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Teaching Note: Valuation Using the Adjusted Present Value (APV) Method vs. Adjusted Discount Rate (ADR) Method (Theory)

0. Introduction

In Chapter 18 of Brealey and Myers, we learned that the value of an investment (e.g., a capital budgeting project, a division of a company or an entire company) is not independent of the investment's capital structure. In the context of a *trade-off theory of the capital structure,* using debt in the financial structure of the investment offers tax shields via corporate-tax deductible interest payments rather than fully corporate-taxable returns to equityholders. On the other hand, having more debt in the capital structure increases the likelihood that the firm will have trouble meeting its interest and principal obligations; this increases the present value of future expected leverage related costs.[1] How do we calculate the value of the investment when financial structure matters?

There are two general approaches to valuation with side-benefits of financing: the Adjusted Present Value (APV) method and the Adjusted Discount Rate (ADR) method. The APV method says to value the asset first *as if it were all equity financed*, then add in the tax shields associated with debt financing (or in the financial structure in general) and subtract out the present value of

[1] The generic moniker *leverage related costs* refers to the direct and indirect costs of bankruptcy, as well as potential operating difficulties a company endures when their customers, suppliers, competitors, managers and employees believe that financial distress has a given probability of occurring.

anticipated future leverage related costs associated the financial structure of the asset. Modigliani-Miller's formula for the value of the firm, adjusted for corporate and personal taxes, and including a term, PV(E(LRC's), for leverage related costs is an example of the APV approach to valuation:[2]

$$V_L = V_U + T^* D - PV(E(LRC's)) \tag{1}$$

where V_L refers to the value of an asset (or a firm) with debt in its financial structure, V_U refers to the value of the same asset if it had no debt in its financial structure, D is the market value of debt in the asset's financial structure and T*D is the present value of all future tax shields associated with the debt in the asset's capital structure, assuming that the asset has D dollars of market value of debt in the capital structure *in perpetuity*.

[2]For a discussion of the impact of personal taxes on the leverage decision, see Brealey and Myers, Chapter 18; for an application of the integrated (corporate and personal) tax model to the CAPM and the cost of equity capital see my *Financial Leverage and the Cost of Equity Capital: Teaching Note.*

Lastly, T* is the "gains from leverage" term which incorporates the tax advantage of debt at the corporate level and the relative tax disadvantage of debt at the personal level.[3] In the formulation of equation (1), V_U is calculated as the present value of all the future Free Cash Flows from Operations of the asset, discounted at the required rate of return on the equity of the asset *if it had no debt in the financial structure*, r_{eU}.

An alternative approach is to incorporate the financing-side benefits (tax shields) and costs (LRC's) into the discount rate rather than adding them on separately; the discount rate is generically referred to as an adjusted discount rate (ADR). The application of the ADR approach is to calculate V_L directly by calculating the present value of all the future Free Cash Flows from Operations of the asset discounted at the ADR. Notice that the cash flows discounted are the same *unlevered* cash

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[3] Again, the reader is referred to *Financial Leverage and the Cost of Equity Capital: Teaching Note*, for a derivation of T*. To see that T*D is the present value of the net tax shields offered by perpetual debt with market value of D, assume that the debt instrument pays annual interest payments of Int in perpetuity. The required rate of return on the debt, r_d, may be less than or greater than the stated rate (or coupon rate) on the debt, but the market value of the debt, D is equal to Int/r_d. If the firm were to substitute a marginal amount of debt for equity in the financial structure such that the company would pay out $1 more in interest each year rather than to equityholders, the dollar of interest would not be taxed at the corporate level (interest is tax deductible) but would be taxed at the personal level at a tax rate of τ_{pd}, ending up as $\$1(1-\tau_{pd})$ after corporate and personal taxes; had it remained a dollar paid to equity holders, it would be taxed at the corporate level, at τ_c, and again at the personal level, at τ_{pe}, ending up as $\$1(1-\tau_c)(1-\tau_{pe})$. For every dollar of interest paid per year, the net tax shield is given by:

$$(1 - \tau_{pd}) - (1 - \tau_c)(1 - \tau_{pe})$$

If we multiply the above amount by the annual interest and calculate the present value of all future tax shields, PVTS, as an perpetuity, we get:

$$PVTS = \frac{[(1 - \tau_{pd}) - (1 - \tau_c)(1 - \tau_{pe})]Int}{(1 - \tau_{pd})\ r_d}$$

Notice that the annual tax shield has the risk characteristics of the debt and is an *after-personal tax cash flow*; as such the discount rate used to discount the annual tax shield is the *after-personal tax required rate of return on debt*. The latter formula can be rewritten as T*D, where T* is given by:

$$T^* = \left[1 - \frac{(1 - \tau_c)(1 - \tau_{pe})}{(1 - \tau_{pd})}\right]$$

recognizing that $D = Int/r_d$.

flows used to calculate V_U in the APV method, whereas they are calculating V_L in the ADR method --- the value of the tax shields and LRC's associated with financing decisions are implicitly being included in the adjusted discount rate.

