Supplementary Appendix to "The Risk Premia Embedded in Index Options"

Torben G. Andersen* Nicola Fusari† Viktor Todorov‡

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Contents

*Department of Finance, Kellogg School of Management, Northwestern University, Evanston, IL 60208; NBER, Cambridge, MA; and CREATES, Aarhus, Denmark; e-mail: t-andersen@northwestern.edu.

†The Johns Hopkins University Carey Business School, Baltimore, MD 21202; e-mail: nicola.fusari@jhu.edu.

‡Department of Finance, Kellogg School of Management, Northwestern University, Evanston, IL 60208; e-mail: v-todorov@northwestern.edu.

A The Non-Linear Factor Structure of Option Surfaces

In a standard option pricing model, the Black-Scholes implied volatility is given by $\kappa(k, \tau, \mathbf{S}_t, \theta)$ where $\mathbf{S}_t$ is the state vector driving the option surface dynamics ($\mathbf{S}_t$ equals $(V_{1,t}, V_{2,t}, U_t)$ for our three-factor model in the paper). This makes the option surface obey a *nonlinear* factor structure. The goal of this section is to illustrate in a simple setting the amount of this nonlinearity and further illustrate the extra dynamics in the option surface induced by it.

We use simulated data from the Heston model for this analysis. We recall the Heston model is a one-factor constrained version of our general stochastic volatility model in (3.1) in the paper with no price and volatility jumps. The parameters in the simulation are set to: $\theta_1 = 0.0225$, $\kappa_1 = 4$, $\sigma_1 = 0.3$ and $\rho_1 = -0.9$ (for simplicity we assume the same model under $\mathbb{P}$ and $\mathbb{Q}$). We simulate the process over a period of five years and, at the end of every week on the simulated trajectory, we compute the theoretical price of an OTM put option with fixed moneyness and maturity equal to 0.1 years. We report results for two alternative ways of defining moneyness: fixed moneyness of $K/F_{t,t+\tau} = 0.9$ and volatility adjusted moneyness of $\log(K/F_{t,t+\tau})/(\sqrt{\tau}\sqrt{V_{1,t}}) = -2$.

For each of the days in the simulated sample and for the two different ways of fixing the moneyness, we then compute the derivative of the theoretical Black-Scholes implied volatility with respect to the current level of the stochastic variance, i.e., we compute

$$\frac{\partial \kappa(k, \tau, V_{1,t}, \theta)}{\partial V_{1,t}}.$$

The results are reported on Figures A.1 and A.2. As seen from the figures, the sensitivity of the option implied volatility with respect to current spot variance is rather nontrivial, regardless of the method of fixing the moneyness, even in the very simple Heston stochastic volatility model. This variability plotted on Figures A.1 and A.2 is essentially ignored when conducting linear factor analysis on the the Black-Scholes implied volatility surface.

B Additional Results and Robustness Checks for the Parametric Modeling of the Option Panel

B.1 Estimation Results for the General Model

The model is given by:

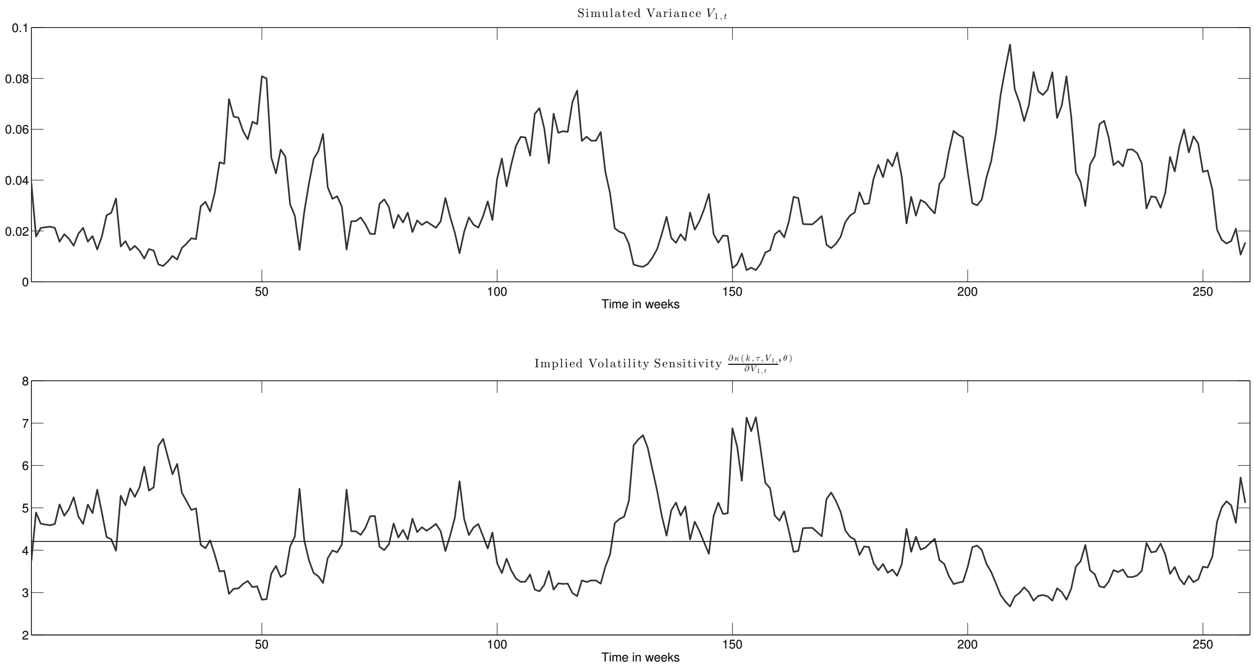

Figure A.1: **Sensitivity of Option Implied Volatility to the level of Volatility in the Heston Model: Case of Fixed Moneyness**. Top Panel: simulated variance path from the Heston model. Bottom Panel:sensitivity of OTM Put option's implied volatility with respect to the the current level of variance, for a fixed level of moneyness $K/F_{t,t+\tau} = 0.9$. The straight line on the bottom panel corresponds to the average value of the derivative in the sample.

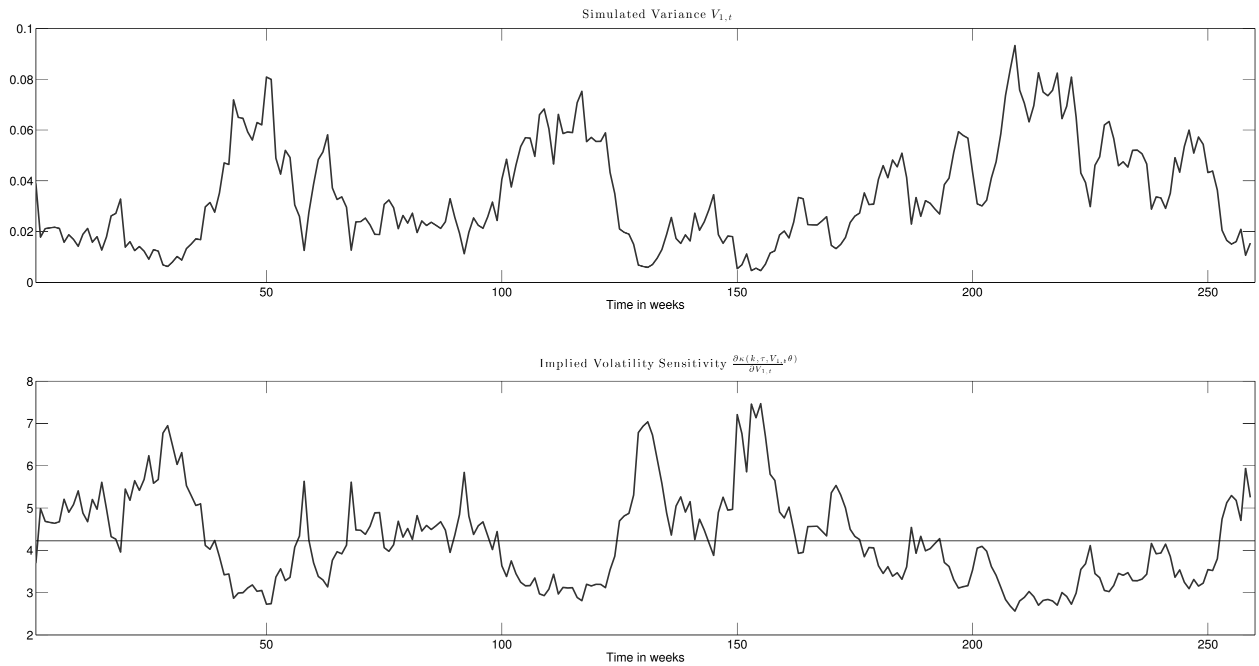

Figure A.2: **Sensitivity of Option Implied Volatility to the level of Volatility in the Heston Model: Case of Volatility-Adjusted Moneyness**. Top Panel: simulated variance path from the Heston model. Bottom Panel: sensitivity of OTM Put option's implied volatility with respect to the the current level of variance, for a volatility adjusted level of moneyness $\log(K/F_{t,t+\tau})/(\sqrt{\tau}\sqrt{V_{1,t}}) = -2$. The straight line on the bottom panel corresponds to the average value of the derivative in the sample.

