

Review session

- 1) Term structure
- 2) Beta problem
- 3) Promised vs. expected return
- 4) Pro-formas
- 5) Option portfolio problem
- 6) Real option problem
- 7) Dividend/Share repurchase

1) Term structure. One year government bonds are currently yielding 5.5 percent. The current yield on two year government bonds is 6.5 percent. Historically, the return on one year government bonds has averaged 3.6 percent while the return on two year government bonds has averaged 4.6 percent. You can assume that one year government bonds are risk free.

A) Is the  $\beta$  of two year government bonds zero? Explain.

Historically the return on two year government bonds has exceeded the return on one year government bonds by 1 percent (4.6 - 3.6). Given the government bond markets are competitive and efficient, this implies that investors in two year government bonds bear systematic risk and are compensated for this risk. Since investors in two year government bonds earn the positive risk premium (1 percent), the  $\beta$  of two year government bonds must be positive.

The fact that the current two year (6.5%) is greater than the current one year (5.5%) does not imply that the two year government bond  $\beta$  is positive.

B) What is the two year risk free rate?

The return on the two year government bonds is the two year risk free rate plus the two year risk premium. We know from above that the risk premium on two year government bonds is 1 percent. Since the return on two year government bonds is 6.5%, the two year risk free rate is 5.5%.

C) You are considering investing your signing bonus in a one year t-bill. However, you won't receive your signing bonus for a year. Thus you would like to purchase a t-bill one year from now. What does the market expect the promised yield on this one year t-bill will be? This is the t-bill which will be sold one year from now and will payoff two years from now.

The two year risk free rate is the expected return an investor would earn by investing in risk free assets for two years. The risk free asset is the one year government bond (the t-bill). Thus an investor who invested in one year government bonds and then rolled their investment over into another one year t-bill would earn 5.5% per year (see B).

The current one year t-bill rate is 5.5%. Since the average of this year and next year's one year t-bill rate is 5.5% also, this implies that the market expects next year's t-bill rate to be 5.5% too. The market doesn't expect short term (one year) rates to rise. The term structure is upward sloping because of the risk premium, not because short term rates are expected to rise.

- 2) Nike has \$1B in Shoe assets, and \$150M in outstanding debt.  $\beta_{shoe}$  is 1.5 and  $\beta_D$  is .3. The market price of risk,  $E[r_m - r_f]$  is 8.5 percent. The risk free rate is 5%. How much cash should NIKE hold such that  $r_E=13.5\%$ ?

**Solution:**

For  $r_E=13.5\%$ , we need  $\beta_E = 1$

$$(13.5=5+\beta_E *8.5)$$

Remember that  $A=L+E$ , so

$$Shoes + C = D + E$$

$$1 + C = .15 + E$$

$$E = .85 + C$$

We have one equation and 2 unknowns. How do we solve for C?

Use again the fact that assets= liabilities, solve the following equation

$$\frac{C}{C + \$1B} \beta_{cash} + \frac{\$1B}{C + \$1B} \beta_{shoe} = \frac{D}{D + E} \beta_D + \frac{E}{D + E} \beta_E$$

For  $r_E=13.5$ , we HAVE  $\beta_E = 1$

$$\frac{C}{C + \$1B} 0 + \frac{\$1B}{C + \$1B} 1.5 = \frac{D}{D + E} .3 + \frac{E}{D + E} 1.0$$

Solve this using  $E = .85 + C$

$$\$1.5B = (.15)(.3) + E$$

$$\$1.5B = (.15)(.3) + .85 + C$$

$$\$1.5B = (.045) + .85 + C$$

$$1.5 - .045 - .85 = C$$

$$C = .605$$

Note that  $E=1.455$

Double check the balance sheet:

	L+E	=	A
E =	1.455		1.0
D=	.15		.605
TOTAL	1.605		1.605

3) Expected vs. promised

Assume the following:

I=\$1,000

T=30 years

CF(if solvent)=\$25,000

a) If  $\beta = .6$ ,  $r_f=5\%$  and  $(r_m - r_f)=8.5\%$  what is the probability of default? Assume principal, but no interest is paid upon default.

Denote  $p$ = probability of default

$$1. \quad 1,000(1 + r_{\text{exp}})^{30} = (1 - p) * 25,000 + p(1,000)$$

$$2. \quad r_{\text{exp}} = .05 + .6 * .085 = .101$$

Plug 2 into 1

$$1,000(1.01)^{30} = (1 - p) * 25,000 + p(1,000) = 25,000 - 24,000p$$

$$p = 29.5\%$$

b) What is  $r_{\text{promised}}$  ?

