Jump Tails, Extreme Dependencies, and the Distribution of Stock Returns*

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Abstract

We provide a new framework for estimating the systematic and idiosyncratic jump tail risks in financial asset prices. Our estimates are based on in-fill asymptotics for directly identifying the jumps, together with Extreme Value Theory (EVT) approximations and methods-of-moments for assessing the tail decay parameters and tail dependencies. On implementing the procedures with a panel of intraday prices for a large cross-section of individual stocks and the S&P 500 market portfolio, we find that the distributions of the systematic and idiosyncratic jumps are both generally heavy-tailed and close to symmetric, and show how the jump tail dependencies deduced from the high-frequency data together with the day-to-day variation in the diffusive volatility account for the "extreme" joint dependencies observed at the daily level.

Keywords: Extreme events, jumps, high-frequency data, jump tails, non-parametric estimation, stochastic volatility, systematic risks, tail dependence.

JEL classification: C13, C14, G10, G12.

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1 Introduction

Tail events and non-normal distributions are ubiquitous in finance. The earliest comprehensive empirical evidence for fat-tailed marginal return distributions dates back more than half a century to the influential work of Mandelbrot (1963) and Fama (1965). It is now well recognized that the fat-tailed unconditional return distributions first documented in these, and numerous subsequent, studies may result from time-varying volatility and/or jumps in the underlying stochastic process governing the asset price dynamics. Intuitively, periods of high-volatility can result in seemingly "extreme" price changes, even though the returns are drawn from a normal distribution with light tails, but one with an unusually large variance; see e.g., Bollerslev (1987), Mikosch and Starica (2000), and the empirical analyses in Kearns and Pagan (1997) and Wagner and Marsh (2005) pertaining to the estimation of tail parameters in the presence of GARCH effects. On the other hand, the aggregation of multiple jump events over a fixed time interval will similarly result in fat-tailed asset return distributions, even for a pure Lévy-type jump processes with no dynamic dependencies; see, e.g., Carr et al. (2002). As such, while fundamentally different, these two separate mechanisms will both manifest themselves in the form of apparent "tail" events and leptokurtic marginal return distributions.[1]

These same general issues carry over to a multivariate context and questions related to "extreme" dependencies across assets. In particular, it is well documented that the correlations between equity returns, both domestically and internationally, tend to be higher during sharp market declines than during "normal" periods;[2] see e.g., Longin and Solnik (2001) and Ang and Chen (2002). Similarly, Starica (1999) documents much stronger dependencies for large currency moves compared to "normal-sized" changes, while Jondeau (2010) based on an explicit parametric model reports much stronger tail dependence on the downside for several different equity portfolios.

In parallel to the marginal effects discussed above, it is generally unclear whether these increased dependencies in the tails are coming from commonalities in time-varying volatilities across assets and/or common jumps. Poon et al. (2004), for instance, report that "devolatilizing" the daily returns for a set of international stock markets significantly reduces the joint tail dependence, while Bae et al. (2003) find that time-varying volatility and GARCH effects can not fully explain the counts of coincident "extreme" daily price moves observed across international equity markets. More closely related to

[1]Importantly, these different mechanisms also have very different pricing implications and risk premia dynamics, as recently explored by Bollerslev and Todorov (2011).

[2]The use of simple linear correlations as a measure of dependence for "extreme" observations has been called into question by Embrechts et al. (2002), among others.

the present paper, recent studies by Bollerslev et al. (2008), Jacod and Todorov (2009), and Gobbi and Mancini (2009), based on high-frequency data and nonparametric methods, have all argued for the presence of common jump arrivals across different assets, thus possibly inducing stronger dependencies in the "extreme."

In light of these observations, one of the goals of the present paper is to separate jumps from volatility to more directly assess the "extreme" dependencies inherent in the jump tails. Motivated by the basic idea from asset pricing finance that only non-diversifiable systematic jump risks should be compensated, we further dissect the jumps into their systematic and idiosyncratic components. This decomposition in turn allows us to compare and contrast the behavior of the two different jump tails and how they impact the return distributions.[3]

Our estimation methodology is based on the idea that even though jumps and time-varying volatility may have similar implications for the distribution of the returns over coarser sampling frequencies, the two features manifest themselves very differently in high-frequency returns. Intuitively, treating the volatility as locally constant over short time horizons, it is possible to perfectly separate jumps from the price moves associated with the slower temporally varying volatility through the use of increasingly finer sampled observations. Empirically, this allows us to focus directly on the high-frequency "filtered" jumps. Relying on the insight from Bollerslev and Todorov (2010) that regardless of *any* temporal variation in the jump intensity, the jump compensator for the "large" jumps behaves like a probability measure, we non-parametrically estimate the decay parameters for the univariate jump tails using a variant of the Peaks-Over-Threshold (POT) method.[4]

Going one step further, we characterize the extreme joint behavior of the "filtered" jump tails through non-parametric estimates of Pickands (1981) dependence function as well as the residual tail dependence coefficient of Ledford and Tawn (1996, 1997). The Pickands dependence function succinctly characterizes the dependence of the limiting bivariate extreme value distribution. When the latter has independent marginals, the residual tail dependence coefficient further discriminates among the dependencies that disappear in the limit.[5] We implement several different estimators for the Pickands

[3]In a related context, Barigozzi et al. (2010) have recently explored a factor structure for disentangling the total realized variation for a large panel of stocks into a single systematic component and remaining idiosyncratic components, while Todorov and Bollerslev (2010) propose a framework for the estimation of separate continuous and jump CAPM betas.

[4]The POT method for characterizing extremes dates back to Fisher and Tippett (1928). It has been formalized more recently by Balkema and de Haan (1974) and Pickands (1975); for general textbook discussions see also Embrechts et al. (2001) and Jondeau et al. (2007).

[5]In technical terms, the Pickands dependence function captures asymptotic tail dependence, while

dependence function and the residual tail dependence coefficient. Together with the estimated decay parameters for each of the underlying univariate extreme distributions, these summary measures effectively describe the key features of the bivariate joint tail behavior.[6]

Our actual empirical analysis is based on high-frequency observations for fifty large capitalization stocks and the S&P 500 aggregate market portfolio spanning the period from 1997 through 2010. We find that the number of "filtered" idiosyncratic jumps exceeds the number of systematic jumps for all of the stocks in the sample, and typically by quite a large margin. Nonetheless, the hypothesis of fully diversifiable individual jump risk is clearly not supported by the data, thus pointing to more complicated dependence structures in the tails than hitherto entertained in most of the existing asset pricing literature.[7]

Even though the assumption of "light" Gaussian jump tails can not necessarily be rejected for many of the individual estimates, the combined evidence for all of the stocks clearly supports the hypothesis of heavy jump tails. Our estimates for the individual jump tail decay parameters also suggest that the tails associated with the systematic jumps are slightly fatter than those for the idiosyncratic jumps, albeit not uniformly so. Somewhat surprisingly, we also find that the right tail decay parameters for both types of jumps in quite a few cases exceed those for the left tail.

Our estimates of various dependence measures reveal a strong degree of tail dependence between the market-wide jumps and the systematic jumps in the individual stocks. This therefore calls into question the assumption of normally distributed jumps previously used in the asset and derivatives pricing literature.

Further, comparing our high-frequency based estimation results with those obtained from daily returns, we find that the latter indicate much weaker tail dependencies. Intuitively, while the estimates based on the daily returns represent the tail dependence attributable to both systematic jumps and common volatility factors, both of which may naturally be expected to be associated with positive dependence, the idiosyncratic jumps when aggregated over time will tend to weaken the dependence. In contrast, by focusing directly on the high-frequency "filtered" systematic and idiosyncratic jumps,

the residual tail dependence coefficient captures pre-asymptotic tail dependence.

[6]For a general textbook discussion of the relevant concepts, see, e.g., Coles (2001) and Beirlant et al. (2004). Existing applications of these ideas have primarily been restricted to climatology and insurance. Steinkohl et al. (2010), for instance, have recently employed this approach to characterize the asymptotic dependence for high-frequency wind speeds across separate geographical locations.

[7]The mere existence of market-wide jumps, of course, refutes the hypothesis of fully diversifiable jump risk as in Merton (1976). The estimates reported in, e.g., Eraker et al. (2003), also suggest large risk premia for systematic jump risk.

we are able to much more accurately assess the true extreme jump tail dependencies, and assess how the different effects impart the dependencies in the lower-frequency daily returns.

The rest of the paper is organized as follows. Section 2 introduces the formal setup and assumptions. Section 3 outlines the statistical methodology and econometric procedures, beginning in Section 3.1 with the way in which we disentangle jumps from continuous prices moves, followed by a discussion of our univariate tail estimation procedures in Section 3.2, and the framework that we rely on for assessing the joint jump tail dependencies in Section 3.3. Section 4 presents the results from an extensive Monte Carlo simulation study designed to assess the properties of the different estimators in an empirically realistic setting. Section 5 summarizes our main empirical results, starting in Section 5.1 with a brief description of the data, followed by our findings pertaining to the individual jump tails in Section 5.2, and the bivariate jump tail dependencies in Section 5.3. Section 6 concludes.

2 Formal Setup and Assumptions

We will work with a total of $M+1$ financial asset prices. The individual assets will be enumerated $1, ..., M$, while the aggregate market portfolio will be indexed by 0.[8] The dynamics for the log-price for the j'th asset is assumed to follow the generic semimartingale process,

$$dp_t^{(j)} = \alpha_t^{(j)} dt + \sigma_t^{(j)} dW_t^{(j)} + \int_{\mathbb{R}} x \mu^{(j)}(dt, dx), \quad j = 0, ..., M, \tag{2.1}$$

where $\alpha_t^{(j)}$ and $\sigma_t^{(j)}$ are locally bounded processes, $W_t^{(j)}$ denote possibly correlated Brownian motions, and $\mu^{(j)}(ds, dx)$ are integer-valued random measures that capture the jumps in $p_t^{(j)}$ over time dt and size dx.[9]

Our main focus centers on the behavior of the jumps in the individual assets; i.e., the $\mu^{(j)}(ds, dx)$ measures for $j = 1, ..., M$. We will further categorize these jumps as being either systematic or idiosyncratic depending upon their association with the market-wide jumps, or $\mu^{(0)}(ds, dx)$. As we show below, as long as the systematic market factor is

[8]The M individual assets do not comprise the full market, so that $p_t^{(0)}$ isn't simply given by a weighted average of the $p_t^{(j)}$ $j = 1, ..., M$ prices.

[9]Equation (2.1) implicitly assumes that the jumps are of finite variation. This assumption only restricts the behavior of the very small jumps, and has no practical implications for our subsequent analysis of the jump tails. We also implicitly assume that $\alpha_t^{(j)}$ and $\sigma_t^{(j)}$ both satisfy sufficient integrability conditions.

assumed to be directly observable, such a decomposition can easily be formally justified and implemented empirically.[10]

To more rigorously set out our procedures, let

$$\mathcal{T}^{(j)}_{[0,T]} = \left\{ s \in [0,T] : \Delta p^{(j)}_s \neq 0 \right\}, \quad j = 0, ..., M,$$

where $\Delta p^{(j)}_s \equiv p^{(j)}_s - p^{(j)}_{s-}$, denotes the set of jump times for asset j. The $\mathcal{T}^{(j)}_{[0,T]}$ sets may in theory be infinite, but countable, as the jump processes may be infinitely active.[11] Note that in a standard one-factor market model $\mathcal{T}^{(0)}_{[0,T]} \subset \mathcal{T}^{(j)}_{[0,T]}$, although this need not be the case in general.

Further denote with $\mu^{(j,0)}(ds, dx)$ the jump measure for asset j for the jumps that occur at the same time as the market-wide jumps; i.e., at times restricted to the intersection of $\mathcal{T}^{(j)}_{[0,T]}$ and $\mathcal{T}^{(0)}_{[0,T]}$. Similarly, let $\mu^{(j,j)}(ds, dx)$ denote the jump measure for the asset j jumps that occur at times restricted to the set $\mathcal{T}^{(j)}_{[0,T]} \setminus \left\{ \mathcal{T}^{(0)}_{[0,T]} \cap \mathcal{T}^{(j)}_{[0,T]} \right\}$. Then by definition

$$\mu^{(j)}(ds, dx) \equiv \mu^{(j,0)}(ds, dx) + \mu^{(j,j)}(ds, dx), \quad j = 1, ..., M.$$

In parallel, denote with $\mu^{(0,j)}(ds, dx)$ the jump measure for the aggregate market jumps that arrive at the same time as the jumps in asset j; i.e., the counting measure for the systematic jumps restricted to the subset $\mathcal{T}^{(0)}_{[0,T]} \cap \mathcal{T}^{(j)}_{[0,T]}$.

In addition, denote the compensators, or jump intensities, for $\mu^{(j,0)}(ds, dx)$, $\mu^{(j,j)}(ds, dx)$ and $\mu^{(0,j)}(ds, dx)$ by $dt \otimes \nu^{(j,0)}_t(dx)$, $dt \otimes \nu^{(j,j)}_t(dx)$ and $dt \otimes \nu^{(0,j)}_t(dx)$, respectively, where $\nu^{(j,0)}_t(dx)$, $\nu^{(j,j)}_t(dx)$ and $\nu^{(0,j)}_t(dx)$ are some nonnegative measures satisfying the condition

$$\int_{\mathbb{R}} (x^2 \wedge 1)\nu^{(j,0)}_t(dx) + \int_{\mathbb{R}} (x^2 \wedge 1)\nu^{(j,j)}_t(dx) + \int_{\mathbb{R}} (x^2 \wedge 1)\nu^{(0,j)}_t(dx) < \infty,$$

for any $t > 0$. The main goal of the paper in essence amounts to characterizing the tail properties of $\nu^{(j,0)}_t(x)$, $\nu^{(j,j)}_t(x)$ and $\nu^{(0,j)}_t(x)$.

Rather than doing so directly, for theoretical reasons explained in Bollerslev and Todorov (2010), we will do so for their images under the following mappings

$$\psi^+(x) = \begin{cases} e^x - 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \qquad \psi^-(x) = \begin{cases} 0, & x \geq 0 \\ e^{-x} - 1, & x < 0 \end{cases}. \tag{2.2}$$

This in effect transforms the logarithmic jumps Δp_s into $\frac{P_s - P_{s-}}{P_{s-}}$, or functions thereof, akin to switching from discrete-time logarithmic returns to arithmetic returns. In practice, of course, for the actually observed jumps, the difference between Δp_s and $\frac{P_s - P_{s-}}{P_{s-}}$ is very small.

