Supplementary Appendix to: "Tails, Fears and Risk Premia"

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A. Estimation under $\mathbb{P}$

I. Scaling of Returns

Our estimation under $\mathbb{P}$ relies on high-frequency intraday returns. To alleviate the concerns about the sensitivity of our results to the use 5-minute versus coarser daily data in our identification of the jump tails, we redid much of the estimation based solely on daily returns. In summarizing these additional results we begin by showing that our current separation of volatility from jumps based on the high-frequency data is indeed broadly consistent with the decomposition obtained with daily frequency data, and that the corresponding intraday jump-adjusted returns correctly scales to the daily level.[1]

The top panel in Figure A.1 shows a Gaussian QQ-plot for the daily returns, as formally defined by the sum of the corresponding intraday returns. The daily returns are clearly far from normally distributed. This, of course, is well known and directly attributable to: (i) the presence of jumps, and (ii) time-varying stochastic volatility.

Before illustrating how the use of high-frequency data allows us to forcefully separate these two effects in a manner that gives rise to empirically sound daily return distributions, the second panel in Figure A.1 shows the quantiles of the daily returns standardized by the Realized Variance over the day, or $\sqrt{RV_t}$, against the normal quantiles. Comparing the first two panels, the distribution of the RV-standardized daily returns is obviously much closer to the normal distribution than is the distribution of the raw returns. However, there is still some notable deviations from normality visible in the tails. There are essentially two reasons for this, both of which are related to the presence of jumps in the returns:[2]

- When jumps are present, $RV_{t+1} \approx \int_t^{t+1} V_s ds + martingale$. The martingale term will be zero

[1] By the basic statistical principle that more data is always better (see, e.g., the pertinent discussion of high-frequency data based volatility estimation in Zhang, Mykland, and Aït-Sahalia, 2005), the high-frequency data, if used correctly, will *always* result in more accurate estimates. As such, it wouldn't be reasonable to expect the estimates obtained across different frequencies to perfectly coincide. In particular, as discussed in more detail below, the jump identification based on the coarser daily data will invariable be clouded somewhat by the intraday continuous price variation.

[2] Both of these reasons concern how we account for the time-varying volatility. Of course, even if we have correctly accounted for it, the standardized returns should not be normally distributed but rather fat-tailed. The main point of the discussion here is that in presence of jumps, how to correctly account for stochastic volatility is far from obvious, and in particular Realized Variance cannot be the right scaling factor for the returns when "cleaning" from stochastic volatility.

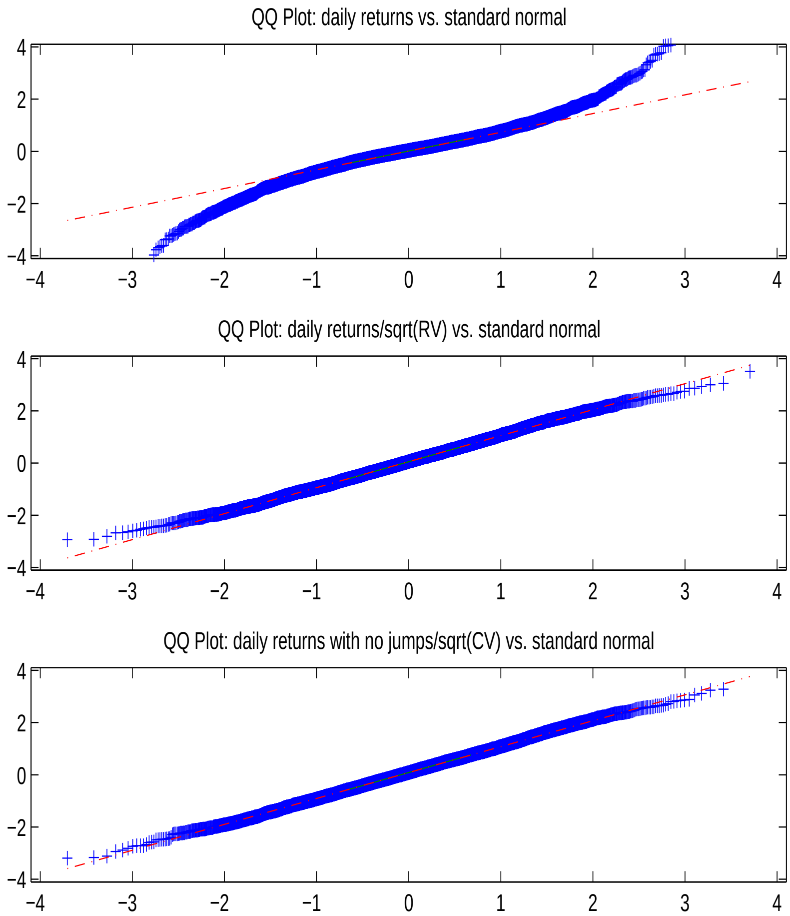

Figure A.1: *QQ-plots of daily returns against the normal distribution. The empirical quantiles for the returns are on the y-axis, while the quantiles for the standard normal are on the x-axis. The top panel gives the plot for the raw daily returns; the middle panel corresponds to the daily returns standardized by the square-root of the daily realized variation; the bottom panel gives the daily returns with the intrdaily jumps removed and divided by the square-root of the daily quadratic variation of the diffusive price component.*

when there are no jumps, in which case RV_{t+1} correctly measures the integrated volatility over the day, $\int_t^{t+1} V_s ds$, rendering the RV-standardized returns Gaussian. However, for days with jumps the additional martingale term implies that RV_{t+1} is not the correct scaling factor.

