Asymmetric disassembly and robustness in declining networks

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Mechanisms that enable declining networks to avert structural collapse and performance degradation are not well understood. This knowledge gap reflects a shortage of data on declining networks and an emphasis on models of network growth. Analyzing >700,000 transactions between firms in the New York garment industry over 19 years, we tracked this network’s decline and measured how its topology and global performance evolved. We find that favoring asymmetric (disassortative) links is key to preserving the topology and functionality of the declining network. Based on our findings, we tested a model of network decline that combines an asymmetric disassembly process for contraction with a preferential attachment process for regrowth. Our simulation results indicate that the model can explain robustness under decline even if the total population of nodes contracts by more than an order of magnitude, in line with our observations for the empirical network. These findings suggest that disassembly mechanisms are not simply assembly mechanisms in reverse and that our model is relevant to understanding the process of decline and collapse in a broad range of biological, technological, and financial networks.

Research on the dynamics and robustness of complex networks (1–3) has emphasized the study of global network growth, identifying assembly mechanisms such as preferential attachment (4, 5), vertex fitness (6), vertex duplication (7), and fractal network growth (8), which, at the macroscopic level, generate stable topological characteristics despite large fluctuations in the microscopic network parameters. In addition, mechanisms have been proposed that can produce stable topological metrics in networks of constant size (9–12). Notably, methods based on percolation theory have significantly contributed to our understanding of how robust the static network structures generated by these assembly mechanisms are to fragmentation under random and targeted attack (8, 13–15).

Less is known about the dynamics and robustness of networks under sustained decline. In declining networks, new nodes and links may be added over time but the net process is a progressive loss of nodes and links. Hence, although assembly mechanisms may come into play, the emphasis must be on which disassembly mechanisms help preserve the topological characteristics and performance of the network. For example, Alzheimer’s research examines how degradation in mental performance can be related to sustained decline. In an ecological context, analogous questions arise regarding the vulnerability of food webs to habitat loss and fragmentation (17, 18). In social and economic systems, network decline has raised questions about the preservation of social capital (19), resource allocation in developing economies (20), the prevention of financial collapse (21), effective coordination among suppliers (22), the restructuring of political systems (23), and lock-in into inferior technological standards (24).

We analyze the interorganizational network that makes up the famous New York City Garment Industry (NYGI) (25, 26), which has persistently shrunk over the last 19 years. In this network, nodes correspond to designers and contractors that are linked through coproductions of annual runs of lines of clothing. Designers design clothing and contractors fabricate it, but their roles often overlap because the industry’s low-cost and quick-to-market conditions entwine design and production (25, 27). Our data include virtually all 10,000 plus firms and their >700,000 bilateral exchanges, circa 1985–2003, as recorded by the Union of Needle Trades and Industrial and Textile Employees (UNITE) (see Methods). Although these data resemble nation-to-nation commodity flow or interbank payment data (28–31), there are important differences. The links are not directional as in commodity chains, which are typified by flows from raw to finished goods along distinct stages of production. Instead, in our data, links are primarily reciprocal. They reflect the coproduction process between designers and contractors at one stage in the production process. Also, we have a measure of network performance not found in trade data: the fraction of bilateral transactions with errors per annum, which directly gauges the network’s loss of functionality.

In this article, we show that the topological robustness of a declining network depends critically on how disassembly and assembly processes act in conjunction with each other. We define topological robustness as the ability of a network subject to both contraction and growth processes to resist quick fragmentation and to preserve the stability of key metrics characterizing the network topology. Our analysis and model indicates that in a declining network the growth process corresponding to partial recovery follows preferential attachment (PA) (4), and the contraction process favors the preservation of asymmetric links (namely, interactions between high-degree nodes and low-degree nodes). We label this contraction process asymmetric disassembly and show that it is associated with stable topological and functional features in a declining network. Song et al. (8) showed that for growing networks disassortativity generates self-similar structures that play a crucial role in preventing network fragmentation. Our model complements these findings by identifying disassembly mechanisms that avert network fragmentation in a distinct class of network problems corresponding to sustained decline.

Empirical Results and Discussion

Topological robustness and network contraction. The starting point for our analysis charts changes in the topology and performance

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of the NYGI network as it shrinks in size [see supporting information (SI) Fig. S1 and Table S1]. The findings suggest that in a declining network, topological robustness can be preserved despite massive turnover and a substantial net loss of nodes and links. The persistence of topological robustness as the network evolves coincides with a stable measure of network performance.