[10] The current analysis could also quite easily be extended to situations with more than one observable systematic risk factor, including e.g., the popular Fama-French portfolios.

[11] This has no practical implication for our statistical analysis, however, as we focus on the "large" jumps, of which there are always a finite number in a finite sample.

For the implementation of our estimation strategy, we will further assume that the jump compensators $\nu_t^{(j,0)}(x)$ and $\nu_t^{(j,j)}(x)$ satisfy

$$\begin{aligned}\nu_t(dx)^{(j,d)} &= (\varphi_t^{+(j,d)}1_{\{x>0\}} + \varphi_t^{-(j,d)}1_{\{x<0\}})\nu^{(j,d)}(dx), \quad j=1,...,M, \quad d=0,j,\\ \nu_t(dx)^{(0,j)} &= (\varphi_t^{+(j,0)}1_{\{x>0\}} + \varphi_t^{-(j,0)}1_{\{x<0\}})\nu^{(0,j)}(dx), \quad j=1,...,M,\end{aligned} \tag{2.3}$$

where $\varphi_t^{\pm(j,0)}$ and $\varphi_t^{\pm(j,j)}$ are nonnegative-valued stochastic processes with càdlàg paths.[12] The separability of the jump compensators into time and jump size in equation (2.3) is trivially satisfied for almost all of the parametric jump models hitherto analyzed in the literature, including the popular affine jump-diffusion class of models advocated by, e.g., Duffie et al. (2000).

Next, denote the tail jump intensities by $\overline{\nu}_\psi^{\pm(j,d)}(x) = \int_{\psi^\pm(u)\geq x}\nu^{\pm(j,d)}(du)$, for $x\in\mathbb{R}_+$. We will then assume that for some (and hence any) $x>0$ and $u>0$, the ratio[13]

$$\frac{\overline{\nu}_\psi^{\pm(j,d)}(u+x)}{\overline{\nu}_\psi^{\pm(j,d)}(x)}, \tag{2.4}$$

is in the domain of attraction of an extreme value distribution and satisfies a second-order condition as in, e.g., Smith (1987).[14] Recall, see, e.g., Theorem 1.2.5 of de Haan and Ferreira (2006), that a distribution function F is defined to be in the domain of attraction of an extreme value distribution if and only if for some positive valued function f,

$$\lim_{u\uparrow x^*}\frac{1-F(u+xf(u))}{1-F(u)} = (1+\xi x)^{-1/\xi}, \tag{2.5}$$

where $1+\xi x>0$, and x^* denotes the endpoint of the distribution; i.e., $x^*=\sup\{x: F(x)<1\}$. The case $\xi>0$ corresponds to heavy-tailed distributions, $\xi=0$ defines light tails with infinite end points (e.g., the normal), while $\xi<0$ corresponds to short-tailed distributions with finite end points (e.g. the uniform). In the empirically most relevant heavy-tailed case, the extreme value approximation amounts to assuming that $\overline{\nu}_\psi^{\pm(j,d)}(x)$ are regularly varying at infinity functions; i.e., $\overline{\nu}_\psi^{\pm(j,d)}(x) = x^{-\beta^{\pm(j,d)}}L^{\pm(j,d)}(x)$, where $\beta^{\pm(j,d)}>0$ corresponds to $1/\xi$ in (2.5), and $L^{\pm(j,d)}(x)$ are slowly varying at infinity functions.

[12] Note that equation (2.3) implicitly assumes that the temporal variation in the jump intensities for asset j and the market portfolio constrained to the set $\mathcal{T}_{[0,T]}^{(0)}\cap\mathcal{T}_{[0,T]}^{(j)}$ are the same. This restriction is unavoidable for the class of time-changed Lévy processes.

[13] Note that although the jump intensity $\overline{\nu}_\psi^{\pm(j,d)}(x)$ is not a distribution function, the ratio is.

[14] The condition effectively restricts the deviations from the power law tail behavior. It is well-known that second order terms in the tails can generate finite sample biases in the estimation of tail parameters. Our estimation procedure, as explained in more detail in Section 3 below, is based on the Peaks-Over-Threshold method, which as shown in Smith (1987), Section 4, is location-invariant and generally less biased than the popular Hill estimator. Methods for reducing the bias of the Hill estimator have been discussed in, e.g., Baek and Pipiras (2010), and some of the references therein.

To facilitate the discussion of our assumptions needed for the systematic jump tail dependencies, we will let $\nu^{(j)}_{\text{syst}}(\mathbf{x})$ for $\mathbf{x} \in \mathbb{R}^2 \setminus (0,0)$ denote a measure with marginals $\nu^{(j)}_{\text{syst}}([x \times (-\infty,+\infty)]) = \nu^{(j,0)}(x)$ and $\nu^{(j)}_{\text{syst}}([(-\infty,+\infty) \times x]) = \nu^{(0,j)}(x)$ for $x \in \mathbb{R}$, respectively. This measure controls the time-invariant part of the jump compensator of the jumps in asset j and the market constraint to the common set $\mathcal{T}^{(0)}_{[0,T]} \cap \mathcal{T}^{(j)}_{[0,T]}$. Generalizing the univariate tail measures to a vector $[x_1,\ x_2] \in \mathbb{R}^2_+ \setminus (0,\ 0)$, we denote the corresponding jump tail intensity by $\overline{\nu}^{\pm(j)}_{\text{syst},\psi}([x_1,\ x_2]) = \int_{\psi^{\pm}(u_1) \geq x_1\ \cup\ \psi^{\pm}(u_2) \geq x_2} \nu^{\pm(j)}_{\text{syst}}(d[u_1,\ u_2])$.[15]

We will then assume that for some (and hence any) $\mathbf{x} \in \mathbb{R}^2 \setminus (0,0)$ and $\mathbf{u} \in \mathbb{R}^2_+$, the ratio

$$\frac{\overline{\nu}^{\pm(j)}_{\text{syst},\psi}(\mathbf{u}+\mathbf{x})}{\overline{\nu}^{\pm(j)}_{\text{syst},\psi}(\mathbf{x})}, \tag{2.6}$$

is in the domain of attraction of a mutlivariate extreme value distribution and satisfies certain second order conditions as in, e.g., Einmahl et al. (1997). Recall, see, e.g., Theorem 6.2.1 of de Haan and Ferreira (2006), that a bivariate distribution function F, with marginals F_i in the domain of attraction of $\exp(-(1+\xi_i x)^{-1/\xi_i})$ for $i = 1,2$, is defined to be in the domain of attraction of a multivariate extreme value distribution G if and only if for every $x, y > 0$,

$$\lim_{u\to\infty} \frac{1 - F(U_1(u \cdot x), U_2(u \cdot y))}{1 - F(U_1(u), U_2(u))} = \int_0^{\pi/2} \left(\frac{1 \wedge \tan(\theta)}{x} \vee \frac{1 \wedge \cot(\theta)}{y} \right) \Phi(d\theta) \Big/ \log G(0,0), \tag{2.7}$$

where $U_i(\cdot)$ for $i = 1,2$ denote the inverse of the functions $x \to 1/(1 - F_i(x))$ that standardize the marginals to belong to the domain of attraction of $\exp(-1/x)$, and the distribution function $\Phi(\cdot)$ is concentrated on $[0,\pi/2]$ and satisfies the terminal condition $\int_0^{\pi/2}(1 \wedge \tan(\theta))\Phi(d\theta) = \int_0^{\pi/2}(1 \wedge \cot(\theta))\Phi(d\theta) = 1$. Following Einmahl et al. (1997), $\Phi(\cdot)$ is commonly referred to as the spectral, or angular, measure of the extreme value distribution. It accounts for the tail dependence between the two components and together with the extreme value distributions for the marginals completely characterizes the bivariate extreme value distribution.

Rather than directly estimating and interpreting the angular extreme value measure, empirically it is more convenient to characterize the tail dependencies through Pickands (1981) dependence function. This function is formally defined from $\Phi(\cdot)$ as

$$A(u) = \int_0^{\pi/2} ((1-u)(1 \wedge \tan(\theta)) \vee u(1 \wedge \cot(\theta)))\, \Phi(d\theta), \quad u \in [0,1]. \tag{2.8}$$

[15]Formally, this definition only pertains to the quadrants of $\mathbb{R}^2$ for which the signs of the jumps coincide. It would be trivial, albeit notationally more cumbersome, to extend the analysis to jumps of opposite signs. However, those cases are practically irrelevant.

The function $A(u)$ is convex and restricted to lie in the unit triangle; i.e., $u \vee (1 - u) \leq A(u) \leq 1$, with endpoints $A(0) = A(1) = 1$. The lower bound of the triangle $u \vee (1-u)$ corresponds to perfect dependence, while the upper bound of unity is obtained for asymptotically independent variables; see, e.g., the discussion in Coles (2001) and Beirlant et al. (2004). In particular, as first pointed out by Sibuya (1960), a bivariate normal distribution with correlation less than unity has asymptotically independent tails and implies $A(u) = 1$ for all $u \in [0, 1]$.

Going one step further, the overall degree of asymptotic dependence may be conveniently summarized in terms of the single *extreme tail-dependence coefficient*,

$$\chi = \lim_{u \to 1^-} \mathbb{P}\left(F_1(x) > u | F_2(y) > u\right), \tag{2.9}$$

originally proposed by Sibuya (1960); see also the more recent discussion in Coles et al. (1999). Intuitively, this measure gives the probability of observing an "extreme" observation in one of the series given that the other series is also "extreme." For two asymptotically independent series with $A(u) \equiv 1$ it follows that $\chi = 0$. More generally, it is possible to show that $\chi = 2\left(1 - A(\frac{1}{2})\right)$, so that the extreme tail-dependence coefficient is directly related to the value of Pickands dependence function at one-half.

The above characterization of the tail dependence via the limiting multivariate extreme-value distribution does not discriminate between distributions with asymptotically independent tails and $\chi = 0$. Yet, in practice it might be interesting to further differentiate the dependencies depending upon the rate at which they disappear in the tails. To this end, we calculate the additional statistic proposed by Ledford and Tawn (1996, 1997),

$$\lim_{u \to \infty} \frac{\mathbb{P}\left(X > U_1(ux),\ Y > U_2(ux)\right)}{\mathbb{P}\left(X > U_1(u),\ Y > U_2(u)\right)} = x^{-1/\eta}, \quad x > 0, \quad \eta \in (0, 1]. \tag{2.10}$$

We will refer to η as the *residual tail dependence coefficient.* A value of $\eta = 1$ is associated with asymptotic dependence (i.e., a bivariate distribution with $\chi \neq 0$), while $\eta < 1$ implies asymptotic independence (i.e., a bivariate distribution with $\chi = 0$). Specifically, for a bivariate normal distribution $\chi = 0$, while $\eta = (1 + \rho)/2$ where ρ denotes the standard correlation coefficient. More generally, taken together the χ and η coefficients succinctly summarizes the characteristics of the dependencies in the tails.[16]

Before we discuss the actual inference procedures that we rely on in quantifying the different theoretical measures outlined above, it is important to stress that all of

[16]Note that our analysis relates explicitly to the cross-sectional dependence inherent in the systematic jumps, as opposed to the temporal dependencies in the returns. In fact, the Pickands dependence function is not necessarily the most informative measure for characterizing dependencies over different points in time; see e.g., the discussion in Hill (2011b,a).

these pertain to "large" jumps and corresponding "extreme" dependencies. We have essentially nothing to say about the dependencies inherent in the "smaller" jumps related to the pathwise properties of the process and the degree of jump activity. An empirical study of these features would be interesting, but it would also necessitate the use of entirely different statistical techniques from the ones that we discuss next.

3 Jump Tail Estimation from High-Frequency Data

We will assume the availability of equidistant price observations for each of the M+1 assets over the discrete time grid $0, \frac{1}{n}, \frac{2}{n}, ..., T$, where $n \in \mathbb{N}$ and $T \in \mathbb{N}$. We will denote the log-price increments over the corresponding discrete time-intervals $[\frac{i-1}{n}, \frac{i}{n}]$ by $\Delta_i^n p^{(j)} = p^{(j)}_{\frac{i}{n}} - p^{(j)}_{\frac{i-1}{n}}$. Our estimation procedures will rely on both increasing sampling frequency and time span; i.e., $n \to \infty$ and $T \to \infty$. Intuitively, we will use in-fill asymptotics, or $n \to \infty$, to non-parametrically separate jumps from continuous price moves, and more conventional long-span asymptotics, or $T \to \infty$, and Extreme Value Theory (EVT) for our inference about the jump tails. We begin with a discussion of the former.

3.1 Separating Jumps from Volatility

In our separation of the price increments into jumps and continuous price moves we take into account both the strongly persistent day-to-day variation and the intraday diurnal patterns in the volatility; see, e.g, Andersen and Bollerslev (1997). In order to do so, for each day, $t = 1, ..., T$, and each asset, $j = 0, 1, ..., M$, in the sample, we first compute the Realized Variation (RV) and Bipower Variation (BV), defined by

$$RV_t^{(j)} = \sum_{i=tn+1}^{tn+n} |\Delta_i^n p^{(j)}|^2, \qquad BV_t^{(j)} = \frac{\pi}{2} \sum_{i=tn+2}^{tn+n} |\Delta_i^n p^{(j)}||\Delta_{i-1}^n p^{(j)}|, \tag{3.1}$$

respectively. Under weak regularity conditions and $n \to \infty$, see e.g., Andersen et al. (2003a) and Barndorff-Nielsen and Shephard (2004, 2006),

$$RV_t^{(j)} \xrightarrow{\mathbb{P}} \int_t^{t+1} (\sigma_s^{(j)})^2 ds + \int_t^{t+1} \int_{\mathbb{R}} x^2 \mu^{(j)}(ds, dx), \qquad BV_t^{(j)} \xrightarrow{\mathbb{P}} \int_t^{t+1} (\sigma_s^{(j)})^2 ds. \tag{3.2}$$

Note that the Bipower Variation consistently estimates only the part of the total variation due to continuous prices moves, or the so-called daily integrated variance.