$$
\begin{aligned}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)\,dt + \sqrt{V_{1,t}}\,dW^{\mathbb{Q}}_{1,t} + \sqrt{V_{2,t}}\,dW^{\mathbb{Q}}_{2,t} + \eta\sqrt{U_t}\,dW^{\mathbb{Q}}_{3,t} + \int_{\mathbb{R}^2} (e^x - 1)\,\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy), \\
dV_{1,t} &= \kappa_1\,(\overline{v}_1 - V_{1,t})\,dt + \sigma_1\sqrt{V_{1,t}}\,dB^{\mathbb{Q}}_{1,t} + \mu_1 \int_{\mathbb{R}^2} x^2\,1_{\{x<0\}}\,\mu(dt, dx, dy), \\
dV_{2,t} &= \kappa_2\,(\overline{v}_2 - V_{2,t})\,dt + \sigma_2\sqrt{V_{2,t}}\,dB^{\mathbb{Q}}_{2,t}, \\
dU_t &= -\kappa_u\,U_t\,dt + \mu_u \int_{\mathbb{R}^2} \left[(1-\rho_u)\,x^2\,1_{\{x<0\}} + \rho_u\,y^2\right]\mu(dt, dx, dy),
\end{aligned}
\tag{B.1}
$$

where $(W^{\mathbb{Q}}_{1,t}, W^{\mathbb{Q}}_{2,t}, W^{\mathbb{Q}}_{3,t}, B^{\mathbb{Q}}_{1,t}, B^{\mathbb{Q}}_{2,t})$ is a five-dimensional Brownian motion with $\mathrm{corr}\left(W^{\mathbb{Q}}_{1,t}, B^{\mathbb{Q}}_{1,t}\right) = \rho_1$ and $\mathrm{corr}\left(W^{\mathbb{Q}}_{2,t}, B^{\mathbb{Q}}_{2,t}\right) = \rho_2$, while the remaining Brownian motions are mutually independent. The jump compensator is given by,

$$
\frac{\nu^{\mathbb{Q}}_t(dx, dy)}{dxdy} = \begin{cases} \left(c^-(t)\cdot 1_{\{x<0\}}\lambda_- e^{-\lambda_-|x|} + c^+(t)\cdot 1_{\{x>0\}}\lambda_+ e^{-\lambda_+ x}\right), & \text{if } y = 0, \\ c^-(t)\lambda_- e^{-\lambda_-|y|}, & \text{if } x = 0 \text{ and } y < 0, \end{cases}
$$

with

$$
c^-(t) = c^-_0 + c^-_1 V_{1,t-} + c^-_2 V_{2,t-} + U_{t-}, \qquad c^+(t) = c^+_0 + c^+_1 V_{1,t-} + c^+_2 V_{2,t-} + c^+_u U_{t-}\,.
$$

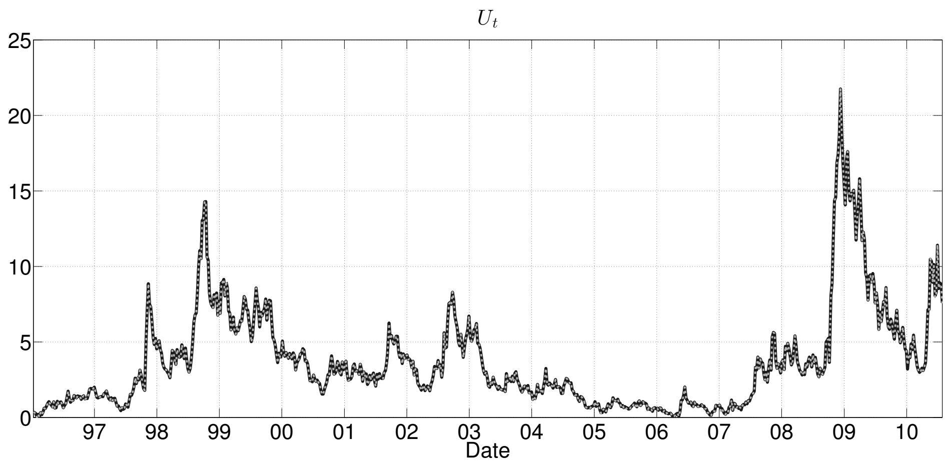

Figure B.1: **Recovered** U. Solid dark line: U corresponding to model (B.1) with no restrictions on coefficients. Dotted light line: U corresponding to model (B.1) with $c^-_0 = c^-_2 = c^+_u = \eta = 0$.

Table B.1: Estimation results for model (B.1).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.962	0.157	σ_2	0.168	0.007	c_1^+	25.178	5.246
$\overline{v}_1$	0.003	0.000	η	0.001	0.013	c_2^-	0.233	3.391
κ_1	10.968	0.204	μ_u	7.3425	29.736	c_2^+	82.151	7.051
σ_1	0.257	0.049	κ_u	0.084	0.146	c_u^+	0.007	0.007
μ_1	12.161	0.261	ρ_u	0.385	5.438	λ_-	25.941	0.212
ρ_2	-0.989	0.036	c_0^-	0.005	0.132	λ_+	36.543	0.826
$\overline{v}_2$	0.010	0.000	c_0^+	0.348	0.029			
κ_2	1.828	0.122	c_1^-	110.324	5.139			

Panel B: Summary Statistics

RMSE	1.71%
Mean jump intensity (yearly) (-/+)	5.55/1.79
Mean jump size (-/+)	3.85%/2.74%

Note: Estimation period is January 1996-July 2010.

B.2 Estimation Results on Subsamples

B.2.1 From January 1996 to December 2006

Table B.2: Estimation results for model (B.1).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.974	0.271	$\overline{v}_2$	0.016	0.000	c_0^+	0.384	0.029
$\overline{v}_1$	0.001	0.000	κ_2	1.677	0.109	c_1^-	176.055	10.385
κ_1	13.028	0.287	σ_2	0.223	0.005	c_1^+	9.098	4.419
σ_1	0.144	0.048	μ_u	9.013	71.215	c_2^+	84.003	7.611
μ_1	10.983	0.347	κ_u	0.162	0.373	λ_-	26.306	0.238
ρ_2	-0.858	0.018	ρ_u	0.887	7.651	λ_+	37.218	0.735

Panel B: Summary Statistics

RMSE	0.99%
Mean jump intensity (yearly) (-/+)	6.65/1.54
Mean jump size (-/+)	3.80%/2.69%

Note: Estimation period is January 1996-December 2006.

B.2.2 From January 2007 to July 2010

Table B.3: Estimation results for model (B.1).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.987	0.081	$\overline{v}_2$	0.007	0.000	c_0^+	0.571	0.066
$\overline{v}_1$	0.004	0.000	κ_2	2.061	0.155	c_1^-	112.519	6.845
κ_1	11.962	0.318	σ_2	0.169	0.011	c_1^+	18.786	4.863
σ_1	0.322	0.034	μ_u	9.163	23.635	c_2^+	87.929	11.474
μ_1	12.630	0.347	κ_u	0.056	0.119	λ_-	25.720	0.353
ρ_2	-0.990	0.054	ρ_u	0.323	3.982	λ_+	35.814	0.995

Panel B: Summary Statistics

RMSE	2.16%
Mean jump intensity (yearly) (-/+)	5.59/1.94
Mean jump size (-/+)	3.89%/2.79%

Note: Estimation period is January 2007-July 2010.