- When jumps are present, even abstracting from the *martingale* component, the scaling by $\sqrt{\int_t^{t+1} V_s ds}$ is generally not correct. In particular, consider the simple example where $X_t = L_{T_t}$, and L denotes β-stable process ($\beta = 2$ corresponds to a Brownian motion) and $T_t = \int_t^{t+1} V_s ds$. Then, by the self-similarity property of stable processes, $X_t \stackrel{d}{=} \left(\int_0^t V_s ds\right)^{1/\beta} X_1$, so that dividing by $\sqrt{\int_0^t V_s ds}$ will not correctly purge the time-varying volatility unless, of course, $\beta = 2$ which is equivalent to no jumps.

As these observations make clear, the presence of jumps implies much more complicating scaling properties of the returns, and importantly that the scaling of the daily returns by the square-root of the integrated variance over the day generally does not provide the correct adjustment for time-varying volatility. Hence, it is not surprising that the QQ-plot for the standardized daily returns in the middle panel still deviates from the normal 45-degree line.

In an effort to show that our high-frequency based analysis appropriately scales to the lower frequency daily level, we therefore removed all of the high-frequency returns identified as jumps and subsequently aggregate those returns to a daily level. We then divided these daily jump-adjusted returns by the Continuous Variation over the day, or $\sqrt{CV_t}$, as measured by the square-root of the Truncated Variation described in the paper. If our high-frequency based procedure correctly identifies the jump component, these daily adjusted standardized returns should be approximately normally distributed.[3] The third panel in Figure A.1 confirms that this is indeed what happens in the data.

The visual impressions from the figure are further underscored by the p-values of more formal Jarque-Bera test for normality for each of the three series:

- Raw daily returns: 0.0010.
- Daily returns standardized by the square-root of RV: 0.0014.
- Daily returns with intraday jumps removed and standardized by the square-root of CV: 0.1442.

[3]The result will be only approximate because of the possible "leverage" effect coming from correlated Brownian motions in price and volatility. The deviation from normality caused by this leverage effect is usually quite small over short intervals like days, see e.g., Andersen et al. (2007).

Thus, only for the third jump-adjusted return series, is the assumption of normality not rejected at conventional levels of significance.

Taken as a whole, the results discussed above therefore indirectly confirm that our high-frequency based procedure does indeed identify the "right" jumps, and scales to daily return distributions that are in line with those implied by the underlying theoretical assumptions and corresponding distributional implications.

II. Jump Tail Estimation

As a further robustness check of our findings with respect to the intraday based extraction of the $\mathbb{P}$ jump tails, we also redid the estimation of the actual tail parameters underlying our analysis using only daily data. In order to do so, we first classified as jumps all of the daily returns (including the overnight period) that were larger in absolute value than three standard deviations, with the current standard deviation, or volatility level, measured by the continuous variation over the previous day using the Truncated Variation measure defined in the paper. This corresponds directly to our extraction of the jumps at the 5-minute frequency. For the returns that were classified as jumps, we then estimated the tail index using the same estimation procedure that we rely on for the high-frequency based results currently reported in the paper.

Restricting the analysis to coarser daily frequency, as opposed to intraday, data invariably handicaps the detection of jumps and the corresponding jump tail estimation. Intuitively, all of the small jumps essentially becomes "invisible."[4] Importantly, of course, our estimation does not rely on the very small jumps. Indeed, one of the primary motivations behind the new non-parametric estimation scheme developed in the paper is exactly the desire to avoid the tight restrictions across jumps of different sizes inherent to more conventional fully parametric approaches. Still, restricting the analysis to only daily data, the continuous within day price variation will obviously mask, even some of the more moderate sized jumps. As a result, we would naturally expect to have to resort to higher truncation levels in the tail estimation to effectively mitigate the impact of the continuous price

[4]Some of the small jumps may be associated with macroeconomic and other readily identifiable news announcements with seemingly relatively short lived effects (see, e.g., the discussion and additional references in Andersen, Bollerslev, and Diebold, 2007). While it is true that many of these small jumps may not be obviously "visible" at the daily level, they are still an important part of the intraday return distribution that defines the daily returns, and part of the actual return distribution faced by investors.

variation.[5] Along these same lines, the more important is the continuous component at the daily level in a relative sense, the more it will bias the estimated tail decay toward looking steeper than it actually is.

These general ideas are all confirmed by Figure A.2, which plots the empirical right and left jump tail estimates based on daily and high-frequency intraday returns. In order to facilitate the interpretation, all of the plots are reported on a log-log scale, so if the extreme-value theory approximation works well, we would expect to see straight lines. While the straight lines are clearly visible for the high-frequency based estimation, even for the more moderate-sized jumps, as expected we need to resort to much higher truncation levels to recover the straight lines when we only use the daily data. Importantly, however, the slopes of the lines are actually very similar across the two sampling frequencies for the largest jump sizes.