Based on these daily realized variation measures, we subsequently estimate the Time-of-Day (TOD) volatility pattern for each of the stocks and the aggregate market by,[17]

$$TOD_i^{(j)} = \frac{n\sum_{t=1}^{T}|\Delta_{i_t}^n p|^2 1\left(|\Delta_{i_t}^n p^{(j)}| \le \tau\sqrt{BV_t^{(j)} \wedge RV_t^{(j)}}n^{-\varpi}\right)}{\sum_{s=1}^{nT}|\Delta_s^n p|^2 1\left(|\Delta_s^n p^{(j)}| \le \tau\sqrt{BV_{[s/n]}^{(j)} \wedge RV_{[s/n]}^{(j)}}n^{-\varpi}\right)}, \;\; i_t = (t-1)n+i, \quad (3.3)$$

where $i = 1, ..., n$, $\tau > 0$ and $\varpi \in (0, 0.5)$ are both constants, and $1(\cdot)$ denotes the indicator function. The truncation of the price increments implied by τ and ϖ in the definition of $TOD_i^{(j)}$ effectively removes the jumps. Hence $TOD_i^{(j)}$ measures the ratio of the diffusive variation over different parts of the day relative to its average value for the day. In the empirical analysis reported below we set $\tau = 2.5$ and $\varpi = 0.49$ (the Monte Carlo simulation evidence reported in Section 4 further corroborates our choice of these truncation levels). Intuitively, this means that we classify as jumps all of the high-frequency price increments that are beyond two-and-a-half standard deviations of a local estimator of the corresponding stochastic volatility. The resulting $TOD_i^{(j)}$'s generally exhibit the well-known U-shaped pattern as a function of i over the trading day.[18]

Relying on a similar approach, we estimate the Continuous Variation over the whole day using a modification of the truncated variation measure originally proposed by Mancini (2009),

$$CV_t^{(j)} = \sum_{i=tn+1}^{tn+n} |\Delta_i^n p^{(j)}|^2 1\left(|\Delta_i^n p^{(j)}| \le \alpha_i^{(j)} n^{-\varpi}\right). \quad (3.4)$$

Consistency and asymptotic normality of this estimator for $n \to \infty$ and appropriate choice of truncation level follow from Mancini (2009) and Jacod (2008). The truncation level $\alpha_i^{(j)}$ that we actually use in separating the "realized" jumps from the continuous price moves is chosen adaptively based on our preliminary estimates of the stochastic volatility over the day together with the within-day volatility pattern. Specifically,

$$\alpha_i^{(j)} = \tau\sqrt{(BV_{[i/n]}^{(j)} \wedge RV_{[i/n]}^{(j)}) * TOD_{i-[i/n]n}^{(j)}}, \quad i = 1, ..., nT, \quad (3.5)$$

with τ and ϖ set to the same values as discussed above. We rely on the difference between the continuous and previously defined realized variation measures,

$$JV_t^{(j)} \;=\; RV_t^{(j)} - CV_t^{(j)} \;\xrightarrow{\mathbb{P}}\; \int_t^{t+1}\int_{\mathbb{R}} x^2 \mu^{(j)}(ds, dx), \quad (3.6)$$

[17]Note, the asymptotic limit of $BV_t^{(j)}$ is always below that of $RV_t^{(j)}$. The trimming $BV_{[i/n]}^{(j)} \wedge RV_{[i/n]}^{(j)}$ is merely a finite-sample adjustment.

[18]Further details concerning our $TOD_i^{(j)}$ estimates are available upon request. This same approach has also recently been used by Bollerslev and Todorov (2011), who do provide a plot of the estimated $TOD_i^{(0)}$ for the aggregate market.

for consistently estimating the total variation attributable to jumps.

We also use the identical truncation approach to directly identify the sets of high-frequency jump increments for each of the assets,

$$\widehat{\mathcal{T}}_{[0,T]}^{(j)} = \left\{ i \in [0, nT] : |\Delta_i^n p^{(j)}| \geq \alpha_i^{(j)} n^{-\varpi} \right\}, \quad j = 0, 1, ..., M. \tag{3.7}$$

Similarly, we define the sets of systematic and idiosyncratic jump times by,

$$\widehat{\mathcal{T}}_{[0,T]}^{(j,0)} = \widehat{\mathcal{T}}_{[0,T]}^{(j)} \cap \widehat{\mathcal{T}}_{[0,T]}^{(0)}, \qquad \widehat{\mathcal{T}}_{[0,T]}^{(j,j)} = \widehat{\mathcal{T}}_{[0,T]}^{(j)} \setminus \left\{ \widehat{\mathcal{T}}_{[0,T]}^{(j)} \cap \widehat{\mathcal{T}}_{[0,T]}^{(0)} \right\}, \quad j = 1, ..., M. \tag{3.8}$$

Armed with these high-frequency based estimates for the times and actual "realized" jumps, we next show how to use these in our estimation of the jump tail characteristics. We begin with the univariate jump tails.

3.2 Univariate Jump Tails

To keep the notation simple, we will focus on the right tail and the systematic jumps. Our estimation of the parameters for the negative and/or idiosyncratic jumps proceed analogous.

The general assumptions about the jump tails set out in Section 2 imply that

$$1 - \frac{\overline{\nu}_{\psi}^{+(j,0)}(u+x)}{\overline{\nu}_{\psi}^{+(j,0)}(x)} \overset{\text{appr}}{\sim} \begin{cases} 1 - \left(1 + \xi^{+(j,0)} u / \eta^{+(j,0)}\right)^{-1/\xi^{+(j,0)}}, & \xi^{+(j,0)} \neq 0, \\ e^{-u/\eta^{+(j,0)}}, & \xi^{+(j,0)} = 0, \end{cases} \tag{3.9}$$

where $u > 0$, $x > 0$ is some "large" value, and $\eta^{+(j,0)} > 0$;[19] for additional discussion of the approximating Generalized Pareto distribution, see, e.g., Embrechts et al. (2001). Now, denote the (re-scaled) scores associated with the log-likelihood function of the Generalized Pareto distribution by,

$$\begin{aligned} \phi_1^+(u, \xi^{+(j,0)}, \eta^{+(j,0)}) &= \frac{1}{\eta^{+(j,0)}} \left(1 - \left(1 + \xi^{+(j,0)}\right) \left(1 + \frac{\xi^{+(j,0)} u}{\eta^{+(j,0)}} \right)^{-1} \right), \\ \phi_2^+(u, \xi^{+(j,0)}, \eta^{+(j,0)}) &= \log \left(1 + \frac{\xi^{+(j,0)} u}{\eta^{+(j,0)}} \right) - \left(1 + \xi^{+(j,0)}\right) \left\{ 1 - \left(1 + \frac{\xi^{+(j,0)} u}{\eta^{+(j,0)}} \right)^{-1} \right\}, \end{aligned} \tag{3.10}$$

where $i = 1, 2$ refer to the derivatives with respect to $\eta^{+(j,0)}$ and $\xi^{+(j,0)}$, respectively. Then, for truncation level $tr_T^{(j,0)}$ increasing to infinity with $T \to \infty$,

$$\int_0^t \int_{\mathbb{R}} \phi_i^+(\psi^+(x) - tr_T^{(j,0)}, \xi^{+(j,0)}, \eta^{+(j,0)}) 1\left(\psi^+(x) \geq tr_T^{(j,0)}\right) \mu^{(j,0)}(ds, dx), \qquad i = 1, 2,$$

[19]Note that with fat tails and $\beta^{+(j,0)} > 0$, as discussed in Section 2, $\eta^{+(j,0)} \equiv \frac{x}{\beta^{+(j,0)}}$ and $\xi^{+(j,0)} \equiv \frac{1}{\beta^{+(j,0)}}$.

behave approximately as martingales. Combined with our previously discussed procedures for directly "filtering" the "large" jumps from the high-frequency data, this in turn allows for the construction of a standard method-of-moments type estimators for the jump tail parameters.

In particular, following Bollerslev and Todorov (2010) the simple-to-implement moment conditions defined from the two martingales above and the jump sets defined in equation (3.8),

$$\sum_{i\in\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}}\begin{pmatrix} \phi_1^+(\psi^+(\Delta_i^n p^{(j)}) - tr_T^{(j,0)}, \xi^{+(j,0)}, \eta^{+(j,0)})1\left(\psi^+(\Delta_i^n p^{(j)}) \geq tr_T^{(j,0)} \vee \psi^+(\alpha_i^{(j)} n^{-\varpi})\right) \\ \phi_2^+(\psi^+(\Delta_i^n p^{(j)}) - tr_T^{(j,0)}, \xi^{+(j,0)}, \eta^{+(j,0)})1\left(\psi^+(\Delta_i^n p^{(j)}) \geq tr_T^{(j,0)} \vee \psi^+(\alpha_i^{(j)} n^{-\varpi})\right) \end{pmatrix}, \tag{3.11}$$

should both be arbitrarily close to zero asymptotically under the joint fill-in and long-span asympotics. Moreover, the precision of the resulting estimator for $\xi^{+(j,0)}$, determined by the asymptotic limiting variance of the moment conditions, may be conveniently expressed as,

$$\widehat{\text{Var}}\left(\widehat{\xi}^{+(j,0)}\right) = \frac{1}{M_T^{+(j,0)}}\left(1 + \widehat{\xi}^{+(j,0)}\right)^2, \tag{3.12}$$

where

$$M_T^{+(j,0)} = \sum_{i\in\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}} 1\left(\psi^+(\Delta_i^n p^{(j)}) \geq tr_T^{(j,0)} \vee \psi^+(\alpha_i^{(j)} n^{-\varpi})\right), \tag{3.13}$$

denotes the actual number of jumps used in the estimation.

In order to actually implement these estimating equations, we obviously need to specify the truncation level $tr_T^{(j,0)}$ for each of the assets, $j = 0, 1, ..., M$. This choice must balance the two opposing effects associated with the use of more jumps in the estimation generally resulting in smaller sampling error, versus the use of more, and hence smaller, jumps resulting in poorer approximation by the EVT distribution in equation (3.9). In the main empirical result reported on below, we set $tr_T^{+(j,0)}$ so that $M_T^{+(j,0)}/T = 0.02$, corresponding to jumps of that size or larger occurring $5-6$ times per year.[20]

Our calculations for the jumps in $p^{(0)}$ that belong to the set $\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}$ proceed in exactly the same fashion. Note, that in the following it is always the case that $M_T^{+(j,0)} = M_T^{+(0,j)}$.

We next turn to a discussion of our multivariate estimation procedures and the empirical strategies that we use for assessing the "extreme" jump tail dependencies.

[20]This choice, of course, directly dictates the accuracy of the estimator for $\xi^{+(j,0)}$ according to the expression in equation (3.12). As noted below, we also experimented with the use of other truncation levels in the empirical analysis, resulting in qualitatively very similar point estimates.

3.3 Jump Tail Dependencies

We will focus our discussion on the estimation of the tail dependencies between the jumps in the aggregate market and the systematic jumps in the individual stocks; i.e., the jumps in $p^{(0)}$ and $p^{(j)}$ that arrive at the same time corresponding to the set $\mathcal{T}^{(0)}_{[0,T]} \cap \mathcal{T}^{(j)}_{[0,T]}$. However, the same basic estimation techniques may be applied to other bivariate series, and we do so for other pairs of returns and jump tails in the empirical section.

Following the discussion in Section 2, our main estimate for χ is derived from an estimate of Pickands dependence function, $A(\cdot)$. More specifically, we follow the approach of Einmahl et al. (1997) by first estimating the underlying spectral measure; see also Steinkohl et al. (2010). For $i \in \widehat{\mathcal{T}}^{(j,0)}_{[0,T]}$, denote

$$\begin{aligned}
\widehat{X}_{i,1} &= \frac{\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}{M_T^{+(j,0)}} \left[1 + \frac{\widehat{\xi}^{+(j,0)}}{\widehat{\eta}^{+(j,0)}}(\psi^+(\Delta_i^n p^{(j)}) - tr_T^{(j,0)})\right]^{1/\widehat{\xi}^{+(j,0)}} 1_{\{\psi^+(\Delta_i^n p^{(j)}) \geq tr_T^{(j,0)}\}} \\
&\quad + \frac{\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}{M_T^{+(j,0)}(i)} 1_{\{\psi^+(\Delta_i^n p^{(j)}) < tr_T^{(j,0)}\}}, \\
\widehat{X}_{i,2} &= \frac{\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}{M_T^{+(0,j)}} \left[1 + \frac{\widehat{\xi}^{+(0,j)}}{\widehat{\eta}^{+(0,j)}}(\psi^+(\Delta_i^n p^{(0)}) - tr_T^{(0,j)})\right]^{1/\widehat{\xi}^{+(0,j)}} 1_{\{\psi^+(\Delta_i^n p^{(0)}) \geq tr_T^{(0,j)}\}} \\
&\quad + \frac{\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}{M_T^{+(0,j)}(i)} 1_{\{\psi^+(\Delta_i^n p^{(0)}) < tr_T^{(0,j)}\}},
\end{aligned} \tag{3.14}$$

where $\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|$ refers to the number of elements in the set $\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}$, $M_T^{+(j,0)}(i)$ refers to the number of elements k in the set $\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}$ for which $\psi^+(\Delta_k^n p^{(j)}) > \psi^+(\Delta_i^n p^{(j)})$ with $M_T^{+(0,j)}(i)$ defined in an analogous way from the jumps $\Delta_k^n p^{(0)}$. Let

$$\widehat{R}_i = \widehat{X}_{i,1} + \widehat{X}_{i,2}, \tag{3.15}$$

denote the sum of the two marginals. An initial estimator for Pickands dependence function is then naturally obtained by,

$$\widehat{A}^{+(j,0)}(u) = \frac{2}{M_T^{+(j,0)}} \sum_{i \in \widehat{\mathcal{T}}^{(j,0)}_{[0,T]}} 1\left(\widehat{R}_i > \widehat{R}_{\left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right| - M_T^{+(j,0)},\ \left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}\right) \frac{\max\left\{(1-u)\widehat{X}_{i,1},\ u\widehat{X}_{i,2}\right\}}{\widehat{R}_i}, \quad u \in [0,1].$$

where $\widehat{R}_{i,\ \left|\widehat{\mathcal{T}}^{(j,0)}_{[0,T]}\right|}$ denotes the i-th order statistics.