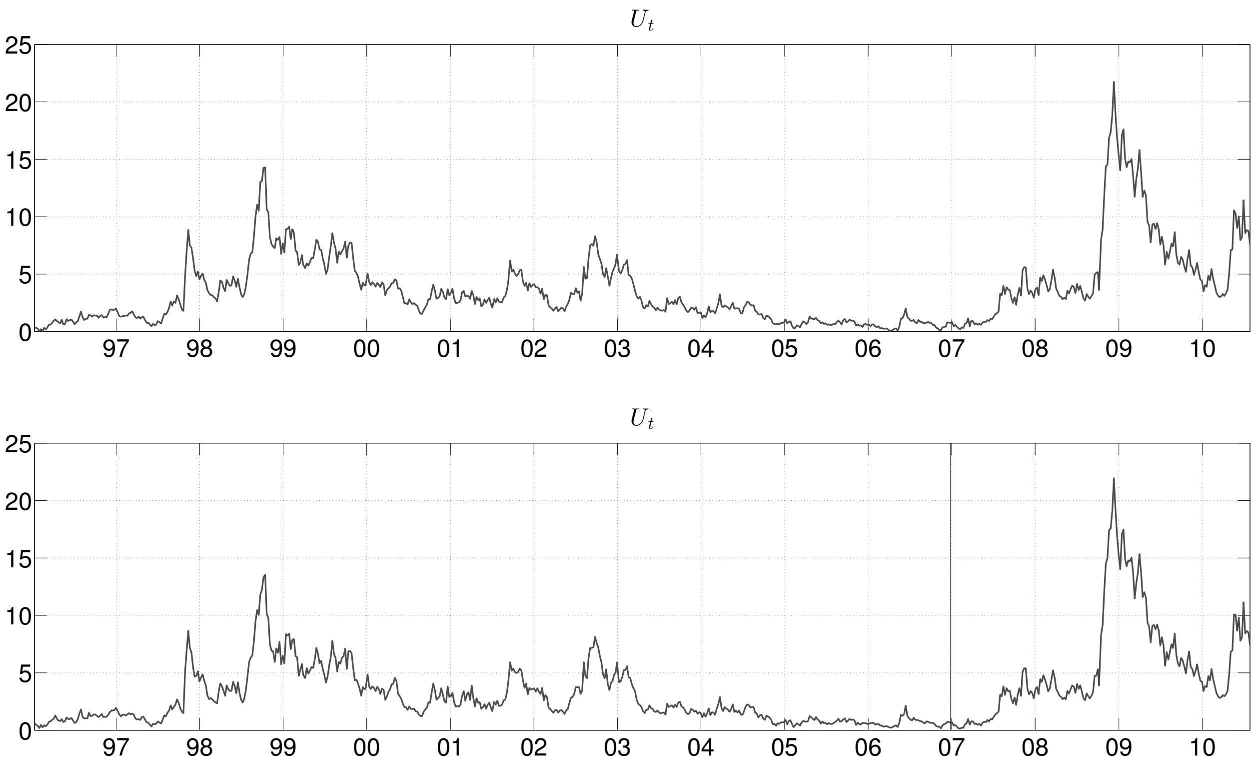

Figure B.2: Top Panel: U from the full sample estimation from January 1996 to July 2010. Bottom Panel: U from the two sub-sample estimations: January 1996 to December 2006 and January 2007 to July 2010.

B.3 Effect of ρ_u Parameter

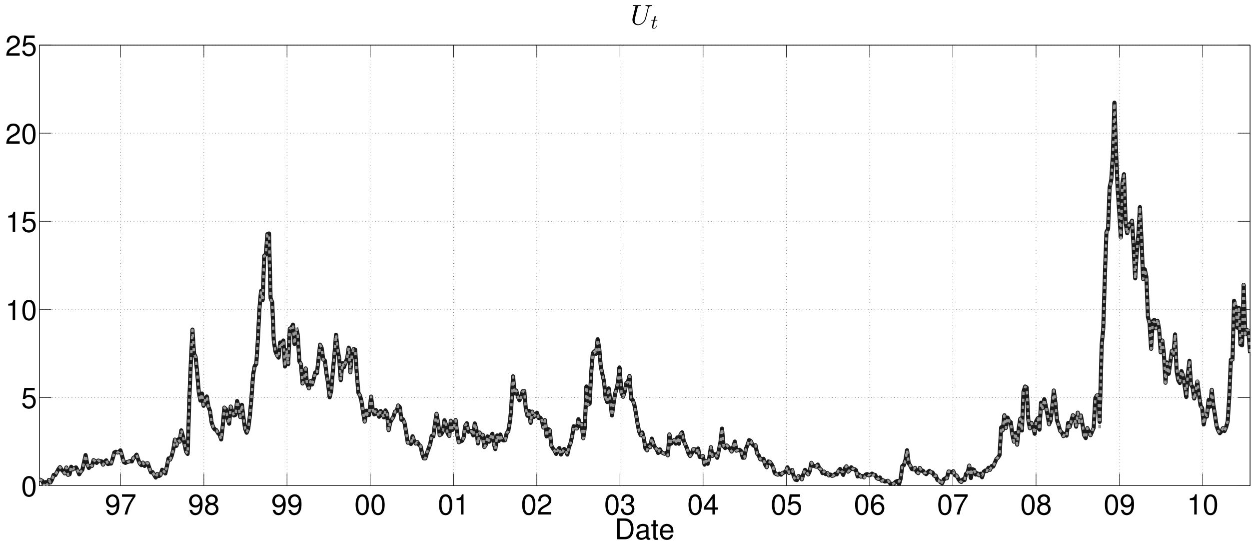

Figure B.3: **The effect of ρ_u on U.** Solid dark line is U given parameter estimates of model (B.1) reported in Table 4 in the paper with ρ_u set to zero. Dotted light line is U given parameter estimates of model (B.1) reported in Table 4 in the paper with ρ_u set to one.

B.4 Effect of Volatility Fit Penalization in Estimation

B.4.1 $\lambda = 0.1$

Table B.4: Estimation results for model (B.1).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.929	0.076	$\overline{v}_2$	0.011	0.000	c_0^+	0.461	0.027
$\overline{v}_1$	0.003	0.000	κ_2	2.091	0.101	c_1^-	110.244	5.327
κ_1	11.066	0.245	σ_2	0.203	0.004	c_1^+	22.415	4.024
σ_1	0.242	0.024	μ_u	5.512	30.187	c_2^+	78.805	5.548
μ_1	12.188	0.254	κ_u	0.267	0.148	λ_-	25.982	0.273
ρ_2	-0.992	0.019	ρ_u	0.480	7.157	λ_+	36.329	0.588

Panel B: Summary Statistics

RMSE	1.71%
Mean jump intensity (yearly) (-/+)	5.54/1.78
Mean jump size (-/+)	3.84%/2.75%

Note: Estimation period is January 1996-July 2010. We set λ in the objective function given in (4.1) in the paper to 0.1.