This teaching note outlines the basic theoretical issues in valuation using the APV and ADR analyses; its companion teaching note, *Teaching Note: Valuation Using the APV method and the ADR Method (Examples)*[4] uses the ideas developed in this note and applies them to project valuation. In section 1 of this note, we outline the assumptions that are made in the the APV approach and the ADR approach to valuation; in section 2, we introduce the most common ADR used for valuation purposes, the weighted average cost of capital; in section 3, we point out how the choice between APV and WACC is complicated by the presence of non-perpetuity cash flows; in section 4 we conclude with a discusssion of the pros and cons of using either approach in general and make specific recommendations of which rule to use for some common circumstances.

1. Assumptions underlying the APV and ADR approaches.

Keeping in mind that the ultimate goal of either approach to valuation, APV or ADR, is to incorporate financing side-benefits and costs into the valuation of assets, what are the assumptions that are made by the two approaches? Can these assumptions be made in an internally consistent fashion, so that you always get the same answer with either method? The important assumptions made in either approach relate to 1. the assumed tax regime underlying the model (i.e., are there only corporate taxes or are there corporate and personal taxes) and 2. the time pattern and riskiness of the corporate debt tax shields. Before we discuss these assumptions, we must address the very sticky

[4]Not yet written. See Inselbag and Kaufold, op. cit., for examples.

issue of leverage related costs.

Leverage related costs: A digression

Leverage related costs clearly affect investment and financing decisions of firms. Strip financing used to finance early LBO's in the late 1970's and early 1980's was touted as a vehicle which reduces the incentive of LBO participants to fight with each other over who will be repaid, because in the strip each investor holds some common, some preferred and some of the debt of the highly levered firm. Having less incentive to break up the company can lead to more unanimity among LBO participants; the company can be left to maximize value rather than trying to convince investors with divergent incentives to keep the firm out of bankruptcy. In an equally dramatic example, when Chrysler was in danger of corporate bankruptcy in 1983, before the government bailout, they lost considerable sales of automobiles from customers who were worried that warranties on cars they might purchase would be worthless and/or they may not be able to service the car at all. Similarly, some of the most-valued employees of a firm with high leverage may choose to go to a competitor with less debt. If your company has a lot of debt in its capital structure, your competitors may view their relatively debt-free balance sheet as a competitive advantage, and may cut prices or invest in product improvements to capture market share from your company. If your levered position makes it difficult to respond to competitive threats like these, you may be much worse off having levered up.

That being said, the precise value of leverage related costs is notoriously difficult to estimate. The market value of corporate securities presumably includes a deduction for the present value of future expected leverage related costs, but extracting the estimate from market price of stocks and bonds is a daunting if not quixotic task. Hence, calculating PV(E(LRC's)) will often not be feasible and we will ignore this term. We will keep in mind in our analysis that our numbers will make it look like more debt is preferred to less debt almost all the time and we must temper these "conclusions" with a healthy skepticism of this result, remembering that we have ignored the potentially important impact of leverage related costs.

We might ask the question, shouldn't the expected costs of financial distress be incorporated into market required rates of return on debt and equity? If the *risk* of incurring financial distress is, in part, a systematic risk, we would expect it to show up in expected rates of return on securities (hence, be a part of the cost of capital). Fama and French (1993) attribute their recent findings, that firm size and the book-to-market ratio are more important than beta in predicting average equity returns, to possible missing factors in the CAPM which may incorporate a bankruptcy or financial distress risk factor in pricing equities. Similarly, some LRC's are the monitoring and bonding costs that are incurred by lenders if they lend you money; they charge the expected level of monitoring and bonding costs through to the company in the form of a higher interest rate (or fees) on the loan. Hence, the borrowing rate of the firm includes an estimate of some of the LRC's incurred by the firm. Hence the costs of different sources of capital include premiums paid to capital providers to compensate them for some expected levels of LRC's.