Solve this equation:

$$(1,000) * (1 + r_{\text{promised}})^{30} = 25,000$$

$$r_{\text{promised}} = 11.3\%$$

3)

Financial statements for Medco are shown below. The 1999 and 2000 numbers are actual and the 2001 numbers are estimates

Income statement					
	2000	2001 (est.)			
Sales	3432	3560			
Costs	450	495			
Depreciation	510	570			
Interest Expense	830	950			
Pre-Tax Income	1642	1545			
Taxes	267	285			
Net Income	1375	1260			
Shareholder distribution	210	250			

Balance sheet, Year-end							
	1999	2000	2001(est.)		1999	2000	2001 (est.)
Current assets	5368	5900		Debt	9000	11000	
Net PPE	9000	11633	11000	Paid in capital	2034	2034	2034
Other assets	0	0	0	Retained Earnings	3334		
Total assets	14368	17533		Total liabilities +equity	14368		

- A) Assume that Medco pays 8% interest on its debt. Further assume that interest expenses are calculated by multiplying the interest rate by the average of the current debt outstanding and the previous year debt outstanding. How much debt will be outstanding at the end of 2001? Show your calculation below (10).

We know that 2001 interest expense is \$950. Since this equals 8% times the average of this years and last years debt outstanding, we can write the following equation:

$$950 = 0.08 \frac{\text{Debt}_{2000} + \text{Debt}_{2001}}{2} = 0.08 \frac{11,000 + \text{Debt}_{2001}}{2}$$

$$\text{Debt}_{2001} = 12,750$$

- B) Fill in the missing entries (all seven double-outlined cells) in Medco 2000 and 2001 balance sheets. Show any necessary calculation below. (10)

(B) The missing entries can be calculated using two relationships. First, Retained Earnings equals last year's retained earnings plus net income minus shareholder distributions. Second, assets equal liabilities plus equity. Correct entries are shown below:

<b>Income statement</b>		
	<b>2000</b>	<b>2001 (est.)</b>
Sales	3432	3560
Costs	450	495
Depreciation	510	570
Interest Expense	830	950
Pre-Tax Income	1642	1545
Taxes	267	285
Net Income	1375	1260
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<b>Balance sheet, Year-end</b>							
	<b>1999</b>	<b>2000</b>	<b>2001(est.)</b>		<b>1999</b>	<b>2000</b>	<b>2001 (est.)</b>
Current assets	5368	5900	9293	Debt	9000	11000	12750
Net PPE	9000	11633	11000	Paid in capital	2034	2034	2034
Other assets	0	0	0	Retained Earnings	3334	4499	5509
Total assets	14368	17533	20293	Total liabilities +equity	14368	17533	20293

4) Financial options

A) Draw the gross payoff diagram for the following portfolio: sell one call option on a share of Ashton Technology Group with a strike price of \$100 and sell one put option on a share of Ashton Technology Group with a strike price of \$100. Both options have one year to expiration. The one year risk free rate is 5.5 percent and the current stock price is 100. Make sure your diagram is clearly labeled.

A) You are selling two options. Thus, your gross payoff will never be positive. If the stock price of Ashton Technology Group ends up above \$100, the put will be worthless. However the person you sold the call to will come and demand a payment of the stock price minus 100 ( $S_t - 100$ ). This is a negative cashflow for you. If the stock price of Ashton Technology Group ends up below \$100, the call will be worthless. However, the person you sold the put to will come and demand a payment of 100 minus the stock price ( $100 - S_t$ ). This is a negative cashflow for you. Your gross payoff is graphed to the right.

B) Would you be willing to pay \$10 for the portfolio described in A)? Assume that you are fully diversified and the  $\beta$  of Ashton Technology Group's equity is 0.8. Hint: If this question seems very complicated, you are approaching it incorrectly or you may want to check your answer to A).

B) This portfolio never produces a positive cashflow next year. Thus you shouldn't be willing to pay any positive price (including \$10) for the investment. In fact, you must be paid to accept this portfolio. If you invest in this portfolio, you will receive the price of the call and the put today.

C) If your information or expectations differ from the market's and you are correct, you can make more than your required rate of return trading securities. What beliefs must an investor have for the portfolio described in A) to be a good bet.

C) An investment in this portfolio gives you a positive cashflow today. You receive money when you sell the put and the call. However, you are committed to payout money tomorrow if the stock price isn't 100. If the stock price rises a lot, you lose a lot. If the stock price falls a lot, you lose a lot. Thus this is a good investment if you think the stock price isn't going to change much. If you think volatility is lower than the market thinks, this is a good speculative investment.

It is not necessary or sufficient that the stock price is \$100 next year for this to be a good speculative investment. If the put and call sell for a total of \$20, for example, any stock price between 80 and 120 will yield a net profit for this portfolio. Conversely, if you and the market knew the stock price was going to be \$100 next year, the portfolio would sell for zero and payoff zero. Not a bad investment, but not a good investment.

## 5) Real option

United Airlines is thinking about beginning perpetual service between Chicago and Buenos Aires. To enter this market, United must purchase a new aircraft. They are considering the purchase of a 757 which can carry 200 passengers or a 747 which can carry 400 passengers. The prices and operating costs of the two aircraft models are shown below.

	Boeing 757	Boeing 747
Purchase Price	\$55 million	\$105 million
Annual Operating Cost	\$15 million	\$20 million
Passenger Capacity	200	400

The primary source of uncertainty facing United is the level of demand. First year demand is known with certainty - 200 round-trip passengers per day. However, demand in subsequent years is uncertain. With 50% probability, next year's demand will increase to 400 round-trip passengers per day and will remain at 400 in all subsequent years. With 50% probability, demand will stay at 200 round-trip passengers per day forever.