The robustness of our high-frequency based findings are further corroborated by the specific tail estimation results based on the daily data reported in Table A.1. As seen from the table, when accounting for the much-higher standard errors, none of the tail parameters are significantly different from the ones based on the high-frequency data currently reported in the paper.[6]

Table A.1: Estimates for $\mathbb{P}$ Tail Parameters using Daily Data

Left Tail			Right Tail		
Parameter	Estimate	St.error	Parameter	Estimate	St.error
$\xi_{\mathbb{P}}^{-}$	0.1871	0.1600	$\xi_{\mathbb{P}}^{+}$	0.0237	0.6835
$100\sigma_{\mathbb{P}}^{-}$	0.8165	0.1807	$100\sigma_{\mathbb{P}}^{+}$	1.0048	0.1881

The truncation levels are fixed at $tr^{-} = e^{-0.03} \approx 0.97$ and $tr^{+} = e^{0.03} - 1 \approx 0.03$. The estimates are based on daily futures prices from January 1990 through June 2007.

As the preceding analysis makes clear, even though the daily estimates are generally subject to greater statistical estimation errors than the high-frequency estimates in the paper, the magnitudes

[5] This follows from the relative speed requirement between the truncation level and the sampling frequency of Theorem 3 in Bollerslev and Todorov (2010). The estimation here is based on the theoretical result of this theorem.

[6] Note, the estimates for $\sigma_{\mathbb{P}}^{\pm}$ are not directly comparable across the two frequencies, as the daily estimation results reported here are based on much higher truncation levels in order to reliably make use of the extreme-value based tail approximations.

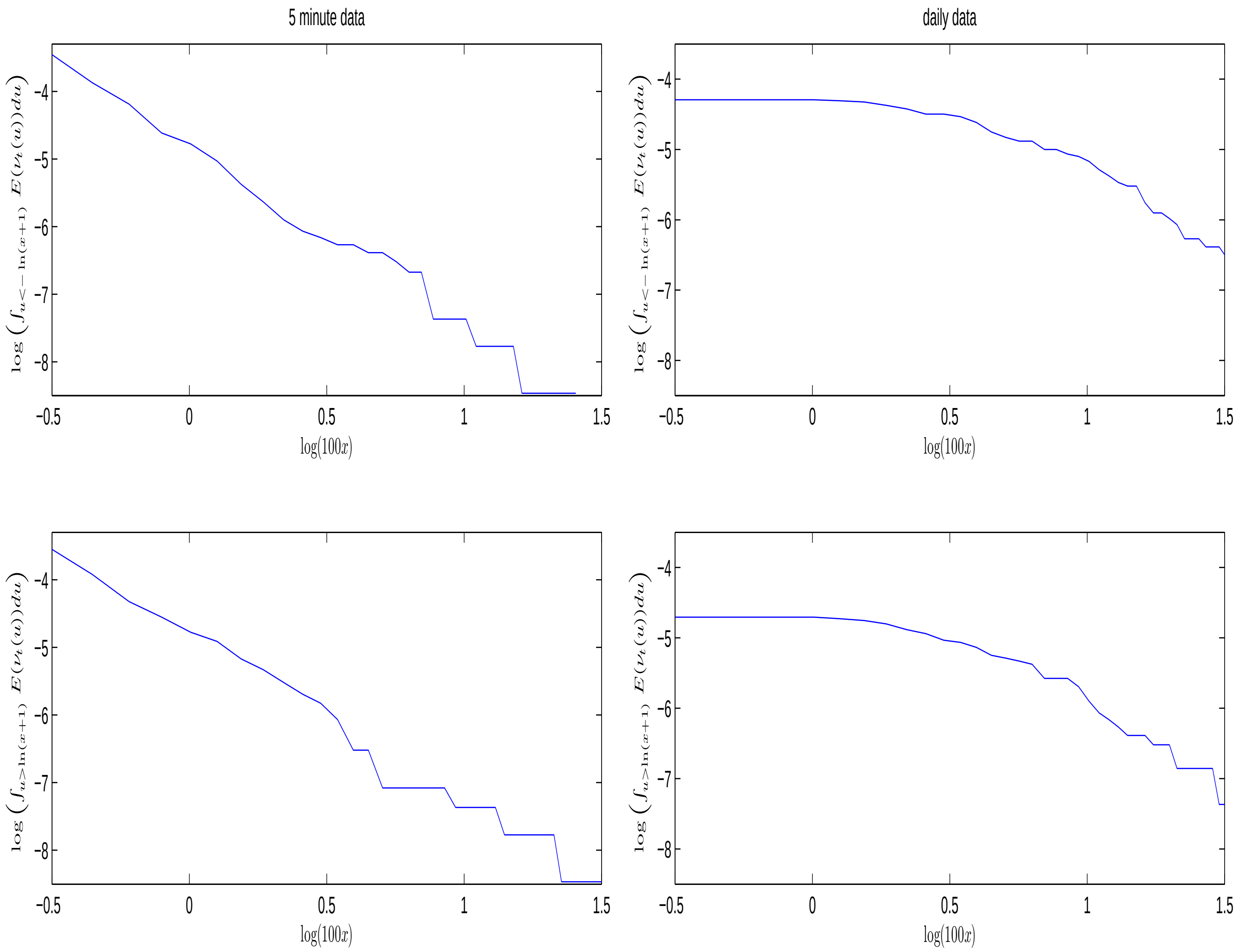

Figure A.2: *Empirical jump tails based on daily and 5-minute data. All of the estimates are reported on a log-log scale.*

of the jump tail frequencies implied from the daily date are in fact quite similar to those obtained from the high-frequency data. To more directly illustrate this point, we report in Table A.2 the implied jump intensities for jumps of different sizes inferred from the daily and intradaily data. As expected the estimated jump intensities based on the daily data are typically smaller than the ones based on the 5-minute data due to the blurring arising from the within day continuous variation. Nonetheless, all of the estimates are well within standard confidence bands of one another.