Topological changes were measured with characteristics that focus on the network’s connectivity and degree distribution and that have been used in prior research on network dynamics (32). Table 1 shows the observed contraction of the largest connected component (LCC) and total number of links L, and the progressive decline in the average degree (k) of nodes. However, as shown in Table 1, the LCC resisted fragmentation in the sense that it contained a high and relatively stable proportion of the total nodes within the empirical network over the years. Focus- ing on the network’s topological features, Table 1 also shows that the average path length (l) and diameter D of the network remain stable. The degree distribution is characterized by a truncated power-law and remains stable over the observation period, until 2000, when finite size effects in the network become stronger (see Table 1 and Fig. S2). Fig. 1A shows how the cumulative degree distribution Pcum(k) varies over the observation period, where for a given year the distribution is calculated using all transactions in that year (see SI Text and Table S2). In addition, we track over time how the average over the degree of all nodes ⟨k⟩ directly connected to a reference node k scales with the degree of that node (32). As shown in Fig. 1B, we find that this relation is defined by ⟨k⟩ ~ kν with ν = −0.5 (see Table 1), which provides strong evidence for disassortativity (i.e., preference of high-degree nodes to connect to low-degree nodes and vice versa) (32) and the persistence of topological features in the network (see also Figs. S3 and S4). Disassortativity is common in biological, technological and financial networks (8, 29, 33, 34), which suggests that asymmetric links may play an important role in network robustness.

To measure changes in performance directly, we examine the network’s error rate, given by the fraction of bilateral transactions that include “refunds.” A refund is a reversal of a transaction between firms that occurs when the original exchange involved an error (e.g., design mistakes, shorted goods, manufacturing errors, etc.). A low fraction of errors signifies high performance. Fig. 1C shows how refunds as a percentage of total transactions changed each year over the observation period. We see that the fraction of links with errors remained reasonably stable until 1998, after which its mean and variability rose significantly. The observed correspondence between changes in the network topology and functional performance suggest that assembly and disassembly mechanisms, structure, and function are strongly related (see Methods).

### Assembly and Disassembly Mechanisms

Building on prior research on the growth and cohesiveness of social networks (35, 36), we first analyze the data to see whether local assembly rules for nodes during global decline follow PA mechanisms. PA assumes that a node’s degree provides a reasonable proxy measure for its fitness when direct performance information is unavailable for nodes (38, 37). In this way, a node’s probability of acquiring new links is linearly proportional to its degree (fitness). We found that newcomer firms enter with low degree (see Fig. S5), and, consistent with past research, PA characterizes how newcomer firms attach to incumbent firms. Fig. 2A shows the relative probability T(t), compared with random firm selection, that a link added at year t connects to an incumbent firm with k(t−1) previous partners. This probability is defined by T(t) ~ k(t)ν with ν = 1.20 ± 0.06 (R2 = 0.81), which shows that the number of newcomers’ links acquired by incumbent firms scales proportionally with the firms’ degree (39).

Shifting our focus to deletion mechanisms for nodes, we found that firms have a probability of deletion inversely proportional to their degree (see Fig. S6), in line with realistic deletion mechanisms found in other dynamic networks (40, 41). Given that the mechanism acting on newcomer nodes that is responsible for the partial growth process mirrors the mechanism by which nodes are deleted as part of the contraction process, we examine whether contraction and recovery processes for links between incumbent nodes follow the same pattern.

Following this logic, we measure the relative probabilities R(t) and R(t) that an incumbent firm with k links in year t−1 will lose and gain new incumbents’ links in year t (i.e., excluding newcomer attachment). We again followed Newman (39) to extract R(t), and found that a firm’s relative probability, compared with random firm selection, of gaining new links by connecting to other incumbent firms scales with the firm’s degree as R(t) ~ k(t) with ν = 0.84 ± 0.04 (R2 = 0.72) (see Fig. S7). For the relative probability of link deletion, we calculate R(t), assuming that links are randomly removed between incumbent firms. Fig. 2B shows that the probability of a firm losing a link decreases in proportion to the firms’ degree as k(t) with ν = −0.41 ± 0.04 (R2 = 0.54) and a significant exception for the least connected firms. Our statistical analysis for links shows that PA seems to explain the assembly rule, whereas the disassembly rule is characterized by an asymmetric process that favors firms with extreme low and high degrees.