We should point out that this does not solve our problem completely. The equity required rate of return will include a risk premium for systematic, non-diversifiable component of **risk** associated with financial distress. This discount rate is used to discount the set of *cash flows* expected to accrue to the shareholders in the future --- many, if not most, of the expected LRC's are in the cash flows, not the discount rate! Similarly, while the rate of interest charged by lenders includes compensation for monitoring costs in addition to the risk premium charged by the bank, this is only one source of leverage related costs.

Therefore, in theory, while we should be attempting to include both the tax shields of financial structure and the leverage related costs of financial structure into our analysis, we will have to sidestep the LRC issue at times. We must always be aware of the qualitative limitations associated with this sin of omission on our conclusions.

APV Approach

The Modigliani-Miller formulation of the value of the levered firm in equation (1) is the simplest example of APV valuation method. For the remainder of this teaching note we will assume there are no leverage related costs associated with debt financing. In essence, we are assuming perfect capital markets except for corporate and personal taxes. With this adjustment, equation (1) becomes:

$$V_L = V_U + T^* D \qquad \textbf{(2)}$$

What assumptions are underlying equation (2)? It assumes there are both corporate and personal taxes; we will be maintaining the corporate and personal taxes assumption throughout the teaching note. The valuation of the future tax shields of debt indicates that the future debt level will be maintained at D in perpetuity. The derivation of the present value of tax shields (see footnote 3) discounts all future debt tax shields at the after-personal tax r_d, indicating that the assumed riskiness of the future tax shields is equivalent to the riskiness of the debt.

The assumptions that the debt tax shields are a perpetuity and are equally risky as the debt cash flows are not the only assumptions that could be made in the context of the APV method. For instance, we could assume the company has D dollars of borrowing today, but will increase its borrowing amount by g = 4% per year, indefinitely. If the future borrowing is as low risk as the current borrowing, then r_d may still be the appropriate discount rate at which to discount the future tax shields and we will get the following APV:[5]

$$V_L = V_U + \frac{[(1 - \tau_{pd}) - (1 - \tau_c)(1 - \tau_{pe})]Int_1}{(1 - \tau_{pd})\, r_d - g} \qquad \textbf{(3)}$$

where the present value of tax shields is calculated under the assumption that, with the dollar amount of debt growing per year by g%, the annual interest tax shield grows at the same rate.

Similarly, we need not assume that the risk level of our future interest tax shields is the same as the riskiness of our debt. Consider a company which intends to keep a target capital structure of, say, 40% debt to total capital. In the future, if the value of the company increases it should borrow more, generating larger interest tax shields; if the value of the firm decreases, it should reduce its borrowing, reducing its interest tax shields. The interest tax shields in this instance are positively correlated (in fact, they are proportional) with the value of the company and thus should be considered to be equally risky as the company's assets. As such, they should perhaps be discounted at a higher rate, e.g., r_{eU}.[6,7]

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[5]If the company increases its borrowing every year, is it likely that the future tax shields will be as low risk as the current tax shields? It depends on whether the value of the firm is increasing at the same pace or greater. If the value of the firm is also increasing at 4% per year, then the debt to total capital ratio of the firm is unchanged and we might argue that the debt risk is unchanged. This is, however, too simplistic perhaps as the next paragraph explains.

[6]Brealey and Myers refer to this possibility in Chapter 19 under the heading "What happens when future debt levels are uncertain" and give the formula for r* derived in Miles and Ezzell, "The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis*, (September, 1980).

ADR Approach

The thing to remember about adjusted discount rates (ADR's) is that they are attempting to do the same thing as the APV, that is, to include the value of future interest tax shields into the value of the assets. Assumptions concerning tax regime (we will consider the personal and corporate tax model only), the timing and riskiness of the future tax shields are made in any adjusted discount rate approach. Unfortunately, these assumptions are usually not as obvious as they are in the APV approach. In theory, however, if you make the same assumptions in an ADR approach as are made in the comparable APV approach, you should get the same calculated value of the assets. We show this fact next in the context of company that generates a perpetuity of Free Cash Flow from Operations.