Revenue is \$50 M annually per 200 daily round-trip passengers. So, revenue will be either \$50M or \$100M in the future. For simplicity, assume that first year cash flows are received immediately, next years cash flows are received one year from now, and so on. Further assume that there is no competition and the appropriate discount rate is 10%.

(A) Expected demand next year (and every year thereafter) is 300 round-trips per day. Using this expected level of demand, calculate the Net Present Value of purchasing the 747. Remember, the first year's revenues and costs occur immediately. (5)

Answer:

If United purchases the 747, it will have to pay \$105 million today. It will receive \$50 million in revenues this year, but will incur \$20 million in operating costs. In subsequent years, expected revenues will increase to \$75 million (=300 daily round trips X \$.25 million per round-trip). Operating costs in future years will remain \$20 million. Therefore, the NPV of the 747 purchase is:

$$NPV_{747} = -105 + 50 - 20 + \sum_{t=1}^{\text{infinity}} \frac{(75 - 20)}{(1 + .10)^t} = \$475 \text{ million}$$

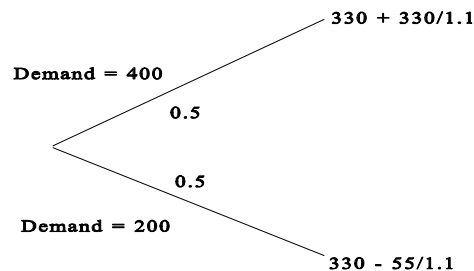
(B) If the 757 is purchased this year, an additional 757 aircraft will have to be purchased next year to meet the expected demand of 300 round-trips per day. Suppose that United's decision to take delivery of the second 757 must be made today, but they pay for the aircraft next year (when it's delivered). Assuming that United commits to taking delivery of the second 757 next year, what is the NPV associated with using 757s to serve the Chicago- Buenos Aires route? (10)

Answer:

The first 757 will be used at 100% capacity no matter what demand is next year and in subsequent years. The NPV for this first jet is:

$$NPV_{1st\ 757} = -55 + 50 - 15 + \sum_{t=1}^{\text{infinity}} \frac{(50 - 15)}{(1 + .10)^t} = \$330 \text{ million}$$

The NPV of the second jet depends on the level of demand next year. If demand increases to 400 daily round-trips, then cashflows from the second jet will replicate those from the first. The only difference is that these cashflows start one year later and so must be discounted one period. If demand remains at 200 daily round-trips, then the second jet will not be flown--United will exercise its real option to ground the jet. By grounding the jet, United saves \$15M in annual operating expenses. United will pay \$55 million in one year for an asset that generates cashflows of zero.



Therefore, the total NPV is:

$$NPV_{Total} = 330 + .5 \frac{330}{(1+.10)} + .5 \frac{-55}{(1+.10)}$$
$$= 330 + 150 - 25 = \$455 \text{ million}$$

- (C) Suppose that Boeing offers to waive the requirement that United commit to the purchase of the second 757 immediately. What is the maximum amount United should pay for this waiver? (15)

Answer:

By not requiring United to commit to purchasing the second 757 before the level of demand is known, Boeing allows the airline to avoid a -\$55 million cashflow. This negative cashflow occurs with 50% probability one year hence. Therefore, the present value of avoiding this cashflow is  $.5 * 55 / 1.1 = \$25$  million. This would increase the NPV of using 757s to serve the Chicago - Buenos Aires route to \$480 million (\$455 million + \$25 million). However, United would not be willing to pay \$25 million for this waiver. Remember that they could buy a 747 with an NPV of \$475 million. Therefore, the maximum that United would be willing to pay for waiver is \$5 million. If they paid \$5 million and were allowed to buy the second 757 only if needed, the 757 alternative would have an NPV of \$475 million (\$480 - \$5). This would be exactly the same as the NPV of the 747.

6) Dividend/Repurchase

Luxottica shares have a stock price (cum-dividend) of \$100 and there are 100,000 shares outstanding. The firm has \$500,000 free cash flow and is planning to distribute all its cash flow to its shareholders.

- a) If Luxottica decides to pay back all the cashflow, what is the dividend per share?

$$500,000 / 100,000 = \$5$$

- b) If instead of distributing dividends Luxottica decides to distribute the cashflow through a share repurchase, how many shares will be repurchased?

$$500,000 / (\$100/\text{share}) = 5,000$$

- c) Derive the stock price ex-dividend, if dividends are distributed

Value of the firm (ex dividend) = Value of the firm (cum dividend) - shareholders distribution

$$\text{Value of the firm (cum dividend)} = \$10,000,000$$

$$\text{Value of the firm (ex-dividend)} = 9,500,000$$

$$\text{Price of the stock (cum dividend)} = 100$$

$$\text{Price of the stock (ex-dividend)} = 9,500,000 / 100,000 = \$95$$

- d) Derive the stock price if the shares are repurchased

$$\text{After 500K repurchase, equity value} = 9,500,000$$

$$\text{Outstanding shares are now} = 95,000$$

$$\text{The price of each share is } \$100$$