Following Beirlant et al. (2004), Section 9.4.1, we further modify this initial estimator,

$$\widetilde{A}^{+(j,0)}(u) = \max\left\{u,\ 1-u,\ \widehat{A}^{+(j,0)}(u) + 1 - (1-u)\widehat{A}^{+(j,0)}(0) - u\widehat{A}^{+(j,0)}(1)\right\}, \quad u \in [0,1].$$

(3.16)

so that it always stays within its lower asymptotic bound of $\max(1-u, u)$ and the upper bound of unity. Using the relationship discussed in Section 2, our primary estimate for the extreme tail-dependence coefficient is simply obtained by evaluating this function at one-half,

$$\widehat{\chi}^{j,0} = 2(1 - \widetilde{A}^{j,0}(1/2)). \tag{3.17}$$

Turning next to the estimation of η. This can easily be accomplished by appealing to the regular variation result in (2.10) in conjunction with a Hill-type estimator (see, e.g., Ledford and Tawn (1996) and Draisma et al. (2004) for a discussion of the relevant ideas),

$$\widehat{\eta}^{+(j,0)} = \frac{1}{M_T^{+(j,0)}} \sum_{i=0}^{M_T^{+(j,0)}-1} \log \left(\frac{T_{\left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right| - M_T^{+(j,0)},\ \left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|}}{T_{\left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right| - i,\ \left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|}} \right), \tag{3.18}$$

where $T_{i,\ \left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|}$ denotes the i-th order statistics of T_i, which in turn is defined for every $i \in \widehat{\mathcal{T}}_{[0,T]}^{(j,0)}$ as

$$T_i = \frac{\left|\widehat{\mathcal{T}}_{[0,T]}^{(j,0)}\right|}{M_T^{+(j,0)}(i) \vee M_T^{+(0,j)}(i)}. \tag{3.19}$$

The two nonparametric estimators for χ and η in equations (3.17) and (3.18), respectively, serve as our main statistics for characterizing the tail dependencies in the empirical analysis. However, in effort to further verify the robustness of our conclusions, we also implemented a few alternative estimators and additional summary type statistics.

In particular, following Davis and Mikosch (2009), the χ coefficient may alternatively be estimated from the so-called extremogram,

$$\widehat{\chi}_E^{j,0} = \frac{\sum_{i \in \widehat{\mathcal{T}}_{[0,T]}^{(j,0)}} 1\left(\psi^+(\Delta_i^n p^{(0)}) \geq tr_T^{(0,j)},\ \ \psi^+(\Delta_i^n p^{(j)}) \geq tr_T^{(j,0)}\right)}{M_T^{+(j,0)}}. \tag{3.20}$$

The extreme tail-dependence coefficient may also be estimated by semiparametric techniques under the assumption of a logistic dependence structure for Pickands dependence function,

$$A_\theta^{(0,j)}(u) = \left(u^{1/\theta} + (1-u)^{1/\theta}\right)^\theta, \quad \theta \in (0,1],$$

where total dependence corresponds to the limiting case $\theta \to 0$, while independence is obtained for $\theta = 1$.[21] We denote the resulting estimate for χ, by

$$\widehat{\chi}_l^{j,0} = 2(1 - \widehat{A}_\theta^{j,0}(1/2)). \tag{3.21}$$

Lastly, we also calculate Kendall's tau τ_k and Spearman's rho ρ_s coefficients for the systematic jump pairs for which each of the individual elements are above the $tr_T^{(j,0)}$ and $tr_T^{(0,j)}$ thresholds, respectively. These two coefficients are, of course, quite widely used in the literature as "distribution-free" dependence measures.

This completes our discussion of the different estimation procedures. Before turning to our discussion of the actual empirical findings based on high-frequency intraday data for a large cross-section of individual stocks, we first summarize the results from a Monte Carlo simulation study designed to assess the reliability of the different estimators and the choice of truncation levels.

4 Monte Carlo

The estimators discussed in the previous section are based on Extreme Value Theory (EVT) approximations, along with in-fill asymptotics, or $n \to \infty$, to non-parametrically identify the jumps, and long-span asymptotics, or $T \to \infty$, for estimating the jump tail dependence parameters. In practice, of course, we do not have access to continuous price records over an infinitely long sample, and it is instructive to consider how the estimators perform in a controlled simulation setting that more closely mimics that of the actual data.

The bivariate model that we use in our simulations of the theoretical market prices $p_t^{(0)}$, and the prices for the individual stock $p_t^{(1)}$, is based on the affine jump-diffusion model used extensively in the empirical asset pricing literature,

$$\begin{aligned} p_t^{(0)} - p_0^{(0)} &= \int_0^t \sqrt{V_s} dW_s^{(0)} + \sum_{s \le N_t} Z_s^{(0)}, \\ p_t^{(1)} - p_0^{(1)} &= \int_0^t \sqrt{V_s} dW_s^{(1)} + \sum_{s \le N_t} Z_s^{(1)}, \\ V_t - V_0 &= 0.0128 \int_0^t (3 - V_s) ds + 0.0954 \int_0^t \sqrt{V_s} dB_s, \end{aligned} \tag{4.1}$$

[21] This is a semiparametric estimator, in the sense that it avoids parametric assumptions about the marginals. Our actual implementation is based on censored maximum likelihood techniques following Ledford and Tawn (1996), and we refer to that paper for additional details.

where $W_t^{(0)}$, $W_t^{(1)}$ are Brownian motions with $\mathrm{Corr}(W_1^{(0)}, W_1^{(1)}) = 0.4$,[22] both independent from the Brownian motion B_t, while conditional on $\{V_t\}_{t\geq 0}$, $N_t \sim$ Poisson $\left(\frac{\lambda}{6}\int_0^t V_s ds + 0.5(1-\lambda)t\right)$ The persistence of the square-root volatility process and the volatility-of-volatility parameter are both taken directly from Eraker et al. (2003), with the mean parameter adjusted to reflect the median diffusive volatility of the individual stocks in our empirical application. As for the conditionally Poisson distributed jumps, we fix λ at 0 or 1 resulting in constant and time-varying jump intensities, respectively, both of which imply an average unconditional intensity corresponding to a jump every other day. The distribution of the jump sizes, $(Z_s^{(0)},\ Z_s^{(1)})$, $s = 1, 2, ...$, are assumed to be i.i.d. with CDF,

$$G(x_0, x_1) = C_\theta(F(x_0), F(x_1)), \tag{4.2}$$

where $F(\cdot)$ refers to the CDF of a truncated at 0.2 zero-mean normal variable with standard deviation equal to 0.91,[23] and the Gumbel-Hougard (logistic) copula,

$$C_\theta(u_0, u_1) = \exp\left(-\left[(-\log u_0)^{1/\theta} + (-\log u_1)^{1/\theta}\right]^\theta\right), \quad u_0, u_1 \in [0, 1], \tag{4.3}$$

directly links jumps. As such, the extreme dependence between the systematic jumps is simply controlled by the value of θ, with $\theta = 1$ corresponding to independence, and complete dependence arising for $\theta \to 0$.

All of our results are based on a total of 1,000 replications, with each of the replications consisting of 252×10 "days", or 10 "years", of simulated data. We record the prices at 77 equidistant times within each "day", corresponding to a 5-minute sampling frequency, as employed in our actual empirical analysis.

We also allow for market microstructure noise in some of the simulations, by assuming that at observation times $i = 0,\ \frac{1}{n},\ \frac{2}{n}, ..., T$, instead of observing $p_i^{(0)}$ and $p_i^{(1)}$, we actually observe their "contaminated" counterparts

$$p_i^{(0)*} = p_i^{(0)} + \epsilon_i^{(0)}, \qquad p_i^{(1)*} = p_i^{(1)} + \epsilon_i^{(1)}, \tag{4.4}$$

where $(\epsilon_i^{(0)}, \epsilon_i^{(1)})$ is i.i.d. bivariate normally distributed with correlation 0.5, and variances equal to $\frac{3+0.5}{4\times 76\times(1-0.1)}$. This implies that the noise accounts for 10% of the total return variation at the 5-minute sampling frequency, which is generally in line with the numbers reported in the extensive empirical study of Hansen and Lunde (2006).

[22]This correlation closely matches the average correlation between the jump-adjusted five-minute returns of the individual stocks and the aggregate market in our panel data.

[23]These parameters imply an expected squared jump size equal to 1.0, which again closely matches that of the actual data analyzed below. This jump specification is also similar to the one previously used by Jacod and Todorov (2009).

The results of the Monte Carlo study are summarized in Table 1 in the form of the median and interquartile range across the 1,000 replications. The first three columns pertain to the infeasible case where we directly observe $(Z_s^{(0)},\ Z_s^{(1)})$, $s = 1, 2, ...$, and thus do not need to first "filter" the jumps from the discretely observed high-frequency returns. The first column gives the results for the extreme tail dependence parameter equal to $\chi = 0.50$. This is close to the median estimated value for the systematic jumps in the actual data. Columns 2 and 3 give the results for $\chi = 0.25$ and $\chi = 0.75$, respectively. For the cases of high and medium tail dependence, the estimators for χ are all approximately unbiased and reasonably precisely determined. The case of low dependence contains a small upward bias. This holds true regardless of the estimator used for inferring the extreme tail dependence parameter. Comparing the estimator $\widehat{\chi}$ in equation (3.17) based on Pickands dependence function with the one based on the extremogram $\widehat{\chi}_E$ in equation (3.20), the two estimators are generally close, with the former being marginally more precise. The semiparametric estimator $\widehat{\chi}_l$ based on the known logistic tail dependence function and censored maximum likelihood, not surprisingly, tend to result in the smallest biases overall.

Turning to the residual tail dependence coefficient, all of the cases reported in the table formally imply $\eta = 1$. The estimates of η in the first three columns are indeed close to 1, albeit somewhat downward-biased, with the bias naturally being the most severe in column two for $\chi = 0.25$. All in all, however, the pair of estimates for χ and η generally provide good and reliable diagnostic for gauging the tail dependence inherent the true systematic jumps.

The last two panels in the table report the results for Kendall's tau $\widehat{\tau}_k$ and Spearman's rho $\widehat{\rho}_s$. The true population values of these commonly used statistics are not known in the present setting, and as such it is more difficult to formally judge the results from the simulations. Nonetheless, it is comforting that the ranking of the median values of the estimates across the three different scenarios are in concert with the degree of extreme dependence implied by the values of χ. At the same time, however, the variation in the estimates for τ_k and ρ_s within each of the different scenarios is obviously much greater than the variation in the estimates for χ.

We turn next to a brief discussion of the empirically more realistic cases in columns $4-8$, where we first need to "filter" the jumps from the high-frequency data. For ease of comparisons, we fix the logistic copula parameter such that $\chi = 0.5$ in all of the cases.[24] Columns 4-6 compare the effect on the estimation from the use of different truncation

[24] As already noted, this is close to median estimate for χ for the systematic jumps in the empirical analysis discussed in the next section.

levels in equation (3.5) equal to $\tau = 2.5,\ 2.0,\ 3.0$, respectively. As seen from the table, there is generally very little difference in the estimates across the three different truncation levels. Intuitively, the actual estimates of the different tail parameters are only based on the relatively large sized jumps, while the tuning parameter τ mostly affects the inference about the smaller sized jumps. Indeed, comparing the feasible setups in columns $4-6$ with the infeasible scenario in the first column, it is clear that the "filtering" of the jumps does not materially affect the estimation of the tail dependence measures. The only noticeable difference being the estimates for the residual tail dependence coefficient η, for which the downward bias does increase somewhat relative to its theoretical value of unity.

The results in column 7 for $\lambda = 1$, and jumps with time-varying jump intensity, are almost identical to the ones with constant jump intensity, or $\lambda = 0$, reported in column 4. Intuitively, since our estimates of the tail dependence are based directly on the jumps instead of first aggregating the jumps over time, any temporal dependencies in the jump arrivals should not affect the estimates.

Lastly, we report in column 8 the results based on the "contaminated" $p_i^{(0)*}$ and $p_i^{(1)*}$ prices. The differences between column 4 and 8 are again negligible. Our estimates for the tail dependence measures are all based on the "large" jumps, and the presence of market microstructure "noise" does not materially affect our ability to separate the "large" jumps from the diffusive price moves and the smaller sized jumps.

Taken as a whole, the results in Table 1 confirm that the different estimators for χ all perform well and give rise to similar conclusions. In the empirical results discussed next we therefore only report detailed findings for $\widehat{\chi}$ based on Pickands dependence function coupled with shorter summaries for the other statistics, leaving detailed results for all of the different estimators to a supplementary Appendix.

5 Empirical Results

5.1 Data

Our high-frequency data for the individual stocks was obtained from Price-data. It consists of 5-minute transaction prices for the fifty largest capitalization stocks included in the S&P 100 index with continuous price records from mid 1997 until the end of 2010.[25] The price records cover the trading hours from 9:35 EST to 16:00 EST, for a total of 76 intraday return observations per day. Our proxy for the aggregate market

[25]The actual start date and number of complete trading days available for each of the stocks in the sample differ slightly, ranging from a low of 3,330 to a high of 3,413, with a medium of 3,410 days.

portfolio is based on comparable 5-minute data for the S&P 500 futures index obtained from Tick Data Inc.

Table 2 provides key summary statistics for each of the stocks included in the sample as well as the S&P 500 futures index (SPFU). Not surprisingly, the average continuous variation (CV) for all of the individual stocks far exceeds that of the market. Similarly, the variation attributable to jumps (JV) is also numerically much larger for each of the individual stocks than it is for the market. In terms of the total variation, the share due to jumps ranges from a low of 19.6% to a high of 29.2%, with a median value across all fifty stocks of 21.5%. In contrast, the corresponding number for the aggregate market index equals 16.1%, so that jump risk appears to be relatively more important at the individual stock level.

The last two columns in the table, which report the total number of systematic and idiosyncratic jumps detected for each of the stocks, further corroborates this idea. For almost all of the stocks the total number of jumps exceed the number of market-wide jumps for the S&P 500. These numbers also suggest very high overall jump intensities ranging from slightly more than one jump per day to about one jump every other day.[26] The finding that the individual stocks contain more jumps than the market is consistent with the hypothesis of diversifiable individual jump risk originally put forth by Merton (1976). Of course, the mere existence of jumps at the market level refutes the conjecture that jump risk is entirely firm specific.

Further to this effect, the number of so-called systematic jumps, or jumps in the individual stocks that occur at the same time the market jumps, are clearly non-trivial. Still, it is obviously not the case that when a "large" market jump occurs, it automatically triggers "large" jumps in all of the individual stocks. As such, a simple linear one factor market model appears too simplistic to describe the relation between the individual and market-wide jumps, and in turn the joint dependencies in the jump tails.