Equivalence of APV Approach and ADR Approach: Perpetuity Case and Circularity of Using ADR for Valuation Purposes

The value of the assets of the firm is the present value of the future Free Cash Flow from Operations on the assets of the firm[8]. In our simplified perpetuity case, we will make the assumption that future capital expenditures for growth are equal to zero and that maintenance capital

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The Miles and Ezzel formula given there makes a specific assumption concerning the riskiness of future debt tax shields and makes the assumption that the firm maintains a constant proportion of debt to market value in the capital structure. Issues of risky future debt shields and the Miles Ezzel analysis will be contained in future editions of Appendix II to this note.

[7]Kaplan and Ruback use a valuation method they call *compressed APV* which implicitly assumes that the future tax shields of debt are the same degree of risk as the assets of the firm, using the same logic as Miles-Ezzell, and thus discount both the unlevered cash flows *and the future interest tax shields* by the unlevered cost of equity. The compressed APV method will be taken up in future editions of this note.

[8]See *Project Cash Flows - Teaching Note*.

expenditures and working capital investments (perhaps inflation induced) exactly offset the annual depreciation of the company. In this case, the annual Free Cash Flow from Operations equals Net Operating Profit Less Adjusted Taxes, NOPLAT. For equation purposes, let's assume the pretax Net Operating Profit is given by X and we multiply this by $(1-\tau_c)$ to get NOPLAT. We also assume that the firm will have D dollars in debt in perpetuity and that future tax shields are as risky as the corporation's debt. Using equation (2) to calculate V_L, and remembering that V_U is the present value of the Free Cash Flows from Operation *as if the company were unlevered*, we get:

$$V_L = \left[\frac{X(1 - \tau_c)}{r_{eu}}\right] + T^*D \qquad \textbf{(4)}$$

What we want to show is that an ADR approach that makes the same assumptions as equation (4) will yield the same value of the levered firm. In general, we will follow Brealey and Myers' convention, referring generically to adjusted discount rates as r*. The Modigliani-Miller definition of r* for the perpetual debt tax shields as risky as debt, case is[9]

$$r^* = r_{eu}(1 - T^*(\frac{D}{V_L})) \qquad \textbf{(5)}$$

and the adjusted discount rate valuation of the levered firm is the present value of the Free Cash Flows from Operations discounted at the ADR, r*. The equation for valuing V_L by the ADR approach is

[9]This equation for the adjusted discount rate is in Brealey and Myers, 5th edition, page 535.

$$V_L = \frac{X(1 - \tau_c)}{r^*} \qquad \textbf{(6)}$$

As an aside, equations (5) and (6) indicate that attempting to value the levered firm by the ADR approach involves a **circularity problem**: if our goal is to estimate the value of the firm, V_L, by discounting the cash flows of the firm by r*, but r* is a function of D/V_L, then we need to know V_L in order to estimate V_L, which means that this method is inherently circular. The ADR method is often used in situations where it is assumed that the company will attempt to maintain a particular debt policy over time, and that that policy may differ from the current capital structure. For example, if the company is expected to maintain a 30% debt to total capital ratio over time, D/V_L may be a more accurate measure of the availability of tax shields than in the APV case. It should also be remembered that all of the inputs to the valuation formula (e.g., X, τ_c, etc.) are *forecasts*, and are not always very accurate forecasts either (not to say they are biased, but that there may be substantial variance in the estimates). In practical situations where the company may be expected to follow a consistent capital structure policy, the WACC may be the most accurate estimate in your valuation analysis! In summary, you should be aware that any valuation of assets using the ADR method (including WACC analyses) involves a circularity problem, but that you should not necessarily lose sleep over this fact.

To show that equations (4) and (6) yield the same value for V_L, divide both sides of (4) by V_L to get

$$1 = \left[\frac{X(1 - \tau_c)}{r_{eU} \; V_L}\right] + \left[T^*(\frac{D}{V_L})\right] \qquad \textbf{(7)}$$

Multiply both sides of equation (7) by r_{eU},

$$r_{eU} = \left[\frac{X(1 - \tau_c)}{V_L}\right] + \left[T^* r_{eu}(\frac{D}{V_L})\right] \qquad \textbf{(8)}$$

Subtracting the second term on the right hand side of (8) from both sides and factoring r_{eU} out of each term on the resulting left hand side yields:

$$r_{eU}\left(1 - T^*(\frac{D}{V_L})\right) = \left[\frac{X(1 - \tau_c)}{V_L}\right] \qquad \textbf{(9)}$$

Recognizing that the left hand side of (9) is the MM definition of r*, and isolating V_L on the left hand side yields the following expression for V_L:

$$V_L = \frac{X(1 - \tau_c)}{r^*} \qquad \textbf{(10)}$$

which is the same as equation (6) for the ADR method of calculating V_L.

