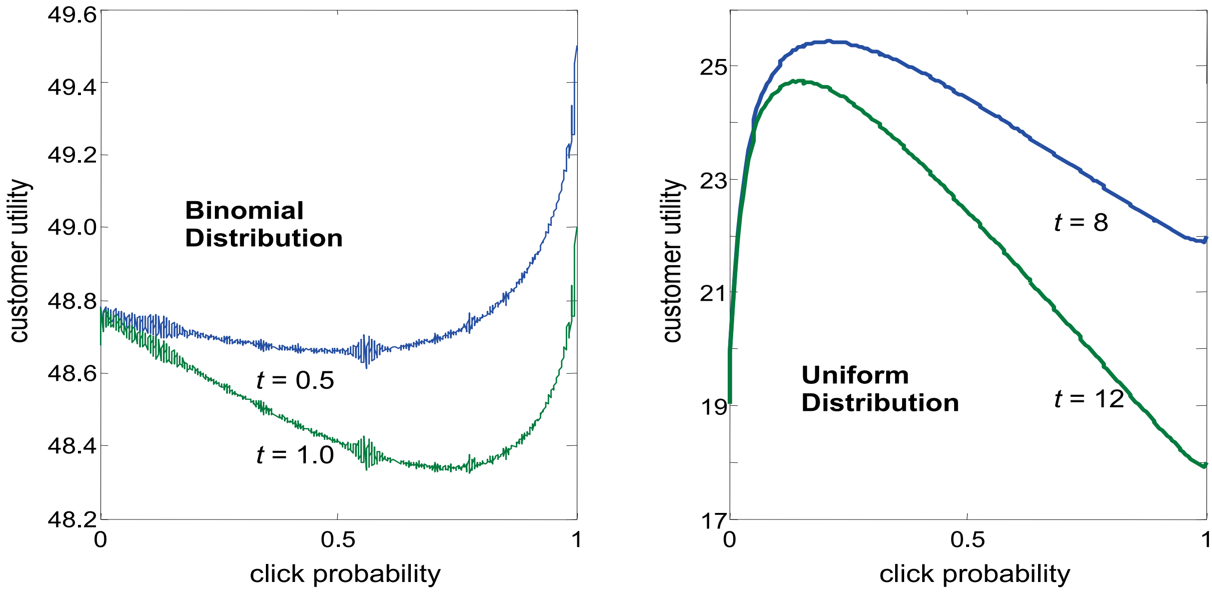
$$\Pi(\xi^*)=(p-c)\mu\left[1-\frac{t}{(v-p)d}\sqrt[d-1]{\frac{t}{(v-p)d}}\right]<\Pi^*=(p-c)\mu.$$

Hence, the firm is strictly worse off. This conclusion is also general, since in mixed strategy equilibria, the firm still has supply-demand mismatches. The question is what happens to social welfare. The expected social welfare for the mixed-strategy equilibrium is defined as $W(\xi^*) = \Pi(\xi^*) + \mu U(\xi^*)$. Recall that the social welfare with perfect ADI is simply $W^* = \Pi^* + \mu U^* = (v-c-t)\mu$. Some calculation yields $W(\xi^*) = W^* + \mu t \sqrt[d-1]{\frac{t}{(v-p)d}} \frac{(v-p)(d-1)-(p-c)}{(v-p)d}$. Hence, if $d > \overline{d} = \frac{p-c}{v-p} + 1$, then social welfare is strictly improved if the mixed-strategy equilibrium emerges. Otherwise, social welfare decreases. The reason is that, as the expected number of customers in the market becomes large, the customers' utility gain over the clicking cost due to customers' mixing exceeds the firm's profit loss due to supply-demand mismatches.

We summarize the following general insights from this simple example: First, there may not exist a mixed-strategy equilibrium for any strictly positive inconvenience cost, which stands in contrast to the result in Proposition 1. Second, if there exists a mixed-strategy equilibrium, then customers are better off while the firm is worse off in this imperfect-ADI equilibrium compared to the equilibrium that yields perfect ADI. Third, the social welfare comparison depends on the expected demand: For a sufficiently large market, imperfect-ADI equilibria yield strictly higher social welfare simply because the welfare gain from customers is greater than the welfare loss from the firm.

Next, we turn to an extensive numerical study to investigate the existence of mixed-strategy equilibria for three typical distributions: (1) Demand D follows a binomial distribution. Our numerical examples cover a wide range of parameters in terms of the critical fractile β, the inconvenience cost t and the parameters of the demand distribution. Surprisingly, for all our numerical examples, we have not found a single example where a mixed strategy equilibrium exists. We present a representative example for the following data: $\beta = 0.4$, $v = 90$, $p = 40$, and demand D follows the binomial distribution $\mathbf{B}(200, 0.7)$ with sample size 200 and "success probability" 0.7. The left panel in Figure 1 plots the customer utility as a function of the click probability for different inconvenience costs and shows that: First, the customer utility is not necessarily continuous with respect to the click probability due to the fact that the optimal quantity $q^*(X)$ is an integer that depends on the click probability discontinuously. Second, when the click probability is low, the customer utility can be lower than the utility without clicking; when the click probability is high, higher click probability brings higher utility. (2) Demand D follows a Poisson distribution. We have similar findings in these cases, and we have not found a single example where a mixed-strategy equilibrium exists. (3) Demand D follows a uniform distribution over the finite set $\{0, 1, 2, ..., d\}$. In these cases, depending on the parameters, there are examples where no mixed-strategy equilibria

Figure 1 **Examples of Mixed-strategy Equilibria Existence (right panel with $\beta = 0.4$, $v = 90$, $p = 60$, $D \sim \mathbf{U}\{0,1,2,...,179\}$) and Non-Existence (left panel with $\beta = 0.4$, $v = 90$, $p = 40$, $D \sim \mathbf{B}(200, 0.7)$).)**



exist and examples where mixed-strategy equilibria do exist. The right panel in Figure 1 shows a representative example where a mixed-strategy equilibrium exists for the following data: $\beta = 0.4$, $v = 90$, $p = 60$, and $d = 179$. It shows that: First, for either one of the two fixed click costs, there exists a unique mixed strategy equilibrium. Second, the equilibrium click probability decreases in the click cost.

Our numerical study demonstrates that the existence of a mixed strategy equilibrium depends crucially on the nature of the demand distribution. Intuitively, increasing the click probability increases the availability benefit but also the inconvenience cost. While the marginal click cost is a constant t, the marginal availability benefit can be higher or lower than t. For the left graph in Figure 1, when the click probability is low (e.g., below 0.5), then the marginal availability benefit is smaller than t. However, when the click probability is high (say above 0.9) then the marginal availability benefit is greater than t. Hence, no mixed-strategy equilibria exist. For the right graph in Figure 1, the reverse trend holds, so that we can find $\xi^* \in (0,1)$ that maximizes the customer utility.

To summarize our equilibrium analysis: Whether the firm can obtain any ADI crucially hinges on the inconvenience cost t. When $t \le \bar{t}$, there exists a pure-strategy strong equilibrium where perfect ADI is obtained. When $t \ge \bar{t}\mathbb{E}\max\{0, D - q^*\}$, there exists a pure-strategy strong equilibrium where no ADI is obtained. Imperfect ADI may be obtained when customers choose to click with some non-degenerate probability to click in equilibrium. However, as demonstrated by our numerical study, predicting the existence of such mixed-strategy equilibria is difficult as it depends not only on the click cost, but also on the underlying demand distribution. Therefore, for the remainder, we will focus on pure strategies.

Remark. So far the simple model allowed us to focus on the following simple assumption: a customer's decision to visit a website is driven by the tradeoff between his inconvenience cost of clicking t versus how his click will affect the firm's production quantity and the availability of the product. This assumption is made intentionally: We believe that click tracking is novel because it significantly reduces the inconvenience cost t for consumers compared to traditional information sharing channels such as market surveys, telephone calls, and so on. Our analysis in the simple model clearly shows the crucial role that the inconvenience cost t plays in customer incentives to share their demand information, and thus demonstrates the unique advantage of click tracking as an information sharing channel between the firm and its customers. In other words, click tracking is unique in that, the clicking cost t is relatively small. We also refer to click tracking of strategic customers as "strategic clicks" to emphasize that such clicks are provided endogenously by strategic customers. In what follows, we follow Ellison and Ellison (2005) and Fay et al. (2009) and assume that t ***is strictly positive yet arbitrarily small***. This assumption also allows us to isolate and focus on **other factors** (noise, preference learning, pricing) that may affect customers' incentives to click.

4. Random Conversion, Noisy Clicks, and Learning

4.1. Random Conversion and Noisy Clicks

In the simple model, every click necessarily leads to a purchase. However, in reality, some customers who click do not "convert," i.e., they visit the website without purchasing the product eventually. This means that in practice the clicks data is *noisy* in that the firm cannot perfectly distinguish *buyers* from *non-buyers* who click without purchasing the product. Such noisy clicks provide the firm with *imperfect* ADI, which often results from customers' *valuation uncertainty* of the product when they click. Recall that, in the simple model, mixed strategy equilibria can model imperfect ADI. However, the number of clicks always provides a *lower* bound for the number of purchases since every click necessarily leads to a purchase in that model, which is not always realistic. Typically, customers search online or offline to *learn* more information about the product. After they learn enough information, their valuation or willingness-to-pay of the product is realized before purchasing the product. Only if their valuation turns out to be higher than the price, will they buy.

In this section, we are interested in how the "noise" of clicks affects customers' incentives to click. To isolate this effect, we will first assume that customer valuation uncertainty is resolved even without clicking (e.g., time itself and alternative learning channels can resolve this uncertainty). After that focus on noise, we will study the impact of preference learning from clicking.

To focus on the effect of noisy clicks, we continue with the simple model where the selling price p is exogenously given, but add valuation uncertainty. To analyze the impact of noisy click data, we distinguish between a large population of a random number N of homogenous strategic customers with uncertain valuation V and the *actual* number of buyers D. Note that customers are homogenous ex ante, i.e., before clicking. However, they are ***heterogenous*** ex post, i.e., after clicking, their valuation realizations may be different. We assume that N is approximately normally distributed with mean μ_N and standard deviation σ_N. Using the continuous distribution for the number of discrete customers is an *approximation* purely for the sake of analytical tractability. Denote the coefficient of variation of N by $COV_N = \frac{\sigma_N}{\mu_N}$. Before clicking, the prior valuation V has distribution function $G(.)$ and density $g(.)$ over the support $[v_L, v_H]$, where $p > v_L$.[11] Let the mean of V be μ_V and standard deviation be σ_V. After her valuation uncertainty is resolved, a customer buys if $v \geq p$, and does not buy otherwise.

The sequence of events is similar as before: At the beginning of the sales season, customers decide whether to click. Then the firm observes the number of clicks and uses Bayesian updating to forecast demand. In contrast to the simple model, the firm can no longer infer the exact number of realized demand from the clicks, but only the *potential demand*.

To characterize strong Nash equilibria in this game, it is useful to first go through the following preliminary analysis.

If all customers click, then $X = N$ and the firm knows the market size. Suppose $N = n$ clicks are observed, then the number of buyers D follows a binomial distribution with parameters $\overline{G}(p)$ and n. For a large n, this binomial distribution can be approximated by a normal distribution with mean $\mathbb{E}(D|n) = n\overline{G}(p)$ and variance $Var(D|n) = n\overline{G}(p)G(p)$. Note that the coefficient of variation is $COV(D|n) = \sqrt{\frac{G(p)}{n\overline{G}(p)}}$. If $G(p) = 0$, $COV(D|n) = 0$, and demand information is *perfect*. As $G(p)$ becomes larger, the demand information is *less informative* or *noisier*. Upon observing n clicks, the firm solves its newsvendor problem by stocking quantity $q^*_{D|n} = n\overline{G}(p) + z\sqrt{n\overline{G}(p)G(p)}$. The firm's expected profit is $\Pi = \mathbb{E}_N[\Pi(N)] = (p-c)\mu_N\overline{G}(p) - p\phi(z)\mathbb{E}_N\left[\sqrt{N\overline{G}(p)G(p)}\right]$.

If none of the customers click, then the firm can only use its prior demand distribution. Since the conditional random variable $D|N$ is approximately normally distributed and N is normally

[11] Note that §3 includes the case when $p \leq v_L$.

distributed, the unconditional demand D also approximately follows a normal distribution with mean $\mu_D = \mathbb{E}(D) = \mathbb{E}\left[\mathbb{E}(D|N=n)\right] = \mu_N \overline{G}(p)$ and variance $\sigma_D^2 = \mu_N \overline{G}(p)G(p) + \sigma_N^2 \overline{G}^2(p)$. The firm thus uses its optimal newsvendor stocking quantity $q_D^* = \mu_N \overline{G}(p) + z\sqrt{\mu_N \overline{G}(p)G(p) + \sigma_N^2 \overline{G}^2(p)}$. The firm's expected profit is $\Pi_0 = (p-c)\mu_N \overline{G}(p) - p\phi(z)\sqrt{\mu_N \overline{G}(p)G(p) + \sigma_N^2 \overline{G}^2(p)}$. Note that σ_N partially measures the imperfection of strategic clicks as ADI. If $\sigma_N = 0$, then clicking or not clicking would not make a difference for the firm. As σ_N is large, clicking provides more demand information for the firm.

With a fixed price, customers are only concerned with availability. The fill rate when they click is $s_C = \mathbb{E}_N\left\{\frac{\mathbb{E}\min\{D|N, q_{D|N}^*\}}{\mathbb{E}(D|N)}\right\}$, otherwise the fill rate is $s_N = \frac{\mathbb{E}\min\{D, q_D^*\}}{\mathbb{E}(D)}$. Define $(V-p)^+ = \max\{0, V-p\}$. Clicking is better than no clicking if $U = s_C \mathbb{E}(V-p)^+ - t \geq U_N = s_N \mathbb{E}(V-p)^+$.

To state the strong Nash equilibria, we denote

$$t_0 = [\phi(z) - (1-\beta)z]\sqrt{\frac{G(p)}{\overline{G}(p)}}\left[\sqrt{\frac{1}{\mu_N} + \frac{\overline{G}(p)}{G(p)}COV_N^2} - \mathbb{E}\sqrt{\frac{1}{N}}\right]\mathbb{E}(V-p)^+,$$

and

$$t_1 = [\phi(z) - (1-\beta)z]\sqrt{\frac{G(p)}{\overline{G}(p)}}\left[\mathbb{E}\sqrt{\frac{1}{N-1}} - \mathbb{E}\sqrt{\frac{1}{N}}\right]\mathbb{E}(V-p)^+,$$

For convenience, we define $\bar{t}_I = \min\{t_0, t_1\}$ as a similar threshold to $\bar{t}$, but when clicks are noisy.

Proposition 2. *If and only if $t \leq \bar{t}_I$, a strong Nash equilibrium exists, in which all customers click, and the firm produces quantity $q_{D|X}^* = X\overline{G}(p) + z\sqrt{X\overline{G}(p)G(p)}$ upon observing X clicks. Furthermore, in equilibrium, the value of observing noisy clicks is strictly positive,*

$$\Delta\Pi = \Pi - \Pi_0 = p\phi(z)\sqrt{\overline{G}(p)G(p)}\left[\sqrt{\mu_N + \frac{\overline{G}(p)}{G(p)}\sigma_N^2} - \mathbb{E}\sqrt{N}\right] > 0. \tag{1}$$

For the equilibrium in Proposition 2 to exist, the threshold $\bar{t}_I$ must be strictly positive. While it is guaranteed that $\bar{t} > 0$ in the simple model, $\bar{t}_I$ can be negative. Indeed, given that $t_0 > 0$, $\bar{t}_I > 0$ if and only if

$$\sqrt{\frac{1}{\mu_N} + \frac{\overline{G}(p)}{G(p)}COV_N^2} > \mathbb{E}\sqrt{\frac{1}{N}}. \tag{2}$$

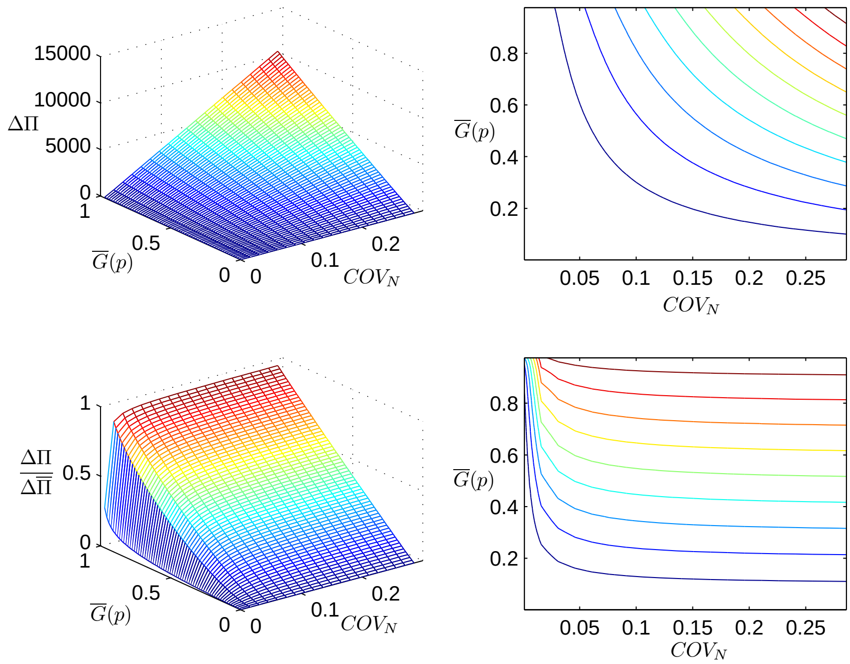
Condition (2) highlights the key factors that induce strategic customers to click: Jensen's inequality yields $\sqrt{\frac{1}{\mu_N}} = \sqrt{\frac{1}{\mathbb{E}(N)}} < \mathbb{E}\sqrt{\frac{1}{N}}$ since the function $\sqrt{\frac{1}{x}}$ is strictly convex. Therefore, the population size N needs to be highly uncertain (large coefficient of variation COV_N), or the purchasing probability $\overline{G}(p)$ needs to be high for strategic customers to click.