Table A.2: Estimated Mean Jump Intensities under $\mathbb{P}$

Jump Size	Daily Data	5-Minute Data
$> 7.5\%$	0.0044 (0.0253)	0.0098 (0.0142)
$> 10\%$	0.0005 (0.0064)	0.0050 (0.0083)
$> 20\%$	0.0000 (0.0001)	0.0010 (0.0022)
$< -7.5\%$	0.0010 (0.0024)	0.0036 (0.0048)
$< -10\%$	0.0004 (0.0012)	0.0017 (0.0026)
$< -20\%$	0.0000 (0.0001)	0.0002 (0.0005)

The jump intensities are reported in annualized units. The jump sizes are in percentage changes in the price level. Standard errors for the estimates are reported in parentheses.

B. Estimation under $\mathbb{Q}$

I. Monte Carlo Results

We begin our discussion of the additional robustness results pertaining to the $\mathbb{Q}$ tail estimation, by providing a more detailed account of the Monte Carlo evidence for the four different models discussed in the revised version of paper. In addition to the results summarized therein, we here report the results across a wider range of times-to-maturity and moneyness. The exact estimates for the $LT_t^{\mathbb{Q}}(k)$ and $RT_t^{\mathbb{Q}}(k)$ measures for the Merton model with time-invariant tails are reported in Tables B.1-B.2. The results for the three other models with time-varying jump tails are reported in Tables B.3-B.8, in the form of the median tail estimates, along with the corresponding Median Absolute Deviation (MAD), across a total of 5,000 simulated days. In addition, Figure B.1 provides a scatter plot of the standardized true Q-tails against the estimated Q-tails for the same sample of $5,000$ simulated days. The Monte Carlo simulations show:

- If the Q-jump distribution is skewed to the left, the left jump tail will generally be estimated very accurately, while there will be somewhat larger upward biases in the estimation of the right jump tail. Still, the biases for the right tail are relatively small in an absolute sense. This conclusion holds across all of the four different models, regardless of whether the jumps are of the Merton or tempered stable type.

- Consistent with the intuition discussed in the previous section, the tail estimates are systematically upward biased, as the diffusive price variation invariably clouds the identification of the tails. This holds true for both the right and the left tails.

- Our Investor Fears index depicted in Figure 5 in the paper, essentially ends up differencing the left and right jump tails. Thus, some of the upward bias in each of the two tails will naturally cancel out in that calculation. However, since, the right tail is generally less precisely estimated and also slightly more biased then the left tail, if anything, this will tend to induce a small downward bias in the estimated Investor Fears index.

- As seen from Figure B.1, the dynamic dependencies in the Q-jump tails can be inferred very accurately from our estimates of the $LT_t(k)$ and $RT_t(k)$ measures. Importantly, this implies that our key assumption that the temporal variation in the Q-jump tails is affine in the diffusive variation is both readily identifiable and possibly refuteable in the data.

- Not surprisingly, the quality of the tail estimates deteriorate the closer to the money and the longer lived are the options used in the estimation. However, the biases for the typical options that we rely on in the actual estimation are relatively small.

The additional estimation results reported in the next section, based on a monthly subsample of shorter lived options, speak further to this last point above.

Table B.1: Model A Left Tail Estimation

Time to Maturity in Days	Moneyness					
	$k = \ln(0.9250)$		$k = \ln(0.9125)$		$k = \ln(0.9000)$	
	True	Estimate	True	Estimate	True	Estimate
5	0.0672	0.0672	0.0573	0.0575	0.0484	0.0488
10	0.0672	0.0675	0.0573	0.0578	0.0484	0.0492
15	0.0672	0.0684	0.0573	0.0583	0.0484	0.0496
20	0.0672	0.0695	0.0573	0.0589	0.0484	0.0501

The table reports the estimates from out-of-the-money options for $LT_t(k)$ for Model A.

Table B.2: Model A Right Tail Estimation

Time to Maturity in Days	Moneyness					
	$k = \ln(1.0750)$		$k = \ln(1.0875)$		$k = \ln(1.1000)$	
	True	Estimate	True	Estimate	True	Estimate
5	0.0277	0.0289	0.0236	0.0247	0.0201	0.0211
10	0.0277	0.0314	0.0236	0.0262	0.0201	0.0222
15	0.0277	0.0358	0.0236	0.0287	0.0201	0.0238
20	0.0277	0.0408	0.0236	0.0320	0.0201	0.0260

The table reports the estimates from out-of-the-money options for $RT_t(k)$ for Model A.

Table B.3: Model B Left Tail Estimation

Time to Maturity in Days	Moneyness								
	$k = \ln(0.9250)$			$k = \ln(0.9125)$			$k = \ln(0.9000)$		
	True	Median	MAD	True	Median	MAD	True	Median	MAD
5	0.0572	0.0584	0.0087	0.0488	0.0493	0.0071	0.0412	0.0410	0.0069
10	0.0576	0.0577	0.0095	0.0491	0.0502	0.0078	0.0415	0.0409	0.0057
15	0.0580	0.0593	0.0070	0.0494	0.0492	0.0074	0.0418	0.0437	0.0072
20	0.0583	0.0593	0.0082	0.0498	0.0507	0.0069	0.0421	0.0426	0.0063

The table reports summary statistics for the estimation of $LT_t(k)$ based on out-of-the-money options computed with simulated data from Model B. The length of the simulated series is $5,000$ days. The true value is computed from the parametric specification of the jump tail under $\mathbb{Q}$ and the realization of the volatility process. MAD denotes the Median Absolute Deviation.