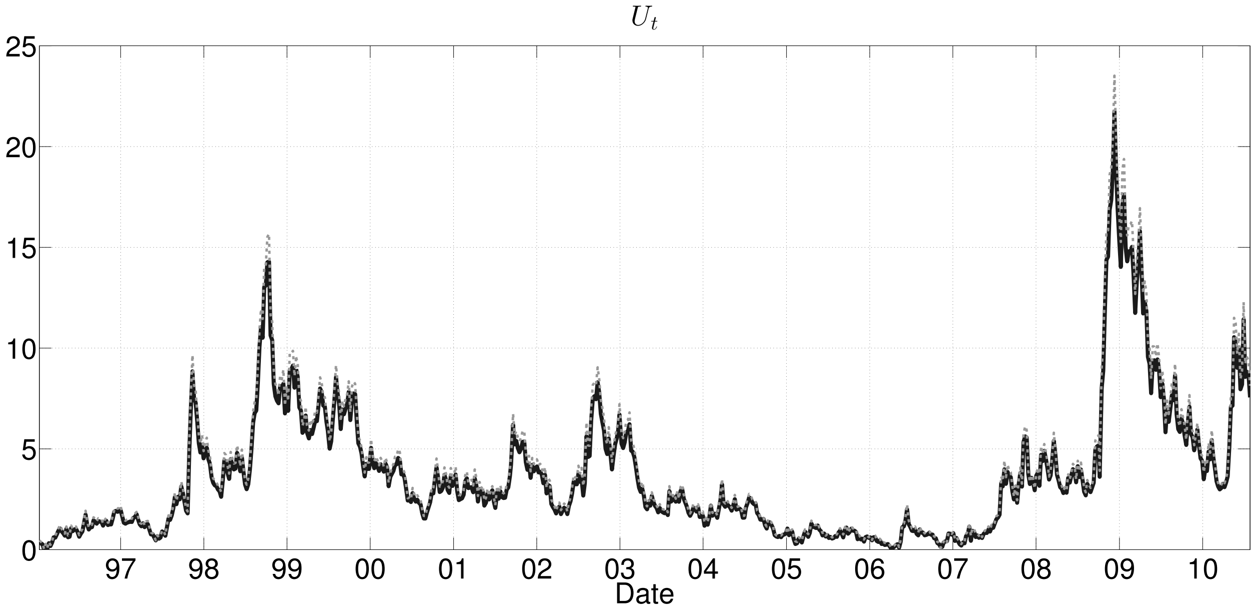

Figure B.4: Solid dark line is U based on estimation with $\lambda = 0.2$ and dotted light line is U based on estimation with $\lambda = 0.1$. Estimation period is January 1996 to July 2010.

B.4.2 $\lambda = 0.3$

Table B.5: Estimation results for model (B.1).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.948	0.075	$\overline{v}_2$	0.008	0.000	c_0^+	0.368	0.026
$\overline{v}_1$	0.003	0.000	κ_2	1.865	0.086	c_1^-	106.353	4.498
κ_1	10.896	0.210	σ_2	0.163	0.004	c_1^+	13.742	3.751
σ_1	0.251	0.026	μ_u	6.188	25.739	c_2^+	80.797	5.612
μ_1	12.023	0.239	κ_u	0.076	0.123	λ_-	25.984	0.234
ρ_2	-0.987	0.023	ρ_u	0.467	5.220	λ_+	36.747	0.604

Panel B: Summary Statistics

RMSE	1.75%
Mean jump intensity (yearly) (-/+)	5.72/1.63
Mean jump size (-/+)	3.85%/2.72%

Note: Estimation period is January 1996-July 2010. We set λ in the objective function given in (4.1) in the paper to 0.3.

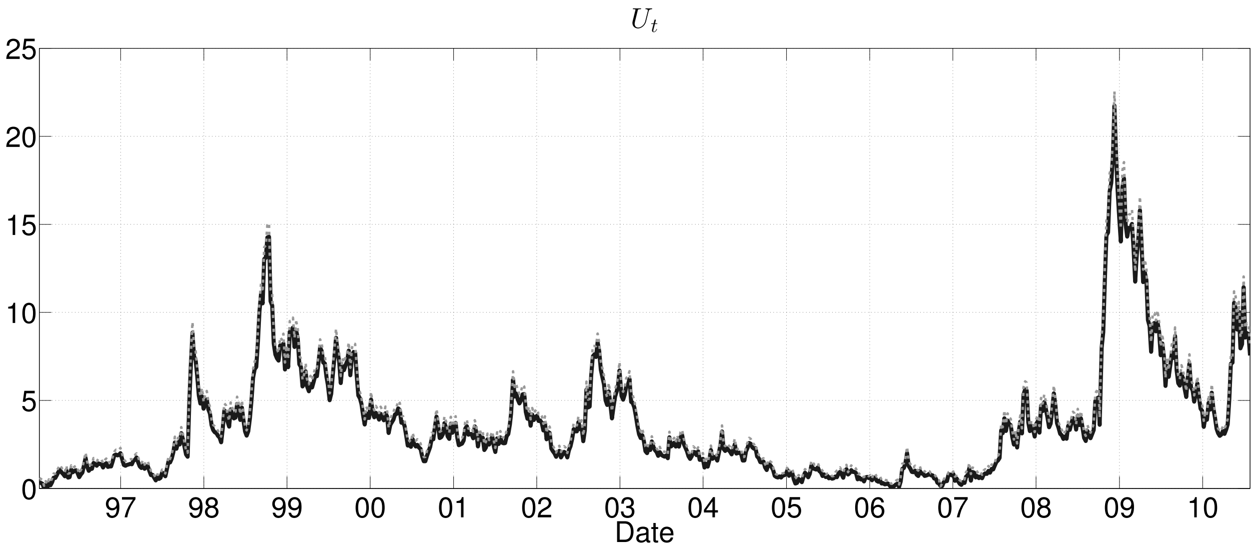

Figure B.5: Solid dark line is U based on estimation with $\lambda = 0.2$ and dotted light line is U based on estimation with $\lambda = 0.3$. Estimation period is January 1996 to July 2010.

B.5 Alternative Models and/or Constrained Versions of the Model

B.5.1 Estimation Results for 1FGSJ Model

1FGSJ Model is given by:

$$\begin{aligned}\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)\,dt + \sqrt{V_{1,t}}\,dW_{1,t}^{\mathbb{Q}} + \int_{\mathbb{R}^2} (e^x - 1)\,\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy), \\ dV_{1,t} &= \kappa_1\,(\overline{v}_1 - V_{1,t})\,dt + \sigma_1\sqrt{V_{1,t}}\,dB_{1,t}^{\mathbb{Q}} + \int_{\mathbb{R}^2} y\mu(dt, dx, dy),\end{aligned} \tag{B.2}$$

where $(W_{1,t}^{\mathbb{Q}}, B_{1,t}^{\mathbb{Q}})$ is a two-dimensional Brownian motion with corr $\left(W_{1,t}^{\mathbb{Q}}, B_{1,t}^{\mathbb{Q}}\right) = \rho_1$.
The risk-neutral compensator for the jump measure is

$$\nu_t^{\mathbb{Q}}(dx, dy) = \left\{ (c_0 + c_1 V_{1,t-}) \frac{e^{-\frac{(x-\mu_x-\rho_j y)^2}{2\sigma_x^2}}}{\sqrt{2\pi}\sigma_x} \frac{e^{-y/\mu_v}}{\mu_v} 1_{\{y>0\}} \right\} dx \otimes dy.$$

Table B.6: Estimation results for the 1FGSJ model (B.2).

Panel A: Parameter Estimates								
Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.899	0.009	c_0	0.011	0.002	σ_x	0.138	0.001
$\overline{v}_1$	0.028	0.001	c_1	20.517	0.347	μ_v	0.025	0.001
κ_1	0.686	0.017	μ_x	-0.076	0.001	ρ_j	-0.621	0.048
σ_1	0.154	0.003						

Panel B: Summary Statistics	
RMSE	3.14%
Mean jump intensity (yearly)	0.63
Mean jump size	−9.25%

Note: Estimation period is January 1996-July 2010.

