Option pricing

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Options

- Review of concepts
- Methodology for pricing options
  - Binomial model
- Comparative statics
- When is it optimal to exercise options?

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Terminology

- American vs. European options
- Long vs. Short. “Long” positions have positive payoffs, “Short” positions have negative payoffs
- Exercise price vs. market price ⇒ when to exercise and payoff
- Out-of-the money vs. in the money
- Payoffs, and intrinsic value.
- Up-front Premium, market value or price of the option

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Option pricing

- Objective: calculate the up-front Premium, market value or price of the option
- For a call option C
- For a put option P
- C and P depends on a number of variables
  - underlying stock price S_t
  - exercise price X
  - time to expiration (T-t)
  - dividends
  - interest rates
  - volatility

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Option pricing an overview

- Owning a option gives you a choice
- Time Value of Information
- Time value of money
- No arbitrage conditions affect the minimum price of an option
  - The value of the option must exceed its intrinsic value
  - The value of the option must exceed the current stock price minus the present value of the strike price.

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The value of the option must exceed its intrinsic value

$$(\text{Price of a call})_t \geq Max[S_t - X, 0]$$

$$(\text{Price of a put})_t \geq Max[X - S_t, 0]$$

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The value of the option must exceed the current stock price minus the present value of the strike price

Price of a call> S_t-PV[X]

Portfolio A	Cash outflow today (Cost of portfolio)	Cash inflow tomorrow	
		$S_T > X$	$S_T < X$
Call option (1)	$C(S_t, X, T\text{-}t)$	$S_T - X$	0

Portfolio B	Cash outflow today (Cost of portfolio)	Cash inflow tomorrow	
		$S_T > X$	$S_T < X$
Long a Stock	S_t	S_T	S_T
Short risk free bond	-PV[X]	-X	-X
Borrow and buy stock (2)	S_t – PV[X]	$S_T - X$	$S_T - X < 0$

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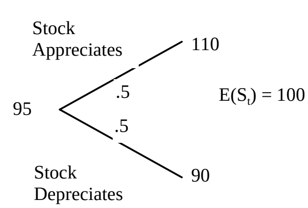
Binomial pricing methodology

- If payoffs are identical, prices should be identical

Example: Price a call option on IBM with X=100

r_f=0

X=100



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What is the cost of the option?

	Payoff		Cost of the portfolio
	S_t= 90	S_t= 110	
Long call	0	10	?

Build another replicating portfolio that has the same payoff as the call option:

	Payoff		Cost of the portfolio
	S_t= 90	S_t= 110	
Long call	0	10	?

	Payoff		Cost of the portfolio
	S_t= 90	S_t= 110	
Long Δ stocks	Δ 90	Δ 110	95Δ
Short B bonds	-B(1+ r_f)	-B(1+ r_f)	-B
Total	Δ90- B(1+ r_f)	Δ110 -B(1+ r_f)	95Δ-B

110Δ - B(1+ r_f) = 10
90Δ - B(1+ r_f) = 0

Pricing options

$110\Delta - B(1 + r_f) = 10$

$90\Delta - B(1 + r_f) = 0$

$\Delta = 0.5$ and $B = 45$

$C = \Delta S - B = (0.5)(95) - 45 = \$2.50$

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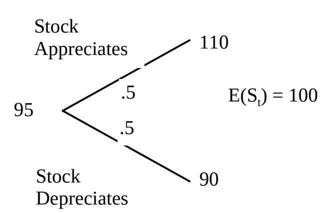
Effect of interest rate on option price

Example:

$r_f = 5\%$

$X = 100$



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Build another replicating portfolio that has the same payoff as the call option:

	Payoff		**Cost of the portfolio**
	S_t= 90	S_t= 110	
Long call	0	10	?

	Payoff		**Cost of the portfolio**
	S_t= 90	S_t= 110	
Long Δ stocks	Δ 90	Δ 110	95Δ
Short B bonds	-B(1+ r_f)	-B(1+ r_f)	-B
Total	Δ90- B(1+ r_f)	Δ110 -B(1+ r_f)	95Δ-B

$110\Delta - B(1+ r_f) = 10$
$90\Delta - B(1+ r_f) = 0$

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Pricing options

Δ= 0.5 and B = 42.86
C = ΔS - B = (0.5)(95) - 42.86 = $4.64

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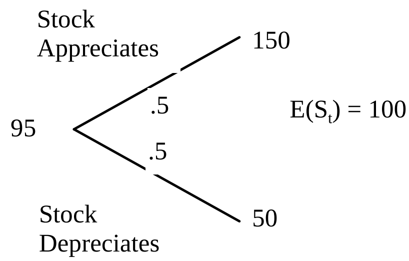
Effect of volatility on Option values

Example:

r_f=0

X=100



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Effect of volatility on Option values

	Payoff		**Cost of the portfolio**
	S_t=50	S_t=150	
Long call	0	50	C

	Payoff		**Cost of the portfolio**
	S_t=500	S_t=150	
Long Δ stocks	Δ 50	Δ 150	95Δ
Short B bonds	-B	-B	-B
Total	Δ50- B	Δ150 -B	95Δ-B

50Δ - B=0

150Δ - B = 50

B=25 Δ=1/2

C= $22.50

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Determinant of option value (Comparative statics)

	Effect of Increase on	
	Call Option	**Put Option**
Underlying Asset Price	Increase	Decrease
Exercise Price	Decrease	Increase
Volatility	Increase	Increase
Interest Rates	Increase	Decrease
Dividends	Decrease	Increase
Time to Expiration	?	?

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The optimal time to exercise American call options

$$Value\ \ Alive = C(S_t, X, t, T) \geq S_t - PV[X] > S_t - X = Value\ \ Dead$$

1. Time value of money
2. Time value of information
3. Dividends

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What did we learn?

- Payoffs of options and options’ porfolios
- Application to corporate securities
- Pricing
  - how to use “no arbitrage con.” to derive prices
  - what option prices depend upon (comparative statics)
  - intuitive feel for how options are priced
- What variables affect optimal exercise time?

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