We have shown that using MM's definition of the adjusted cost of capital will derive the same company value, V_L, as the APV method *if you are careful to make the same assumptions*. This is true in general: **if you are careful to make exactly the same assumptions in the ADR analysis as in an APV analysis, you will get the same value of the firm.**

We must be careful, however: the APV formulas (1) and (2) are more general than the simple perpetuity case that we used to show the equivalence of the two rules. In the general APV analysis, V_U is the present value of all future Free Cash Flows from Operations, but they need not be in the form of a perpetuity. MM's r* formula for ADR *does require perpetuity of Free Cash Flows from Operations*. Taggart (1989) shows that for any set of assumptions concerning the tax regime, the timing and riskiness of debt tax shield cash flows and asset cash flows, there is an ADR which will give the same value of the levered firm as an APV analysis. He also shows that the weighted average cost of capital (WACC), correctly formulated, is equal to the appropriate ADR in each circumstance.[10] A more recent paper Inselbag and Kaufold (1997), shows the same types of results and has several numerical examples demonstrating the equivalence of the two approaches.[11]

In summary, both the APV method and the ADR method require assumptions concerning 1. the tax regime; 2. the timing of future interest tax shields of debt financing; and 3. the riskiness of future tax shields of debt financing. The most common formula for the APV value of levered assets, equations (1) or (2), assumes perpetual debt, the interest tax shields on which are as risky as the debt. We have shown that the APV method and ADR method (using MM's formulas) are equivalent under these assumptions and if future Free Cash Flow from Operations is a perpetuity.

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[10]Taggart, Robert A. Jr., NBER Working Paper #3074.

[11]Inselbag, Isik and Howard Kaufold, *Two DCF Approaches for Valuing Companies under Alternative Financing Strategies (and how to choose between them)*, Journal of Applied Corporate Finance, Spring 1997.

2. Weighted Average Cost of Capital (WACC)

The most common discount rate in practice is the weighted average cost of capital, which is defined as:[12]

$$WACC = r_d(1-\tau_c)\left(\frac{D}{D+E}\right) + r_{eL}\left(\frac{E}{D+E}\right) \qquad \textbf{(11)}$$

where D/D+E is the debt capacity of the asset being valued, expressed as a percentage of the value of the investment, D+E; r_d is the required return on the debt capacity of the investment if it were a stand alone firm; the required return on the assets of the investment, r_{eU}, is relevered to the debt capacity of the asset, r_{eL}; lastly, τ_c is the corporate tax rate, reflecting the corporate tax deductibility of corporate interest payments. In Chapter 17 of Brealey and Myers, this was referred to as r_a, the required rate of return on the assets of the firm and was shown, in the absence of taxes and transactions costs, to be independent of the amount of leverage in the firm. With taxes, the required return on the assets of the firm is *not* independent of leverage, but is explicitly a function of how the firm is financed (through the value of tax shields). Just as tax shields add to the value of the firm in equations (1) and (2), tax shields associated with debt financing *reduce the cost of capital (WACC) to the firm*.

Investigating the terms in the WACC, we see that it is a capital structure-weighted average of the cost of debt to the firm (acknowledging the fact that the interest is tax-deductible) and the cost

[12] We have to be careful here, we are not advocating the use of a single company-wide cost of capital to use as a hurdle rate for individual projects. We are talking about using project-specific WACC's: the project is assumed to have some *debt capacity*, which is expressed as a percentage of the value of the project; the project's WACC reflects the project's asset beta via r_{eU}, which is levered up to the debt capacity of the investment; the r_d is the required return on the debt if the project were a stand-alone firm.

of equity to the firm. r_{eL} and r_d are both before-personal tax required rates of return and, hence, include the tax premium charged in each market for personal taxes. Also, the cost of equity is the *levered* cost of equity. Interpreting (11) is simple enough: Think of the asset as a mini-firm with its own capital structure (D/D+E) if the asset (firm) is financed with D dollars in debt which costs the firm $r_d(1-\tau_c)$ after tax and with E dollars in equity which costs the firm r_{eL}, then the marginal cost of capital is the asset's WACC. The weighted average cost of capital is an example of an adjusted discount rate, r*, and as such would be the discount rate to apply to the future Free Cash Flow from Operations of the asset for valuation purposes. In the perpetual cash flow case, WACC would replace r* in the denominator of (6) to value the levered assets:

$$V_L = \frac{X(1 - \tau_c)}{WACC} \qquad \textbf{(12)}$$

Will this method yield the same answer as the APV in equation (2) or (4)? Will the WACC give the same adjusted discount rate as the MM formula for r* (equation (5))? If they are different, which is preferable? We now show the following result:

. In the perpetuity case (perpetuity of cash flows, perpetual debt and debt tax shields as risky as the debt), the WACC will equal MM's adjusted cost of capital and the value of the firm calculated via the ADR method of equation (6) will be the same as via the APV method of equation (4), which is the perpetual cash flow version of equation (2).