B.5.2 Estimation Results for 2FGSJ Model

2FGSJ Model is given by:

$$
\begin{aligned}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)\,dt + \sqrt{V_{1,t}}\,dW^{\mathbb{Q}}_{1,t} + \sqrt{V_{2,t}}\,dW^{\mathbb{Q}}_{2,t} + \int_{\mathbb{R}^2} (e^x - 1)\,\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy), \\
dV_{1,t} &= \kappa_1\,(\overline{v}_1 - V_{1,t})\,dt + \sigma_1\sqrt{V_{1,t}}\,dB^{\mathbb{Q}}_{1,t} + \int_{\mathbb{R}^2} y\mu(dt, dx, dy), \\
dV_{2,t} &= \kappa_2\,(\overline{v}_2 - V_{2,t})\,dt + \sigma_2\sqrt{V_{2,t}}\,dB^{\mathbb{Q}}_{2,t},
\end{aligned}
\tag{B.3}
$$

where $(W^{\mathbb{Q}}_{1,t}, W^{\mathbb{Q}}_{2,t}, B^{\mathbb{Q}}_{1,t}, B^{\mathbb{Q}}_{2,t})$ is a four-dimensional Brownian motion with $W^{\mathbb{Q}}_{1,t} \perp W^{\mathbb{Q}}_{2,t}$, $W^{\mathbb{Q}}_{1,t} \perp B^{\mathbb{Q}}_{2,t}$, and $W^{\mathbb{Q}}_{2,t} \perp B^{\mathbb{Q}}_{1,t}$, while $\operatorname{corr}\left(W^{\mathbb{Q}}_{1,t}, B^{\mathbb{Q}}_{1,t}\right) = \rho_{d,1}$ and $\operatorname{corr}\left(W^{\mathbb{Q}}_{2,t}, B^{\mathbb{Q}}_{2,t}\right) = \rho_{d,2}$.
The risk-neutral compensator for the jump measure is

$$
\nu^{\mathbb{Q}}_t(dx, dy) = \left\{ (c_0 + c_1 V_{1,t-} + c_2 V_{2,t-})\,\frac{e^{-\frac{(x-\mu_x-\rho_j y)^2}{2\sigma_x^2}}}{\sqrt{2\pi}\sigma_x}\,\frac{e^{-y/\mu_v}}{\mu_v}\,1_{\{y>0\}} \right\} dx \otimes dy.
$$

Table B.7: Estimation results for the 2FGSJ model (B.3).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.969	0.031	$\overline{v}_2$	0.008	0.000	c_2	6.363	0.297
$\overline{v}_1$	0.009	0.000	κ_2	17.976	0.782	μ_x	-0.193	0.003
κ_1	1.341	0.021	σ_2	0.074	0.054	σ_x	0.063	0.002
σ_1	0.138	0.006	c_0	0.006	0.004	μ_v	0.047	0.001
ρ_2	-0.742	0.525	c_1	25.557	0.575	ρ_j	0.638	0.023

Panel B: Summary Statistics

RMSE	2.56%
Mean jump intensity (yearly)	0.62
Mean jump size	−16.30%

Note: Estimation period is January 1996-July 2010.

B.5.3 Estimation Results for 2FESJ Model

2FESJ Model is given by:

$$\begin{aligned}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)\,dt + \sqrt{V_{1,t}}\,dW_{1,t}^{\mathbb{Q}} + \sqrt{V_{2,t}}\,dW_{2,t}^{\mathbb{Q}} + \int_{\mathbb{R}^2} (e^x - 1)\,\widetilde{\mu}^{\mathbb{Q}}(dt, dx),\\
dV_{1,t} &= \kappa_1\,(\overline{v}_1 - V_{1,t})\,dt + \sigma_1\sqrt{V_{1,t}}\,dB_{1,t}^{\mathbb{Q}} + \mu_1 \int_{\mathbb{R}^2} x^2\,1_{\{x<0\}}\,\mu(dt, dx),\\
dV_{2,t} &= \kappa_2\,(\overline{v}_2 - V_{2,t})\,dt + \sigma_2\sqrt{V_{2,t}}\,dB_{2,t}^{\mathbb{Q}},
\end{aligned} \tag{B.4}$$

where $(W_{1,t}^{\mathbb{Q}}, W_{2,t}^{\mathbb{Q}}, B_{1,t}^{\mathbb{Q}}, B_{2,t}^{\mathbb{Q}})$ is a four-dimensional Brownian motion with $W_{1,t}^{\mathbb{Q}} \perp W_{2,t}^{\mathbb{Q}}$, $W_{1,t}^{\mathbb{Q}} \perp B_{2,t}^{\mathbb{Q}}$, and $W_{2,t}^{\mathbb{Q}} \perp B_{1,t}^{\mathbb{Q}}$, while $\operatorname{corr}\left(W_{1,t}^{\mathbb{Q}}, B_{1,t}^{\mathbb{Q}}\right) = \rho_{d,1}$ and $\operatorname{corr}\left(W_{2,t}^{\mathbb{Q}}, B_{2,t}^{\mathbb{Q}}\right) = \rho_{d,2}$.
The risk-neutral compensator for the jump measure is

$$\nu_t^{\mathbb{Q}}(dx, dy) = \left\{\left(c^- 1_{\{x<0\}}\lambda_- e^{-\lambda_-|x|} + c^+ 1_{\{x>0\}}\lambda_+ e^{-\lambda_+ x}\right)\right\} dx,$$

$$c^- = c_0^- + c_1^- V_{1,t-} + c_2^- V_{2,t-}, \qquad c^+ = c_0^+ + c_1^+ V_{1,t-} + c_2^+ V_{2,t-}\,.$$

In our estimation we set c_0^- to zero.

Table B.8: Estimation results for 2FESJ model (B.4).

Panel A: Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	−0.762	0.091	κ_2	0.169	0.014	c_2^+	52.795	4.597
$\overline{v}_1$	0.004	0.000	σ_2	0.129	0.002	λ_-	16.943	0.070
κ_1	12.831	0.227	c_0^+	2.315	0.063	λ_+	51.818	0.220
σ_1	0.247	0.031	c_1^-	69.208	1.856	μ_1	6.262	0.143
ρ_2	−0.945	0.009	c_1^+	13.161	11.356			
$\overline{v}_2$	0.059	0.004	c_2^-	97.656	1.091			

Panel B: Summary Statistics

RMSE	2.07%
Mean jump intensity (yearly) (-/+)	2.6/3.36
Mean jump size (-/+)	−5.9%/1.93%

Note: Estimation period is January 1996-July 2010.

B.5.4 Estimation Results for 3FESJ-V Model

3FESJ-V Model is given by:

$$
\begin{aligned}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)\,dt + \sqrt{V_{1,t}}\,dW^{\mathbb{Q}}_{1,t} + \sqrt{V_{2,t}}\,dW^{\mathbb{Q}}_{2,t} + \sqrt{V_{3,t}}\,dW^{\mathbb{Q}}_{3,t} + \int_{\mathbb{R}^2} (e^x - 1)\,\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy),\\
dV_{1,t} &= \kappa_1 (\overline{v}_1 - V_{1,t})\,dt + \sigma_1 \sqrt{V_{1,t}}\,dB^{\mathbb{Q}}_{1,t} + \mu_1 \int_{\mathbb{R}^2} x^2\, 1_{\{x<0\}}\,\mu(dt, dx, dy),\\
dV_{2,t} &= \kappa_2 (\overline{v}_2 - V_{2,t})\,dt + \sigma_2 \sqrt{V_{2,t}}\,dB^{\mathbb{Q}}_{2,t},\\
dV_{3,t} &= -\kappa_3\, V_{3,t}\,dt + \mu_3 \int_{\mathbb{R}^2} \left[(1-\rho_3)\,x^2\, 1_{\{x<0\}} + \rho_3\, y^2\right] \mu(dt, dx, dy),
\end{aligned}
\tag{B.5}
$$

where $(W^{\mathbb{Q}}_{1,t}, W^{\mathbb{Q}}_{2,t}, B^{\mathbb{Q}}_{1,t}, B^{\mathbb{Q}}_{2,t})$ is a four-dimensional Brownian motion with $W^{\mathbb{Q}}_{1,t} \perp W^{\mathbb{Q}}_{2,t}$, $W^{\mathbb{Q}}_{1,t} \perp B^{\mathbb{Q}}_{2,t}$, and $W^{\mathbb{Q}}_{2,t} \perp B^{\mathbb{Q}}_{1,t}$, while $\text{corr}\left(W^{\mathbb{Q}}_{1,t}, B^{\mathbb{Q}}_{1,t}\right) = \rho_{d,1}$ and $\text{corr}\left(W^{\mathbb{Q}}_{2,t}, B^{\mathbb{Q}}_{2,t}\right) = \rho_{d,2}$.
The risk-neutral compensator for the jump measure is

$$
\nu^{\mathbb{Q}}_t(dx) = \left\{\left(c^- 1_{\{x<0\}} \lambda_- e^{-\lambda_-|x|} + c^+ 1_{\{x>0\}} \lambda_+ e^{-\lambda_+ x}\right) 1_{\{y=0\}} + c^- 1_{\{x=0,\ y<0\}} \lambda_- e^{-\lambda_-|y|}\right\} dx \otimes dy,
$$

$$
c^- = c^-_0 + c^-_1 V_{1,t-} + c^-_2 V_{2,t-} + c^-_3 V_{3,t-}, \qquad c^+ = c^+_0 + c^+_1 V_{1,t-} + c^+_2 V_{2,t-} + c^+_3 V_{3,t-}\,.
$$

In our estimation we set c^-_0 and c^+_3 to zero.