Table 1 Numerical Experiments of Imperfect ADI

μ_N	COV_N	$\overline{G}(p)$	Percentage of cases where strategic customers will click
30	0.001:0.001:0.3	0.001:0.001:0.99	97.28%
50	0.001:0.001:0.3	0.001:0.001:0.99	98.38%
100	0.001:0.001:0.3	0.001:0.001:0.99	99.21%
200	0.001:0.001:0.3	0.001:0.001:0.99	99.62%
300	0.001:0.001:0.3	0.001:0.001:0.99	99.77%
500	0.001:0.001:0.3	0.001:0.001:0.99	99.88%
1000	0.001:0.001:0.3	0.001:0.001:0.99	99.99%
2000	0.001:0.001:0.3	0.001:0.001:0.99	99.99%
4000	0.001:0.001:0.3	0.001:0.001:0.99	99.99%
5000	0.001:0.001:0.3	0.001:0.001:0.99	100%
10000	0.001:0.001:0.3	0.001:0.001:0.99	100%
60000	0.001:0.001:0.3	0.001:0.001:0.99	100%
100000	0.001:0.001:0.3	0.001:0.001:0.99	100%
1000000	0.001:0.001:0.3	0.001:0.001:0.99	100%
10000000	0.001:0.001:0.3	0.001:0.001:0.99	100%

Notes. For each value of μ_N, there are 297,000 number of parameter cases.

Figure 2 The value of imperfect ADI increases as population size is more uncertain and valuations are higher ($p = 10, c = 6, \mu_N = 10000$)



To investigate the parameter regimes where this inequality holds, we conducted a numerical study. As detailed in Table 1, when $\mu_N > 5000$, strategic customers are *always* willing to click. Even when μ_N is small, in more than 97% percentage of cases they are willing to click. The few cases they are not willing to click are when μ_N, COV_N and $\overline{G}(p)$ are *all* small. This is intuitive: If the potential market size is small, fairly certain, and it is most likely that each customer will not purchase, it does not make sense to click given a strictly positive inconvenience cost.

We also conducted a numerical study of the value of imperfect ADI $\Delta\Pi$ expressed in equation (1) as a function of the underlying parameters. One representative example is shown in Figure 2 which shows that $\Delta\Pi$ is increasing in both σ_N and $\overline{G}(p)$. Moreover, the numerical study suggests that $\Delta\Pi$ increases fairly linearly in each parameter separately.

The upper bound value is obtained if $\overline{G}(p) = 1$ when clicks provide perfect ADI: $\Delta\overline{\Pi} = p\phi(z)\sigma_N$. Hence, the relative value of strategic clicks can be defined as: $\frac{\Delta\Pi}{\Delta\overline{\Pi}} = \sqrt{\overline{G}(p)G(p)}\left[\sqrt{\frac{\mu_N}{\sigma_N^2}+\frac{\overline{G}(p)}{G(p)}} - \frac{\mathbb{E}\sqrt{N}}{\sigma_N}\right]$, which is shown as a function of the purchasing probability $\overline{G}(p)$ and the coefficient of variation COV_N in Figure 2. Thus, also the relative performance gain is increasing in both σ_N and $\overline{G}(p)$.

In conclusion, if the clicks are noisy but the total population is large and volatile and the customers' purchasing probability is not too small, then strategic customers are willing to click. The resulting noisy clicks always provide strictly positive benefit for the firm. The main finding from this section is that ***customer incentives to click are fairly robust to noise***.

4.2. Preference Learning by the Firm from Clickstreams

In the previous section, customer valuation uncertainty naturally introduces noise to ADI. We made two assumptions in that section: First, the firm only learns the *quantity*, i.e., the size of the potential demand by observing clicks. Hence, the posterior belief of the purchasing probability of each customer who has clicked is the same as the prior belief, i.e., $\overline{G}(p)$. In other words, the firm does not learn anything about a customer's valuation from her clicking behavior. Second, clicking does not affect customers' valuation *realization* or *distribution*.

In this section and next, we relax both assumptions by incorporating firm and customer preference-learning respectively. This relaxation is desirable, because customers typically go through "clickstreams" rather than a single abstract "click" (which has been the focus in the previous sections). **Clickstreams record the sequence of clicks which may reflect customer heterogeneity.** Such clickstreams provide the firm a platform to learn customer valuation (For example, regression equations can be used to predict the purchasing probability associated with each individual's clickstream). Meanwhile, each customer can learn more about the firm's product offerings, and thus her valuation of the product, by browsing the firm's informational website.

In this section, we incorporate firm preference-learning to study its impact on our findings in the previous section. To this end, it is useful to recognize the following two extreme cases: First, assume noise but *no* preference learning from clickstreams. Then we are back in the setting of

§4.1. Upon observing n clicks, the firm solves its newsvendor problem by stocking quantity $q^*_{D|n} = n\overline{G}(p) + z\sqrt{n\overline{G}(p)G(p)}$. Second, suppose there is *perfect* preference learning, so that the firm learns the valuation of each clicking customer. Then the firm knows D exactly for any observed number of clicks $N = n$, and we are back to the simple model in §3. We are interested in the case when the firm's learning is *imperfect*. The reason is that the preference heterogeneity among potential customers is typically *partially* revealed in their clickstreams, and the firm may use statistical regressions or other methods (such as data mining and artificial intelligence) to capture this heterogeneity.

We model this imperfect preference-learning by the firm as follows for a given realization of the potential demand $N = n$: The demand D follows the normal distribution $N(n\overline{G}(p), n\overline{G}(p)G(p))$ as before. The novel part is that customers' clickstream data provides the firm with a noisy signal S. The distribution of this signal conditional on the true demand D is approximately $N(D, \eta)$, where $\eta \geq 0$ measures the degree of noise in the signal from clickstream data. When $\eta = 0$, the signal is the same as the demand so that the preference learning is *perfect*. When $\eta = \infty$, the signal is completely *uninformative*. We can link this abstract model to the practice of using a random utility model (see Huang and Van Mieghem 2011) as follows: Notice that the true demand $D = \sum_{i=1}^n \mathbf{1}_i$, where $\mathbf{1}_i = 1$ if customer i purchases, otherwise, $\mathbf{1}_i = 0$. Denote customer i's clickstream by $\mathbf{X}_i$, which is a vector capturing online click behavior including the IP address, the number of visits, visit duration, pages browsed, and so on. Then, the associated random utility for customer i from purchasing the product is $U_i(\mathbf{X}_i)$ while her utility of not purchasing is normalized to zero. Define $\widehat{\mathbf{1}}_i = \mathbf{1}_{\{U_i(\mathbf{X}_i)>0\}}$, then we can write $S = \sum_{i=1}^n \widehat{\mathbf{1}}_i$. Hence, the signal S is a noisy indicator of demand D. The conditional expectation of the demand D upon observing the signal S is: $\widetilde{D} = \mathbb{E}(D|S) = \frac{\eta n\overline{G}(p) + Sn\overline{G}(p)G(p)}{\eta + n\overline{G}(p)G(p)}$, since one can prove that $D|S$ follows the normal distribution $N\left(\frac{\eta n\overline{G}(p) + Sn\overline{G}(p)G(p)}{\eta + n\overline{G}(p)G(p)}, \left(\frac{1}{\eta} + \frac{1}{n\overline{G}(p)G(p)}\right)^{-1}\right)$. (For brevity, we omit the proof and refer readers to DeGroot 1970.) We can also obtain that $\widetilde{D}$ follows a normal distribution with mean $n\overline{G}(p)$ and variance $\frac{[n\overline{G}(p)G(p)]^2}{\eta + n\overline{G}(p)G(p)}$. And the conditional demand $D|\widetilde{D}$ follows the normal distribution with mean $\widetilde{D}$ and variance $\frac{\eta n\overline{G}(p)G(p)}{\eta + n\overline{G}(p)G(p)}$.

Equivalently, we can model this imperfect preference-learning by *directly* introducing the noisy signal $\widetilde{D}(N)$ (which is just an affine transformation of $S(N)$ as we have seen) of the true demand D when $X = N$ clicks are observed, and $\widetilde{D}(N)$ is the mean of the noisy prediction of the demand. Formally, we have $D(N) = D|N = \widetilde{D}(N) + \varepsilon(N)$, and $\mathbb{E}(\varepsilon(N)) = 0$. Given any number of clicks $X = N$, we make the following assumptions: (1) The distribution of the noisy signal $\widetilde{D}$ is a normal distribution with mean $N\overline{G}(p)$ and variance $\frac{[N\overline{G}(p)G(p)]^2}{\eta + N\overline{G}(p)G(p)}$, where $\eta \in [0, \infty]$ is a parameter that measures the noise of the demand signal. (2) The error term ε follows the normal distribution with mean zero and variance $\frac{\eta N\overline{G}(p)G(p)}{\eta + N\overline{G}(p)G(p)}$. Based on these two assumptions, the conditional demand $D|\widetilde{D}$

upon observing signal $\widetilde{D}$ follows the normal distribution with mean $\widetilde{D}$ and variance $\frac{\eta N\overline{G}(p)G(p)}{\eta+N\overline{G}(p)G(p)}$. Within this framework, when there is no preference learning, i.e., $\eta=\infty$, we have $D(N)|\widetilde{D}(N)$ follows the normal distribution with mean $N\overline{G}(p)$ and variance $N\overline{G}(p)G(p)$ so that the signal is most noisy, and we are back to the base model of noisy clicks in §4.1. When there is perfect preference learning, i.e., $\eta=0$, we have $D(N)=\widetilde{D}(N)$ so that there is no noise in the signal, and we are back to the simple model in §3. The signal noise parameter η manifests how API determines the quality of ADI.

We are now ready to investigate how the signal noise parameter η affects customers' incentive to click. Given the customer expected utility by clicking as a function of η: $U(\eta)=s_C(\eta)\mathbb{E}(V-p)^+-t$, whether customers are willing to click depends on how $s_C(\eta)$ behaves. We can derive the service level in closed form, as shown in Proposition 3:

PROPOSITION 3. *The service level* $s_C(\eta)=1-[\phi(z)-(1-\beta)z]\,\mathbb{E}\sqrt{\frac{G(p)}{N\overline{G}(p)\left(1+\frac{N\overline{G}(p)G(p)}{\eta}\right)}}$ *is strictly decreasing in* η*.*

Proposition 3 implies that the customer expected utility $U(\eta)$ from clicking is strictly decreasing in η. Hence, for any finite η, $U(\eta)$ is higher than $U(\infty)$, which corresponds to the base model of noisy clicks in §4.1. Hence, the conclusion is that, ***firm preference-learning strengthens our previous finding*** (i.e., customers have more incentives to click so that the equilibrium in Proposition 2 remains), and it brings benefits to all parties.

4.3. Preference Learning by Customers from Clickstreams

Now we turn to customer preference-learning, which can be modeled in two different ways: (1) Before clicking, customers know their distribution $G(.)$ of random valuation V. After clicking, they learn their actual valuation v. Hence, clicking purely resolves customer valuation uncertainty without affecting customer *intrinsic* valuation of the product. (2) Clicking changes the valuation distribution. Hence, clicking shifts customers' intrinsic valuation profile or distribution of the product. For example, learning information from the website may make customers' valuation more dispersed. Depending on the nature of the website and the product, one way of modeling customer preference-learning may be more appropriate than the other. We study these two types of learning one by one.

Suppose clicking is the only way for customers to resolve their valuation uncertainty. We are interested in how this type of customer learning affects our previous results. If no customers click, their utility $U_{0N}=s_{0N}\max\{0,\mathbb{E}(V-p)\}$. Hence, if $\mu_V\geq p$, then all potential customers actually purchase the product, and thus the firm's optimal service rate $s_{0N}=1-COV_N\,[\phi(z_\beta)-(1-\beta)z_\beta]$.

Each customer's expected utility is $U_{0N} = s_{0N}(\mu_V - p)$ and the firm's expected profit is $\Pi_{0N} = (p-c)\mu_N - p\phi(z_\beta)\sigma_N$. Otherwise, if $\mu_V < p$, no potential customers purchase the product and the firm stocks zero quantity, in which case customers' incentive to click is clearly strengthened by preference learning. Assuming $\mu_V \geq p$, the specified equilibrium in Proposition 2 remains if $U \geq U_{0N}$, i.e.,

$$\mathbb{E}_N\left[1-\sqrt{\frac{G(p)}{N\overline{G}(p)}}\left[\phi(z_\beta)-(1-\beta)z_\beta\right]\right]\mathbb{E}(V-p)^+ - t \geq \left[1 - COV_N\left[\phi(z_\beta)-(1-\beta)z_\beta\right]\right](\mu_V - p), \quad (3)$$

which holds only if COV_N is large and $\overline{G}(p)$ is large. If inequality (3) does not hold, then customers may not be willing to click, although there is learning benefit. The managerial insight is that, when clicking becomes the only channel of resolving valuation uncertainty, click tracking is valuable only if the population is sufficiently volatile and customers' purchasing probabilities are high. This condition is in line with inequality (2) which induces customers to click in the absence of customer learning. We state the discussion above as Proposition 4.

PROPOSITION 4. *Suppose $\mu_V \geq p$. If and only if inequality (3) holds, all customers clicking is a strong Nash equilibrium. A sufficient condition for inequality (3) is*

$$\sqrt{\frac{\overline{G}(p)}{G(p)}COV_N^2} > \mathbb{E}\sqrt{\frac{1}{N}}. \quad (4)$$

Inequality (4) can be equivalently written as follows:

$$\overline{G}(p) > \frac{\left(\mathbb{E}\sqrt{\frac{1}{N}}\right)^2}{COV_N^2 + \left(\mathbb{E}\sqrt{\frac{1}{N}}\right)^2},$$

which clearly highlights that our result in Proposition 2 is strengthened only for a high purchasing probability or a highly volatile distribution of potential customers.

We now investigate the second type of customer preference learning, where visiting the informational website adds additional random utility ε to each customer's ex ante utility V. One can think of V as each customer's initial *latent* utility, and ε as her incremental learning utility from the informational website. After a customer browses the website, both her latent utility V and learning utility ε are realized, and we denote these realizations as v and ϵ respectively. If $v + \epsilon \geq p$, she would buy the product, otherwise, she won't. For analytical convenience, denote $\widetilde{V} = V + \varepsilon$, and its distribution $\widetilde{G}$.

There are two plausible assumptions we will make: First, we may assume that the expectation of the learning utility is zero, i.e., $\mathbb{E}(\varepsilon) = 0$, so that browsing the website does not add more value to the customers *on average*. Then, $\widetilde{G}$ is a mean preserving spread of G, which is equivalent to the second order stochastic dominance, i.e., G second-order stochastically dominates $\widetilde{G}$. Second and alternatively, we may assume that any realization of ε is non-negative, so that the learning utility is always positive. This implies that $\mathbb{E}(\varepsilon) > 0$, and that $\widetilde{G}$ first-order stochastically dominates (Mas-Colell et al. 1995) G. One can think of the website under this assumption is "better" (from the customers' perspective) than the one under the previous assumption.

We first assume that the distribution $\widetilde{G}$ is private information, i.e., not known to the firm. Only the distribution G is known to and used by the firm, i.e., the firm is not aware of the customer-learning from clicking. Then the customer utility $\widetilde{U} = s_C\mathbb{E}(V + \varepsilon - p)^+ - t$ by clicking. We are interested in comparing the utility with learning $\widetilde{U}$ to the utility U without learning. The following proposition shows that customer learning is always beneficial to customers under either of the two assumptions. Hence, customer-learning reinforces the equilibrium result in §4.1.

Proposition 5. *Assume that the distribution $\widetilde{G}$ is not known to the firm. If either $\mathbb{E}(\varepsilon) = 0$ or $\varepsilon \geq 0$, then $\widetilde{U} \geq U$ and all customers clicking remains a strong Nash equilibrium.*

Suppose the distribution $\widetilde{G}$ is also known to the firm, we are interested in how such customer-learning affects the equilibrium outcomes. First, customers compare $U_N = s_N\mathbb{E}(V - p)^+$ with $\widetilde{U}' = \widetilde{s}_C\mathbb{E}(V + \varepsilon - p)^+ - t$, where $\widetilde{s}_C = \mathbb{E}_N\left[1 - \sqrt{\frac{\widetilde{G}(p)}{N(1-\widetilde{G}(p))}}\,(\phi(z_\beta) - (1-\beta)z_\beta)\right]$. Recall that $\widetilde{U} = s_C\mathbb{E}(V + \varepsilon - p)^+ - t$. It is useful to compare $\widetilde{U}'$ with U. We provide sufficient conditions under which the customers' incentive to click will be reinforced as follows.