In order to more clearly visualize the different types of jump sets, we plot in Figure 1 the 5-minute logarithmic prices for three separate days for IBM, as a representative stock, and the S&P 500 market portfolio. For ease of comparison, we normalize the logarithmic price at the beginning of the day to zero across all of the panels. The top panel shows the intraday prices on October 29, 2002, a day where the aggregate market jumped but IBM did not. The jump in the market obviously occurred at 10:00EST, and is readily associated with a disappointing reading of the Consumer Confidence Index

[26] With infinitely activity jumps, the total number of "significant" jumps will naturally be expected to increase to infinity for ever increasing sampling frequency, and these numbers need to be interpreted accordingly.

released at that exact time.[27] The middle panel shows the prices on January 3, 2001, a day with a systematic jump in IBM. The timing of the systematic jumps is again readily associated with the surprise cut in the Federal Funds Rate announced at 13:10EST on that day. The final third panel shows February 26, 2008, when the board of directors for IBM announced at 11:00EST that they had authorized $15 billion in additional funds for stock repurchases, resulting in an idiosyncratic jump in IBM, but no discernable discontinuities in the within-day prices for the aggregate market.

We continue next with a discussion of our estimation results pertaining to the idiosyncratic and systematic jump tail distributions.

5.2 Marginal Jump Tails

Our estimation results for the scalar tail decay parameter ξ for each of the marginal jump tail distributions are reported in Table 3. The relevant truncation levels for the different tails are determined by the equivalent of 0.02 times the daily sample sizes, along with $\tau = 2.5$ in equation (3.3). Corresponding asymptotic standard errors for each of the individual estimates are immediately available from the formula in equation (3.12).

Looking first at the results for the S&P 500 market portfolio, both of the jump tails are heavy with the right tail decaying at a slower rate than the left, or $\widehat{\xi}^{+(0,0)} > \widehat{\xi}^{-(0,0)} > 0$. This is consistent with the empirical evidence reported in Bollerslev and Todorov (2010), and directly refutes the popular compound Poisson jump model with normally distributed jump sizes that have been used extensively in the existing literature.

Turning to the results for the systematic jumps in the individual stocks, most of the point estimates for ξ are positive, again indicating heavy-tailed jump distributions. In parallel to the results for the market, for several of the stocks the estimate for the right tail appears larger than the left.[28] Of course, given the relatively low number of observations invariably available for the estimation of the jump tails, many of the estimates are not significantly different from zero when judged by their individual standard errors of approximately $M^{-1/2} \approx 0.121$ under the null hypothesis of light tails. Taken as a whole, however, the cross-sectional evidence clearly suggests that the systematic jumps are heavy-tailed.[29] At the same time, the dispersion in the estimates again suggests that

[27] Andersen et al. (2003b) and Andersen et al. (2007a), among many others, have previously studied the relationship between regularly scheduled macroeconomic news announcements and jumps and/or large price movements in asset prices.

[28] Related empirical evidence for overall larger right tails in half-hourly raw returns for various sector indexes has recently been reported by Straetmans et al. (2008).

[29] Related to this, Kelly (2010) has recently explored ways in which to increase the efficiency of tail index estimation by pooling the estimates across different stocks. His estimates, however, are based on coarser daily frequency returns and not the jump tails per se, and do not explicitly differentiate between

the relationship between the individual and market-wide jumps is not well described by a simple one factor market model, which would imply identical systematic jump tail decay parameters across all of the stocks.

The estimates for the idiosyncratic jump tails are reported in the last two columns of the table. Almost all of the point estimates are again positive, and for many of the stocks are bellow those for the systematic jump tails. Also, in parallel to the systematic jump tails, the tail decay parameters for the right tails in quite a few cases dominate those for the left, indicative of greater upside potential than downside firm-specific risks.

Meanwhile, as previously noted, given the relatively short time span and limited number of "tail" observations underlying the estimation, all of the point estimates are admittedly somewhat imprecise.[30] To check the robustness of the results, we therefore redid the estimation for the idiosyncratic jump tails based on a truncation level equivalent to a total of 200 jump tail observations, implying a smaller asymptotic standard error under the null of $\xi = 0$ approximately equal to $M^{-1/2} \approx 0.071$. The resulting estimates are generally fairly close to the ones based on the larger truncation level, with medium estimates of 0.177 and 0.212 for the right and left tail decay parameters, respectively, compared to the values of 0.187 and 0.214 reported in the table.[31]

To more directly illustrate the estimation results, we plot in Figure 2 the relevant jump tail estimation for IBM together with the actually observed "moderate" to "large" sized jumps. To facilitate the visual comparisons, the tails are plotted on a double logarithmic scale.[32] As is evident from the figure, the overall magnitude of the idiosyncratic jump tails in the bottom two panels dominate the systematic ones depicted in the top two panels. At the same time, the corresponding estimates for ξ are all quite similar, except for the right systematic jump tail shown in the top right panel, which appears to decay at a somewhat slower rate. The generally excellent fits afforded by the estimated

the systematic and idiosyncratic parts of the tails.

[30] Thirteen-and-a-half years might, of course, not seem like a short time span, but it leaves us with only 60-70 observations. As discussed above, this choice of truncation is essentially dictated by a usual bias-variance tradeoff, with the use of lower truncation levels resulting in more observations and everything else equal more accurate estimates on the one hand, but potentially larger deviations from the asymptotic approximations and therefore larger biases on the other.

[31] Further details concerning these robustness checks are available upon request. We also experimented with the use of lower truncation levels for the estimation of the systematic jump tails. However, the total number of systematic jumps for each of the stocks defined by the set in (3.8) and the relatively high threshold level in (3.5) naturally limit the total number of systematic jumps, as reported in Table 2. As such, the jumps identified as systematic are truly "large" in a joint sense, and the jumps actually used in the estimation much "deeper" in the tails than a naive comparison of their number relative to the total number of systematic jumps would suggest.

[32] The flat lines for the actually observed jumps at $-8.14 \approx log(1/3, 413)$ correspond to the occurrence of one jump of that particular size in the sample. Similarly, for the other apparent lines at $log(j/3, 413)$ for integer j.

solid lines for the actually observed jump tails, also directly underscore the accuracy of the marginal EVT approximation underlying our estimation procedures.[33]

5.3 Systematic Jump Tail Dependencies

5.3.1 High-Frequency Dependencies

Before we discuss the general set of estimation results pertaining to all of the fifty stocks, it is instructive to again consider the jump tail dependencies that we are after by looking at IBM as a representative stock. To this end, we plot in Figure 3 the pairs of realized positive and negative systematic jumps for IBM and the S&P 500 market portfolio. The figure clearly reveals a strong positive association between the systematic jumps in the stock and the jumps in the market index. Visual inspection also suggests that for IBM this association might be slightly stronger for the negative than the positive jumps, albeit not overwhelmingly so.[34]

Of course, we are primarily interested in the "extreme" tail dependencies, and the probability/intensity of observing a "large" jump in one of the individual stocks given that the market jumped by a "large" amount. As discussed above, this probability follows directly from Pickands dependence function. Our estimates of that function for the negative (solid line) and positive (dashed line) systematic IBM and market-wide jumps are plotted in Figure 4. Both of the estimated curves are far below unity, as would be implied by independent tails, and much closer to the lower bound of perfect dependence as indicated by the triangle. Moreover, while simple visual inspection of the aforementioned scatter plot in Figure 3 seemingly points to somewhat stronger dependencies for the negative jump tails, the non-parametrically estimated "extreme" dependence functions are fairly close throughout most of the support. The corresponding estimates for the tail-dependence coefficients obtained by evaluating the functions at one-half together with the formula for χ in equation (3.17) equal 0.625 and 0.655 for the right and left tails, respectively. Hence, counter to the naive impression from Figure 3 and many stories in the popular financial press about various "doomsday scenarios," our formal high-frequency based estimates actually suggest very similar asymptotic tail dependencies during sharp market rallies, or positive jumps, and periods of steep market declines,

[33]Importantly, the new procedures would also allow us to meaningfully extrapolate the behavior of the jump tails and corresponding "extreme" jump quantiles to levels which would be impossible to accurately estimate with standard parametric approaches and lower-frequency, say daily, data; for further discussion along these lines see Bollerslev and Todorov (2010).

[34]The simple linear correlations for the jump pairs depicted in the two panels equal 0.856 and 0.746 for the negative and positive jumps, respectively.

or negative jumps.[35]

To further help gauge the magnitude of the estimated dependencies, we also include in Figure 4 estimates of Pickands dependence function based on the raw high-frequency 5-minute returns. These functions are systematically higher, and the resulting tail dependencies lower, than the ones based on the systematic jump tails. Intuitively, the dependence in the raw returns manifests several features in the underlying latent bivariate semimartingale process that describes the joint dynamics of the two price series. On the one hand, the presence of common, or systematic, jumps tends to produce strong tail dependencies, as directly evidenced by the previously discussed estimates. On the other hand, the presence of idiosyncratic jumps tends to weaken the tail dependencies. Similarly, pure diffusive price moves formally imply asymptotic independence. At the same time, however, the presence of time-varying stochastic volatility will tend to generate tail dependence through periods of high volatility. As further discussed below, the joint influence of all of these separate effects in turn combine to account for the weaker tail dependencies observed with the raw high-frequency returns.

These specific results for IBM carry over to the rest of the stocks in the sample. In particular, turning to Table 4, the first two columns in the table show the estimated asymptotic tail dependencies for the raw 5-minute returns for each of the fifty stocks. These estimates are generally fairly low. The results also closely mirror those obtained by restricting the sample to only those 5-minute returns that are classified as jumps, or the set $\widehat{\mathcal{T}}^{(j)}_{[0,T]}$. By contrast, the estimated dependence coefficients for the systematic jump tails, or the returns in the set $\widehat{\mathcal{T}}^{(j,0)}_{[0,T]} = \widehat{\mathcal{T}}^{(j)}_{[0,T]} \cap \widehat{\mathcal{T}}^{(0)}_{[0,T]}$, are all very high ranging from 0.455 to 0.662. The estimates are also surprisingly close to symmetric for most of the stocks, and if anything slightly larger for the right tails.

These conclusions based on $\widehat{\chi}$ are further corroborated by the summary results reported in Table 5 for the alternative statistics discussed in Section 3.3.[36] As seen from the table, the estimates for the extreme tail-dependence coefficient χ using the extremogram and the semiparametric setting based on a logistic copula for the tail dependence lead to essentially the same results as the ones reported in Table 4. For instance, the median

[35]This, of course, also contrasts with many of the estimates reported in the existing empirical finance literature based on daily, or coarser frequency, data and standard correlation based measures, or parametric GARCH type models allowing for time-varying dynamic correlations, which typically point to stronger dependencies on the downside; see, e.g., Engle (2009) and the many references therein. We will try to reconcile these differences below by directly attributing the tail dependencies in daily returns to systematic jumps and commonalities in volatilities.

[36]To conserve space, we only report the median and interquartile range of the estimates obtained across all of the fifty stocks. Detailed estimates for each of the individual stocks are given in a supplementary Appendix available upon request.

values of $\widehat{\chi}_E^+$ and $\widehat{\chi}_E^-$ for the systematic jump tails equal 0.603 and 0.574, respectively, compared to 0.574 and 0.547 for $\widehat{\chi}^+$ and $\widehat{\chi}^-$ in Table 4.

The estimates for the coefficient of residual tail dependence η are all reasonably close to 1, as formally implied by $\chi > 0$, with the downward biases relative to unity comparable to those documented in the Monte Carlo simulation study. Although more difficult to interpret from a formal theoretical perspective, Kendall's tau $\widehat{\tau}_k$ and Spearman's rho $\widehat{\rho}_s$ also both achieve their largest average values for the systematic jump tails. This again is in concert with the results from the simulations.

All in all, these results clearly support the notion that most of the "extreme" joint dependencies reside in the systematic jump tails. Building on this idea, we next show how to identify and isolate the effect of common time-varying stochastic volatility as another separate source of tail dependence in lower-frequency daily returns.

5.3.2 Daily Dependencies

We continue to rely on the high-frequency data for explicitly "filtering" out the jumps in the daily returns and variation measures. In particular, for each asset, $j = 0, 1, ..., M$, and day, $t = 1, ..., T$, in the sample, the part of the daily returns associated with continuous price moves are naturally estimated by the sum of the intraday high-frequency returns that are not classified as jumps,

$$\mathbf{z}_t^{(j)} = \sum_{i=(t-1)n+1}^{tn} \left[\begin{pmatrix} \Delta_i^n p^{(j)} \\ \Delta_i^n p^{(0)} \end{pmatrix} \mathbf{1} \begin{pmatrix} |\Delta_i^n p^{(j)}| \le \alpha_i^{(j)} n^{-\varpi} \\ |\Delta_i^n p^{(0)}| \le \alpha_i^{(0)} n^{-\varpi} \end{pmatrix} \right]. \tag{5.1}$$

Under the assumption of finite variation jumps and weak additional regularity conditions, it follows readily from the expression for the general semimartingale process in equation (2.1) that for $n \to \infty$,

$$\mathbf{z}_t^{(j)} \xrightarrow{\mathbb{P}} \begin{pmatrix} \int_{t-1}^t \alpha_s^{(j)} ds + \int_{t-1}^t \sigma_s^{(j)} dW_s^{(j)} ds \\ \int_{t-1}^t \alpha_s^{(0)} ds + \int_{t-1}^t \sigma_s^{(0)} dW_s^{(0)} ds \end{pmatrix}. \tag{5.2}$$

The first integrals on the right-hand-side associated with the drifts in the individual stock and aggregate market prices are both negligible, and will not affect the estimated daily tail dependencies. Further, assuming the diffusive volatilities to be constant and the Brownian motions not perfectly correlated, the terms associated with the second integrals would be jointly normally distributed and hence result in asymptotically independent tails. Consequently, any tail dependence between the two components in $\mathbf{z}_t^{(j)}$ is directly attributable to time-varying stochastic volatility.

Going one step further, it is possible to non-parametrically "remove" the effect of the stochastic volatility by standardizing $\mathbf{z}_t^{(j)}$ with an estimator of its quadratic variation.