This result is proved in Appendix I. Because the WACC and MM's r* are equivalent, the APV method and the WACC method will yield the same value of the levered assets in the perpetuity cash flow, perpetual debt case.

3. Non-perpetuity cash flows

In non-perpetuity cases, we have to be careful what we mean by using APV approach or a WACC approach. Equations (1) and (2) do not assume the cash flows from the firm are perpetuities, but they do assume perpetual debt, i.e., that the market value of the debt outstanding is always equal to D. This assumption can be relaxed, allowing for any time pattern of debt amounts, repayments, new issues, etc. But the analyst must be careful to reflect appropriately the riskiness of the debt tax shield cash flows given the time pattern of expected operating cash flows and debt amounts.

Similarly, the typical application of WACC in valuations is to use **one WACC**, reflecting the target proportion of debt in the capital structure over time. Irrespective of the time pattern of cash flows, if the amount of debt (as a proportion of the value of the firm) is not a constant over time, using one WACC is not appropriate. If you calculate one WACC and use it to value a stream of future cash flows, you are assuming that the target debt to value ratio will be roughly the same throughout the time period of the cash flows. The WACC method can be modified to calculate the appropriate WACC for each period of a DCF analysis, and if this analysis is done correctly, we can show that the value of the firm is the same as you would get using APV (with the same assumptions). In general, however, when the debt is not going to be (roughly) a constant proportion of the value of the asset, it is very complicated to enforce the absolute consistency of assumptions between the two approaches.

The choice, therefore, between using APV or WACC for valuing assets when capital structure matters, is more a matter of convenience than theory and we would make the following conclusions:

1. In project analyses where the debt policy is not expected to remain roughly a constant proportion of the value of the asset, it is preferable to use the more versatile APV method to accomodate the changing amounts of debt in the future. The APV method allows the analyst to forecast the amounts of debt at each future date and, hence, the magnitude, timing and riskiness of the interest tax shields on the debt.
2. In project analyses where the debt policy is assumed to remain roughly a fixed proportion of the value of the asset, the WACC method is much easier to apply. The APV method requires an estimate of the value of the project at each date, calculating the implied debt level and the associated tax shield for each future date. The WACC method will in this case, as in every case, still suffer from the circularity problem associated with adjusted discount rates, but we will appeal to the notion of its being a target debt to value ratio as we did in section 2.

Using a single WACC with non perpetuity cash flows: WACC assumes a constant proportion of debt to value

The perpetuity case is the only case where a constant amount of debt in perpetuity is equivalent to a constant proportion of debt to value. Consider a firm that generates a perpetuity of future expected flows of $100,000 per year. Each cash flow is risky, but the expected value of every future cash flow is $100,000. Let's assume, for simplicity, that there are no taxes and the appropriate discount rate for this asset's cash flows is 10%. The value of the asset is $1,000,000 = $100,000/.10. Suppose they have $50,000 in debt financing the asset and will maintain this level perpetually. $50,000 is 5% of the value of the asset today.

Will it be 5% of the value of the asset next year? As long as our assumption about the future cash flows remaining a perpetuity is correct and the discount rate remains at 10%, it will. Next year's value of the asset will still be $1,000,000 and $50,000 in debt is still 5% of the value of the asset.

On the other hand, if the annual expected cash flow is not constant then $50,000 in debt will not be 5% of the value of the asset in each future period. For example, suppose the annual expected cash flow is growing at 5% per year. The value of the asset is $100,000/(.10-.05) = $2,000,000. $50,000 in debt is currently 2.5% of the value of the asset. But next year the value of the asset will be $100,000(1.05)/(.10-.05) = $2,100,000. So, next year, $50,000 will be 2.38%, a lower percentage, of the value of the firm.