Proposition 6. *Assume that the distribution $\widetilde{G}$ is known to the firm. If either $\mathbb{E}(\varepsilon) = 0$ and $p \geq \mu_V$, or $\varepsilon \geq 0$, then $\widetilde{U}' \geq U$ and all customers clicking remains a strong Nash equilibrium.*

The intuition behind Proposition 6 is as follows: Under the first assumption, the customers' purchasing probability under customer learning is higher, so that it is profitable for the firm to improve its service rate, which also brings benefits to the customers. Under the second assumption, customers always benefit from learning, which consequently improves their purchasing probability and thus the firm's service level.

However, under the first mean-preserving-spread condition, if $p < \mu_V$, then we have $\widetilde{s}_C < s_C$. From the customers' perspective, clicking brings the learning benefit, however, the service level decreases. Hence, customers have a tradeoff between the learning benefit and the service level loss, in which case the equilibrium outcome in Proposition 2 may not continue to be a strong Nash equilibrium. However, it remains a Nash equilibrium.

In conclusion, in the presence of customer preference-learning, customer incentives to click are reinforced under certain conditions. In contrast, firm preference-learning always strengthens our previous results.

5. Pricing

So far, we have studied posted price models where the essential tradeoffs for strategic customers involve weighing the benefits of clicking (in inducing higher product availability or learning) against three factors: the cost of clicking, noise, and preference learning. We found that customers are *typically* willing to click. Now we investigate how robust these findings are when the firm can *price dynamically* and customer clicks may induce higher price. We identify two settings where customers may not be willing to click: *price-sensitive demand* and *markdown pricing*. For the sake of brevity and focus, we summarize the main conclusions here while relegating the detailed analysis to the Online Supplement.

When demand is price-sensitive, and the firm sets price after observing clicks, customers face the tradeoff between price and availability. Interestingly, we show in the Online Supplement 2 that the demand functional form plays a crucial role: While customers are still willing to click in the presence of multiplicative demand, the result may *reverse* when demand is additive. To induce customers to click, we propose *price commitment*, which essentially brings us back to the simple model.

When markdown pricing is possible or frequent, customers may strategically wait for the markdown period to enjoy a lower price. In that case, customers may prefer the firm to have a poor forecast of demand so that the chance of overstocking for the firm is high. In the Online Supplement 3, we characterize the equilibria depending on customer valuations in the markdown period. When customers are not willing to click, we propose *product personalization* which increases customer valuation and thus induces them to click. This demonstrates how the value of click tracking increases by collaboration between operations and marketing.

6. Value of Strategic Clicks: Comparing to Traditional Operations & Marketing Strategies

The last significant part of our study involves evaluating the value of click tracking of strategic customers, especially when comparing to traditional operations and marketing strategies. For fair comparisons, we distinguish two different settings based on whether customers' valuation is certain or uncertain when they make their clicking decision. The aim of this comparison is to further understand how well this novel click tracking technology performs relative to other strategies.

Notice that the production lead time is typically related to the profile of the customer valuation volatility. In our model, the clicking decision is always made earlier than the purchasing decision, and the time difference is at least the production lead time. If the production lead time is long, then the time of clicking is much earlier than the time of consumption. Hence, at the time of clicking, customer valuation uncertainty is typically high. In contrast, if the production lead time is short, then the time of clicking is closer to the time of consumption. Hence, at the time of clicking, customer valuation is much less volatile or even certain.

When customers' valuation is certain, we compare strategic clicks with quantity commitment and availability guarantees, studied in Su and Zhang (2009), and quick response studied extensively in the literature (cf. Fisher and Raman 1996, Iyer and Bergen 1997, Cachon and Swinney 2009 and references therein). We refer readers to the Online Supplement 4 and 5. The main finding is that strategic clicks outperform all the other strategies studied in the literature given that clicks provide perfect demand information from strategic customers in equilibrium. A more realistic evaluation is conducted below when customer valuation is uncertain.

6.1. Comparison with Quantity Commitment, Availability Guarantees and Quick Response

When customer valuation is uncertain, we conduct an analytical study, as detailed in the Online Supplement 6. We use a numerical study to compare the value of different strategies. A subset of representative results are shown in Table 2, where c_2 is the quick-response production cost, h is the physical hassle cost, and w is the cost of compensation when using availability guarantees. These numerical examples suggest that the results from the certain-valuation case are robust. In the majority of the cases, noisy clicks outperform the traditional strategies. This implies that the *efficiency effect* (meaning reducing the supply-demand mismatches) dominates the *strategic effect* (meaning merely relying on commitment power to influence customer behavior). Only when the premium cost of quick response is sufficiently small, can quick response outperform noisy clicks. Only when the demand variation is extremely low, for example, the coefficient of demand is less than 0.001, can the strategic effect dominate the efficiency effect. (Obviously, when demand is certain, there is no value/need in using any of these strategies.)

6.2. Comparison with Advance Selling

Advance selling (also called pre-order strategy) is the marketing practice of selling a product at a time preceding consumption (Shugan and Xie 2000, 2004; Xie and Shugan 2007). Xie and Shugan

Table 2 Comparison of Values (in % Increment) of Different Practices with Uncertain Valuation

Parameters ($u = h,\ t = 0,\ w = 0,\ c = 0.1,\ \overline{v} = 1, \mu_N = 10^5$.)	COV_N	Quantity Commitment	Availability Guarantees (Upper Bound)	Quick Response	Noisy Clicks
$c_2 = c + 0.06, h = 0.01$					
$\mathbb{P}(\overline{v}) = 0.45$	0.20	0.0073%	0.0118%	2.73%	4.09%
$\mathbb{P}(\overline{v}) = 0.30$	0.20	0.0049%	0.0119%	2.78%	4.12%
$\mathbb{P}(\overline{v}) = 0.20$	0.20	0.0129%	0.0122%	2.83%	4.18%
$\mathbb{P}(\overline{v}) = 0.10$	0.20	0.0103%	0.0129%	3.02%	4.38%
$\mathbb{P}(\overline{v}) = 0.45$	0.10	0.0081%	0.0116%	1.35%	1.97%
$\mathbb{P}(\overline{v}) = 0.20$	0.10	0.0064%	0.0000%	1.38%	1.98%
$\mathbb{P}(\overline{v}) = 0.10$	0.10	0.0115%	0.0127%	1.49%	2.05%
$c_2 = c + 0.06, h = 0.02$					
$\mathbb{P}(\overline{v}) = 0.55$	0.25	0.0109%	0.0361%	3.49%	5.25%
$\mathbb{P}(\overline{v}) = 0.50$	0.25	0.0132%	0.0363%	3.53%	5.29%
$\mathbb{P}(\overline{v}) = 0.55$	0.02	0.0117%	0.0116%	0.28%	0.36%
$c_2 = c + 0.001, h = 0.02$					
$\mathbb{P}(\overline{v}) = 0.40$	0.20	0.0130%	0.0243%	4.26%	4.23%
$c_2 = c + 0.001, h = 0.01$					
$\mathbb{P}(\overline{v}) = 0.30$	0.005	0.0064%	0.0000%	0.15%	0.0426%
$\mathbb{P}(\overline{v}) = 0.30$	0.002	0.0067%	0.0000%	0.11%	0.0079%
$\mathbb{P}(\overline{v}) = 0.30$	0.001	0.0067%	0.0000%	0.11%	0.0020%
$\mathbb{P}(\overline{v}) = 0.30$	0.0005	0.0068%	0.0000%	0.10%	0.0005%

(2007) argue that offering advance sales can improve profit because advance selling separates purchase from consumption. This creates buyer uncertainty about their future product/service valuation and removes the seller's information disadvantage (caused by the buyer knowing more about their own valuation than the seller does).

From an operations perspective, advance selling is another mechanism of ADI, and thus allows the firm to better match supply with demand. Given that both advance selling and click tracking can provide a firm ADI, how do they compare?

We build a stylized model of advance selling capturing both the valuation uncertainty feature and ADI feature as in the previous section. Consistent with the literature (cf. Gundepudi et al. 2001, Shugan and Xie 2007, Yu et al. 2007, Prasad et al. 2010 and references therein), suppose there are two time epochs: The first period is the advance selling period, which is equivalent to the time when strategic customers have to decide whether or not to click (recall the model in §4.1). The second period is the regular selling (consumption) period. For brevity, we assume that the regular selling price $p_2 = p$ is exogenously given, but the advance-selling price p_1 is a decision variable for the firm. Strategic customers must decide whether to commit to purchasing in the advance selling period or delay to the regular period. Based on how many pre-orders are received, the firm determines its production quantity.

For convenience, denote the coefficient of variation of the demand D by $COV_D = \frac{\sigma_D}{\mu_D} =$

$\sqrt{\frac{\mu_N \overline{G}(p)G(p)+\sigma_N^2\overline{G}^2(p)}{\mu_N^2\overline{G}^2(p)}}$ and the value of advance selling over regular selling (i.e., selling in a single period with price p) by $\Delta\Pi_A$. While using click tracking never hurts, the firm can lose profit by using advance selling since the advance-selling price p_1 charged has to induce strategic customers to purchase in advance (Prasad et al. 2010), as shown in the following lemma.

Lemma 1. ***(Prasad et al. 2010)*** *$\Delta\Pi_A > 0$ if and only if*

$$\mu_V > p\overline{G}(p) + cG(p) + s_N \int_p^{v_H} (v-p)g(v)dv - p\phi(z)\sqrt{\frac{\overline{G}(p)G(p)}{\mu_N} + \overline{G}^2(p)COV_N^2}.$$

Based on Lemma 1, we can compare click tracking and advance selling as follows.

Proposition 7. *If inequality (2) holds, then $\Delta\Pi > \Delta\Pi_A$ if and only if*

$$\mu_V < p\overline{G}(p) + cG(p) + s_N \int_p^{v_H} (v-p)g(v)dv - \frac{p\phi(z)}{\mu_N}\sqrt{G(p)\overline{G}(p)}\mathbb{E}\sqrt{N}.$$

Otherwise, $\Delta\Pi > \Delta\Pi_A$ if and only if

$$\mu_V < p\overline{G}(p) + cG(p) + s_N \int_p^{v_H} (v-p)g(v)dv - p\phi(z)\sqrt{\frac{\overline{G}(p)G(p)}{\mu_N} + \overline{G}^2(p)COV_N^2}.$$

When $G(p) = 0$ and consumers are *certain* about their own valuation (which exceeds the price p), it is always optimal for the firm to adopt advance selling and customers are always willing to purchase in advance to eliminate any stockout risk. This special case essentially reduces to the simple model in §3. This suggests that, in the absence of valuation uncertainty, advance selling and strategic clicks are equivalent, i.e., they yield the same benefit for the firm *ceteris paribus*.

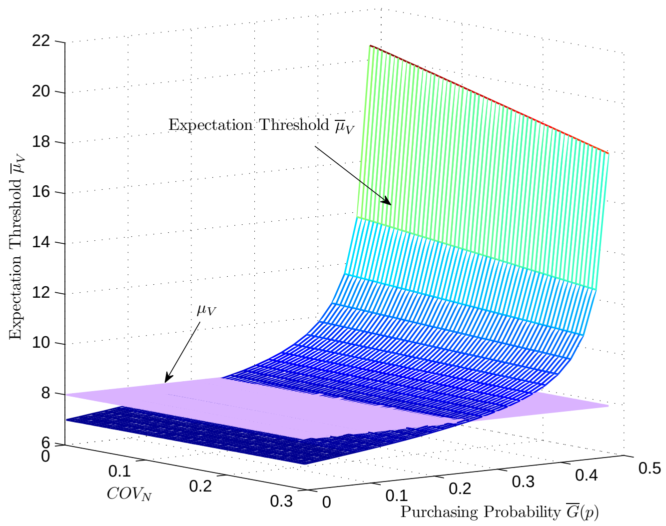
When $G(p) > 0$ and there is *valuation uncertainty*, strategic clicks and advance selling differ in profitability. Proposition 7 says that when customers' expectation of the valuation is low, strategic clicks outperforms advance selling; otherwise, advance selling can outperform strategic clicks by exploiting the benefit of the high expectation and gaining ADI.

To gain some intuition about Proposition 7, we let

$$\overline{\mu}_V(\overline{G}(p), \mu_N, \sigma_N) = p\overline{G}(p) + cG(p) + s_N \int_p^{v_H} (v-p)g(v)dv - \frac{p\phi(z)}{\mu_N}\sqrt{G(p)\overline{G}(p)}\mathbb{E}\sqrt{N} \qquad (5)$$

be the *expectation threshold* of customer valuation below which click tracking is preferred over advance selling. We are interested in how this threshold depends on the purchasing probability $\overline{G}(p)$ and the variation of the potential demand, i.e., the customer population, measured by COV_N. We performed a numerical study fixing μ_V and μ_N while varying other parameters, and one representative example is shown in Figure 3. The numerical example suggests that $\overline{\mu}_V(\overline{G}(p), \mu_N, \sigma_N)$

Figure 3 **The Expectation Threshold $\overline{\mu}_V$ ($p = 10, c = 7, \mu_V = 8, \mu_N = 80000$, V is uniformly distributed in $[v_L, v_H]$, where $v_L = \mu_V + \frac{p-\mu_V}{2\overline{G}(p)-1}, v_H = 2\mu_V - v_L$.)**



is increasing in the purchasing probability $\overline{G}(p)$ but *not necessarily monotone* in the coefficient of variation COV_N. It is interesting to observe that $\overline{\mu}_V(\overline{G}(p), \mu_N, \sigma_N)$ is increasing in COV_N when $\overline{G}(p)$ is small while decreasing in COV_N when $\overline{G}(p)$ becomes large. This observation suggests the following: As each customer is more likely to buy the product in the regular-selling period, click tracking is more likely to be preferred. However, more uncertainty of potential demand favors noisy clicks when $\overline{G}(p)$ is small, while it favors advance selling when $\overline{G}(p)$ is large. Indeed, both click tracking and advance selling reduce demand uncertainty, but which demand uncertainty reduction of the two strategies is more beneficial crucially depends on the purchasing probability $\overline{G}(p)$. From Figure 3, it is also interesting to notice that the expectation threshold is more sensitive to the purchasing probability than to the coefficient of variation of the population. This suggests that, for any given expected valuation of the product μ_V, when customers are more likely to purchase the product in the regular period, click tracking tend to be more valuable to the firm than advance selling.

We offer an intuitive explanation of the difference between click tracking and advance selling as follows. Advance selling mostly benefits from consumers' valuation uncertainty. One necessary condition to reap the benefits is that consumers have sufficiently high expectation about their valuation and thus have incentives to *commit* to purchasing early to secure availability (and thus eliminate stockouts). In contrast, click tracking benefits from taking advantage of consumer strategic behavior in that stockouts are costly for both firms and consumers. Click tracking does not rely on consumers to commit to purchase early, hence, high expectation of valuation on the part of consumers is *not* necessary. When customer expectation of product valuation is fixed, higher purchasing probabilities make click tracking more desirable. This suggests that click tracking is better than pre-orders when selling popular products. However, higher variation of potential demand can favor either strategy depending on customer purchasing probabilities.

Another advantage of click tracking over advance selling is that *ex ante* consumer welfare is *strictly* improved when click tracking is used while it remains unchanged when advance selling is used. Each customer's expected utility in equilibrium under advance selling is $U_A = \mu_V - p_1^* = s_N \mathbb{E}\max\{V-p,0\}$, while her expected utility under strategic clicks is $U = s_C \mathbb{E}\max\{V-p,0\} - t$. We have $U > U_A$ since the cost t is sufficiently small. Therefore, click tracking brings "win-win" outcomes for the firm and its customers, while advance selling can *only* benefit the firm. Furthermore, the *ex post* consumer welfare can be negative (due to low valuation realizations) under advance selling, while it can *never* be negative under click tracking.