Table B.4: Model B Right Tail Estimation

Time to Maturity in Days	Moneyness								
	$k = \ln(1.0750)$			$k = \ln(1.0875)$			$k = \ln(1.1000)$		
	True	Median	MAD	True	Median	MAD	True	Median	MAD
5	0.0236	0.0250	0.0038	0.0201	0.0211	0.0031	0.0171	0.0176	0.0031
10	0.0237	0.0261	0.0050	0.0202	0.0224	0.0038	0.0172	0.0182	0.0027
15	0.0239	0.0297	0.0046	0.0204	0.0233	0.0042	0.0173	0.0205	0.0039
20	0.0240	0.0328	0.0062	0.0205	0.0263	0.0047	0.0174	0.0212	0.0039

The table reports the same summary statistics as in Table B.3 pertaining to the estimation of $RT_t(k)$.

Table B.5: Model C Left Tail Estimation

Time to Maturity in Days	Moneyness								
	$k = \ln(0.9250)$			$k = \ln(0.9125)$			$k = \ln(0.9000)$		
	True	Median	MAD	True	Median	MAD	True	Median	MAD
5	0.0488	0.0482	0.0071	0.0442	0.0448	0.0068	0.0401	0.0415	0.0068
10	0.0491	0.0530	0.0078	0.0445	0.0472	0.0066	0.0404	0.0407	0.0062
15	0.0494	0.0491	0.0076	0.0448	0.0485	0.0074	0.0407	0.0406	0.0056
20	0.0498	0.0572	0.0094	0.0451	0.0497	0.0067	0.0409	0.0434	0.0057

The table reports summary statistics for the estimation of $LT_t(k)$ based on out-of-the-money options computed with simulated data from Model C. The length of the simulated series is $5,000$ days. The true value is computed from the parametric specification of the jump tail under $\mathbb{Q}$ and the realization of the volatility process. MAD denotes the Median Absolute Deviation.

Table B.6: Model C Right Tail Estimation

Time to Maturity in Days	Moneyness								
	$k = \ln(1.0750)$			$k = \ln(1.0875)$			$k = \ln(1.1000)$		
	True	Median	MAD	True	Median	MAD	True	Median	MAD
5	0.0145	0.0149	0.0023	0.0126	0.0133	0.0021	0.0110	0.0118	0.0020
10	0.0146	0.0181	0.0033	0.0127	0.0148	0.0024	0.0111	0.0121	0.0020
15	0.0147	0.0187	0.0040	0.0128	0.0169	0.0034	0.0112	0.0128	0.0021
20	0.0148	0.0267	0.0065	0.0128	0.0197	0.0039	0.0112	0.0152	0.0027

The table reports the same summary statistics as in Table B.5 pertaining to the estimation of $RT_t(k)$.

Table B.7: Model D Left Tail Estimation

Time to Maturity in Days	Moneyness								
	$k = \ln(0.9250)$			$k = \ln(0.9125)$			$k = \ln(0.9000)$		
	True	Median	MAD	True	Median	MAD	True	Median	MAD
5	0.0488	0.0501	0.0071	0.0427	0.0425	0.0061	0.0375	0.0381	0.0074
10	0.0491	0.0517	0.0082	0.0430	0.0445	0.0062	0.0378	0.0405	0.0062
15	0.0494	0.0508	0.0065	0.0433	0.0452	0.0062	0.0380	0.0408	0.0070
20	0.0498	0.0576	0.0082	0.0435	0.0467	0.0066	0.0383	0.0422	0.0056

The table reports summary statistics for the estimation of $LT_t(k)$ based on out-of-the-money options computed with simulated data from Model D. The length of the simulated series is $5,000$ days. The true value is computed from the parametric specification of the jump tail under $\mathbb{Q}$ and the realization of the volatility process. MAD denotes the Median Absolute Deviation.

Table B.8: Model D Right Tail Estimation

Time to Maturity in Days	Moneyness								
	$k = \ln(1.0750)$			$k = \ln(1.0875)$			$k = \ln(1.1000)$		
	True	Median	MAD	True	Median	MAD	True	Median	MAD
5	0.0145	0.0162	0.0025	0.0119	0.0128	0.0020	0.0099	0.0108	0.0022
10	0.0146	0.0190	0.0039	0.0120	0.0147	0.0025	0.0100	0.0125	0.0023
15	0.0147	0.0217	0.0040	0.0121	0.0170	0.0032	0.0101	0.0139	0.0032
20	0.0148	0.0298	0.0062	0.0122	0.0203	0.0042	0.0101	0.0165	0.0032

The table reports the same summary statistics as in Table B.7 pertaining to the estimation of $RT_t(k)$.