Considering our results on assembly and disassembly rules for links in the context of socioeconomic networks, differences in the nature of the information available to firms creating new links or breaking existing links may account for the observed assembly and disassembly mechanisms. In assembly, the use of a firm’s degree as a proxy for fitness when forming new links

<table>
<thead>
<tr>
<th>Year</th>
<th>LCC (%)</th>
<th>L</th>
<th>⟨k⟩</th>
<th>⟨l⟩</th>
<th>D</th>
<th>α</th>
<th>kmin</th>
<th>kmax</th>
<th>ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>3,249 (95)</td>
<td>7,250</td>
<td>4.46</td>
<td>4.22</td>
<td>14</td>
<td>2.44</td>
<td>4</td>
<td>50</td>
<td>−0.52</td>
</tr>
<tr>
<td>1988</td>
<td>2,410 (94)</td>
<td>5,504</td>
<td>4.56</td>
<td>4.16</td>
<td>12</td>
<td>2.38</td>
<td>4</td>
<td>47</td>
<td>−0.49</td>
</tr>
<tr>
<td>1991</td>
<td>1,880 (93)</td>
<td>3,981</td>
<td>4.23</td>
<td>4.15</td>
<td>11</td>
<td>2.31</td>
<td>4</td>
<td>56</td>
<td>−0.55</td>
</tr>
<tr>
<td>1994</td>
<td>1,135 (93)</td>
<td>1,917</td>
<td>3.37</td>
<td>4.3</td>
<td>10</td>
<td>2.36</td>
<td>3</td>
<td>48</td>
<td>−0.52</td>
</tr>
<tr>
<td>1997</td>
<td>842 (91)</td>
<td>1,450</td>
<td>3.44</td>
<td>4.06</td>
<td>10</td>
<td>2.38</td>
<td>4</td>
<td>54</td>
<td>−0.55</td>
</tr>
<tr>
<td>2000</td>
<td>449 (87)</td>
<td>716</td>
<td>3.18</td>
<td>4.01</td>
<td>9</td>
<td>2.20</td>
<td>2</td>
<td>—</td>
<td>−0.54</td>
</tr>
<tr>
<td>2003</td>
<td>190 (84)</td>
<td>228</td>
<td>2.4</td>
<td>4.07</td>
<td>10</td>
<td>2.09</td>
<td>1</td>
<td>—</td>
<td>−0.56</td>
</tr>
</tbody>
</table>

LCC represents the total number of firms in the network’s largest connected component along with the observed percentage in parentheses of the LCC in the empirical network. L gives the number of connections in the LCC, ⟨k⟩ is the average number of connections per firm in the network, ⟨l⟩ is the average path length, and D is the diameter of the network (maximum distance between two firms). Here α, kmin, and kmax correspond to the power-law exponent and the minimum and maximum cut-off values respectively, where the function Pcum(k) = k(−1−ν) is validated by the Kolmogorov–Smirnov goodness-of-fit test and following Newman (42) for the calculations (see SI Text, section 1). Finally, the table shows the exponent ν of the nearest-neighbors average connectivity of nodes degree k as defined by ⟨k⟩ ~ kν.
makes sense because gathering direct performance data are costly when there is no existing link that can act as a conduit for quality information. By contrast, during disassembly there is already a link in place. Through this link the quality of a trading relationship can be judged based on firm-specific, first-hand information taking into account the history of past interactions. Because disassembly decisions can be based on direct data from “the horse’s mouth” rather than a proxy, the degree—a general index of a node’s fitness—is less valuable than the firm-specific information garnered through a link. This means that the lowest degree firms could be excellent collaborators who have become well adapted to an exclusive, high-quality relationship, an argument supported by field work that has found that asymmetric links are indicative of good working partnerships in competitive markets like the NYGI (27, 37).

Following this logic, we introduce a simple measure \( \delta_i \) to capture the level of asymmetry of a link connecting two nodes with degree \( k_i \) and \( k_j \), respectively, where \( \delta_i = (k_j - k_i)^2/(k_j + k_i)^2 \). This measure produces values over the interval \([0, 1]\), where \( \delta_i = 1 \) is a maximally asymmetric link, and is useful for comparisons because it rescales the absolute differences in degree. Looking at the empirical data to see how the frequency of refunds varies across links, we found a negative correlation between the asymmetry level of a link \( \delta_i \) and the probability of a refund occurring \( \beta_i \). This probability was calculated by a probit regression on \( \delta_i \) (\( P < 0.001 \)) of the form \( \beta(\delta_i) = \frac{1}{1 + e^{-\left( a + b \delta_i \right)}} \), where

\[ a = -0.832 \quad \text{and} \quad b = -1.11 \quad (\text{see Fig. S8}) \]

The model was validated by the Homer–Lemeshow goodness-of-fit test with \( P = 0.224 \). This suggests that declining networks that favor asymmetric links increase the likelihood of preserving the network’s functional relationships.