The effect of non-constant cash flows is that the assumption of a constant percentage of debt to value is different than an assumption of constant level of debt. In a tax shield framework, if you assume a constant level of debt in the APV framework, the present value of tax of all future tax shields is T*D. If the firm is experiencing growth in cash flows (really if the value of the firm is growing), then if the firm maintains a constant proportion of debt to value and their current level of debt is D, then the present value of tax shields will be larger than T*D. If you use the WACC method (with a single WACC) to value levered assets, then the value of the asset will exceed the APV value (if you use T*D to estimate the present value of all future tax shields).

We can modify the APV analysis to accomodate the assumption of a constant debt to value ratio and make the analysis consistent with this assumption. But in the general case, this is substantially more work than using the "single" WACC method and the "single" WACC method will do a very reasonable job of incorporating the tax shield value into the levered value of the assets.

How reasonable is it to assume that the debt to value ratio will remain roughly constant over time? Some companies explicitly target their use of debt in the capital structure, although they often do this by using a book value debt to capital target or by selecting debt instruments to maintain desired interest coverage ratios or by utilizing a level of debt which allows them to maintain a desired bond rating. Even though these policies are not exactly the same as keeping a constant market debt to value ratio, they indicate that firms often attempt to maintain a consistent debt policy.

4. Conclusions

In general, the APV and ADR approaches to valuation are different methods of attempting to incorporate the value of tax shields associated with debt financing (and, more generally, to include the net present value of financing side benefits and costs) into the valuation of an asset. Any such attempt will be forced to make assumptions regarding 1. the tax regime (no taxes, corporate taxes only, corporate and personal taxes); 2. the timing of the cash flows of the investment and the timing of the interest tax shields on debt (constant level of perpetual debt, debt at a constant percentage of market value of the asset, or an arbitrarily changing amount of debt in future periods); 3. the riskiness of the future debt tax shields.

This note has limited its focus to the corporate and personal taxes case, considering the others as special cases in this framework. We have largely limited our attention to the case where the riskiness of future debt tax shields is the same risk level as the debt. We have been concerned mostly with the timing of the future interest tax shields. The "single" WACC version of the ADR model assumes constant proportion of debt in the capital structure, whereas in the APV model, the amount and timing of future interest tax shields must be modeled explicitly.

As general rules of thumb:

1. **In project analyses where the debt in the capital structure can be assumed to be roughly a constant proportion to value, it is easier to use the "single" WACC analysis.**

The APV approach would require an estimate of the project value at each date to calculate the amount of debt at its attendant tax shield at each date.

2. **In project analyses where the debt policy is not expected to be a constant proportion of the value of the asset, it is easier to use the more versatile APV analysis to accomodate the changing amounts of debt in the future. The analyst does have to forecast the amounts of debt at each future date and its tax shields.**

The utility of the APV method is especially important in situations involving high leverage: LBO analysis, leveraged recapitalizations, large-scale tender offer repurchases, venture capital analyses, project financing, etc. The reason being that in these HLT's (highly leveraged transactions), the companies are using all excess free cash flow to pay down excessive principal levels. Because there will be large scheduled paydowns of principal and excess paydowns of principal (in good outcomes), the "single" WACC method, assuming the current capital structure will be maintained, will grossly overstate the present value of future tax shields on debt.

A final caveat: we have to remember that most analyses of levered firm valuation, APV or WACC, ignore, because of the difficulty of obtaining good estimates, leverage related costs. As such, these analyses will usually overstate the value added by levering up the assets of the firm.

Appendix I. Proof that WACC equals MM's adjusted cost of capital for the perpetual cash flow, perpetual debt case.

We have to make use of some results from my teaching note, *Financial Leverage and the Cost of Equity Capital: Teaching Note*. The relevering formula (equation 22 in that teaching note) for the cost of equity in a corporate and personal tax regime, with risk free debt and perpetual cash flows is repeated here:

$$r_{eL} = r_{eU} + [(r_{eU} - r_{fe})\ (1 - T^{*})]\ \frac{D}{E} \tag{A1}$$

where T* is the effective present value of tax shields offered per dollar of debt and is given by

$$T^{*} = \left[1 - \frac{(1 - \tau_c)(1 - \tau_{pe})}{(1 - \tau_{pd})}\right] \tag{A2}$$

and r_{fe} is what the risk free rate would be if it were taxed as an equity instrument. In an after-personal tax equilibrium, a debt security and an equity security *of equivalent risk* would be priced to earn an equal after-personal tax expected rate of return: i.e.,