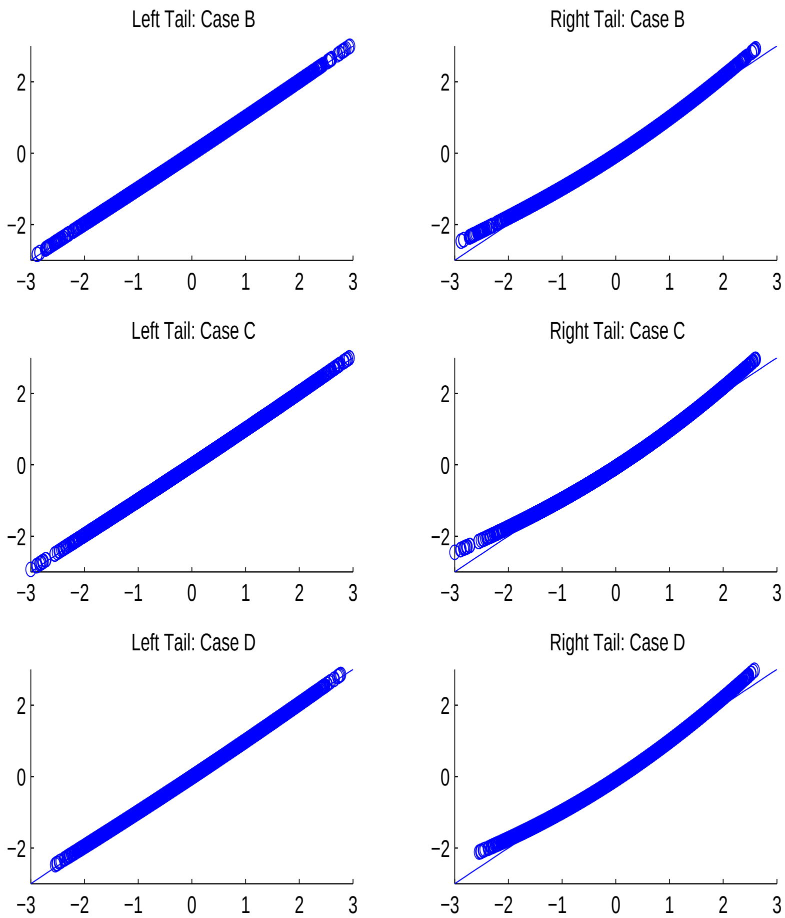

Figure B.1: *Scatter plots of true versus estimated Q-jump tails. The x- and y-axis give the standardized true and estimated jump tails, respectively. The estimates are based on options that are* 8.75% *out-of-the-money with* 15 *business days to maturity.*

II. Subsample Analysis

To alleviate the concerns about the sensitivity of our key findings with respect to the maturity time of the options used in our jump tail estimation, we re-calculated the Investor Fears index with only monthly data.[7] Specifically, by restricting our sample to the 5-th business day of each month, the median time-to-maturity of the options is reduced to 7 business days, compared to the median of 14 business days for the daily sample underlying the estimation results currently reported in the paper.

Comparing the resulting monthly time series estimates for the Investor Fears index in Figure B.2 to the daily Investor Fears index in Figure 5 in the paper (included here for convenience), the two time series are obviously very close, exhibiting the same basic dynamic features and readily identifiable

[7] There are a total of 156 month in the sample.