Table B.9: Estimation results for 3FESJ-V model (B.5)

Panel A: Parameter Estimates								
Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.888	0.025	σ_2	0.093	0.008	c_2^-	146.355	1.315
$\overline{v}_1$	0.015	0.001	μ_3	5.583	0.369	c_2^+	26.180	36.532
κ_1	4.053	0.146	κ_3	17.296	0.623	c_3^-	116.395	4.642
σ_1	0.347	0.011	ρ_3	0.470	0.028	λ_-	17.162	0.105
ρ_2	-0.989	0.058	c_0^+	6.301	0.205	λ_+	78.369	0.425
$\overline{v}_2$	0.003	0.000	c_1^-	0.362	1.556	μ_1	3.244	0.486
κ_2	1.287	0.043	c_1^+	292.907	49.268			

Panel B: Summary Statistics	
RMSE	1.86%
Mean jump intensity (yearly) (-/+)	2.72/9.26
Mean jump size (-/+)	$-5.83\%/1.28\%$

Note: Estimation period is January 1996-July 2010.

C Option Pricing

C.1 Pricing Method

In this section we describe how we obtain the option prices for the most general model outlined in equation (B.1). Since the model belongs to the class of affine models, the option prices can be obtained using standard Fourier methods. We employ here the Fourier-cosine series expansion introduced by Fang and Oosterlee (2008). To apply the method, all we need is the conditional characteristic function of the log-prices. The latter is not available in closed form, but it can be easily obtained through the solution of a system of Ordinary Differential Equations (ODEs), as described in Section 2.3 of Duffie et al. (2000) and Theorem 2.7 of Duffie et al. (2003).

More specifically, for $y_t = \log(X_t)$ and $u \in \mathbb{C}$, we define

$$g(u, y_t, V_{1,t}, V_{2,t}, U_t, \tau) = \mathbb{E}_t^{\mathbb{Q}}[e^{uy_{t+\tau}}].$$

Since, the model in (B.1) is in the affine class, we have

$$g(u, y_t, V_{1,t}, V_{2,t}, U_t) = e^{\alpha(u,\tau)+\beta_1(u,\tau)V_{1,t}+\beta_2(u,\tau)V_{2,t}+\beta_3(u,\tau)U_t+uy_t}.$$

for some coefficients $\alpha(u,\tau)$, $\beta_1(u,\tau)$, $\beta_2(u,\tau)$, and $\beta_3(u,\tau)$ to be determined. The Feynman-Kac theorem implies that $g(u, y_t, V_{1,t}, V_{2,t}, U_t)$ solves (we omit the subscript t for the state variables and the prime indicates the derivative with respect to time-to-expiration τ):

$$\begin{aligned} &-\alpha' - \beta_1' V_1 - \beta_2' V_2 - \beta_3' U_t + u[r_t - \delta_t - \frac{1}{2}(V_1 + V_2 + \eta^2 U) - c_t^-(\Theta^{nc}(u,0,0) - 1) - \\ &\quad c_t^+(\Theta^p(u,0,0) - 1)] + \beta_1(k_1(\bar{v_1} - V_1)) + \beta_2(k_2(\bar{v_2} - V_2)) - k_u U_t \beta_3 + \\ &\quad \frac{1}{2}u^2(V_1 + V_2 + \eta^2 U) + \frac{1}{2}\sigma_1^2 V_1 \beta_1^2 + \frac{1}{2}\sigma_2^2 V_2 \beta_2^2 + \beta_1 u \sigma_1 \rho_1 V_1 + \beta_2 u \sigma_2 \rho_2 V_2 + \\ &\quad c_t^-(\Theta^{nc}(u,\beta_1,\beta_3) - 1) + c_t^-(\Theta^{ni}(0,0,\beta_3) - 1) + c_t^+(\Theta^p(u,0,0) - 1) = 0, \end{aligned} \tag{C.1}$$

where

$$\Theta^{nc}(q_0, q_1, q_3) = \int_{-\infty}^{0} e^{q_0 z + q_1 \mu_1 z^2 + q_3(1-\rho_u)\mu_u z^2} \lambda^- e^{z\lambda^-} dz \tag{C.2}$$

$$\Theta^{ni}(q_3) = \int_{-\infty}^{0} e^{q_3 \rho_u \mu_u z^2} \lambda^- e^{z\lambda^-} dz \tag{C.3}$$

$$\Theta^{p}(q_0) = \int_{0}^{+\infty} e^{q_0 z} \lambda^+ e^{-z\lambda^+} dz. \tag{C.4}$$

For $\Theta^{nc}(q_0, q_1, q_3)$ we have

$$\begin{aligned} \Theta^{nc}(q_0, q_1, q_3) &= \lambda^- \int_{-\infty}^{0} e^{z^2(q_1\mu_1 + q_3(1-\rho_u)\mu_u + \frac{2(q_0+\lambda^-)}{2(q_1\mu_1 + q_3(1-\rho_u))}(q_1\mu_1 + q_3(1-\rho_u)\mu_u)z} dz \\ &= \lambda^- e^{\frac{(q_0+\lambda^-)^2}{4(q_1\mu_1+q_3(1-\rho_u)\mu_u)}} \int_{-\infty}^{0} e^{(q_1\mu_1+q_3(1-\rho_u)\mu_u)\left(z + \frac{q_0+\lambda^-}{2(q_1\mu_1+q_3(1-\rho_u)\mu_u)}\right)^2}, \end{aligned} \tag{C.5}$$

and defining:

$$a = -(q_1\mu_1 + q_3(1-\rho_u)\mu_u)), \quad b = \frac{q_0 + \lambda^-}{2(q_1\mu_1 + q_3(1-\rho_u)\mu_u))}$$

we obtain:

$$\begin{aligned}\Theta^{nc}(q_0, q_1, q_3) &= \lambda^- e^{ab^2}\int_{-\infty}^{0} e^{-a(z+b)^2}dz = \lambda^- e^{ab^2}\int_{-\infty}^{b} e^{-ay^2}dy \\ &= \lambda^- e^{ab^2}\frac{1}{\sqrt{a}}\int_{-\infty}^{\sqrt{a}b} e^{-(\sqrt{a}y)^2}d(\sqrt{a}y) = \lambda^- e^{ab^2}\frac{1}{\sqrt{a}}erf(\sqrt{a}b),\end{aligned} \tag{C.6}$$

where $erf(.)$ is the complex error function. $\Theta^{ni}(q_3)$ can be computed following the steps in equations (C.5) and (C.6), while $\Theta^p(q_0)$ is given by the characteristic function of the exponential distribution.