$$r_{fe}\ (1 - \tau_{pe}) = r_{fd}\ (1 - \tau_{pd}) \tag{A3}$$

The discussion in this teaching note, however, has allowed for risky debt, and the adjustments to the equation (A1) and (A3) for risky debt are intuitive enough. If the firm has risky debt with a before-personal tax required rate of return r_d, then the risky required rate of return should be used in (A1) rather than the risk-free rate. But the rate to be used in (A1) still has to be a rate of return *consistent with equity taxation*. The equity instrument with the same risk as the debt being considered would have a before-personal tax required rate of return we call $r_{d,e}$, which is given by:

$$r_{d,e}\ (1 - \tau_{pe}) = r_d\ (1 - \tau_{pd}) \tag{A4}$$

$r_{d,e}$, remember is an equity required rate of return; the d refers to its risk, which is equal to the risk of the debt of the project being considered, and the e refers to its taxation. (A4) merely says that on an after-personal tax basis, the equity instrument would have to earn the same rate of return as the equal risk debt. Plugging this value into (A1) for r_{fe}, we get

$$r_{eL} = r_{eU} + [(r_{eU} - r_{d,e})\ (1 - T^*)]\ \frac{D}{E} \tag{A5}$$

What we want to show is that the WACC, as given by (11) is equal to MM's adjusted cost of capital (5) in the perpetuity case. Repeating WACC, and noting that D+E = V, we get equation (A6):

$$WACC = \left(\frac{D}{V}\right)\ r_d\ (1 - \tau_c) + \left(\frac{E}{V}\right)\ r_{eL} \tag{A6}$$

Substitute from (A5) into (A6) for r_{eL}:

$$\left(\frac{D}{V}\right)\ r_d\ (1 - \tau_c) + \left(\frac{E}{V}\right)\ [r_{eU} + (r_{eU} - r_{d,e})\ (1 - \tag{A7}$$

Multiplying the (E/V) term in the second term on the right hand side of (A7) through the terms in brackets yields:

$$\left(\frac{D}{V}\right) r_d\ (1 - \tau_c) + \left[\left(\frac{E}{V}\right) r_{eU} + (r_{eU} - r_{d,e})\ (1 - \right. \tag{A8}$$

Expanding the bracketed term on the right hand side of (A8) and collecting terms in r_{eU}:

$$WACC = \left(\frac{D}{V}\right) r_d\ (1 - \tau_c) + \left[\left(\frac{E}{V}\right) r_{eU} + \left(\frac{D}{V}\right) r_{eU} - T^*\left(\frac{D}{V}\right) \right. \tag{A9}$$

E/V plus D/V equals one, so the first two terms in the bracketed term on the right hand side of (A9) sum to r_{eU}; separating terms in r_{eU} and putting them in one bracketed term, leaving the rest of the terms in a second bracketed term yields:

$$= \left[r_{eU}\left(1 - T^*\left(\frac{D}{V}\right)\right)\right] + \left[\left(\frac{D}{V}\right) r_d\ (1 - \tau_c) - \left(\frac{D}{V}\right)\ (1 - \right. \tag{A10}$$

The first bracketed term on the right hand side of (A10) is MM's adjusted cost of capital formula, which is what we are attempting to show that WACC is equal to in the perpetuity case. Clearly the two are only equal if the second bracketed term on the right hand side of (A10) is zero. Lucky for us, it is! To show this, see that solving (A4) for $r_{d,e}$ yields $r_{d,e} = r_d(1 - \tau_{pd})/(1 - \tau_{pe})$; also, see from (A2) that $1 - T^*$ is equal to $(1 - \tau_c)(1 - \tau_{pe})/(1 - \tau_{pd})$. Substituting these expressions for $r_{d,e}$ and $(1 - T^*)$ into (A10) yields:

$$= MM's\ r^* + \left(\frac{D}{V}\right) \left[r_d\ (1 - \tau_c) - \frac{(1 - \tau_c)\ (1 - \tau_{pe})}{(1 - \tau_{pd})}\ \frac{(1 -}{(1} \right. \tag{A11}$$

Cancelling the $(1-\tau_{pe})$ and $(1-\tau_{pd})$ terms in (A11) clearly shows that the bracketed term is zero, and we have shown what we intended to, that

$$WACC = MM's\ r^* = r_{eU}\left(1 - T^*\left(\frac{D}{V}\right)\right) \tag{A12}$$

in the perpetuity cash flow, perpetual debt case.