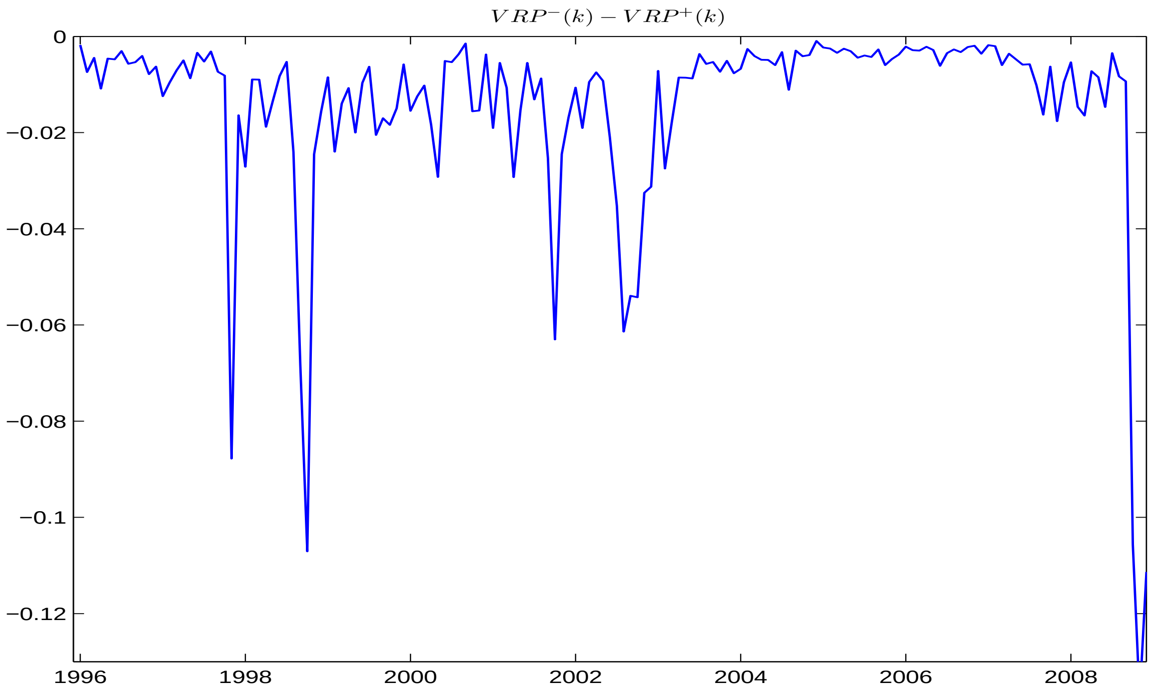

Figure B.2: *Investor Fears with monthly data and shorter maturity options.*

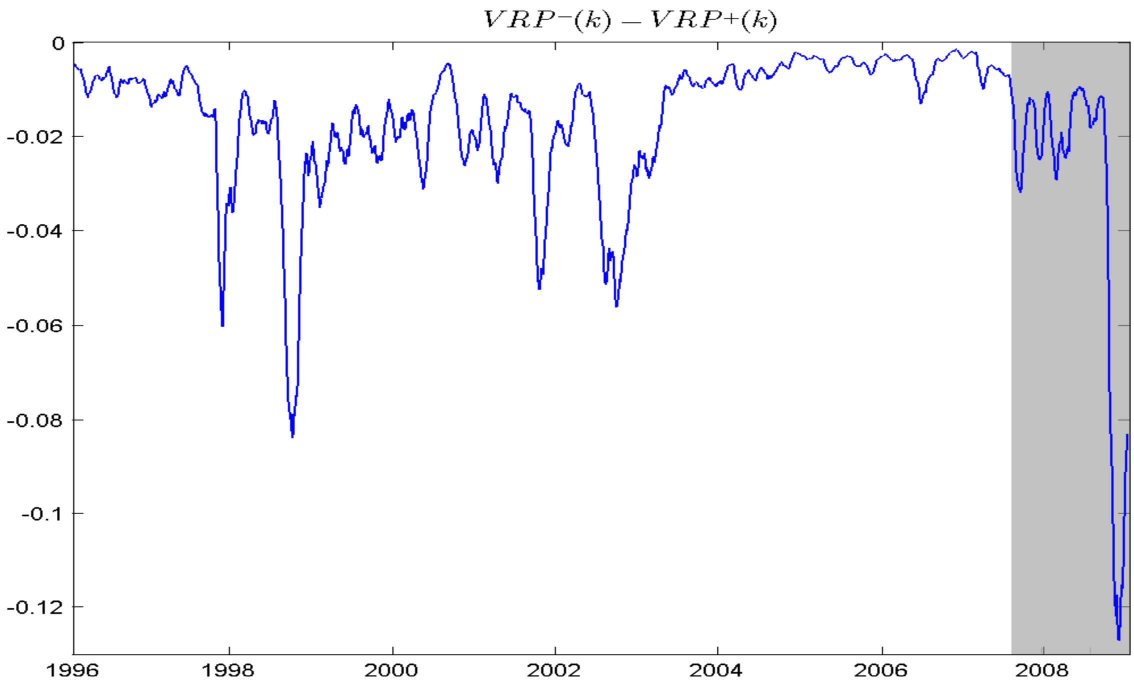

Figure 5: *Investor Fears.*

peaks and throughs. As such, this again confirms the robustness of the key findings reported in the paper with respect to the underlying approximating assumptions and data used in the estimation.

III. Sensitivity to levels of moneyness used in estimation

As a final robustness check for our $\mathbb{Q}$ tail estimation, we check the sensitivity of our result with respect to the levels of moneyness of the options used in the estimation. In the paper we use moneyness of 0.9250, 0.9125 and 0.9000 for the left tail and 1.0750, 1.0875 and 1.100 for the right tail. Here we redo the estimation but using moneyness of 0.9350, 0.9225 and 0.9100 for the left tail and 1.0650, 1.0775 and 1.090 for the right tail, i.e., we shift the moneyness of the options used in the estimation with 1% closer to 1.0000.

The results for the $\mathbb{Q}$ tail parameters for the new set of moneyness are reported in Table B.9 below. Comparing with our original estimates reported in Table 7 in the paper, we see that there are no statistically significant changes. In particular, the left tail continues to be significantly fatter than the right tail.

Table B.9: Estimates for $\mathbb{Q}$ Tail Parameters

The estimates are based on options data from January 1996 through June 2007 and levels of moneyness of 0.9350, 0.9225 and 0.9100 for the left tail and 1.0650, 1.0775 and 1.090 for the right tail.

Left Tail			Right Tail		
Parameter	Estimate	St.error	Parameter	Estimate	St.error
$\xi_{\mathbb{Q}}^{-}$	0.3095	0.0195	$\xi_{\mathbb{Q}}^{+}$	0.1118	0.0183
$\sigma_{\mathbb{Q}}^{-}$	0.0430	0.0013	$\sigma_{\mathbb{Q}}^{+}$	0.0218	0.0010

We next compare the implications of the new estimation in this Section, based on the closer to moneyness options, with the original one for the tail components of the equity and variance risk premia. First, in Table B.10 we compare the risk-neutral mean jump intensities for various jump sizes. As we see from the table, the results are virtually the same when the two different sets of moneyness are used in the estimation.

Next, in Table B.11 we compare the estimates for the jump tail components of the risk-neutral variance (which are reported for the original estimation in Table 2 of the paper). As we see from the table, the results for the estimation with the two different sets of moneyness are the same in a statistical sense.