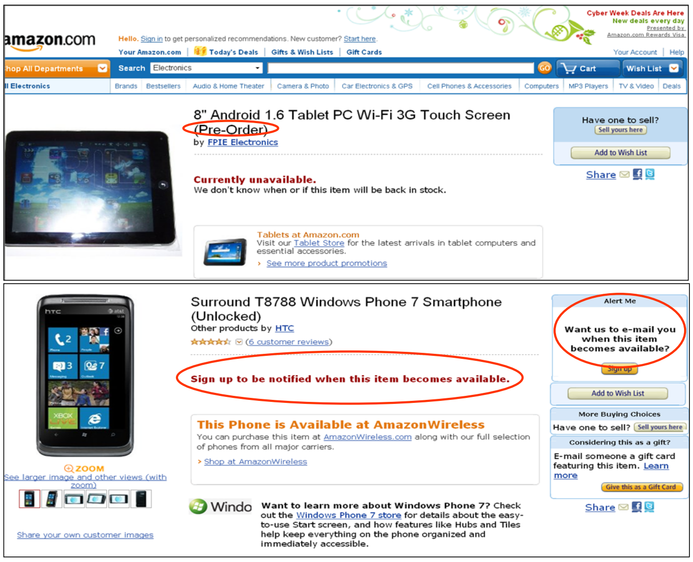
While advance selling has been practiced for quite some time, click tracking is fairly new. Our forward-looking comparison suggests that click tracking is promising in the future when customers are strategic, yet both practices can co-exist. Indeed, Figure 4 provides an example from Amazon.com where the company takes pre-orders (i.e., advance selling) for some products while inviting customers to be notified for others, such as "Want us to e-mail you when this item becomes available?" and "Sign up to be notified when this item becomes available." This practice is akin to click tracking. Observing Figure 4, one may tend to claim that the Smartphone is more "popular" than the Tablet PC touch screen, then Amazon's practice is by chance already aligned with our suggestion that click tracking is preferred when selling popular products. Clearly, rigorous empirical validation is needed and would be valuable for future research.

7. Discussion

We found that click tracking of strategic customers can be of great value to the firm. This technology can provide a better match of supply with demand than other operations and marketing strategies and brings win-win outcomes for both firm and customers. Contrary to the conventional wisdom that strategic customer behavior is typically a peril for firms, we demonstrate its *promise* in the context of click tracking.

Obviously, this theoretical result must be put in perspective: In practice, implementing click tracking may be difficult for traditional brick-and-mortar retailers who can more easily adopt

Figure 4 Advance Selling versus Click Tracking: A Casual Observation



quantity commitment and availability guarantees. No strategy fits all settings. Click tracking can be implemented in manufacturing and retailing by firms facing newsvendor problems. Wherever advance selling is used to reduce supply-demand mismatches, click tracking can be used. For example, new product releases, retailing of new novels, DVDs, and video games, which have short selling seasons. Click tracking can also be used for products that require customers to search online to collect or learn more information in advance or specify a certain level of customization.

Theoretically, our model and insights are applicable to settings where the supply-demand mismatch problem is present, especially to settings where: *a*) The mismatch problem is significant such as with new products whose demand is volatile and difficult to predict, or with established products whose consumers vary over time which also results in significant demand uncertainty; *b*) The availability concern is pronounced for customers such as settings where customers' hassle costs are significant and customers' outside options are limited. Hence, the product market competition should be moderate; *c*) Practically, each customer in the market should readily have access to the Internet and the firm's website to ensure the inconvenience cost of clicking is indeed small. In these settings, our findings suggest that the firm should inform customers that click tracking is used to collect advance demand information, and explain its benefits, i.e., *train* its customers to be strategic.

Like other economic models, our models are stylized and do not account for many practical and implementation issues. For example, while clear identification of customers in the B2B setting where we conducted our empirical study (that motivates this game-theoretic study) is not an issue, it becomes more difficult in B2C (i.e., business to consumers) settings. One person may use different computers, so identification from IP addresses becomes unreliable. What makes it even worse is that competitors may purposely generate fraud clicks. However, B2C firms can pro-actively *ask* customers to reveal more reliable information of their identities (e.g. email address, home address, or even credit card information). As we have seen in Figure 4, Amazon.com has already been implementing this strategy. Our forward-looking study offers a novel rationale for such a practice by showing that strategic customers are typically *willing* to share their demand information (in terms of their interest in the product), and it could be better than traditionally-used pre-orders for both the firm and its customers.

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Online Supplement

for "The Promise of Strategic Customer Behavior: On the Value of Click Tracking"

Online Supplement 1. Proofs

Proof of Proposition 1. (i) We first show the "if" part: If $t \leq \bar{t}$, we first verify that the equilibrium specified is a strong Nash equilibrium by showing that no deviations of any customer-firm coalition is profitable.

Suppose the firm deviates by producing another quantity $q \neq X^*$, then the firm is strictly worse off by mismatching supply with demand. Suppose a single customer deviates and does not click, then $X = D - 1$ and the firm would produce $q = D - 1$ number of products according to its equilibrium strategy. Hence, the deviating customer's expected utility $U_N = (v-p)s(q) = (v-p)\frac{\mathbb{E}\min\{D,q\}}{\mathbb{E}(D)} = (v-p)\frac{\mu-1}{\mu}$. The customer's expected utility of not deviating is $U^* = v - p - t$. Note that $t \leq \bar{t}$ is equivalent to $U^* \geq U_N$. Similar to one customer deviating, several customers jointly deviating by not clicking cannot be profitable either.

Suppose the firm and one potential customer jointly deviate by a preplay communication *before* the potential customer enters the market. Hence, the firm stocks the quantity other than the number of observed clicks, and this single potential customer does not click. In this case, whether this potential customer who deviates enters the market or not is not known by the firm. Hence, the firm has no way to perfectly match supply with demand since the number of customers is still random. Therefore, the firm strictly loses profits by such a deviation. Similarly, the firm and several potential customers jointly deviating cannot be profitable either. Hence, we have shown that the equilibrium specified is indeed a strong Nash equilibrium.

Now we show the "only if" part: If the equilibrium specified exists, it is necessary that $t \leq \bar{t}$. Suppose it were not, i.e., $t > \bar{t}$, then a single customer has a profitable deviation by not clicking to save her inconvenience cost of clicking, based on our argument above. This completes the proof of part (i).

(ii) We first verify whether the equilibrium specified is indeed a Nash equilibrium. First, given the firm's strategy, no single customer has any profitable deviation by clicking since the click cost t is strictly positive. Second, given all customers' strategies, the firm receives no demand information and faces a newsvendor problem: The firm's optimal production strategy is q^*. Hence, this equilibrium is verified.

To prove strong if $t \geq \bar{t}\mathbb{E}\max\{0, D - q^*\}$, we must verify that no subset of players has an incentive to deviate. First, no single customer or any subset of customers has any incentive to deviate by clicking, given the firm's strategy of producing the newsvendor quantity for the same reason in the previous paragraph. Second, the firm itself has no profitable deviation given that no customers click, again, for the same reason in the previous paragraph.

It remains to verify whether the firm and some customer(s) can jointly deviate profitably. Note that, for any customer, an upper bound on the utility gain of clicking is to induce the firm to guarantee the availability, i.e., $\Delta \overline{U} = (v-p)\left[1 - \frac{\mathbb{E}\min\{D,q^*\}}{\mathbb{E}(D)}\right] = \bar{t}\mathbb{E}\max\{0, D-q^*\}$. If the click cost is greater than this value, it does not make sense for any customer to click. Not clicking would be the best strategy for any customer. Hence, we complete the proof. ■

Proof of Proposition 2. First note that the fill rate $s(\beta) = 1 - COV_D[\phi(z_\beta) - (1-\beta)z_\beta]$ using Winkler et al. (1972). Plugging this into s_C and s_N, we have, $s_C = \mathbb{E}_N\left[1 - \sqrt{\frac{G(p)}{N\overline{G}(p)}}(\phi(z_\beta) - (1-\beta)z_\beta)\right]$, while $s_N = 1 - \sqrt{\frac{\mu_N \overline{G}(p)G(p) + \sigma_N^2 \overline{G}^2(p)}{\mu_N^2 \overline{G}^2(p)}}(\phi(z_\beta) - (1-\beta)z_\beta)$, where $\beta = \frac{p-c}{p}$.

Let us first show the "if" part: If $t \le \bar{t}_I$, a strong equilibrium exists, in which all customers click with probability one. We verify it is indeed a strong equilibrium by checking whether any coalition of the players has any profitable deviations.

By definition, the firm does not have any incentive to deviate by using production quantities other than $q^*_{D|n}$ upon observing n clicks given that each customer clicks with probability one.

Suppose a single customer deviates by not clicking, then her deviating utility is $U_{N1} = s_{C1}\mathbb{E}\max\{0, V-p\}$, where $s_{C1} = \mathbb{E}_N\left[1 - \sqrt{\frac{G(p)}{(N-1)\overline{G}(p)}}(\phi(z_\beta) - (1-\beta)z_\beta)\right]$. Her equilibrium utility $U = \mathbb{E}\max\{0, V-p\}s_C - t$. If $U \ge U_{N1}$, which is equivalent to $t \le t_1$. Hence, if $t \le \bar{t}_I$, no single customer has any profitable deviation. We can use the same argument to show no coalition of customers has any profitable deviation, given that $\mathbb{E}\sqrt{\frac{1}{N-k}} - \mathbb{E}\sqrt{\frac{1}{N}} > \mathbb{E}\sqrt{\frac{1}{N-1}} - \mathbb{E}\sqrt{\frac{1}{N}}$ for any $k > 1$.

Suppose the firm and a single customer jointly deviate, i.e., a single customer does not click and the firm produces quantities other than the one specified in the proposition. Then, the firm has no way to exactly know the true demand distribution $\mathbb{P}(D|N=n)$ given that the market size is random. Hence, the optimal quantity $q^*_{D|n}$ cannot be chosen by the firm, which results in strictly profit losses by definition. Hence, the firm has no incentives to deviate. The argument also demonstrates that the firm has no incentives to deviate with any number of customers. Therefore, the equilibrium specified is indeed a strong Nash equilibrium.

Now, we show the "only if" part: If the equilibrium specified exists, then it is necessary that $t \le \bar{t}_I$. If this condition is not satisfied, either one single customer or the coalition of all the customers has incentives to deviate.

Finally, the conclusion that the firm has strictly positive profit increment is immediate by Jensen's inequality, $\sqrt{\mu_N} > \mathbb{E}\sqrt{N}$ since function $\sqrt{x}$ is strictly concave. ■

Proof of Proposition 3. The service level as a function of η can be written as

$$s_C(\eta) = 1 - [\phi(z) - (1-\beta)z]\mathbb{E}_S\left[COV_{D|S}(\eta)\right],$$

using Winkler et al. (1972). Notice that, $D|S$ follows the normal distribution $N\left(\frac{\eta n\overline{G}(p)+Sn\overline{G}(p)G(p)}{\eta+n\overline{G}(p)G(p)}, \left(\frac{1}{\eta}+\frac{1}{n\overline{G}(p)G(p)}\right)^{-1}\right)$. After some mathematical manipulations, we have this expression in Proposition 3. ■

Proof of Proposition 4. The proof to show the strong Nash equilibrium specified in Proposition 2 remains in this proposition is similar to the proof of Proposition 2, by checking no coalition of the players has any profitable deviation. Hence, we omit the details for brevity. The sufficient condition is obtained since we have assumed that t is strictly positive but arbitrarily small. ■

Proof of Proposition 5. First, we want to show $\mathbb{E}\max\{0, V+\epsilon-p\} \geq \mathbb{E}\max\{0, V-p\}$ if either of the two conditions holds.

If the first condition holds, then $\widetilde{G}$ is a mean preserving spread of G, which implies that $\widetilde{G}(v) \geq G(v)$ for all $v < \mu_V$, and $\widetilde{G}(v) \leq G(v)$ for all $v > \mu_V$ (Mas-Colell et al. 1995). Notice that

$$\mathbb{E}\max\{0, V+\epsilon-p\} = \int_p^{v_H}(v-p)d\widetilde{G}(v) = (v_H - p) - \int_p^{v_H}\widetilde{G}(v)dv,$$

where the last equality is due to integration by parts. Similarly, we have

$$\mathbb{E}\max\{0, V-p\} = (v_H - p) - \int_p^{v_H} G(v)dv.$$

Hence,

$$\widetilde{U} - U = \int_p^{v_H}[G(v) - \widetilde{G}(v)]dv \geq 0$$

if $p \geq \mu_V$. If $p < \mu_V$, we have

$$\int_p^{v_H} G(v)dv = v_H - G(p)p - [\mu_V - \int_{v_L}^p vdG(v)] = v_H - \mu_V - \int_{v_L}^p G(v)dv.$$

We know $\int_{v_L}^p G(v)dv \leq \int_{v_L}^p \widetilde{G}(v)dv$ since $p < \mu_V$. Therefore,

$$\int_p^{v_H} G(v)dv \geq \int_p^{v_H}\widetilde{G}(v)dv,$$

which implies that $\widetilde{U} - U \geq 0$.

If the second condition holds, then $\widetilde{G}$ first-order stochastically dominates G, which is equivalent to $\widetilde{G}(v) \leq G(v)$ for any v. Therefore, we obtain $\widetilde{U} - U \geq 0$. Lastly, we can show that there exists a strong Nash equilibrium in which all customers click, which follows from a similar argument for Proposition 2. ■

Proof of Proposition 6. Suppose the first condition holds. Since $\widetilde{G}$ is a mean preserving spread of G, we have $\widetilde{G}(p) \le G(p)$ if $p \ge \mu_V$. This implies that $\widetilde{s}_C \ge s_C$. Hence, we have $\widetilde{U}' \ge \widetilde{U} \ge U$.

Suppose the second condition holds. If $\varepsilon \ge 0$, then $\widetilde{G}(p) \le G(p)$ for any p. Hence, similar to the first case, we have the conclusion. There exists a strong Nash equilibrium in which all customers click, for similar reasons as Proposition 2. ■

Proof of Lemma 1. If every customer purchases in the advance selling period, then her expected utility is $U_1 = \mu_V - p_1$. We want to characterize the strong Nash equilibrium. First, we have to ensure that the coalition of all customers has no profitable deviations. If each customer decides to delay to the regular-selling period, i.e., all customers jointly deviate, then the expected utility is $U_2 = s(q)\mathbb{E}\max\{V - p, 0\} = s(q)\int_p^{v_H}(v-p)g(v)dv$, where $s(q) = s_N$ is the availability probability when q units are stocked for the regular period. To find the optimal stocking quantity q^* when the customers purchase in the regular period, we first need to find the demand distribution. Let $D(n)$ be this demand when the total realized population is n, then $D(n)$ follows a binomial distribution with mean $n\overline{G}(p)$ and variance $n\overline{G}(p)G(p)$, which can be simply approximated by the normal distribution with the same mean and variance when n is large enough. The unconditional demand D follows the normal distribution with mean $\mu_D = \mu_N\overline{G}(p)$ and variance $\sigma_D^2 = \mu_N\overline{G}(p)G(p) + \sigma_N^2\overline{G}^2(p)$. Then, $q^* = \mu_D + \Phi^{-1}(\frac{p-c}{p})\sigma_D$, and $F(q^*) = \frac{p-c}{p}$. The optimal profit under no advance selling is $\Pi_2 = (p-c)\mu_D - p\phi(\Phi^{-1}(\frac{p-c}{p}))\sigma_D$.

When $U_1 \ge U_2$, all customers in the market are willing to purchase in the advance selling period, in which case the firm's profit is $\Pi_1(p_1) = (p_1 - c)\mathbb{E}(N)$. Let $U_1 = U_2$, we have the maximum advance-selling price the firm can charge $p_1^* = \mu_V - s(q^*)\mathbb{E}\max\{V - p, 0\}$. Hence, the optimal profit under advance selling strategy is $\Pi_1 = [\mu_V - s(q^*)\mathbb{E}\max\{V - p, 0\} - c]\mu_N$. Hence, $\Delta\Pi_A = \Pi_1 - \Pi_2 = [\mu_V - s(q^*)\mathbb{E}\max\{V - p, 0\} - c]\mu_N - (p-c)\mu_D + p\phi(\Phi^{-1}(\frac{p-c}{p}))\sigma_D$. Letting $\Delta\Pi_A > 0$ and simplifying yields the desired result. When this inequality holds, every customer in the market purchases the product in the advance-selling period, and the firm is able to perfectly match supply with demand. It is straightforward to verify that no single customer or the firm has incentives to deviate in this specified equilibrium: Each customer only purchases the good at the advance selling period if price $p_1 \le p_1^*$, the firm charges price p_1^* and stocks the quantity according to the number of pre-orders and stocks zero for the regular-selling period.