Specifically, let

$$\widetilde{\mathbf{z}}_t^{(j)} = \left\{ \sum_{i=(t-1)n+1}^{tn} \left[\begin{pmatrix} (\Delta_i^n p^{(j)})^2 & \Delta_i^n p^{(j)} \Delta_i^n p^{(0)} \\ \Delta_i^n p^{(j)} \Delta_i^n p^{(0)} & (\Delta_i^n p^{(0)})^2 \end{pmatrix} \mathbf{1} \begin{pmatrix} |\Delta_i^n p^{(j)}| \leq \alpha_i^{(j)} n^{-\varpi} \\ |\Delta_i^n p^{(0)}| \leq \alpha_i^{(0)} n^{-\varpi} \end{pmatrix} \right] \right\}^{-1/2} \mathbf{z}_t^{(j)}, \tag{5.3}$$

where the estimator for the daily quadratic variation is based on a multivariate version of the truncated Continuous Variation measure defined in equation (3.4).[37] Then, in analogy to the results discussed above, it follows that for $n \to \infty$,

$$\widetilde{\mathbf{z}}_t^{(j)} \xrightarrow{\mathbb{P}} \begin{pmatrix} \langle p_s^{(j)}, p_s^{(j)} \rangle_{(t-1,t]}^c & \langle p_s^{(j)}, p_s^{(0)} \rangle_{(t-1,t]}^c \\ \langle p_s^{(j)}, p_s^{(0)} \rangle_{(t-1,t]}^c & \langle p_s^{(0)}, p_s^{(0)} \rangle_{(t-1,t]}^c \end{pmatrix}^{-1} \begin{pmatrix} \int_{t-1}^t \alpha_s^{(j)} ds + \int_{t-1}^t \sigma_s^{(j)} dW_s^{(j)} ds \\ \int_{t-1}^t \alpha_s^{(0)} ds + \int_{t-1}^t \sigma_s^{(0)} dW_s^{(0)} ds \end{pmatrix}, \tag{5.4}$$

where $\langle p_s^{(j)}, p_s^{(0)} \rangle_{(t-1,t]}^c$ refers to the continuous part of the quadratic covariation between $p^{(j)}$ and $p^{(0)}$ over the $(t-1, t]$ daily time interval. As before, the impact of the drift terms may be ignored, so that the non-parametrically "devolatilized" pairs of returns $\widetilde{\mathbf{z}}_t^{(j)}$ should be approximately bivariate standard normally distributed.[38]

Motivated by these ideas, the right panel in Figure 4 plots Pickands dependence functions for each of the bivariate IBM series $\mathbf{z}_t^{(IBM)}$ and $\widetilde{\mathbf{z}}_t^{(IBM)}$. Our estimates are based on exactly the same estimation procedures and truncation levels as the ones described for the jump tails in Sections 3.2 and 3.3. Per the discussion above, we would expect the left and right tail functions corresponding to $\widetilde{\mathbf{z}}_t^{(IBM)}$ to be close to unity. The two curves in the figure confirm this, thus indirectly underscoring the accuracy of our empirical approximations, and the minimal influence imparted by the finite-sample measurement errors and "leverage effect."

Further elaborating on these results, the wedge between the estimated dependence functions for $\mathbf{z}_t^{(IBM)}$ and $\widetilde{\mathbf{z}}_t^{(IBM)}$ directly reveals the effect of the diffusive stochastic volatility on the overall tail dependence. As seen from the figure, this wedge is obviously non-trivial, and shows that time-varying volatility is indeed responsible for some of the asymptotic tail dependence between the daily individual stock returns and the return

[37]Note, this estimator is guaranteed to be positive semi-definite by construction.

[38]The approximation comes from the need to estimate the quadratic variation and a possible "leverage effect," or negative correlation between the within-day stochastic volatility and price innovations. The latter should have only minimal effect, and if anything, result in slightly stronger negative tail dependencies. A related univariate standardization approach has been proposed by Andersen et al. (2007b), and further explored empirically by Andersen et al. (2010), who confirm that the jump-adjusted "devolatilized" returns for a sample of individual stocks are approximately univariate standard normally distributed.

on the aggregate market portfolio. The figure also points to slightly stronger, albeit not by much, tail dependencies in the positive (dashed line) than the negative (solid line) jump-adjusted "devolatilized" returns. This slight difference and reversal vis-a-vis the results for the raw daily returns may in part be attributed to the within-day "leverage effect." Meanwhile, comparing the two panels in Figure 4 and the magnitudes therein, the systematic jump tails are clearly associated with much stronger "extreme" dependencies than the ones induced by the more slowly moving daily diffusive volatility.

To further corroborate these specific results for IBM, we report in Table 6 the estimated tail dependence coefficients χ for the raw daily returns, the jump-adjusted returns $\mathbf{z}_t^{(j)}$, and the jump-adjusted "devolatilized" returns $\widetilde{\mathbf{z}}_t^{(j)}$, for each of the fifty stocks in the sample. For comparison purposes, we also include the results where we ignore the temporal variation in the continuous covariation and only standardize the jump-adjusted returns by their respective univariate continuous variation measures; i.e., the series obtained by restricting the off-diagonal elements in the matrix in equation (5.3) to be zero.[39]

Looking first at the results for the raw daily returns, the estimated tail-dependence coefficients are generally quite close across all of the fifty stocks, with median values of 0.317 and 0.336 for the positive and negative tails, respectively. These numbers are, of course, somewhat larger than the median dependencies estimated with the raw 5-minute returns, but they are still dwarfed by the estimates for the systematic jump tails. Removing all of the "large" jumps from the daily returns reduces slightly the average dependence-coefficient estimates. This is consistent with the aforemention dependence coefficients for the high-frequency "filtered" jumps reported in Table 4, which are slightly higher than those for the raw 5-minute returns. The results for the univariate "devolatilized" returns reported in the next pair of columns, confirm that some of the extreme tail dependence may indeed be ascribed to time-varying volatility. For most of the stocks, the univariate standardization reduces $\widehat{\chi}^-$ and $\widehat{\chi}^+$ by more than a third relative to the estimates for the jump-adjusted returns $\mathbf{z}_t^{(j)}$.[40] Meanwhile, standardizing the jump-adjusted returns by the full realized continuous covariation matrix to explicitly account for the temporal variation in the diffusive covariance risk as well, effectively eliminates all of the remaining dependencies, and results in asymptotically independent tails.

[39] As previously noted, Andersen et al. (2010) have recently argue that a closely related univariate standardization scheme results in approximate univariate standard normal distributions empirically.

[40] This is also consistent with the earlier empirical evidence in Poon et al. (2004), who report that standardizing daily international equity index returns by simple univariate parametric GARCH models tends to reduce the estimated tail dependencies.

The results for the alternative tail dependence coefficients summarized in Table 7 again corroborate the more detailed results for all of the individual stocks and $\widehat{\xi}$ discussed above. In particular, the median values of $\widehat{\xi}_l$ and $\widehat{\xi}_E$ for the jump-adjusted bivariate "devolatilized" returns are all close to zero, as are the median values for $\widehat{\tau}_k$ and $\widehat{\rho}_s$. As noted above, the residual tail dependence coefficient for a bivariate normal distribution with correlation $\rho = 0$ equals $\eta = (1+\rho)/2$. Hence, the values of $\widehat{\eta}$ close to 0.5 in the last two columns provide further evidence that the standardized returns are indeed $i.i.d.$ bivariate normal distributed. These results also indirectly suggest that any within-day dynamic "leverage effects" must be fairly small.

Taken as whole, the empirical results reported in Tables 6 and 7 show how the new high-frequency based procedures developed here allow us to "dissect" the generic semimartingale representation in equation (2.1), and effectively assess the role of the different components in generating tail dependencies.

6 Conclusion

We propose a new set of statistical procedures for dissecting the jumps in individual asset returns into idiosyncratic and systematic components, and for estimating the joint distributional features of the corresponding jump tails. Our estimation techniques are based on in-fill and long-span asymptotics, together with extreme value type approximations. On applying the estimation methods with a large panel of high-frequency data for fifty individual stocks and the S&P 500 market portfolio, we find that the idiosyncratic and systematic jumps are both generally heavy tailed. We also find strong evidence for asymptotic tail dependence between the individual stocks and the market index, with most of it directly attributable to the systematic jump tails, and strong dependencies between the sizes of the simultaneously occurring jumps. Further building on the same techniques, we document non-trivial joint tail dependencies in longer horizon daily returns, and show how that dependence may be ascribed to the high-frequency systematic jumps and commonalities in the interdaily temporally varying stochastic volatility.

As such, our empirical findings highlight the importance of the new estimation framework for better understanding and more accurately modeling tail and systemic risk events, like the ones experienced during the recent financial crises. The estimation techniques developed here could also be usefully applied with high-frequency data from different countries in the study of "extreme" international market linkages and contagion-type effects. We leave further empirical investigations along these lines for future work.

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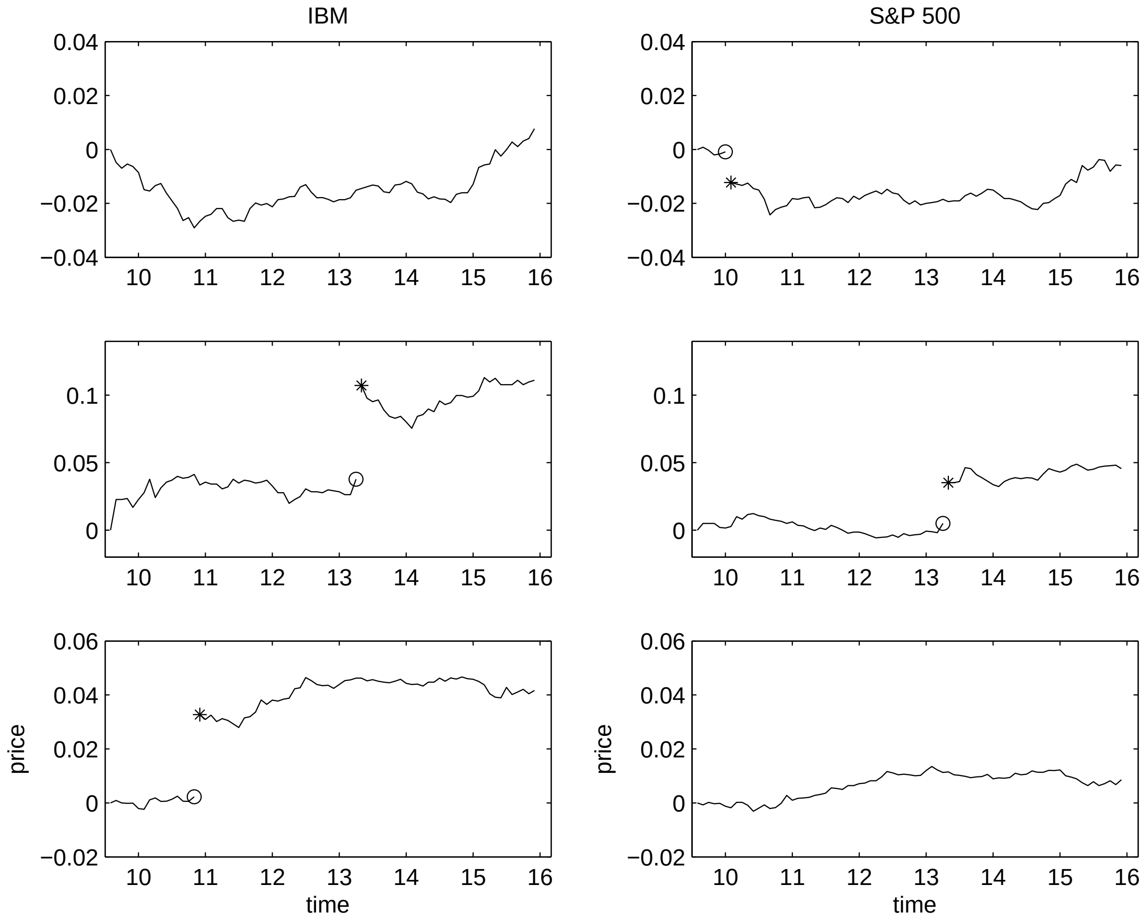
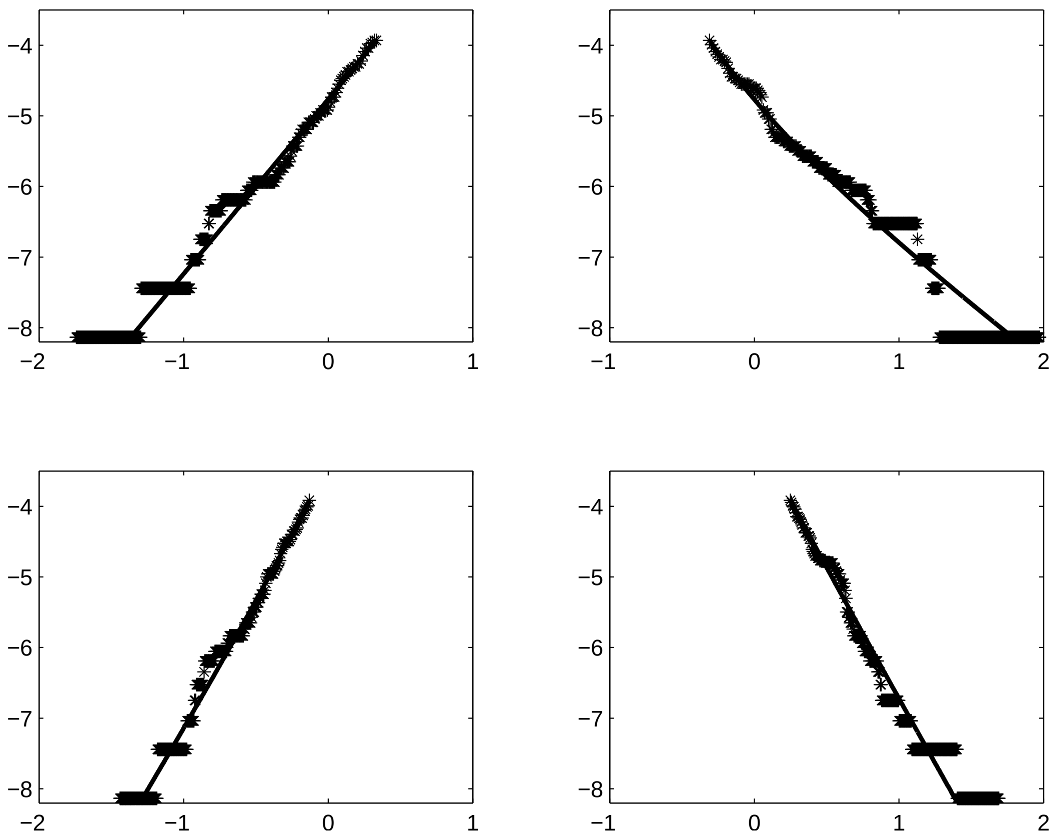
Figure 1: *IBM and Market-Wide Jumps*