Generalized Model of Contraction

Guided by our empirical evidence, we constructed a simple model for network contraction that combines two main processes: asymmetric disassembly, which represents the contracting process, and PA, which represents the partial recovery process in our network (see Fig. S9 for a graphical representation of the model). In its general form, the model applies to undirected networks; however, an extension to directed networks is provided in SI Text and Figs. S10 and S11. The inputs for the model in each time step are the number of nodes deleted (DF) and created (NF), and the number of links deleted (DL) and created (NL) between incumbent nodes, all of which are given directly by the empirical data (see Table S3). The simulation model is initialized using the empirical network of 1985.

The first step in the model is motivated by the empirical observation that the network experiences a sustained net loss of firms each period. This generates the need for a rule on node deletion.

\( i \). Node deletion is treated probabilistically with the inverse network connectivity providing a reasonable proxy for unob-
The function on a log-log scale the relative probability of link deletion corresponds to the function \( T(\tau) \sim k(\tau)^{-v} \) with \( v = 1.2 \), which is a fair approximation to a linear preferential attachment giving the finite size of the network.

The deviating points in the tail of the distribution of the empirical LCC. Fig. 3 shown in Fig. 2. A flat distribution for the relative probability of link deletion replicates the finding of topological robustness. We focus first on the recovery process. Fig. 3A-D summarize the results. The data suggest that when the correct disassembly and ensemble mechanisms are combined, the model accounts well for the empirically observed behavior and replicates the finding of topological robustness. We focus first on the recovery process. Fig. 3A-D shows the agreement between the actual empirical and model-generated values for the degree distribution when we use PA assembly mechanisms, as in the original model, and when we use random assembly mechanisms instead. Under random assembly, steps iib and iii of our model are modified so that incumbent and newcomer nodes attach to a randomly selected node with probability \( P(m) = (k_m + 1)/(\sum_i k_i + 1) \). The number of nodes added (NF) each time step is given empirically by the new number of firms that appear in the dataset the following year, and corresponds approximately to 15% of the LCC.

Simulation Results and Discussion

Fig. 3A-D show the agreement between the actual empirical and model-generated values for the degree distribution when we use PA assembly mechanisms, as in the original model, and when we use random assembly mechanisms instead. Under random assembly, steps iib and iii of our model are modified so that incumbent and newcomer nodes attach to a randomly selected node with probability \( P(m) = (k_m + 1)/(\sum_i k_i + 1) \). The number of nodes added (NF) each time step is given empirically by the new number of firms that appear in the dataset the following year, and corresponds approximately to 15% of the LCC.

The third step corresponds to the creation of new links between newcomer and incumbent firms. This accounts for the parallel regrowth process observed in the network.

The second step captures the fact that incumbent firms make two choices with respect to managing their links in each yearly production cycle: first, they can remove a link that connected them to a partner in the previous time period; second, they can replace a removed link with a connection to a new partner.

Asymmetric disassembly and PA consists of removing and replacing links. For simplicity, we assume that links are randomly selected and are subjected to the following rules:

iia. Removing. In line with our empirical findings, selected links are removed with a probability defined by \( P_{\text{kept}} = 1 - (k_i - k_j)^2/(k_i + k_j)^2 \), which is given by the asymmetry level of the link. The number of links removed (DL) is taken from the empirical data as the interactions in a given year that do not recur in the following year, and corresponds to a 25% of the LCC.

iib. Replacing. For each link \( k_i \) that is removed, one randomly chosen node (either \( i \) or \( j \)) replaces that link with probability \( q \), otherwise both nodes lose the link. When replacing a link, the chosen node \( i \) or \( j \) selects a new node \( m \) following PA, with a probability \( P(m) = (k_m + 1)/(\sum_i k_i + 1) \). The probability \( q \) was empirically fixed at \( q = 0.9 \), which corresponds to the average percentage of new links created between incumbent firms (NL/DL) over the entire observation period. Lower \( q \) values lead to a more rapid fragmentation of the network (see Fig. S12).