Finally, let us verify that this is a strong Nash equilibrium by checking no subset of agents has any profitable deviations. Suppose one customer deviates by not purchasing in the advance-selling period, then she cannot get any good in the regular selling period given the firm and other customers' strategies. Similarly, no coalitions of customers have any profitable deviations. It remains to check whether any coalitions of some customers and the firm have any profitable deviations. Suppose a single customer and the firm jointly deviate: The customer purchases in the second period, and the firm uses an alternative price p_1 or production quantity. Suppose the firm uses an alternative production quantity but still uses price p_1^*, then the firm faces supply-demand mismatches by this deviating and thus strictly worse off. Suppose the firm uses an alternative price p_1 while still keeps its equilibrium production quantity decision, then the deviating customer cannot get any good in the regular period. Hence, the customer has no incentive to deviate. Suppose the firm deviate in both pricing and quantity decisions, and the customer deviates by purchasing in the regular period. Then, it has be the case that the firm stocks at least one unit in the regular period for the deviating customer who may enter the market. The deviating customer is strictly better off by being secured a good at the regular period. However,the firm is strictly worse off due to the supply-demand mismatches. Similar arguments show that no subsets of customers and the firm have joint profitable deviations. Hence, the specified equilibrium is indeed a strong Nash equilibrium. ■

Proof of Proposition 7. We have the firm's profit when advance selling is used, $\Pi_1 = [\mu_V - s_N \int_p^{v_H} (v-p)g(v)dv - c]\mu_N$. When inequality (2) holds, we have the value of strategic clicks in Proposition 2; otherwise, that value is zero. Straightforward comparison yields the results. ■

Online Supplement 2. Strategic Clicks with Price-sensitive Demand

One essential feature of the simple model and the model of noisy clicks is that the demand distribution does not depend on price, and thus we treat price as an external parameter. A natural extension of the standard newsvendor model is to introduce price effects. As argued in Petruzzi and Dada (1999), the newsvendor model where stocking quantity and selling price are set simultaneously provides an excellent vehicle for examining how operational problems interact with marketing issues to influence decision-making at the firm level. We now investigate what happens if demand is price sensitive: When clicking, customers now must tradeoff the benefit of stockout elimination with the risk of *contingent pricing.*

Demand now is a random function of price: $D(p, \epsilon)$, where ϵ is a random variable with support $[A, B]$. Notice that, as always in this paper, customers are discrete, and continuous distributions are used only for analytical tractability. Let $F(x, p)$ denote the probability that $D \leq x$ for a price p. The demand-price relationship $D(p, \epsilon)$ is used commonly in the economics and operations literature to represent the market demand at an aggregate level. Meanwhile, we study customer purchasing behavior at an individual level, where customers are *heterogeneous*. In specific, we assume customer i has a deterministic valuation v_i, for $i = 1, 2, ..., D(p, \epsilon)$. How to link the aggregate level and individual level is not the focus of this paper, and we refer readers to Deaton and Muellbauer (1980) and Petruzzi and Dada (1999) for discussions.

The game is the same as before, except that price now is the firm's decision variable, and it sets the stocking quantity and price simultaneously. Following Petruzzi and Dada (1999), we investigate two extensively used forms of the demand function $D(p, \epsilon)$: the additive demand and the multiplicative demand.

Online Supplement 2.1. Additive Demand

Demand is defined as $D(p, \epsilon) = y(p) + \epsilon$ in the additive case (Mills 1959), where $y(p)$ is a decreasing function that captures the dependency between demand and price. In particular, let $y(p) = a - bp$, $a > 0, b > 0$, which represents a linear demand curve commonly used in economics literature. The demand parameters a and b must be estimated from the experience, and may not be known to the customers. However, to isolate the role of the demand functional form, we ignore the information asymmetry here.

Petruzzi and Dada (1999, p.185) offers us the optimal stock quantity q^* and price p^* of the firm in the absence of ADI, and the optimal price p_ϵ in the presence of perfect ADI. Let $p^0 = \mathbb{E}(p_\epsilon)$ be the expected price when demand uncertainty is resolved before the firm makes decisions. Let $x^* = q^* - y(p^*)$ and $\Theta(x^*) = \int_{x^*}^{B} (u - x^*) f(u) du$. We can actually use a natural refinement of strong Nash equilibrium, the strong perfect equilibrium (Rubinstein 1980) in this two-stage game. Clearly, all the equilibria found here are both Nash and strong Nash without using this refinement.

Proposition A-1. *With linear demand functions:*
(1) There is a strong Nash equilibrium where all customers click, and the firm sets price p_ϵ *and stocks the number of clicks* $q_\epsilon = X(p_\epsilon, \epsilon)$*. However, this equilibrium is not a strong perfect equilibrium if* $1 - \left[1 + \frac{\Theta(x^*)}{2b(v_i - p^0 - t)}\right] \frac{\mathbb{E}\min\{D(p^*, \epsilon), q^*\}}{\mathbb{E}[D(p^*, \epsilon)]} \leq 0$ *for each customer* i*.*
(2) There is a strong perfect equilibrium where no customers click, the firm sets price p^* *and stock quantity* q^* *on the equilibrium path, but sets price* p_ϵ *and stocks the number of clicks* $q_\epsilon = X(p_\epsilon, \epsilon)$ *off the equilibrium path, if* $1 - \left[1 + \frac{\Theta(x^*)}{2b(v_i - p^0 - t)}\right] \frac{\mathbb{E}\min\{D(p^*, \epsilon), q^*\}}{\mathbb{E}[D(p^*, \epsilon)]} \leq 0$ *for each customer* i*.*

Proof. (1) We first check the specified equilibrium is a strong Nash equilibrium. Suppose one customer deviates while fixing other players' strategies, then this customer will be strictly worse off due to availability decrease. No coalition of customers has any profitable deviation for the same reason. It is clear that in this equilibrium, the firm obtains its maximum profit. Any deviation in its equilibrium price or quantity cannot be profitable. Hence, we have shown no coalitions involving the firm can be profitable. In sum, the equilibrium is indeed a strong Nash equilibrium.

We now show why this equilibrium is not a strong perfect equilibrium under the given condition. In the subgame where no customers click in the first stage, the firm's optimal strategy is to use stocking quantity q^* and price p^*. Given the firm's optimal stocking and pricing strategy, customer i has the expected utility $U_{iN} = (v_i - p^*)\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]} = \left[v_i - p^0 + \frac{\Theta(z^*)}{2b}\right]\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]}$.

In the specified equilibrium where all customers click, customer i can obtain a unit of the product with probability one, and her expected utility is $U_i = \mathbb{E}(v_i - p_\epsilon) - t = v_i - p^0 - t$.

If $v_i - p^0 - t \le \left[v_i - p^0 + \frac{\Theta(z^*)}{2b}\right]\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]}$, i.e., $1 - \left[1 + \frac{\Theta(z^*)}{2b(v_i - p^0 - t)}\right]\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]} \le 0$ for all customer i, then all the customers have incentives to deviate. Hence, the equilibrium specified is not perfect.

(2) It is straightforward to verify this result based on the argument in part (1). ∎

It is indeed possible that the condition in Proposition A-1 is satisfied since $p^* \le p^0$ even if t is arbitrarily small, in which case not all customers are willing to click in a strong perfect equilibrium. The tradeoff is about whether availability improvement outweighs price increment or not.

Online Supplement 2.2. Multiplicative Demand

Demand is defined as $D(p,\epsilon) = y(p)\epsilon$ in the multiplicative case (Karlin and Carr 1962), where $y(p) = ap^{-b}$, $a > 0, b > 1$. Then Theorem 2 in Petruzzi and Dada (1999, p.186) specifies the optimal stock quantity q^* and price p^* of the firm. As before, define $p^0 = \mathbb{E}(p_\epsilon)$. Note that in this multiplicative demand case, we have $p^* \ge p^0$, *opposite* to the result for the additive cases. We have the following result.

Proposition A-2. *With multiplicative demand functions, there is a strong perfect equilibrium in which all customers click, and the firm sets price p_ϵ and stocks the number of clicks $q_\epsilon = X(p_\epsilon, \epsilon)$.*

Proof. First, we can show the equilibrium specified is a strong Nash equilibrium, using a similar argument as the proof of Proposition A-1. To show this equilibrium is perfect, we have to show it is a strong Nash equilibrium in every subgame.

If they all click, the firm has perfect ADI, which enables the firm to charge p_ϵ conditional on realization of ϵ. It is straightforward to show that $p_\epsilon = p^0$ for all $\epsilon \in [A, B]$. Hence, customer i's expected utility of clicking is $U_i = \mathbb{E}(v_i - p_\epsilon) - t = (v_i - p^0) - t$. On the other hand, if all of them do not click, then customer i's expected utility is $U_{iN} = (v_i - p^*)\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]}$. Clearly, $U_{iN} \le U_i$ if $(v_i - p^*)\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]} \le v_i - p^0 - t$, i.e., $1 - \frac{v_i - p^*}{v_i - p^0 - t}\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]} \ge 0$, which is always satisfied for sufficiently small t, since $p^* \ge p^0$. It remains to check subgames where only a proper subset of customers click, and the firm uses Bayesian updating to obtain the posterior demand distribution. Given that the click cost t is assumed to be arbitrarily small, this type of deviations cannot be profitable since the availability strictly decreases. Hence, the specified equilibrium is a strong perfect equilibrium. ■

The explanation follows from comparing the prices that the firm charges to certain-demand cases. The demand variance and coefficient of variation are increasing in p for the additive case; but decreasing in p for the multiplicative case. Hence, for the additive case, the firm is willing to charge a lower price to decrease demand variability when demand is uncertain; while for multiplicative case, the firm prefers to charge a higher price to decrease demand variability when demand is uncertain. Thus, in the former case, consumers may not be willing to click, while they will in the multiplicative case.

In sum, we have the following managerial insights from the two models of price-sensitive demand: Strategic customers face the tradeoff between *availability improvement* and *price increment* when they make their clicking decision. Whether strategic clicks as ADI are valuable critically depends on the *functional form* of the demand function. Thus, the firm must carefully investigate how uncertainties enter into the price-dependent demand function, possibly using historical data. Driver and Valletti (2003) discuss which demand form is more realistic. They suggest that the multiplicative demand model seems more appropriate since the price elasticity of demand is constant regardless of the demand realization in the multiplicative case. Interestingly, their discussion favors the click tracking technology here.

Online Supplement 2.3. Value of Price Commitment

When the price increment outweighs the value of the availability improvement, strategic customers may not be willing to click, and strategic clicks provide no value to the firm. One strategy the firm can use to induce them to click is to credibly commit to a price *ex ante*. Then customers know that even if they click, the price is not going to be increased. Examples of price commitment can take various forms. For example, some firms, such as BestBuy and Gap, are using the best-price policy, i.e., refund customers any price difference if price were decreased.

Now we are interested in the case when the condition in part (2) of Proposition A-1 is satisfied, so that customers are not willing to click. We are interested in whether the firm can induce customers to click via price commitment, and the implication of doing so.

Suppose the firm can credibly commit to a single price p *ex ante*, i.e., before customers decide to click or not. Then we are essentially back to our posted-price model, the demand function would be $D(p,\epsilon)$. We now focus on the equilibria where everybody clicks or nobody clicks. From each customer's perspective, if she does not click, her expected utility would be $U_{iN} = (v_i - p)\frac{\mathbb{E}\min\{D(p,\epsilon),q\}}{\mathbb{E}[D(p,\epsilon)]}$, where $q = F^{-1}_{D(p,\epsilon)}(\frac{p-c}{p})$. If she does click, then their expected utility would be $U_i = v_i - p - t$. Hence, we have $U_i > U_{iN}$ because t is sufficiently small, so that all customers are willing to click. Then, the firm's problem is to choose the optimal p to maximize its profit in the first place. We have the following result under price commitment.

Lemma A-1. *Suppose that the firm can commit to a price when demand is price-sensitive. An equilibrium in which all customers are willing to click exists, and in equilibrium, the firm's price* p^c *and quantity* q^c *satisfy* $p^c \in \arg\max\,(p-c)\mathbb{E}[D(p,\epsilon)]$, *and* $q^c = D(p^c,\epsilon)$. *Moreover, the firm's equilibrium profit is* $\Pi^*_{PC} = (p^c - c)\mathbb{E}[D(p^c,\epsilon)]$ *and customer i's expected utility is* $U^*_{iPC} = v_i - p^c - t$.

Proof. Direct checking of the equilibrium definition. ■

We can easily compare the outcome with the case when no price commitment is used. In that case, customers are not willing to click. The firm's profit is simply $\Pi^*(p^*,q^*) = p^*\mathbb{E}\min\{D(p^*,\epsilon),q^*\} - cq^*$, and customer i's utility is $U^*_i = (v_i - p^*)\frac{\mathbb{E}\min\{D(p^*,\epsilon),q^*\}}{\mathbb{E}[D(p^*,\epsilon)]}$. Direct comparison leads to the following conclusion.

Proposition A-3. *Firm profit with price commitment is higher, i.e.,* $\Pi^*_{PC} > \Pi^*(p^*,q^*)$.

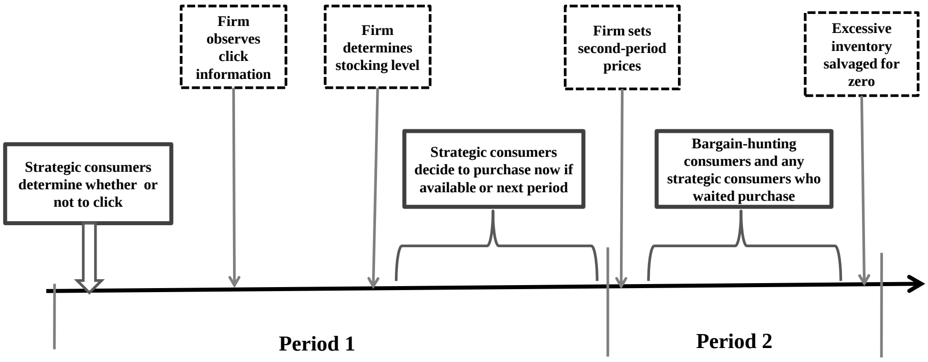
Proof. Direct comparison yields the result. ■

While this is good news for the firm, customers can be worse off under price commitment, because the commitment power allows the firm to extract more surplus from the consumers, similar in spirit to quantity commitment. The additional novelty here is that the commitment power induces consumers to *first* provide ADI and *then* purchase the product since availability would be guaranteed due to their clicking. In contrast, quantity commitment only induces more consumers to purchase without reducing any demand uncertainty.

Online Supplement 3. Strategic Clicks with Markdown Pricing

The simple models illustrate how strategic customers can "coordinate" with the firm to achieve a win-win outcome: strategic customers are willing to click to provide ADI to the firm in order to increase the product availability, which reduces the uncertainty of demand for the firm when click cost is small. A natural question is, is this result robust? Do strategic customers always prefer the firm to face a *certain* demand? We address these questions in this section in the context of a two period model with markdown pricing. The key tradeoff faced by customers now is the *stockout loss* versus *overstock benefit*: Providing ADI becomes a "double-edged sword" since it improves the

Figure A-1 Sequence of Events



availability probability while reducing the markdown probability. If the overstock benefit outweighs the stockout loss, strategic customers are not willing to provide ADI. The following model formalizes this intuition.

Online Supplement 3.1. Clicking Behavior in a Two-period Model

The first-period price p is exogenously given and fixed, but the second-period markdown price is endogenously determined by the firm. We first study the case in which there is only one type of strategic customers whose second-period valuation of the product is $\overline{v}$ (commonly known) and whose first-period valuation is v_1 (commonly known), where $\overline{v} \leq p \leq v_1$. There are a random number D of strategic customers in the first period. For analytical tractability, we assume D is approximately uniformly distributed over $[0, b]$, where $b > 0$. In the second period, the firm has to salvage its remaining inventory at value $v_L < c$. To explicitly account for inventory salvaging in the second period, we introduce bargain-hunting customers who only purchase the product when it is on sale in the second period, having valuation $v_L < c$. There are infinitely many of them in the market, and they do not click at all. Consistent with the literature, we assume that the strategic customers are satisfied first when both types of customers request for a unit. The timing of the game is shown in Figure A-1. For analytical tractability, we focus on the *symmetric* pure-strategy equilibrium where the same type of customers choose the same pure strategy. We characterize the Nash equilibrium as follows.

Proposition A-4. *(i) If $\overline{v} \in (c + \frac{v_1 - p}{v_L}, p)$, then there exists an equilibrium in which all strategic customers are not willing to click; (ii) If $\overline{v} \in (c, c + \frac{v_1 - p}{v_L}]$, then there exists an equilibrium in which all strategic customers are willing to click.*

Proof. We first analyze the case when click tracking technology were not used. The strategic customers can either purchase in the first period if there is product available or wait to purchase in the second period for a possibly lower price. Formally, let $\widehat{v} \in \{\overline{v} - \epsilon, \overline{v}\}$, where $\epsilon(> 0)$ is arbitrarily small and $\widehat{v}$ is the threshold value strictly above which the strategic customers whose second-period valuation is, prefer to wait for second-period selling, while all other customers would like to purchase in the first period.

If $\widehat{v} = \overline{v} - \epsilon$, then strategic customers prefer to wait to purchase in the second period. The firm has no sales in the first period. When the demand of strategic customers is D, the first-period stocking quantity q and second-period sale price s are chosen, its second-period revenue function is

$$R(D,s,q) = \begin{cases} \overline{v}\min\{D,q\} & \text{if } s = \overline{v} \\ v_L q & \text{if } s = v_L \end{cases}$$

It is clear that the optimal sale price as a function of demand D (fixing q) is

$$s^*(D) = \begin{cases} \overline{v} & \text{if } D > \frac{v_L}{\overline{v}} q \\ v_L & \text{if } D \leq \frac{v_L}{\overline{v}} q \end{cases}$$

Then the firm's expected profit is $\pi(q, \overline{v} - \epsilon) = \mathbb{E}[-cq + \overline{v}\min\{D,q\}1_{\{D > \frac{v_L}{\overline{v}} q\}} + v_L q 1_{\{D \leq \frac{v_L}{\overline{v}} q\}}]$. Let q_1^* be the maximizer of $\pi(q, \overline{v} - \epsilon)$. We can solve it analytically using FOC for general demand distributions. For special cases, for example, when D is uniformly distributed in $[a,b]$, we have $q_1^* = \frac{[b\overline{v} - av_L - (b-a)c]\overline{v}}{\overline{v}^2 - v_L^2}$. Strategic consumers' expected payoff is $U_1(\overline{v} - \epsilon) = (\overline{v} - v_L)\mathbb{P}\{D \leq \frac{v_L}{\overline{v}} q_1^*\} = (\overline{v} - v_L)\frac{\frac{v_L}{\overline{v}} q_1^* - a}{b-a} > v_1 - p$. The last inequality is from the assumption that in equilibrium, strategic customers prefer to wait to purchase in the second period. In this case, suppose the firm uses the click tracking technology to collect ADI, are strategic customers willing to click? If they click, then they provide perfect ADI to the firm, so the firm would order $q(D) = D$ and price $\overline{v}$ in the second period. Therefore, their ex ante payoff is $U_1^c(\overline{v} - \epsilon) = \max\{v_1 - p - t, -t\} \approx v_1 - p$ since $t \approx 0$. Clearly, we have $U_1(\overline{v} - \epsilon) > U_1^c(\overline{v} - \epsilon)$, so consumers are not willing to click. Since we assumed $a = 0$, then we know if $\overline{v} \in (c + \frac{v_1 - p}{v_L}, p)$, there exists an equilibrium strategic customers are not willing to click.