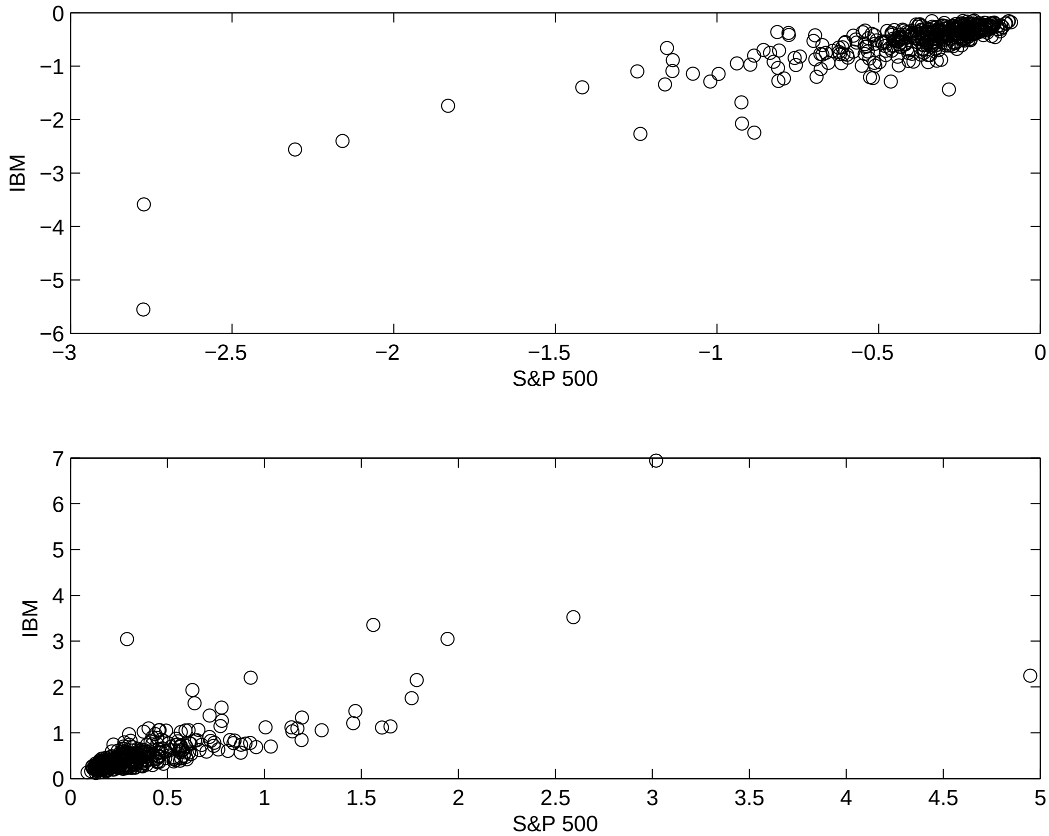
Note: The figure shows the 5-minute logarithmic prices for IBM and the S&P 500 futures index for October 29, 2002 (top panel), January 3, 2001 (middle panel), and February 26, 2008 (bottom panel). The logarithmic prices are normalized to zero at the beginning of each day.

Figure 2: *IBM Jump Tails*



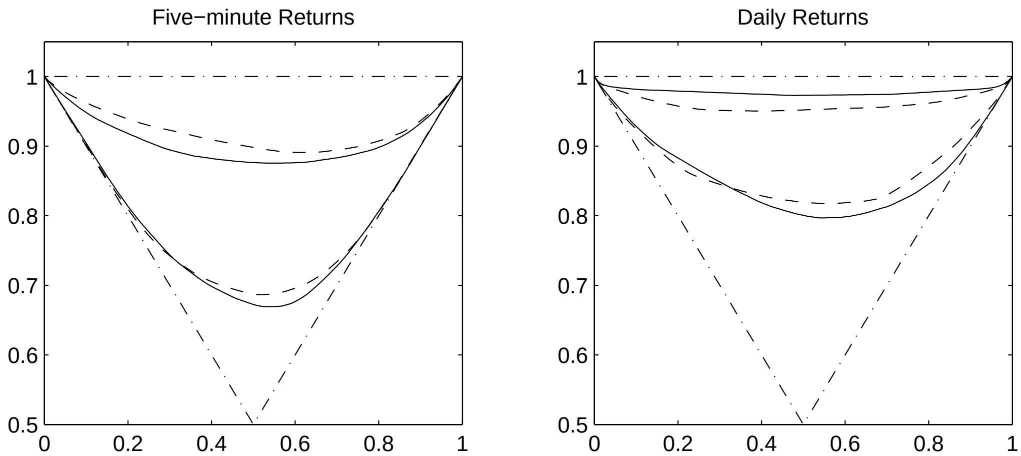
Note: The figure shows the estimated (solid line) and actually observed (stars) systematic (top two panels) and idiosyncratic (bottom two panels) negative (left two panels) and positive (right two panels) jump tails for IBM. The tails are plotted on a double logarithmic scale. All of the jumps are extracted from 5-minute returns spanning 1997 through 2010.

Figure 3: *IBM Systematic Jumps*



Note: The figure shows the scatter of systematic negative (top panel) and positive (bottom panel) jumps in IBM and the S&P 500 market portfolio. The jumps are extracted from 5-minute returns spanning 1997 through 2010.

Figure 4: *Pickands Dependence Functions for High-Frequency and Daily IBM Returns*



Note: The figure shows estimates of Pickands dependence function for IBM and the S&P 500 market portfolio. The left panel is based on 5-minute returns (top two curves) and the systematic jump tails (bottom two curves). The right panel is based on daily returns (bottom two curves) and the jump-adjusted "devolatilized" daily returns (top two curves) denoted by $\widetilde{\mathbf{z}}_t^{(j)}$ in the main text. The dashed (solid) lines correspond to the positive (negative) tails. The jumps are extracted from 5-minute returns and the sample spans the period from 1997 through 2010.

Table 1: *Monte Carlo Simulations*

	Design							
	1	2	3	4	5	6	7	8
				$\widehat{\chi}$				
true value	0.500	0.250	0.750	0.500	0.500	0.500	0.500	0.500
25th quantile	0.481	0.266	0.724	0.477	0.467	0.489	0.478	0.482
50th quantile	0.518	0.295	0.753	0.511	0.499	0.524	0.509	0.514
75th quantile	0.549	0.328	0.775	0.545	0.535	0.556	0.542	0.544
				$\widehat{\chi}_l$				
true value	0.500	0.250	0.750	0.500	0.500	0.500	0.500	0.500
25th quantile	0.443	0.215	0.704	0.424	0.428	0.420	0.416	0.400
50th quantile	0.493	0.261	0.748	0.476	0.479	0.478	0.467	0.467
75th quantile	0.537	0.318	0.774	0.517	0.519	0.521	0.513	0.511
				$\widehat{\chi}_E$				
true value	0.500	0.250	0.750	0.500	0.500	0.500	0.500	0.500
25th quantile	0.480	0.240	0.720	0.480	0.460	0.500	0.480	0.480
50th quantile	0.520	0.280	0.760	0.500	0.500	0.520	0.520	0.520
75th quantile	0.560	0.320	0.780	0.560	0.540	0.560	0.560	0.560
				$\widehat{\eta}$				
true value	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25th quantile	0.890	0.789	0.926	0.828	0.839	0.812	0.820	0.790
50th quantile	0.948	0.853	0.968	0.884	0.895	0.868	0.873	0.844
75th quantile	1.002	0.920	1.007	0.939	0.954	0.923	0.924	0.896
				$\widehat{\tau}_k$				
true value	-	-	-	-	-	-	-	-
25th quantile	0.298	0.158	0.504	0.259	0.257	0.255	0.243	0.234
50th quantile	0.352	0.225	0.556	0.321	0.319	0.318	0.307	0.300
75th quantile	0.406	0.284	0.606	0.381	0.380	0.381	0.376	0.362
				$\widehat{\rho}_s$				
true value	-	-	-	-	-	-	-	-
25th quantile	0.378	0.189	0.645	0.335	0.333	0.337	0.321	0.316
50th quantile	0.451	0.289	0.713	0.422	0.420	0.421	0.405	0.404
75th quantile	0.528	0.380	0.765	0.499	0.496	0.502	0.497	0.487

Note: The table reports the results from the Monte Carlo simulation study detailed in Section 4 based on a total of 1,000 replications. All of the estimates are for the positive systematic jump tails. The eight different designs are defined as follows: 1: $\lambda = 0$, $\chi = 0.50$, true jumps; 2: $\lambda = 0$, $\chi = 0.25$, true jumps; 3: $\lambda = 0$, $\chi = 0.75$, true jumps; 4: $\lambda = 0$, $\chi = 0.50$, $\tau = 2.5$; 5: $\lambda = 0$, $\chi = 0.50$, $\tau = 2.0$; 6: $\lambda = 0$, $\chi = 0.50$, $\tau = 3.0$; 7: $\lambda = 1$, $\chi = 0.50$, $\tau = 2.5$; 8: $\lambda = 0$, $\chi = 0.5$, $\tau = 2.5$, and $p_i^{(j)*}$, $j = 0, 1$.

Table 2: *Summary Statistics*

	CV		JV		Jump Counts	
Ticker	Mean	st.dev.	Mean	st.dev.	Systematic	Idiosyncratic
AAPL	6.070	6.948	1.688	3.986	560	4941
GE	2.859	5.368	0.702	2.776	1002	4063
WMT	2.311	2.861	0.634	2.179	765	4309
IBM	2.082	2.819	0.508	1.687	768	3953
PG	1.624	2.278	0.452	1.660	703	4362
T	2.805	3.887	0.753	1.660	605	4895
JNJ	1.358	1.950	0.385	1.101	720	4874
JPM	4.123	8.821	1.029	3.064	787	4354
WFC	3.825	9.655	0.996	3.355	833	4566
ORCL	5.563	7.366	1.284	2.510	653	4488
KO	1.637	2.101	0.418	0.827	773	4720
PFE	2.243	2.609	0.609	1.409	673	4371
C	5.951	21.04	1.538	6.532	744	4110
BAC	4.659	14.13	1.151	4.442	799	4358
INTC	4.210	5.121	0.743	1.681	666	3613
SLB	4.050	5.671	0.851	1.436	445	4352
CSCO	4.565	6.224	0.895	2.312	667	3902
MRK	2.053	2.795	0.676	2.874	652	4701
PEP	1.913	2.393	0.520	1.493	595	4788
HPQ	3.600	4.528	1.073	2.508	681	4777
MCD	2.148	2.924	0.591	1.088	586	4928
AMZN	10.50	16.22	2.926	5.744	483	4886
QCOM	6.698	10.27	1.618	3.205	572	4302
OXY	3.483	5.983	0.879	1.599	511	5167
UTX	2.224	3.266	0.652	1.852	758	4923
F	5.166	17.40	1.800	6.495	534	5794
MMM	1.809	2.583	0.509	1.501	802	4693
CMCSA	4.338	5.469	1.792	2.606	651	8082
CAT	3.079	4.539	0.799	1.337	678	4843
HD	3.075	4.105	0.819	2.083	722	4606
FCX	5.741	9.484	2.047	3.098	400	8128
AMGN	3.281	4.284	0.901	1.632	595	4924
MO	2.088	2.558	0.763	2.873	526	5030
BA	2.685	3.387	0.713	1.428	622	4648
CVS	2.662	3.849	0.996	2.939	546	6227
EMC	5.931	7.489	1.409	3.196	567	4520
DD	2.711	3.552	0.669	1.520	729	4475
BMY	2.331	2.928	0.794	2.402	652	5225
HON	2.801	4.095	0.919	2.396	743	5430
NKE	2.554	3.152	0.936	1.910	503	5508
MDT	2.060	2.920	0.788	2.204	567	5541
UNH	2.880	4.257	1.044	2.294	441	5634
DOW	3.021	4.321	0.911	2.757	677	5189
CL	1.811	2.369	0.574	1.191	620	5344
TXN	5.460	6.196	1.261	2.498	609	4146
BK	4.260	12.20	1.287	5.521	825	5036
HAL	5.746	9.138	1.467	3.103	383	4759
WAG	2.590	3.027	0.812	1.829	617	5193
LOW	3.409	4.157	1.032	2.110	629	4921
SO	2.000	2.474	0.500	1.161	555	5259
min	1.358	1.950	0.385	0.827	383	3613
max	10.50	21.04	2.926	6.532	1002	8128
25th	2.229	2.921	0.671	1.608	567	4397
50th	2.950	4.207	0.865	2.249	652	4816
75th	4.318	6.767	1.131	2.923	727	5184
SPFU	1.050	2.022	0.201	0.765	4625	

Note: The sample period for the fifty stocks range from mid 1997 through December 2010, for a minimum of 3,330 to a maximum of 3,413 daily observations.

Table 3: *Jump Tail Decay Parameters*

	Systematic		Idiosyncratic	
Ticker	$\widehat{\xi}^{+}$	$\widehat{\xi}^{-}$	$\widehat{\xi}^{+}$	$\widehat{\xi}^{-}$
AAPL	0.137	0.335	0.503	0.002
GE	0.361	0.242	0.167	0.335
WMT	-0.042	0.119	0.395	0.273
IBM	0.646	0.411	0.282	0.306
PG	0.180	0.205	0.340	0.243
T	0.400	0.248	0.020	0.264
JNJ	-0.034	0.396	0.597	0.225
JPM	0.163	0.386	0.124	0.241
WFC	0.306	0.217	-0.159	0.074
ORCL	0.212	0.264	-0.092	-0.179
KO	0.180	0.208	0.317	0.342
PFE	0.094	0.316	0.153	0.221
C	0.481	0.371	-0.036	-0.002
BAC	0.887	0.389	0.112	0.259
INTC	0.443	0.185	0.330	0.109
SLB	0.111	0.253	0.132	0.217
CSCO	0.327	0.353	0.148	0.137
MRK	0.248	0.428	0.241	0.516
PEP	0.077	0.619	0.171	0.416
HPQ	0.335	0.097	0.047	-0.009
MCD	0.247	0.112	0.116	0.211
AMZN	0.176	-0.122	-0.137	0.204
QCOM	0.120	-0.128	0.121	-0.052
OXY	0.126	0.239	0.158	0.395
UTX	0.045	0.435	0.462	0.202
F	0.310	0.435	0.293	0.457
MMM	0.312	0.173	0.466	-0.043
CMCSA	-0.030	0.022	0.067	0.109
CAT	0.210	-0.097	0.051	0.106
HD	0.266	0.259	0.216	0.251
FCX	0.259	-0.026	0.190	0.151
AMGN	0.178	0.047	-0.098	0.083
MO	0.402	0.295	0.185	0.242
BA	0.132	0.391	0.221	0.220
CVS	0.268	0.337	0.246	0.365
EMC	0.218	0.213	0.205	-0.260
DD	-0.048	0.331	0.065	-0.021
BMY	-0.040	0.372	-0.003	0.549
HON	0.323	0.188	0.295	0.204
NKE	0.021	0.215	0.397	0.104
MDT	0.065	0.194	0.169	0.448
UNH	0.360	0.559	-0.004	0.073
DOW	0.402	0.252	0.301	0.227
CL	0.103	0.227	0.281	0.283
TXN	0.320	0.168	0.259	0.036
BK	0.281	0.131	0.427	0.254
HAL	0.007	0.042	0.434	0.143
WAG	0.435	0.265	0.504	0.200
LOW	0.279	0.321	0.053	0.098
SO	0.323	0.237	0.333	0.493
min	-0.048	-0.128	-0.159	-0.260
max	0.887	0.619	0.597	0.549
25th	0.113	0.176	0.078	0.100
50th	0.232	0.245	0.187	0.214
75th	0.323	0.349	0.313	0.270
SPFU	0.364	0.207		

Note: The estimated jump tail decay parameters are based on M = 0.02*T jump observations extracted from 5-minute returns spanning 1997 through 2010.