The second step captures the fact that incumbent firms make two choices with respect to managing their links in each yearly production cycle: first, they can remove a link that connected them to a partner in the previous time period; second, they can replace a removed link with a connection to a new partner.

ii. Asymmetric disassembly and PA consists of removing and replacing links. For simplicity, we assume that links are randomly selected and are subjected to the following rules:
process that follows asymmetric disassembly (blue circles) best corresponds with the observed values over time, whereas random assembly (black triangles) and random link removal (red crosses) clearly do not reproduce the resistance to fragmentation of the LCC. Random node deletion (green squares) leads to an even faster network collapse.

Finally, we studied the network’s robustness to topological changes generated by asymmetric disassembly and the three alternative contraction mechanisms. Fig. 3D shows for the nearest-neighbor average connectivity defined by $k_{nn}(t)$, the ratio between the model-generated and empirical exponent $v$, when the network is subjected to asymmetric disassembly (blue circles), random link removal (red crosses), random recovery (black triangles), and random node deletion (green squares).

Conclusions

Despite a large body of research on network growth, the complementary process of decline has been given short shrift. In both the case of growth and decline, a key issue is robustness. The specific network of firms in the NYGI that we have analyzed and modeled exhibits a remarkably robust topology and stable performance measures while undergoing severe decline. Ultimately the network collapses into a highly centralized configuration characterized by elevated error rates and reduced overall performance, suggesting the existence of a threshold of minimally functional connectivity.

At the microscopic level, the observed network robustness can be linked to the enhanced functional and structural preference for asymmetric interactions. This result has an interesting counterpart in ecological network research on how species may be linked together (34) and respond to realistic extinction sequences (41). Our simulation studies enable us to extend our findings beyond the specific empirical context that we have focused on and show that alternative combinations of assembly and disassembly processes lead to a more rapid network fragmentation and changes in structural features. By augmenting studies of network growth with a general model of contraction and examining the consequences for topological robustness, we hope to open up new directions for research on network dynamics and robustness.

Methods

**NYGI Dataset.** This is longitudinal empirical dataset on the interfirm network of the New York City garment industry from 1985 to 2003. Our data include $\sim$700,000 transactions from January 1985 to December 2003 for 10,000 firms that collaborated in the production of clothing. A link exists between two firms if they coproduce a garment. For example, the typical production process begins with a designer that develops a line of clothing. Each garment in the line is made into a sample prototype, which is disassembled into its component parts such as shelves, collars, waistbands, and so forth. The components of the
sample are then sent by the designer to contractors that cut components from fabric in lots large enough to be mass produced. The cut fabric is then sent by the designer to sewing contractors that sew the fabric together into the garments that are sold directly to consumers at retailers. Links are typically undirected. Information and finances flow reciprocally as part of the production process, rather than directionally. That is, although the designer may produce the original design and sample prototype, the contractors in the network often add design changes that simplify production or enhance efficiency, which makes a final design a reciprocal effort. In our data, all designers and contractors are in the same finished goods stage of the production process; there are no downstream raw fabric suppliers (that only sell cloth) or upstream retailers (that only buy finished goods) in the data. All firms are free to make connections of their own choice; there is no governing body that suggests or mandates connections. Each transaction in the data is associated with a volume of exchange or a run. Because a single line of clothing is often produced in several runs, we aggregated runs between designers and contractors into a single link that represents the whole production job. We focused on the largest connected component, which corresponds to 95% of the population. The dataset was collected and made available by the Union of Needle Trades and Industrial and Textile Employees (UNITE). UNITE organizes nearly all the firms in the NYGI and uses a highly reliable record system (27) that requires firms to report all their network contacts quarterly. These self-reports are checked by union auditors for accuracy. Nonunionized firms are typically small, fly-by-night firms that elude discovery because of their brief existence (27).

Network Performance. In the context of the NYGI, we acknowledge that error rates can fluctuate for various reasons. Based on prior research and personal communication with UNITE (43), we viewed errors as primarily due to the network’s failure to govern effective collaborations among interdependent firms. However, error rates may occur for other reasons. For example, adverse selection could lead a biased sample of firms operating in the market or market power could enable some firms to force fake returns. On the basis of our data, we cannot fully discriminate between these different micro mechanisms. Nonetheless, these other factors seem unlikely. Firms that may be victimized can appeal to UNITE, which protects weak firms against abuses of market power. Similarly, with the onslaught of international competition, adverse selection seems unlikely in that the fittest firms are most likely to survive.

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