If $\widehat{v} = \overline{v}$, i.e., no strategic customers wait for second-period selling. Then the firm faces a newsvendor problem with salvage value v_L. The firm maximizes $\pi(q, \overline{v}) = \mathbb{E}[p\min\{q,D\} - cq + v_L(q-D)^+]$. Then $q_2^* = F^{-1}(\frac{p-c}{p-v_L})$. The strategic customers' expected payoff is $U_2(\overline{v}) = (v_1 - p)\frac{\mathbb{E}\min\{D, q_2^*\}}{\mathbb{E}(D)}$ when click tracking technology were not used. Suppose the firm uses click tracking technology, and if the strategic customers visit the website, their expected payoff is $U_2^c(\overline{v}) = \max\{v_1 - p - t, -t\} \approx v_1 - p$, which is greater than $U_2(\overline{v})$. Hence, all strategic customers are willing to click when click tracking is used. ■

Proposition A-4 can be put as follows: The strategic customers whose second period valuation is lower than the valuation threshold $c+\frac{v_1-p}{v_L}$ would purchase in the first period if click tracking were not used. They are willing to click if the technology is used since the availability benefit outweighs the markdown benefit. The strategic customers whose second period valuation is higher than the valuation threshold $c+\frac{v_1-p}{v_L}$ would wait for the markdown season if click tracking were not used. They are not willing to click if the technology is used since markdown benefit outweighs availability benefit.

To check the robustness of our result we extend to multiple customer types. For simplicity, we analyze the two-type case. In the second period, strategic customers' valuations can be either high or low with equal probabilities, $V_2 \in \{\underline{v}, \overline{v}\}$. Only the distribution of V_2 is known to the firm, and clicks provide no advance preference information (API), which here would mean the realization of V_2. We assume $\overline{v} > 2\underline{v}$ and $\overline{v} \gg v_L$ for analytical convenience (which would be clear in the proof of Proposition A-5). Let $D_i, i \in \{H, L\}$, be the number of type-i strategic consumers and $D = D_1 + D_2$, then we have $\mathbb{P}(D_L = n|D) = \mathbb{P}(D_H = n|D) = (\frac{1}{2})^D \binom{D}{n}$. Let q^* be the firm's optimal first-period ordering quantity when click tracking were not used, and $q^*(n)$ be the firm's optimal first-period ordering quantity upon observing n clicks *if* click tracking is used and only one type of strategic consumers click. At the very beginning of the first period, strategic consumers decide whether to click to provide ADI. Full characterization of equilibria is difficult. The following result partially characterizes the equilibrium outcomes.

Proposition A-5. *(i) If all strategic customers would wait to purchase in the second period if click tracking were not used and* $\mathbb{E}\sum_{n=0}^{D}\mathbb{P}(D < \frac{2v_L}{\overline{v}}q^*(n))\mathbb{P}(D_H = n|D) < \mathbb{P}(D < \frac{2v_L}{\overline{v}}q^*)$*, then no strategic customers are willing to click if click tracking is used.*
(ii) If only the high-type customers would wait to purchase in the second period if click tracking were not used and $\mathbb{E}\sum_{n=0}^{D}\min\{D, q^*(n)\}\mathbb{P}(D_L = n|D) > \mathbb{E}\min\{D, q^*\}$*, then only low-type customers are willing to click if click tracking is used.*
(iii) If no strategic customers would wait to purchase in the second-period if click tracking were not used and $\mathbb{E}\sum_{n=0}^{D}\min\{D, q^*(n)\}\mathbb{P}(D_L = n|D) > \mathbb{E}\min\{D, q^*\}$*, then both types of strategic customers are willing to click if click tracking is used.*

Proof. Similar to the proof of Proposition A-4, we first conduct the analysis for the case when click tracking technology were not used. It is clear that there exists $\widehat{v} \in \{\underline{v} - \epsilon, \underline{v}, \overline{v}\}$ for some arbitrarily small $\epsilon > 0$ such that the strategic consumers whose second-period valuation is strictly greater than $\widehat{v}$ wait to purchase in the second period, and the other consumers whose second-period valuation weakly less than $\widehat{v}$ purchase in the first period. A proof for continuous-type cases in similar models is provided in Cachon and Swinney (2009). Note that, we have broken any ties by assuming that the strategic consumers who get zero payoff will choose to purchase in the first period instead. In other words, there are only three possibilities: (1) No strategic customers wait to purchase in the second period; (2) Only high-type customers wait to purchase in the second period; (3) All strategic customers wait to purchase in the second period.

Knowing customers' second period strategy $\widehat{v}$, the firm chooses its second-period markdown price s to maximize its second period revenue $R(s,I)$, where $I=(q-\xi D)^+$ is the inventory left at the end of the first period, and ξ is the fraction of consumers who choose to purchase in the first period. If $\widehat{v}=\underline{v}-\epsilon$, then $\xi=0$; If $\widehat{v}=\underline{v}$, then $\xi=\frac{1}{2}$; If $\widehat{v}=\overline{v}$, then $\xi=1$. Now, we can discuss all the possible second-period profits of the firm for different $\widehat{v}$, the strategic customers' strategies. We discuss all the possible cases as follows.

- If $\widehat{v}=\underline{v}-\epsilon$, then $\xi=0$, so all strategic customers wait for second-period selling. We have the second-period profit function as a function of s, the second-period price,

$$R(s,I)=\begin{cases} s\min\{\frac{1}{2}D,q\} & \text{if } s\in(\underline{v},\overline{v}] \\ s\min\{D,q\} & \text{if } s\in(v_L,\underline{v}] \\ sq & \text{if } s\in(0,v_L] \end{cases}$$

Clearly, the firm only needs to restrict attention to three pricing options in the second period, pricing $\overline{v}$, $\underline{v}$ or v_L. Therefore, we actually have

$$R(s,I)=\begin{cases} \overline{v}\min\{\frac{1}{2}D,q\} & \text{if } s=\overline{v} \\ \underline{v}\min\{D,q\} & \text{if } s=\underline{v} \\ v_Lq & \text{if } s=v_L \end{cases}$$

Let $s^*(D)$ denote the optimal second-period price as a function of the realized demand D. To find $s^*(D)$, we have to compare the values of $R(s,I)$ associated with the three pricing options.

When $\frac{1}{2}D>q$, i.e., $D>2q$, we have $s^*(D)=\overline{v}$.

When $\frac{1}{2}D\le q$, and $D>q$, i.e., $D\in(q,2q]$, we have

$$R(s,I)=\begin{cases} \overline{v}\frac{1}{2}D & \text{if } s=\overline{v} \\ \underline{v}q & \text{if } s=\underline{v} \\ v_Lq & \text{if } s=v_L \end{cases}$$

To compare $\overline{v}\frac{1}{2}D$ and $\underline{v}q$, we have to first solve $\overline{v}\frac{1}{2}D=\underline{v}q$, which reduces to $D_m=d_mq=\frac{2\underline{v}}{\overline{v}}q$, which can be greater than or less than q depending on whether $\overline{v}>2\underline{v}$ or not. For simplicity, and without loss of generality, we assume $\overline{v}>2\underline{v}$, so that we have $D_m=\frac{2\underline{v}}{\overline{v}}q<q$. Therefore, $s^*(D)=\overline{v}$, since $\overline{v}\frac{1}{2}D>\underline{v}q$ when $D\in(q,2q]$.

When $D \leq q$, we have

$$R(s,I)=\begin{cases}\overline{v}\frac{1}{2}D & \text{if } s=\overline{v}\\ \underline{v}D & \text{if } s=\underline{v}\\ v_L q & \text{if } s=v_L\end{cases}$$

Recall that we assumed $\overline{v} > 2\underline{v}$, then $\overline{v}\frac{1}{2}D > \underline{v}D$. Therefore, we only need to compare $\overline{v}\frac{1}{2}D$ with $v_L q$. Let them be equal, we have $D_l = d_l q = \frac{v_L}{\frac{1}{2}\overline{v}} q$. Note that $\overline{v} > 2\underline{v} \geq 2v_L$, we have $D_l = \frac{2v_L}{\overline{v}} q < q$. Hence, if $D < D_l$, then $\overline{v}\frac{1}{2}D < v_L q$, pricing v_L is the optimal choice. If $D \in [D_l, q]$, then $\overline{v}\frac{1}{2}D \geq v_L q$, pricing $\overline{v}$ is the optimal choice. In sum, we have

$$s^*(D)=\begin{cases}\overline{v} & \text{if } D \in [\frac{v_L}{\frac{1}{2}\overline{v}}q, +\infty)\\ v_L & \text{if } D < \frac{v_L}{\frac{1}{2}\overline{v}}q\end{cases}$$

The corresponding second-period profit/revenue is

$$R(s^*(D),I)=\begin{cases}\overline{v}\frac{1}{2}D & \text{if } D \in [\frac{v_L}{\frac{1}{2}\overline{v}}q, +\infty)\\ v_L q & \text{if } D < \frac{v_L}{\frac{1}{2}\overline{v}}q\end{cases}$$

From strategic consumers' perspective, since they prefer to wait for second-period selling, their expected utility of purchasing in the second period should be greater than purchasing one unit for certain in the first period, i.e., $(\overline{v} - v_L)\mathbb{P}(D < \frac{2v_L}{\overline{v}} q^*) > v_1 - p$ and $(\underline{v} - v_L)\mathbb{P}(D < \frac{2v_L}{\overline{v}} q^*) > v_1 - p$.

If click tracking technology is used, then are they willing to click and who? Clearly, that both types of the strategic consumers do not click Pareto-dominates that both of them click for strategic consumers because of the two inequalities above. Let's investigate under what conditions that no consumers are willing to click is still an equilibrium when click tracking is used. Suppose the high-type consumers click, then the firm can condition its ordering quantity on the number of clicks for any realization of demand. The number of clicks provides the firm with some imperfect information about demand. Let D_H be the number of high-type consumers, which is equivalent to the number of clicks the firm observes. Then, upon observing n clicks, the firm has the following inference about demand, $\mathbb{P}(D|D_H = n) = \frac{\mathbb{P}(D; D_H = n)}{\mathbb{P}(D_H = n)} = \frac{\mathbb{P}(D_H = n|D)\mathbb{P}(D)}{\mathbb{P}(D_H = n)} = \frac{\mathbb{P}(D_H = n|D)\mathbb{P}(D)}{\sum_D \mathbb{P}(D_H = n|D)\mathbb{P}(D)} = \frac{(\frac{1}{2})^D \binom{D}{n}\mathbb{P}(D)}{\sum_D (\frac{1}{2})^D \binom{D}{n}\mathbb{P}(D)}$. Define $F_n(x) = \mathbb{P}(D < x|D_H = n)$ be the cumulative distribution function of D conditional on n number of observed clicks. We assume $F_n(x)$ has the monotone scaled likelihood ratio property (see Cachon and Swinney 2009) to guarantee that an equilibrium exists for any n. Let $q^*(n)$ be the firm's optimal first-period ordering quantity upon observing n clicks. Then, if $\mathbb{E}\sum_{n=0}^{D} \mathbb{P}(D < d_l q^*(n))\mathbb{P}(D_H = n|D) < \mathbb{P}(D < d_l q^*) \equiv \mathbb{E}\sum_{n=0}^{D} \mathbb{P}(D < d_l q^*)\mathbb{P}(D_H = n|D)$ is satisfied, no strategic consumers are willing to click in equilibrium. Intuitively, this condition says that the probability of significant overstock with imperfect ADI is smaller than that with no ADI. There seems to be no reason that this condition always holds, but we conjecture that in most cases the condition holds.

- If $\widehat{v} = \underline{v}$, then $\xi = \frac{1}{2}$. In this case, only the high-type consumers wait to purchase in the second period. Hence, the firm only needs to decide on two prices, either $\overline{v}$ or v_L, the second-period profit function is

$$R(s,I) = \begin{cases} \overline{v}\min\{\frac{1}{2}\alpha D, [q - \frac{1}{2}D]^+\} & \text{if } s = \overline{v} \\ v_L[q - \frac{1}{2}D]^+ & \text{if } s = v_L \end{cases}$$

If $[q - \frac{1}{2}D]^+ \leq \frac{1}{2}D$, i.e., $D \geq q$, then clearly, $s^*(D) = \overline{v}$. Otherwise, if $D < q$, we have to compare $\frac{1}{2}D$ with $v_L[q - \frac{1}{2}D]^+$. Let them be equal, we have the critical value $D_n = \frac{2v_L}{\overline{v}+v_L}q < q$. Hence, if $D < D_n$, then $s^*(D) = v_L$; if $D > D_n$, $s^*(D) = \overline{v}$. In sum, we have

$$s^*(D) = \begin{cases} \overline{v} & \text{if } D \in [\frac{2v_L}{\overline{v}+v_L}q, 2q] \\ v_L & \text{if } D < \frac{2v_L}{\overline{v}+v_L}q \end{cases}$$

The corresponding second-period profit of the firm is

$$R(s^*(D),I) = \begin{cases} \overline{v}\frac{1}{2}D & \text{if } D \in [\frac{2v_L}{\overline{v}+v_L}q, 2q] \\ v_L[q - \frac{1}{2}D]^+ & \text{if } D < \frac{2v_L}{\overline{v}+v_L}q \end{cases}$$

If click tracking technology is adopted by the firm, who are willing to click? First, are low-type consumers willing to click? Notice that in equilibrium without such technology used, low-type consumers prefer to purchase in the first period, hence their expected utility is $(v_1 - p)\frac{\mathbb{E}\min\{D,q^*\}}{\mathbb{E}(D)}$. If the following condition holds $\frac{\mathbb{E}\sum_{n=0}^{D}\min\{D,q^*(n)\}\mathbb{P}(D_L=n|D)}{\mathbb{E}(D)} > \frac{\mathbb{E}\min\{D,q^*\}}{\mathbb{E}(D)}$, then clicking improves the product availability. Therefore, if the inequality above hold, low-type consumers are willing to click. Now let's consider the high-type consumers. When click tracking were not used, high-type consumers would wait to purchase in the second period. However, when click tracking is used and this condition hold, then high-type consumers may prefer to purchase in the first period given the firm has a better forecast of demand upon observing demand of low-type consumers. In general, we need to distinguish the following two cases: Case 1: If the following condition hold $\mathbb{E}\sum_{n=0}^{D}(\overline{v} - v_L)\mathbb{P}(D < d_n q^*(n))\mathbb{P}(D_H = n|D) > v_1 - p$, then high-type consumers still wait to purchase in the second-period given that low-type consumers click. If they click, then the LHS of the inequality/condition is zero, since the firm can perfectly match supply with demand given low-type consumers are willing to click. Hence, high-type consumers are not willing to click. Recall that we have assumed that $\overline{v} \gg v_L$ so that this inequality holds. If this assumption were not assumed, then we need to consider Case 2: If this condition does not hold, then actually, merely by the fact that adopting click tracking technology is known to consumers, the firm has already induced high-type consumers to purchase in the first-period (rather than wait to purchase in the second period when click tracking were not used), and furthermore to click to improve product availability at the beginning of the first period.

- If $\widehat{v} = \overline{v}$, then $\xi = 1$. In this case, no strategic consumers wait for second-period purchase, hence, there is essentially no decision to make for the firm in the second period. The firm has to offer price v_L to clear its inventory, $s^*(D) = v_L$ and $R(s^*(D), I) = v_L(q-D)^+$ for this case. If click tracking is used, who are willing to click? Clearly, that both types of strategic consumers click can be one equilibrium, in which case, all strategic consumers are guaranteed to have a unit. If one type of consumers deviate, then there is some strictly positive probability to face stockouts. However, there might be equilibria in which no consumers are willing to click, if clicking cannot improve product availability, formally, if the (mild) condition $\frac{\mathbb{E}\sum_{n=0}^{D}\min\{D,q^*(n)\}\mathbb{P}(D_L=n|D)}{\mathbb{E}(D)} > \frac{\mathbb{E}\min\{D,q^*\}}{\mathbb{E}(D)}$ does not hold.