Finally, (C.1) can be solved by collecting terms containing $V_{1,t}$, $V_{2,t}$, and U_t and setting the coefficients in front of these variables. This leads to the following system of ODEs that we solve numerically:[1]

$$\begin{aligned}\alpha' &= u[r - \delta - c_0^-(\Theta^n(u,0,0)-1) - c_0^+(\Theta^p(u,0,0)-1)] + \beta_1 k_1\bar{v}_1 + \beta_2 k_2 \bar{v}_2 + \\ &\quad c_0^-(\Theta^{nc}(u,\beta_1,\beta_3)-1) + c_0^-(\Theta^{ni}(0,0,\beta_3)-1) + c_0^-(\Theta^p(u)-1) \\ \beta_1' &= u[-\tfrac{1}{2} - c_1^-(\Theta^n(u,0,0)-1) - c_1^+(\Theta^p(u,0,0)-1)] - \beta_1 k_1 + \tfrac{1}{2}u^2\tfrac{1}{2}\sigma_1^2\beta_1^2 + \beta_1 u\sigma_1\rho_1 + \\ &\quad c_1^-(\Theta^{nc}(u,\beta_1,\beta_3)-1) + c_1^-(\Theta^{ni}(0,0,\beta_3)-1) + c_1^-(\Theta^p(u)-1) \\ \beta_2' &= u[-\tfrac{1}{2} - c_2^-(\Theta^n(u,0,0)-1) - c_2^+(\Theta^p(u,0,0)-1)] - \beta_2 k_2 + \tfrac{1}{2}u^2\tfrac{1}{2}\sigma_2^2\beta_2^2 + \beta_2 u\sigma_2\rho_2 + \\ &\quad c_2^-(\Theta^{nc}(u,\beta_1,\beta_3)-1) + c_2^-(\Theta^{ni}(0,0,\beta_3)-1) + c_2^-(\Theta^p(u)-1) \\ \beta_3' &= u[-\tfrac{1}{2}\eta^2 - c_3^-(\Theta^n(u,0,0)-1) - c_3^+(\Theta^p(u,0,0)-1)] - \beta_3 k_u \\ &\quad c_3^-(\Theta^{nc}(u,\beta_1,\beta_3)-1) + c_3^-(\Theta^{ni}(0,0,\beta_3)-1) + c_3^-(\Theta^p(u)-1)\end{aligned} \tag{C.7}$$

All the other models presented in the paper are special cases of the model in (B.1).

C.2 Monte Carlo Exercise

In this section we compare the pricing obtained using the numerical procedure described in Section C.1 with option prices computed via a Monte Carlo simulation using 10,000,000 simulated paths (simulation is done via Euler discretization with length of the discretization interval of 1/5 of a day). We use the same parameters reported in Table 5 in the paper and we set the state vector at the sample median values from tour estimation. Figure C.1 shows the corresponding implied volatility curve for moneyness m between $[-8,\ 4]$ and maturity equal to seven days.

[1] For example in Matlab the function `ode45.m` works very well.

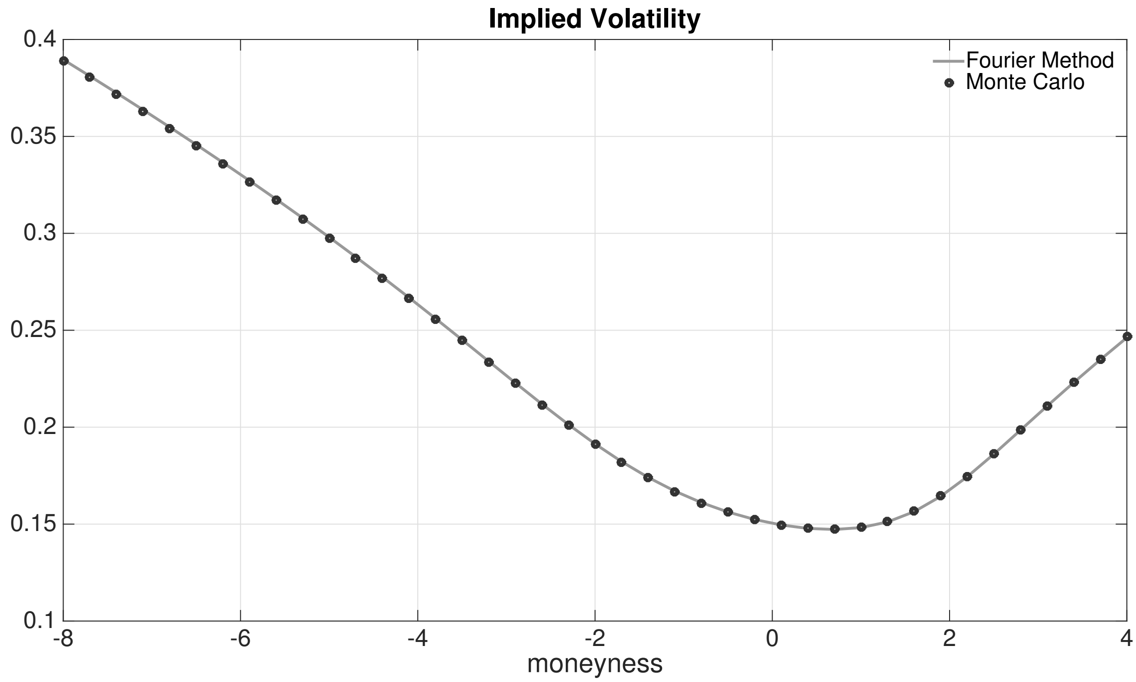

Figure C.1: **Monte Carlo Exercise.** Comparison between the implied volatility curve obtained using the Fourier-cosine series expansion and Monte Carlo simulation with $10,000,000$ simulated paths for the 3FESJ model in Equation B.1. Parameters as in Table 5 and state vector equal to the median of the daily filtered states vectors.

D Additional Details and Results for the Predictive Regressions

D.1 Filtered Jumps from High-Frequency Data

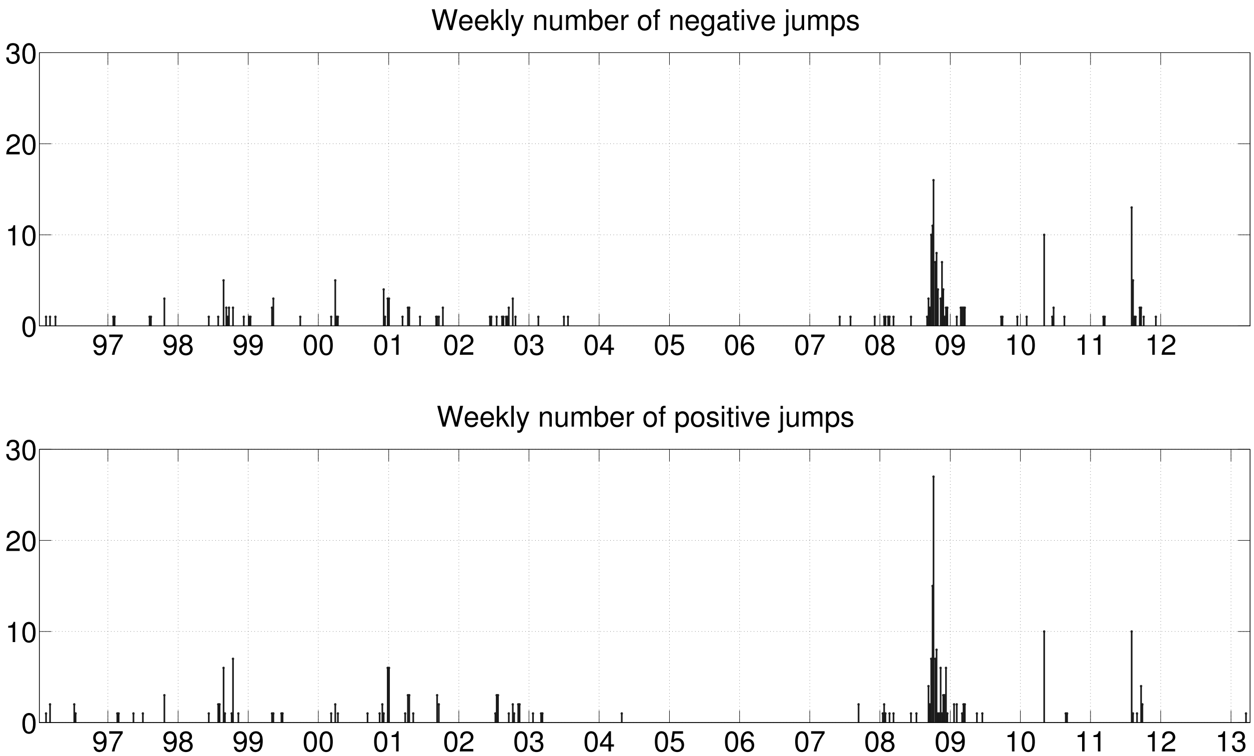

Figure D.1: **Weekly Number of "big" Negative and Positive Market Jumps**. The figure plots $\widehat{LT}^{K,i}_{t,t+\tau}$ and $\widehat{RT}^{K,i}_{t,t+\tau}$, defined in (9.11) in the paper with $K = 0.5\%$ and τ equal to one week. The total number of "big" negative and positive jumps in the sample is 224 and 235 respectively.