Table B.10: Estimated Mean Jump Intensities under $\mathbb{Q}$

Jump Size	Closer to moneyness options	Deeper out-of-the-money options
$> 7.5\%$	0.5695 (0.0415)	0.5551 (0.0443)
$> 10\%$	0.2034 (0.0205)	0.2026 (0.0206)
$> 20\%$	0.0081 (0.0018)	0.0069 (0.0014)
$< -7.5\%$	1.0101 (0.0463)	0.9888 (0.0525)
$< -10\%$	0.5612 (0.0300)	0.5640 (0.0346)
$< -20\%$	0.0888 (0.0077)	0.0862 (0.0084)

The jump intensities are reported in annualized units. The jump sizes are in percentage changes in the price level. Standard errors for the estimates are reported in parentheses. The estimates are based on options data from January 1996 through June 2007. The estimation results in the second column are based on options with levels of moneyness of 0.9350, 0.9225 and 0.9100 for the left tail and 1.0650, 1.0775 and 1.090 for the right tail. The estimation results in the third column are based on options with levels of moneyness of 0.9250, 0.9125 and 0.9000 for the left tail and 1.0750, 1.0875 and 1.100 for the right tail.

Table B.11: Risk-neutral Variance decomposition

	Closer to moneyness options	Deeper out-of-the-money options
$\mathbb{E}\left(\frac{1}{T-t}\int_t^T \int_{x>\ln 1.1} x^2 \mathbb{E}_t^{\mathbb{Q}} \nu_s^{\mathbb{Q}}(dx)ds\right)$	0.0033 (0.0004)	0.0032 (0.0004)
$\mathbb{E}\left(\frac{1}{T-t}\int_t^T \int_{x<\ln 0.9} x^2 \mathbb{E}_t^{\mathbb{Q}} \nu_s^{\mathbb{Q}}(dx)ds\right)$	0.0188 (0.0014)	0.0180 (0.0015)

The average sample estimates are reported in annualized form, with standard errors in parentheses. The estimates are based on options data from January 1996 through June 2007. The estimation results in the second column are based on options with levels of moneyness of 0.9350, 0.9225 and 0.9100 for the left tail and 1.0650, 1.0775 and 1.090 for the right tail. The estimation results in the third column are based on options with levels of moneyness of 0.9250, 0.9125 and 0.9000 for the left tail and 1.0750, 1.0875 and 1.100 for the right tail.

Finally, on Figure B.3 we plot the Investor Fear index based on the $\mathbb{Q}$ tail estimation using the closer to moneyness options. We can compare directly with Figure 5 of the paper, which was reproduced in the previous subsection. As seen from the figure, both the scale and the time-variation of the Investor Fear index do not change compared with our original estimation.

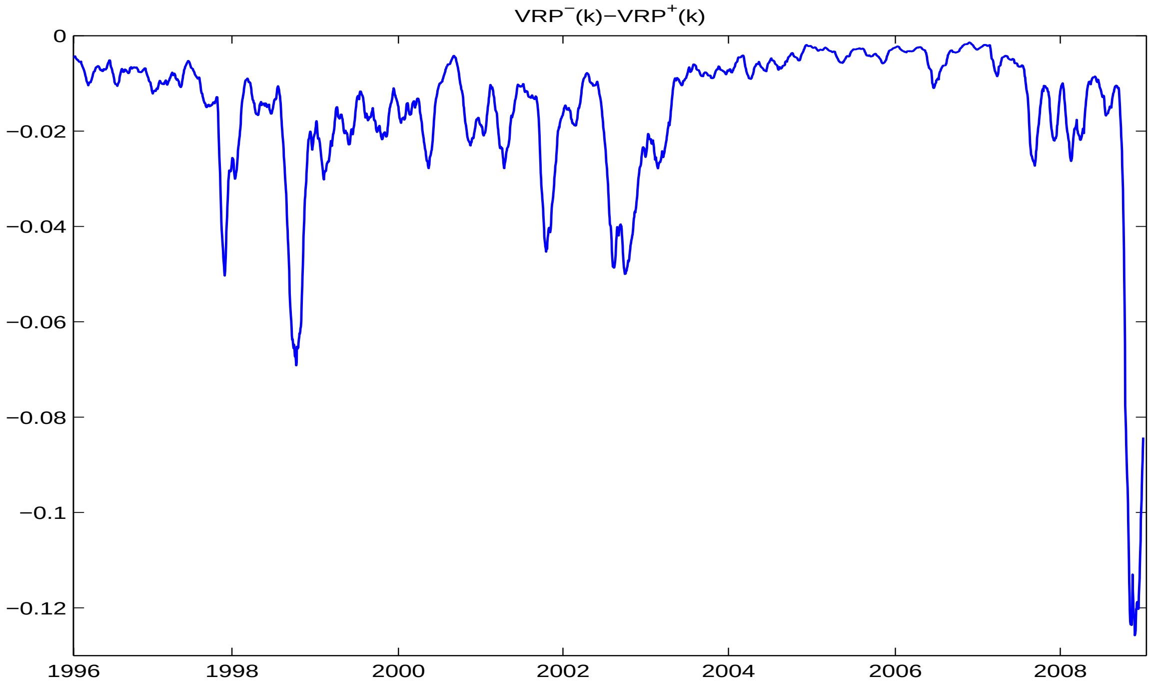

Figure B.3: *Investor Fears based on estimation using options with levels of moneyness of* 0.9350*,* 0.9225 *and* 0.9100 *for the left tail and* 1.0650*,* 1.0775 *and* 1.090 *for the right tail.*

Overall, we can conclude that the key findings in the paper are robust to the choice of the level of moneyness used for the inference of the $\mathbb{Q}$ tail parameters.

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