To find the optimal first-period ordering quantities q^* and $q^*(n)$, one can calculate the expected profit of the firm $\pi(q,\widehat{v}) = \mathbb{E}[p\min\{q,D\} - cq + R(s^*(D),[q-D]^+)]$. Again, one can discuss all the possible cases depending on the values of $\widehat{v}$. If $\widehat{v} = \underline{v} - \epsilon$, then $\xi = 0$, we have $\pi(q,\underline{v}-\epsilon) = \mathbb{E}[p\min\{q,(1-\alpha)D\} - cq + R(s^*(D),q)]$. Plugging in the expressions for $R(s^*(D),q)$, one can find q^* which maximizes this profit function. Similarly, one can do the same thing for other cases. Solving these optimal quantities is not the focus. Hence, we have proved this Proposition. ■

Proposition A-5 continues on the intuitive tradeoff in Proposition A-4: Customers with a higher second-period valuation have less willingness to click in equilibrium since the markdown benefit is more beneficial to them than the availability benefit. In sum, both two-period models suggest that in the settings when markdown pricing is possible and frequent (for instance, when selling nondurable/perishable or fashion goods), using strategic clicks is not always recommended. If many customers wait for second-period selling, then click tracking technology may be of little value.

These results on clicks and markdown pricing are complementary to Cachon and Swinney (2009) in terms of understanding strategic customer behavior in the two different mechanisms: Quick response relies on the firm's own capability to make the markdown probability slim and to induce strategic customers to be more willing to purchase in the first period and thus generate more value. In contrast, click tracking totally depends on consumer strategic behavior to provide value.

Online Supplement 3.2. Value of Product Personalization using Advance Preference Information

We have shown that in a markdown pricing setting strategic customers may not be willing to click. We therefore propose that the firm can use product personalization[12] to induce them to click. Personalized products create higher benefits for customers than standard products because

[12] Personalization broadly includes both "product personalization" and "price personalization," however, in this section, we refer personalization to product personalization only.

they deliver a closer preference fit (Franke et al. 2009). As discussed in the marketing literature (cf. Murthi and Sarkar 2003, Fay et al. 2009 and references therein), clickstream data provides a *learning* platform for personalization. Examples of using clicks as advance information in terms of customer preferences (which we call API) for personalization abound. The e-commerce arena is replete with instances of personalization (cf. Arora et al. 2008). Amazon.com offers customized book recommendations based on customers' online browsing behavior. At Pandora.com, based on the user's previous listening pattern by tracking his online clicking and listening behavior, personalized recommendations are made as to which new releases he would most enjoy (Moser 2006, Fay et al. 2009). Instances of personalization can also be equally (if not more) easily found in offline transactional settings. As Net-Results, a click tracking provider, claims "Net-Results enables your website to 'listen' to a prospect's needs and determine what they are specifically interested in."[13] We simply model personalization as the firm's capability to enhance customer valuation upon observing API from clicks.

We are interested in the welfare implications of personalization using clickstream data.[14] Different from all the values of personalization studied in the literature, we show that it can be used to mitigate the negative effect of customer strategic behavior *purely* from an operations perspective.

We return to the analysis of a single customer type with $\overline{v} \in (c + \frac{v_1 - p}{v_L}, p)$ so that strategic customers are not willing to click. In contrast, if personalization is used, strategic customers' utility by clicking would be $U_1^c(\overline{v} - \epsilon) = v_1 - p - t + e(\overline{v})$, where $e(\overline{v})$ is the enhanced utility due to the firm's product personalization using clicks as API. The customers' utility without clicking is $U_1(\overline{v} - \epsilon) = (\overline{v} - v_L)\mathbb{P}\{D \leq \frac{v_L}{\overline{v}} q_1^*\} = (\overline{v} - v_L)\frac{\frac{v_L}{\overline{v}} q_1^*}{b} = \frac{(\overline{v}-c)v_L}{\overline{v}+v_L}$, where $q_1^* = \frac{b\overline{v}(\overline{v}-c)}{\overline{v}^2 - v_L^2}$. The value of personalization for strategic customers is $V_c = U_1^c(\overline{v} - \epsilon) - U_1(\overline{v} - \epsilon) = v_1 - p - t + e(\overline{v}) - \frac{(\overline{v}-c)v_L}{\overline{v}+v_L} > 0$, provided that $e(\overline{v})$ is high. The value of using strategic clicks for ADI and input for product personalization for the firm is $V_f = (p - c - c_e)\mathbb{E}(D) - \pi(q_1^*, \overline{v} - \epsilon)$, where $\pi(q_1^*, \overline{v} - \epsilon) = \mathbb{E}[-cq_1^* + \overline{v}\min\{D, q_1^*\}1_{\{D > \frac{v_L}{\overline{v}} q_1^*\}} + v_L q_1^* 1_{\{D \leq \frac{v_L}{\overline{v}} q_1^*\}}]$. Apparently $V_f > 0$ provided that the marginal cost of personalization c_e is small. By online personalization (which induces strategic customers to provide ADI), every individual in the society can be better off and social welfare is strictly improved.

The value of using strategic clicks as ADI and input for personalization can be greater than the value of quick response in the presence of strategic consumers, if the cost of personalization is low.

[13] http://www.net-results.com/features/track-website-visitors.php, accessed on Feb. 15, 2010.

[14] As a *BizReport* article states (http://www.bizreport.com/2008/03/study-consumers-want-online-customization.html, accessed on June 26, 2009) "Consumers are no longer satisfied with simply finding a good deal after a long search online. They are also unsatisfied with cookie-cutter products that everyone else has. According to a recent Brand Keys study, today's online consumers want product and service information that is customized to their preferences."

Indeed, while quick response is the firm's own capability of producing products quickly, product personalization is the firm's own capability of improving customers' valuation of the products using clickstream data as API. In markdown pricing settings, each of the two different capabilities can generate significant value for the firm.

Online Supplement 4. Clicks with Endogenous Pricing

Whereas many e-commerce companies post prices *ex ante*, some companies (including the business-to-business (B2B) company we analyzed in detail) can set both quantity and price *after* monitoring clicks. To study the value of click tracking with endogenous pricing and capacity, we study the following game similar as before except that the firm can freely control its price: Customers must first decide whether to click or not, *before* they observe the endogenous price p set by the firm, then physically visit the firm, and experience an in-stock or stockout situation. (Note that the actual stocking quantity q is unobservable to customers.) As before, customers incur the click cost t when clicking. In addition, they incur a hassle cost h, which includes transportation cost, shopping effort and time etc., when buying. We assume $h + t \in (0, v - c)$ to avoid trivialities.

We continue to use the Nash and strong Nash equilibrium, except that the firm now has one more decision to make, i.e., the pricing decision. The following proposition characterizes the strong Nash equilibria.

PROPOSITION A-6. *If and only if* $h + t < v - c$ *and* $t \leq \frac{h}{\mu-1}$*, a unique strong Nash equilibrium exists, where each customer clicks with probability one, the firm produces quantity* $q^*(d) = d$ *if* d *clicks are observed, and charges price* $p^* = v - h - t$ *independent of the number of clicks. Furthermore, firm expected profit in equilibrium is*

$$\Pi^* = (v - c - h - t)\mu. \tag{A-1}$$

Proof. We show the "if" part: If $h + t < v - c$ and $t \leq \frac{h}{\mu-1}$, then the specified equilibrium is indeed a strong Nash equilibrium. Similar to the proof of Proposition A-1, we have to check all the possible deviations: the firm deviates, a single customer deviates, several customers jointly deviate, the firm and one customer jointly deviate, and the firm and several customers jointly deviate.

Suppose the firm deviates unilaterally by either pricing differently or produces differently or doing both, given the customers' strategies fixed. If the firm charges a price lower than $p^* = v - h - t$, then its expected profit would be strictly lower. If it charges a price higher than $p^* = v - h - t$, its profit would be zero given no customer would purchase the product. Now fixing the price $p^* = v - h - t$, the firm clearly would lose profits by producing other quantities than the number of clicks. It remains to check whether the firm has any profitable deviation by pricing and producing differently. Then, the firm chooses $p(D)$ and $q(D)$ to obtain this newsvendor profit: $\Pi(p(D), q(D)) = p(D)\mathbb{E}\min\{q(D), D\} - cq(D)$, which clearly achieves its maximum only if $q(D) = D$ and $p(D) = p^* = v - h - t$. Therefore, the firm has no unilateral profitable deviation.

Suppose a single customer deviates by not clicking. Then, his utility would be $U_N = (v - p^*)\frac{\mu-1}{\mu} - h = t\frac{\mu-1}{\mu} - \frac{h}{\mu}$. If $U = 0 \geq U_N$, i.e., $t \leq \frac{h}{\mu-1}$, then the customer has no profitable deviation. The same arguments tell us that several customers jointly deviating cannot be profitable either.

Suppose the firm and one customer jointly deviates. Note that the firm achieves its first-best by extracting all consumer surplus and perfectly matching demand in the equilibrium specified. Any deviation would be less profitable. Hence, the firm and any number of customers cannot deviate profitably. Hence, we have proved that the equilibrium specified is indeed a strong equilibrium. Now, we have to show the uniqueness, which clearly follows from the arguments above.

Next, we show the "only if" part: If the equilibrium specified exists and is unique, it is necessary that $h + t < v - c$ and $t \leq \frac{h}{\mu-1}$. The condition $h + t < v - c$ is necessary for the firm's profit to be positive, and the condition $t \leq \frac{h}{\mu-1}$ ensures that no single customer has profitable deviation. Hence, we complete the proof. ■

Note that the condition $t \leq \frac{h}{\mu-1}$ implies that the larger the hassle cost h, the less stringent the condition of the click cost t for the firm to obtain perfect ADI from strategic customers.

If $h = 0$ and $t = 0$, then the price postponement model yields the same equilibrium outcome as the posted price model. Next, we keep $h > 0$ so we can compare our results with other strategies analyzed in the literature.

Online Supplement 5. Comparing Strategic Clicks to Other Operations Strategies

In this section, we compare the value of adopting click tracking technology to the value of other operations strategies such as quantity commitment and availability guarantees, studied in Su and Zhang (2009), and quick response studied extensively in the literature. The main reasons for this comparison are: These strategies have the common goal of improving newsvendor firm profit. These comparisons using a unified model help decision making by analyzing which strategy is most effective for given circumstances.

Online Supplement 5.1. Strategic Clicks Compared to Quantity Commitment

Su and Zhang (2009) study "quantity commitment," where the firm commits to a particular quantity *ex ante*, as a strategy to improve profits. They show that by committing to an optimal quantity

q_C^*, the firm is able to increase its endogenous price, quantity and profits. As discussed before, they use the standard "RE equilibrium" (which is actually a Nash equilibrium which implicitly captures "rational expectation") which cannot rule out the possibility that the market is not functional in equilibrium. In fact, the operations literature put the "implementation" issue in the margin and do not discuss why we should ignore this "nonparticipatory equilibrium." Using the strong equilibrium solution concept, interestingly, the nonparticipatory equilibrium is not a strong Nash equilibrium since the grand coalition of all players has profitable deviations. More interestingly, one can easily show that the "RE equilibrium" studied by Su and Zhang (2009) is the unique strong Nash equilibrium. Under quantity commitment, in equilibrium, the firm's expected profit is

$$\Pi_C^* = v\mathbb{E}\min\{D, q_C^*\} - cq_C^* - h\mu, \tag{A-2}$$

where $F(q_C^*) = \frac{v-c}{v}$.

We compare the firm profit using quantity commitment in equation (A-2) with equation (A-1) and show that the firm is strictly better off using strategic clicks if the click cost t is small. For convenience, we denote $t_C = v - c - \frac{v\mathbb{E}\min\{D,q_C^*\}-cq_C^*}{\mu}$. One can verify that $t_C > 0$.

Proposition A-7. *The firm is strictly better off by using strategic clicks over quantity commitment, i.e.,* $\Pi^* > \Pi_C^*$, *if* $t \le t_C$.

Proof. We have $\Pi^* - \Pi_C^* = (v - h - t - c)\mu - [v\mathbb{E}\min\{D, q_C^*\} - cq_C^* - h\mu] > 0$, when $t \le t_C$. ■

The intuition is clear: Strategic clicks can essentially eliminate demand uncertainty, since in equilibrium all strategic customers are willing to click. In contrast, quantity commitment gives the firm the "commitment power" to induce customers to purchase at a higher price, but cannot reduce demand uncertainty. The interesting point is that it is in the best interest of strategic customers to click in this setting of endogenous pricing when the click cost is small.

Online Supplement 5.2. Strategic Clicks Compared to Availability Guarantees

"Availability guarantees" refer to mechanisms in which the firm promises to compensate consumers in the event of a stockout. Examples include transshipment, monetary compensation, gift cards and discount coupons. Su and Zhang (2009) argue that a viable commitment device may not exist in practice to maintain the quantity commitment power and they show that availability guarantees stimulate demand and improve seller profits. Again, there is a unique strong Nash equilibrium, where we let p_G^* and q_G^* denote the optimal endogenous price and quantity under availability guarantees. Firm's profit is $\Pi_G^* = v\mathbb{E}\min\{D, q_G^*\} - cq_G^* - h\mu + (u - w)[\mu - \mathbb{E}\min\{D, q_G^*\}]$, where q_G^*

satisfies $F(q_G^*) = \frac{p_G^* + w - c}{p_G^* + w}$, p_G^* satisfies $(v - p_G^*)\frac{\mathbb{E}\min\{D, q_G^*\}}{\mathbb{E}(D)} + u[1 - \frac{\mathbb{E}\min\{D, q_G^*\}}{\mu}] = h$ and (u, w) denotes the compensation menu.

For convenience, denote $t_G = \frac{(v-h)(\mu - \mathbb{E}\min\{D, q_G^*\}) - c(\mu - q_G^*)}{\mu}$, and one can easily verify that $t_G > 0$. We have compared the performance of using clicks as opposed to availability guarantees in the presence of strategic customers:

Proposition A-8. *The firm is strictly better off by using strategic clicks over availability guarantees, i.e.,* $\Pi^* > \Pi_G^*$*, if* $t \leq t_G$.

Proof. Notice that the upper bound of the value of availability guarantees achieves when $u = h$ and $w = 0$, so $\Pi_G^* < v\mu\min\{D, q_G^*\} - cq_G^* - h\mu + h[\mu - \mathbb{E}\min\{D, q_G^*\}] = (v - h)\mathbb{E}\min\{D, q_G^*\} - cq_G^* \leq (v - c - h - t)\mu = \Pi^*$, if $t \leq t_G$. ■

The intuition is as follows: Strategic clicks can eliminate demand uncertainty, while availability guarantees can only stimulate demand without reducing any demand uncertainty (see Su and Zhang 2009 for detailed intuition on how availability guarantees work).

A numerical study of cross comparison is presented in Table A-1. The numerical results suggest the following insights. Su and Zhang (2009) have proved that commitment and availability guarantees do enhance firm value when selling to strategic consumers. The results here suggest that the *magnitude* of this value of using these strategic measurements can be small, compared to the value of strategic clicks as ADI. Quantity commitment gives the smallest value, while the upper bound value of using availability guarantees (which occurs when consumers' utility from compensation $u = h$ and the firm's cost of the compensation $w = 0$) can be much larger than that of quantity commitment. In contrast, the value of strategic clicks can be more than one hundred fold the upper bound value of using availability guarantees. This suggests that the *efficiency effect* of eliminating supply-demand mismatches can be much more significant than the *strategic effect* of merely influencing customer behavior. Obviously, when considering adopting one of these practices, a firm should not only consider its value but also the application setting and its implementation cost. Typically, the investment cost of quantity commitment or availability guarantees seems small; while click tracking incurs some software investment. Notably, for some traditional brick-and-mortar retailers, implementing the Internet information channel with click tracking can be difficult; while quantity commitment and availability guarantees enjoy more broad applicability. Hence, we are not suggesting that one strategy is better than the others, because that depends on many practical factors excluded in our stylized model. Rather, our results imply that, *ceteris paribus*, if click tracking is a feasible option for a newsvendor firm, it can potentially generate significant value when selling to strategic customers.

Table A-1 Comparison of Values (in % Increment) of Different Practices

Parameters ($u = h = 0.01$, $t = 0$, $w = 0$, $v = 1.$)	Coefficient of Variation	Quantity Commitment	Availability Guarantees (Upper Bound)	Strategic Clicks
$c = 0.4$				
Beta(10,5)	0.18	0.00105%	0.09%	13.03%
Beta(10,50)	0.29	0.00173%	0.20%	23.54%
Beta(10,100)	0.30	0.00173%	0.21%	24.90%
$c = 0.4$				
Gamma(10,50)	0.32	0.00172%	0.23%	26.53%
Gamma(10,5)	0.32	0.00205%	0.23%	26.55%
Gamma(10,1)	0.32	0.00184%	0.23%	26.55%
Gamma(10,0.1)	0.32	0.00187%	0.23%	26.54%
$c = 0.3$				
Gamma(4,2)	0.50	0.00188%	0.25%	36.12%
Gamma(1,2)	1.00	0.00863%	0.93%	109.97%
Gamma(0.6,100)	1.29	0.02400%	1.75%	187.67%
$c = 0.3$				
Lognormal(1,0.01)	0.01	0.02480%	2.55%	177.34%
Lognormal(1,1)	1.31	1.99000%	4.73%	244.48%

In these comparisons, we have implicitly assumed that $h < \overline{h}_C$ and $h < \overline{h}_G$ respectively (see Su and Zhang 2009 for detailed definitions of them). Su and Zhang (2009) prove that $\overline{h}_C \geq \overline{h}_G$. Let $\overline{h}_T = v - c - t$ denote the hassle cost threshold above which the market breaks down in our setting. One can easily prove that $\overline{h}_T > \overline{h}_C \geq \overline{h}_G$ when t is small, so that there are regions where the firm still makes strictly positive profits by using strategic clicks, whereas the market would break down if only either quantity commitment or availability guarantees were used.