Table 4: *High-Frequency Tail-Dependence Coefficients*

	5-min Returns		All Jumps		Systematic Jumps	
Ticker	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$
AAPL	0.114	0.118	0.137	0.145	0.497	0.509
GE	0.274	0.157	0.333	0.212	0.616	0.610
WMT	0.178	0.132	0.254	0.216	0.514	0.563
IBM	0.204	0.247	0.236	0.296	0.625	0.655
PG	0.224	0.124	0.241	0.169	0.552	0.544
T	0.196	0.120	0.216	0.210	0.620	0.583
JNJ	0.235	0.143	0.276	0.173	0.548	0.528
JPM	0.246	0.123	0.344	0.248	0.634	0.576
WFC	0.177	0.097	0.257	0.166	0.555	0.501
ORCL	0.110	0.083	0.143	0.143	0.504	0.455
KO	0.218	0.187	0.239	0.275	0.594	0.592
PFE	0.211	0.144	0.287	0.167	0.590	0.525
C	0.154	0.103	0.245	0.178	0.641	0.506
BAC	0.197	0.105	0.318	0.206	0.563	0.530
INTC	0.211	0.148	0.243	0.206	0.563	0.498
SLB	0.326	0.210	0.304	0.178	0.602	0.549
CSCO	0.207	0.146	0.247	0.209	0.579	0.507
MRK	0.205	0.171	0.226	0.196	0.662	0.577
PEP	0.190	0.126	0.216	0.174	0.585	0.571
HPQ	0.131	0.070	0.199	0.113	0.547	0.550
MCD	0.189	0.093	0.185	0.124	0.554	0.579
AMZN	0.067	0.023	0.082	0.070	0.511	0.470
QCOM	0.081	0.072	0.138	0.114	0.544	0.491
OXY	0.323	0.191	0.279	0.192	0.626	0.518
UTX	0.237	0.148	0.268	0.190	0.607	0.578
F	0.092	0.092	0.117	0.141	0.585	0.574
MMM	0.249	0.150	0.249	0.176	0.559	0.512
CMCSA	0.145	0.086	0.198	0.096	0.477	0.543
CAT	0.224	0.187	0.200	0.235	0.657	0.598
HD	0.248	0.111	0.314	0.202	0.586	0.594
FCX	0.159	0.109	0.165	0.114	0.502	0.506
AMGN	0.139	0.068	0.174	0.072	0.544	0.465
MO	0.119	0.075	0.152	0.103	0.552	0.528
BA	0.254	0.161	0.248	0.224	0.582	0.654
CVS	0.159	0.099	0.186	0.145	0.578	0.580
EMC	0.145	0.067	0.150	0.173	0.493	0.487
DD	0.245	0.158	0.253	0.219	0.625	0.606
BMY	0.131	0.112	0.183	0.123	0.552	0.563
HON	0.212	0.136	0.248	0.175	0.628	0.566
NKE	0.112	0.065	0.099	0.083	0.591	0.612
MDT	0.087	0.057	0.090	0.100	0.523	0.544
UNH	0.190	0.103	0.188	0.119	0.608	0.536
DOW	0.130	0.090	0.151	0.147	0.563	0.512
CL	0.144	0.083	0.175	0.129	0.530	0.524
TXN	0.121	0.072	0.188	0.140	0.564	0.499
BK	0.255	0.116	0.290	0.235	0.650	0.612
HAL	0.148	0.103	0.163	0.123	0.551	0.587
WAG	0.237	0.133	0.255	0.119	0.570	0.526
LOW	0.201	0.167	0.256	0.213	0.600	0.623
SO	0.232	0.142	0.217	0.145	0.581	0.570
min	0.067	0.023	0.082	0.070	0.477	0.455
max	0.326	0.247	0.344	0.296	0.662	0.655
25th	0.140	0.091	0.174	0.123	0.548	0.512
50th	0.193	0.117	0.222	0.171	0.574	0.547
75th	0.230	0.147	0.255	0.206	0.606	0.580

Note: The estimated tail-dependence coefficients for each of the stocks with the S&P 500 market portfolio reported in the three pairs of columns are based on: all of the 5-minute returns; all of the jumps; and the systematic jumps only. The jumps are extracted from the 5-minute returns spanning 1997 through 2010.

Table 5: *Alternative High-Frequency Tail Dependence Coefficients*

	5-min Returns		All Jumps		Systematic Jumps	
	+	-	+	-	+	-
				$\widehat{\chi}_l$		
25th quantile	0.134	0.062	0.149	0.109	0.500	0.471
50th quantile	0.196	0.092	0.120	0.143	0.539	0.517
75th quantile	0.236	0.123	0.248	0.190	0.581	0.565
				$\widehat{\chi}_E$		
25th quantile	0.132	0.061	0.151	0.119	0.559	0.544
50th quantile	0.194	0.088	0.199	0.147	0.603	0.574
75th quantile	0.241	0.118	0.250	0.191	0.633	0.618
				$\widehat{\eta}$		
25th quantile	0.871	0.756	0.888	0.802	0.787	0.788
50th quantile	0.914	0.816	0.936	0.888	0.829	0.833
75th quantile	0.964	0.893	1.004	0.960	0.875	0.884
				$\widehat{\tau}_k$		
25th quantile	0.111	0.054	0.133	0.107	0.259	0.241
50th quantile	0.166	0.118	0.184	0.168	0.312	0.297
75th quantile	0.239	0.178	0.236	0.202	0.364	0.372
				$\widehat{\rho}_s$		
25th quantile	0.102	0.080	0.129	0.118	0.348	0.308
50th quantile	0.172	0.160	0.213	0.213	0.408	0.396
75th quantile	0.273	0.232	0.282	0.276	0.489	0.491

Note: The table reports the median and interquartile range for the different estimators obtained across all of the fifty stocks in the sample based on: the 5-minute returns; all of the jumps; and the systematic jumps only, as further detailed in the note to Table 4.

Table 6: *Daily Tail-Dependence Coefficients*

	Daily Returns		Jump Adj. Returns		Univariate De-vol.		Multivariate De-vol.	
Ticker	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$	$\widehat{\chi}^{+}$	$\widehat{\chi}^{-}$
AAPL	0.190	0.237	0.225	0.226	0.161	0.198	0.090	0.065
GE	0.441	0.516	0.355	0.429	0.232	0.202	0.058	0.061
WMT	0.321	0.235	0.214	0.252	0.140	0.136	0.047	0.040
IBM	0.361	0.399	0.348	0.325	0.207	0.160	0.096	0.054
PG	0.320	0.269	0.241	0.209	0.231	0.118	0.087	0.035
T	0.333	0.344	0.249	0.270	0.157	0.207	0.067	0.093
JNJ	0.296	0.344	0.249	0.302	0.109	0.177	0.051	0.063
JPM	0.473	0.398	0.363	0.290	0.196	0.216	0.075	0.063
WFC	0.397	0.343	0.310	0.336	0.167	0.190	0.062	0.076
ORCL	0.262	0.239	0.263	0.213	0.166	0.112	0.083	0.032
KO	0.296	0.306	0.255	0.299	0.164	0.126	0.073	0.047
PFE	0.271	0.386	0.249	0.327	0.172	0.169	0.061	0.054
C	0.415	0.367	0.314	0.357	0.189	0.236	0.065	0.091
BAC	0.368	0.401	0.279	0.329	0.177	0.206	0.073	0.045
INTC	0.372	0.324	0.314	0.282	0.230	0.216	0.069	0.050
SLB	0.291	0.424	0.231	0.343	0.152	0.126	0.074	0.041
CSCO	0.341	0.295	0.287	0.235	0.186	0.178	0.082	0.043
MRK	0.314	0.436	0.275	0.344	0.144	0.154	0.042	0.042
PEP	0.328	0.264	0.253	0.254	0.110	0.129	0.049	0.057
HPQ	0.312	0.305	0.279	0.245	0.145	0.124	0.063	0.050
MCD	0.228	0.312	0.253	0.209	0.168	0.135	0.068	0.065
AMZN	0.212	0.230	0.188	0.193	0.138	0.182	0.058	0.085
QCOM	0.218	0.242	0.225	0.202	0.162	0.167	0.067	0.056
OXY	0.354	0.407	0.193	0.339	0.118	0.131	0.048	0.063
UTX	0.436	0.373	0.343	0.278	0.160	0.195	0.069	0.065
F	0.321	0.286	0.250	0.314	0.161	0.180	0.088	0.064
MMM	0.383	0.355	0.292	0.405	0.193	0.200	0.047	0.060
CMCSA	0.280	0.335	0.228	0.318	0.147	0.162	0.075	0.074
CAT	0.398	0.363	0.257	0.345	0.190	0.207	0.089	0.045
HD	0.420	0.384	0.328	0.383	0.179	0.187	0.079	0.067
FCX	0.287	0.378	0.207	0.345	0.116	0.148	0.059	0.060
AMGN	0.276	0.303	0.195	0.259	0.124	0.167	0.052	0.052
MO	0.197	0.206	0.167	0.164	0.098	0.157	0.044	0.040
BA	0.375	0.373	0.327	0.288	0.146	0.162	0.068	0.067
CVS	0.272	0.230	0.209	0.217	0.132	0.189	0.057	0.085
EMC	0.282	0.261	0.231	0.245	0.122	0.187	0.042	0.036
DD	0.399	0.398	0.374	0.364	0.189	0.148	0.071	0.084
BMY	0.304	0.337	0.298	0.296	0.145	0.175	0.077	0.058
HON	0.426	0.379	0.314	0.373	0.154	0.199	0.050	0.076
NKE	0.292	0.291	0.227	0.216	0.147	0.187	0.058	0.050
MDT	0.253	0.248	0.241	0.252	0.148	0.178	0.083	0.075
UNH	0.246	0.273	0.210	0.255	0.125	0.119	0.076	0.058
DOW	0.303	0.337	0.292	0.322	0.224	0.189	0.081	0.053
CL	0.253	0.255	0.206	0.221	0.162	0.100	0.077	0.043
TXN	0.284	0.278	0.243	0.222	0.179	0.147	0.087	0.059
BK	0.436	0.411	0.401	0.366	0.135	0.246	0.045	0.106
HAL	0.334	0.350	0.232	0.374	0.115	0.153	0.055	0.051
WAG	0.346	0.253	0.229	0.223	0.075	0.156	0.039	0.069
LOW	0.382	0.356	0.287	0.376	0.157	0.190	0.068	0.071
SO	0.246	0.187	0.225	0.167	0.128	0.143	0.058	0.043
min	0.190	0.187	0.167	0.164	0.075	0.100	0.039	0.032
max	0.473	0.516	0.401	0.429	0.232	0.246	0.096	0.106
25th	0.277	0.265	0.227	0.228	0.136	0.147	0.055	0.047
50th	0.317	0.336	0.251	0.289	0.157	0.172	0.068	0.059
75th	0.374	0.377	0.296	0.342	0.179	0.190	0.076	0.067

Note: The estimated tail-dependence coefficients for each of the stocks with the S&P 500 market portfolio reported in the four pairs of columns are based on: the raw daily returns; the daily returns with the jumps removed denoted by $\mathbf{z}_t^{(j)}$ in the main text; the jump-adjusted $\mathbf{z}_t^{(j)}$ "devolatilized" by the scalar continuous variation measures; and $\mathbf{z}_t^{(j)}$ "devolatilized" by the multivariate continuous covariation measure denoted by $\widetilde{\mathbf{z}}_t^{(j)}$ in the main text. The sample spans the period from 1997 through 2010.

Table 7: *Alternative Daily Tail Dependence Coefficients*

	Daily Returns		Jump Adj. Returns		Univariate De-vol.		Multivariate De-vol.	
	+	-	+	-	+	-	+	-
					$\widehat{\chi}_l$			
25th quantile	0.250	0.244	0.190	0.205	0.071	0.081	0.001	0.001
50th quantile	0.306	0.309	0.230	0.267	0.087	0.100	0.001	0.001
75th quantile	0.370	0.367	0.299	0.332	0.122	0.132	0.020	0.004
					$\widehat{\chi}_E$			
25th quantile	0.265	0.265	0.206	0.227	0.088	0.103	0.015	0.015
50th quantile	0.309	0.324	0.250	0.279	0.112	0.132	0.029	0.015
75th quantile	0.379	0.382	0.301	0.342	0.147	0.165	0.044	0.029
					$\widehat{\eta}$			
25th quantile	0.833	0.813	0.800	0.771	0.580	0.594	0.466	0.436
50th quantile	0.898	0.918	0.874	0.885	0.631	0.652	0.504	0.459
75th quantile	0.960	0.974	0.930	0.986	0.677	0.693	0.536	0.498
					$\widehat{\tau}_k$			
25th quantile	0.180	0.227	0.152	0.193	-0.017	0.010	-0.096	-0.068
50th quantile	0.248	0.302	0.222	0.272	0.048	0.058	-0.045	-0.026
75th quantile	0.315	0.333	0.270	0.324	0.118	0.099	0.042	0.044
					$\widehat{\rho}_s$			
25th quantile	0.226	0.291	0.194	0.232	-0.034	0.014	-0.137	-0.109
50th quantile	0.294	0.378	0.300	0.330	0.060	0.090	-0.072	-0.049
75th quantile	0.391	0.443	0.361	0.422	0.159	0.146	0.046	0.080

Note: The table reports the median and interquartile range for the different estimators obtained across all of the fifty stocks in the sample based on: the raw daily returns; the daily returns with the jumps removed; the jump-adjusted univariate "devolatilized" returns; and the jump-adjusted multivariate "devolatilized" returns, as further detailed in the note to Table 6.