D.2 Predictive Regressions for the 2FESJ Model

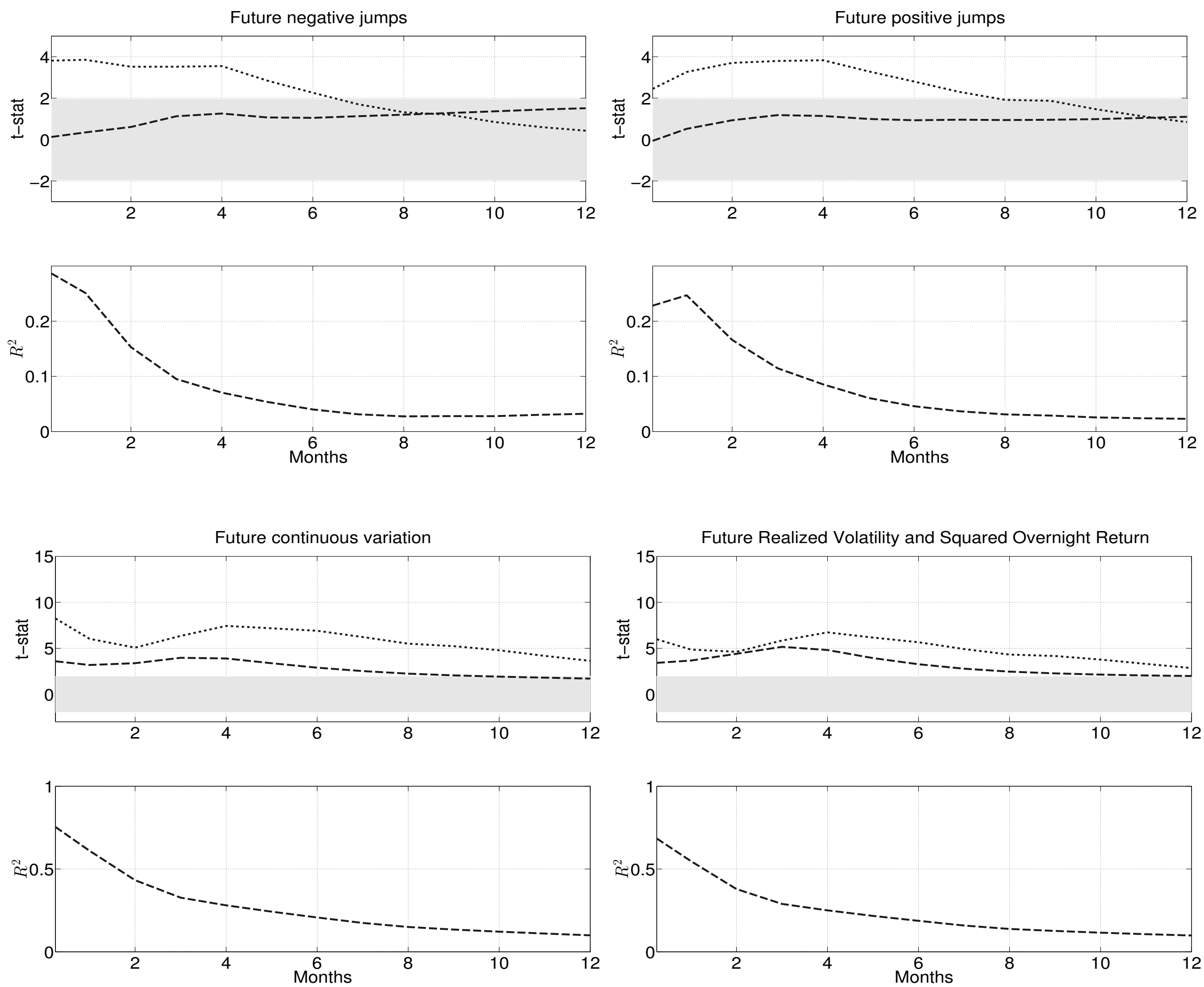

Figure D.2: **Predictive Regressions for Volatility and Jump Risks.** The volatility and jump risk measures are defined in (9.11)-(9.12) in the paper. For each regression, the top panels depict the t-statistics for the individual parameter estimates while the bottom panels indicate the regression R^2. The predictive variables are V_1 (dotted line), V_2 (dashed line).

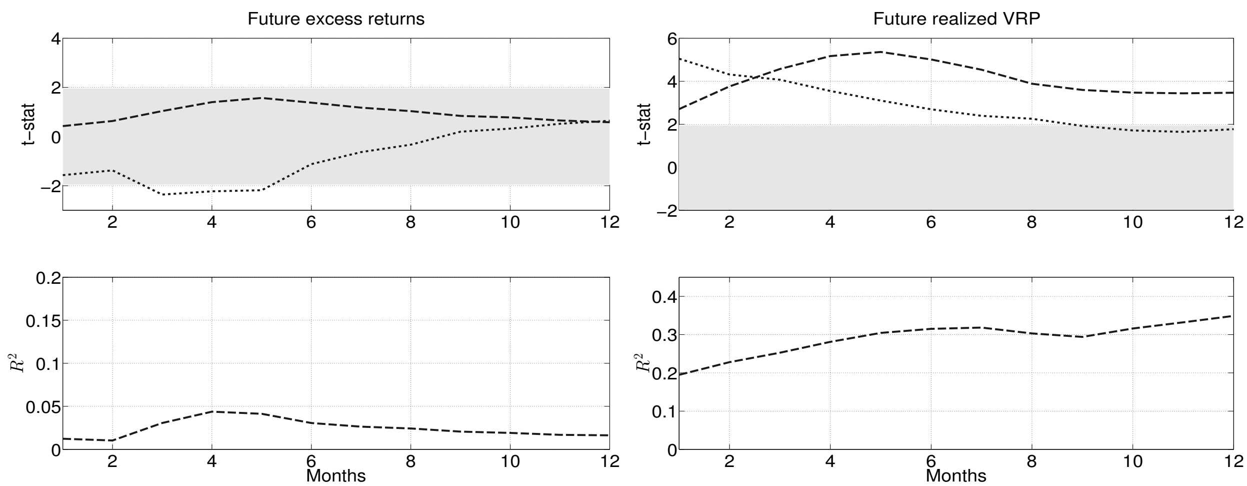

Figure D.3: **Predictive Regressions for Equity and Variance Risk Premia.** For each regression, the top panels depict the t-statistics for the individual parameter estimates while the bottom panels indicate the regression R^2. The predictive variables are V_1 (dotted line), V_2 (dashed line).

D.3 Predictive Regression for Model (B.1) Using CRSP Monthly Returns

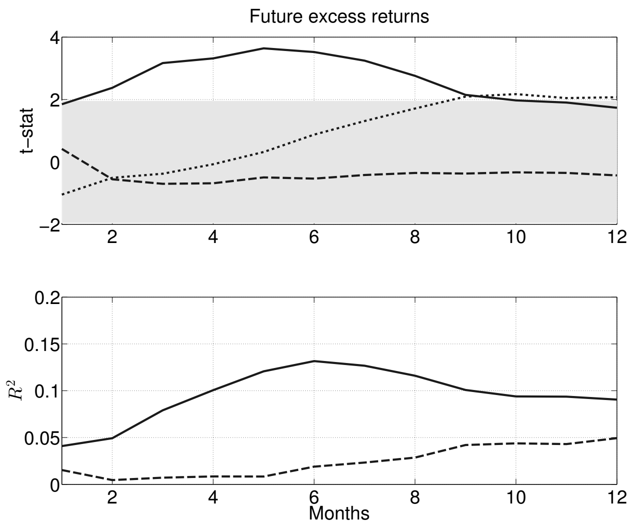

Figure D.4: **Predictive Regression for Future Excess Returns from CRSP**. Monthly returns and the corresponding risk-free rate have been downloaded from Ken French website:`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

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Fang, F. and C. Oosterlee (2008). A Novel Pricing Method for European Options Based on Fourier-CosineSeries Expansions. *SIAM Journal on Scientific Computing 31(2)*, 826–848.