Online Supplement 5.3. Strategic Clicks Compared to Quick Response

Following the literature, we model quick response capability as follows. Before the beginning of the season, the firm can order or produce quantity q at unit cost c_1. After observing demand D, the firm can quickly produce or procure any excess demand $(D-q)^+$ at a higher unit cost $c_2 > c_1$. Consumers know that the firm has quick response capabilities. We have a similar definition of a strong Nash equilibrium as before in this quick response setting.

We have the following result if quick response is adopted by the firm.

Lemma A-2. *Suppose that the firm has quick response capabilities. There exists $\overline{h}_{QR} < v - c_1$, such that a strong Nash equilibrium exists if and only if $h < \overline{h}_{QR}$. In equilibrium, the firm's price $p^*_{QR} = v - h$ and quantity $q^*_{QR} = F^{-1}(\frac{c_2 - c_1}{c_2})$, and the equilibrium firm profit is $\Pi^*_{QR} = \mathbb{E}[(v-h)D - c_2(D - q^*_{QR})^+] - c_1 q^*_{QR}$.*

Proof. Let the search/hassel cost $\overline{h}_{QR}$ satisfy $\Pi^*_{QR} = \mathbb{E}[(v-h)D - c_2(D - q^*_{QR})^+] - c_1 q^*_{QR} = 0$, then as long as the search cost h is less than $\overline{h}_{QR}$, and there is a maxima in the firm profit maximization problem, an equilibrium exists where the market is functional. To solve the optimal initial quantity Π_{QR}^*, plugging the optimal price $p_{QR}^* = v - h$ into the profit function, we have

$$\Pi_{QR}(q) = \mathbb{E}[(v-h)D - c_2(D-q)^+] - c_1 q.$$

Differentiation of $\Pi_{QR}(q)$ using the Leibniz rule yields

$$\Pi'_{QR}(q) = -c_1 + c_2(1 - F(q))$$

First-order condition gives $q_{QR}^* = F^{-1}(\frac{c_2 - c_1}{c_2})$. One can verify the second-order condition is satisfied. Finally, we check whether any coalition has any profitable deviations. First, it is clear that several customers deviate together by not purchasing the product cannot be profitable. Second, if the firm and some customer(s) deviate jointly, then the firm clearly becomes worse off by losing sales. Hence, no coalition of the players has any profitable deviation. The equilibrium specified is indeed a strong Nash equilibrium. ■

It is reasonable to assume that the quick response capability is not too expensive, for example, $c_2 \le p_{QR}^* = v - h$ for the firm. When $c_1 = c$ and $c_2 = v - h$, the firm still enjoys a strictly positive value with quick response versus without quick response, since $\Pi_{QR}^* = v\mathbb{E}\min\{D, q_{QR}^*\} - cq_{QR}^* - h\mathbb{E}\min\{D, q_{QR}^*\} > v\mathbb{E}\min\{D, q^*\} - cq^* - h\mathbb{E}(D) = \Pi^*$, where Π^* is the optimal firm profit without quick response and $q^* < q_{QR}^* = F^{-1}(\frac{v-h-c}{v-h})$ is the optimal newsvendor quantity without response. The intuitive explanation is that, although quick response capability *itself* is not strictly profitable (i.e., $p_{QR}^* - c_2 = 0$), it ensures strategic consumers guaranteed availability so that a higher price can be charged. Indeed, the following corollary shows that, interestingly, the "best" availability guarantee is precisely the "worst" quick response, i.e., the upper bound value of availability guarantees is the same as the lower bound value of quick response.

Corollary 1. *If $u = h$, $w = 0$, $c_1 = c$ and $c_2 = v - h$, $q_{QR}^* = q_G^* = F^{-1}\left(\frac{v-h-c}{v-h}\right)$, then $p_{QR}^* = p_G^* = v - h$ and $\Pi_{QR}^* = \Pi_G^*$.*

Proof. If $u = h$, $w = 0$, then consumers receive the maximum compensation while the firm incurs zero cost by doing that. Clearly, this is the "limit" of the best availability guarantees, under which, we can compute the optimal price $p_G^* = v - h$, the optimal stocking quantity $q_G^* = F^{-1}(\frac{v-h-c}{v-h})$ and the corresponding firm profit $\Pi_G^* = v\mathbb{E}\min\{D, q_G^*\} - cq_G^* - h\mathbb{E}\min\{D, q_G^*\}$. If $c_1 = c$ and $c_2 = v - h$, then the firm's initial unit production cost is the same as that without quick response while the unit quick production cost is the same as the selling price $p_{QR}^* = v - h$. Clearly, this is the "weakest" quick response capability. According to Lemma A-2, this corollary follows. ■

This corollary linking the efficiency effect with the strategic effect appears novel. It illustrates how the efficiency effect from better matching uncertain demand generally dominates the strategic effect from influencing consumer behavior.

To compare quick response with strategic clicks, we have to make assumptions on the relative magnitudes of c, c_1 and c_2. Consistent with the literature (cf. Cachon and Swinney 2009), we assume $c_2 > c$ and $c_1 = c$. For convenience, we denote $t_{QR} = \frac{c_2(D-q^*_{QR})^+ + cq^*_{QR}}{\mu} - c$. Clearly, $t_{QR} > 0$ since $c_2 > c$. Directly comparing the firm profit under quick response (Lemma A-2) with equation (A-1) shows:

Proposition A-9. *Strategic clicks outperform quick response, i.e., $\Pi^* > \Pi^*_{QR}$, if $t \leq t_{QR}$.*

Proof. This result can be proved by simply noting that the firm's profit is monotone decreasing in c_1. When $c_1 = c$, strategic clicks strictly outperform quick response since $c_2 > c$, if $t \leq t_{QR}$. ■

Comparing quick response with strategic clicks, we have two points to make: (1) Strategic clicks merely rely on strategic consumer behavior to match uncertain demand, while quick response requires the firm to incur the higher quick production cost c_2 to do this matching. (2) The two-way (reducing both underage and overage) efficiency effect of using clicks as ADI outweighs the one-way (reducing only underage) effect of using quick response. In addition to these two points, there is an important cost consideration that favors strategic clicks over quick response: The cost to build a quick response capability can be prohibitively high. (Indeed, while the quick response capability of Zara is well known, few firms have been able to copy that capability.) In contrast, click tracking technology is accessible to even small firms, since the software installment and operating cost is relatively low. Again, we are not arguing one strategy is better than the other in practice (where the assumptions of the model are not tested, for example, customers may not be informed or strategic, the production lead time can be too long for the ADI to realize its full value). Indeed, as already discussed, quick response is powerful in the setting of markdown pricing.

Online Supplement 6. Comparing Strategic Clicks with Other Operations Strategies with Uncertain Customer Valuation

In this section, we assume customer valuation V is uncertain *before* physically visiting the seller when any of the traditional strategies is used. The basic setup is essentially the same as in the previous section except the following minor accommodations: First, the valuation distribution $G(V)$ is common knowledge *ex ante*. However, the "*ex ante*" here has different meanings for different strategies. For traditional strategies, i.e., quantity commitment, availability guarantees and quick response, it means "before physically visiting the seller;" for strategic clicks, it means "before clicking." Second, for simplicity, we assume that $G(V)$ is a two-point distribution over $\{\overline{v}, \underline{v}\}$. The reason of working with the two-point distributions would be clear later on. Denote the probability of being of high type by $\mathbb{P}(\overline{v})$. Third, for the purpose of comparisons, we assume the price p is endogenous and the hassle cost is h as before. Fourth, the equilibrium concept we use here is the same as before except that the customer valuation V is uncertain. The distribution $G(V)$ is rationally expected by the firm, but the realization of each customer valuation V after physically visiting the store (or after visiting the firm's website if click tracking is used) is privately known by the customer.

Studying the value of these strategies in the presence of customer valuation *uncertainty* is novel in the literature by itself. As we shall see, unfortunately, we have to lose analytical tractability by introducing this valuation uncertainty in general. Specializing in the special cases when the valuation can only take two values and the demand is normally distributed allows us to have a certain level of mathematical tractability and numerical flexibility.

Online Supplement 6.1. No Strategy

Suppose the firm uses no strategies. We first consider the typical customer's problem. Before the customer physically visits the store, he does not know for sure whether he would eventually buy the product since his valuation is uncertain. The firm charges a price p which coincides with the customer's reservation price by rational expectations. The customer would visit the store if and only if

$$\mathbb{E}\max\{(V-p)1_{\{available\}}, 0\} \geq h, \tag{A-3}$$

where $1_{\{available\}}$ is the indicator function meaning the customer gets a good.

For the given price p, the demand D_p is approximately normally distributed with mean $\mu_D = \mu_N\overline{G}(p)$ and variance $\sigma_D^2 = \mu_N\overline{G}(p)G(p) + \sigma_N^2\overline{G}^2(p)$. One of the several "rules of thumb" for an excellent approximation is: Both $\mu_N\overline{G}(p)$ and $\mu_N G(p)$ must be greater than 5. Note that the firm would rationally choose $p \in (\underline{v}, \overline{v})$ for interesting scenarios such as those in our numerical study later on. Since the valuation distribution is two-point, we have $\overline{G}(p) = \mathbb{P}(\overline{v})$ so that within this price range the demand distribution is effectively price-independent. Working with a general price-dependent demand distribution requires joint optimization over price and quantity. However, we can reduce this joint-optimization problem to a simple single-variable optimization problem only involving price p here. The newsvendor firm would stock the optimal quantity as a function of p,

$$q_p = F_{D_p}^{-1}\left(\frac{p-c}{p}\right). \tag{A-4}$$

The firm chooses price p to maximize its expected profit

$$\Pi_0(p) = p\mathbb{E}\min\{D_p, q_p\} - cq_p, \tag{A-5}$$

subject to the two equilibrium conditions (A-3) and (A-4).

Let $z_p = \Phi^{-1}(\frac{p-c}{p})$. Simplification leads to the following result.

PROPOSITION A-10. *Suppose no strategy is adopted. The equilibrium price p_0^* solves the following constrained optimization problem*

$$\max_p (p-c)\mu_D - p\phi(z_p)\sigma_D$$

s.t.

$$\overline{G}(p)(\overline{v}-p)\left[1 - COV_D\left(\phi(z_p) - \frac{c}{p}z_p\right)\right] \geq h.$$

The optimal quantity $q_0^* = \mu_N \overline{G}(p_0^*) + z_{p_0^*}\sqrt{\mu_N \overline{G}(p_0^*)G(p_0^*) + \sigma_N^2 \overline{G}^2(p_0^*)}$.

The proof is straightforward and omitted for the sake of brevity. Denote the customer expected utility of visiting the store in equilibrium by $u_0^* = \overline{G}(p_0^*)(\overline{v} - p_0^*)\left[1 - COV_D\left(\phi(z_{p_0^*}) - \frac{c}{p_0^*}z_{p_0^*}\right)\right]$ for later use.

Online Supplement 6.2. Quantity Commitment

Under quantity commitment, we can similarly derive the following result.

PROPOSITION A-11. *Suppose the firm adopts the quantity commitment strategy. The equilibrium price* $p_C^* = \overline{v} - \frac{h}{\overline{G}(p_C^*)s(q_C^*)}$, *the optimal quantity* $q_C^* = \mu_N \overline{G}(p_C^*) + z_{\overline{v}}\sqrt{\mu_N \overline{G}(p_C^*)G(p_C^*) + \sigma_N^2 \overline{G}^2(p_C^*)}$, *where* $z_{\overline{v}} = \Phi^{-1}(\frac{\overline{v}-c}{\overline{v}})$, *and the optimal firm profit is* $\Pi_C^* = \overline{v}\mu_D\left[1 - COV_D\left(\phi(z_{\overline{v}}) - \frac{c}{\overline{v}}z_{\overline{v}}\right)\right] - cq_C^* - \frac{h\mu_D}{\overline{G}(p_C^*)}$.

Online Supplement 6.3. Availability Guarantees

We restrict our attention to the "best" availability guarantees, i.e., when the customer utility achieves the maximum, $u = h$, and the cost of compensation in case of stockout is zero, $w = 0$. The condition for customers to visit the store is

$$\mathbb{E}\max\{(V-p)1_{\{available\}}, 0\} + u\mathbb{E}1_{\{V \geq p\}}(1 - 1_{\{available\}}) \geq h. \tag{A-6}$$

We have the following proposition.

Proposition A-12. *Suppose availability guarantees are adopted. The equilibrium price* p_G^* *solves the following constrained optimization problem*

$$\max_p (p-c)\mu_D - p\phi(z_p)\sigma_D$$

s.t.

$$\overline{G}(p)(\overline{v}-p)\left[1 - COV_D(\phi(z_p) - \frac{c}{p}z_p)\right] + \overline{G}(p)COV_D\left(\phi(z_p) - \frac{c}{p}z_p\right)h \geq h.$$

The optimal quantity $q_G^* = \mu_N\overline{G}(p_G^*) + z_{p_G^*}\sqrt{\mu_N\overline{G}(p_G^*)G(p_G^*) + \sigma_N^2\overline{G}^2(p_G^*)}$.

Online Supplement 6.4. Quick Response

Under quick response, the condition for customers to visit the store is

$$\mathbb{E}\max\{V-p,0\} \geq h. \tag{A-7}$$

We have the following result.

Proposition A-13. *Suppose quick response is adopted. The firm's equilibrium price* $p_{QR}^* = \overline{v} - \frac{h}{\overline{G}(p_{QR}^*)}$ *and quantity* $q_{QR}^* = F_{D_{p_{QR}^*}}^{-1}\left(\frac{c_2-c}{c_2}\right)$, *and the equilibrium firm profit is* $\Pi_{QR}^* = p_{QR}^*\mu_D - c_2\mathbb{E}(D_{p_{QR}^*} - q_{QR}^*)^+ - c_1q_{QR}^*$.

Online Supplement 6.5. Strategic Clicks

Suppose the click tracking technology is adopted by the firm. In equilibrium, the firm charges a price p such that all customers click to provide imperfect ADI. Observing the number of clicks $N=n$, the firm approximates the demand by a normal distribution with mean $\mathbb{E}(D|n) = n\overline{G}(p)$ and variance $Var(D|n) = n\overline{G}(p)G(p)$. The condition for customers to click is

$$\mathbb{E}_V\mathbb{E}_N\max\{(V-p)1_{\{N=n,available\}},0\} \geq h+t, \tag{A-8}$$

where the indicator function $1_{\{N=n,available\}}$ denotes the event that both $N=n$ and the customer gets a good.

In any equilibrium (if it exists), the firm has state-dependent price and quantity decisions as functions of the observed number of clicks $N=n$. However, each customer cannot observe the number of clicks and thus can only make his decision by expectation. The firm chooses its optimal price and quantity decisions subject to two conditions: First, each customer prefers to physically visit the firm to purchase the product; Second, each customer prefers to click to provide imperfect ADI for the firm.

The following proposition characterizes the equilibrium outcome.

Proposition A-14. *Suppose click tracking is adopted. The equilibrium price $p^*(n)$ when n clicks are observed solves the following constrained optimization problem*

$$\max_{p(n)} (p(n)-c)n\overline{G}(p(n)) - p(n)\phi(z_{p(n)})\sqrt{\overline{G}(p(n))G(p(n))}\sqrt{n}$$

s.t.

$$\mathbb{E}\overline{G}(p(N))(\overline{v}-p(N))\left[1-\sqrt{\frac{G(p(N))}{\overline{G}(p(N))}}(\phi(z_{p(N)})-\frac{c}{p(N)}z_p)\right]\sqrt{\frac{1}{N}} \geq h+t.$$

and

$$\mathbb{E}\overline{G}(p(N))(\overline{v}-p(N))\left[1-\sqrt{\frac{G(p(N))}{\overline{G}(p(N))}}(\phi(z_{p(N)})-\frac{c}{p(N)}z_p)\right]\sqrt{\frac{1}{N}} \geq u_0^*.$$

The optimal quantity $q^*_{D|n} = n\overline{G}(p^*(n)) + z\sqrt{n\overline{G}(p^*(n))G(p^*(n))}$, *where* $z = \Phi^{-1}\left(\frac{p^*(n)-c}{p^*(n)}\right)$

Notice that here the equilibrium price is a function of observed number of clicks n. Working on this general equilibrium structure is difficult, and we focus on the special case where $p^*(n)$ is actually independent of n in our numerical experiments. Clearly, doing this yields conservative value of noisy strategic clicks, so that we obtain a lower bound value of strategic clicks. The second constraint in this proposition guarantees that customers prefer clicking to non-clicking.