Lecture 4: Supplementary exercises

We know from Finance I, that only systematic risk is priced (rewarded with a higher expected return), because we assume that investors can diversify away (i.e. not bear) idiosyncratic risk. If the intuition of why we used different discount rates in the coffee cafe example has not sunk in, the following set of exercises may help. To complete this exercise you will need to download the Excel spreadsheet sup4.xls from the Corporate Finance web page.

1) Discount rates and systematic risk (version 1). The information from lecture and the spreadsheet is reproduced below. The probability of each state is the probability of preferences times the probability of the state of the economy (see spreadsheet). This question is based on the value of the project when we do not hire the market research firm.

	Future Years					
Preferences	Berkeley	Berkeley	Berkeley	Cleveland	Cleveland	Cleveland
Prob of preferences	80.0%	80.0%	80.0%	20.0%	20.0%	20.0%
State of the economy	High	Medium	Low	High	Medium	Low
Prob of state of economy	25.0%	50.0%	25.0%	25.0%	50.0%	25.0%
Cashflow (year 1-10)	32.0	22.0	12.0	22.0	12.0	2.0
Prob of state	20.0%	40.0%	20.0%	5.0%	10.0%	5.0%

A) The value of the project once we have made the investment is the NPV of the project (20.3) plus the initial investment (100). Thus the value of the project today is 120.3[1] Calculate the value of the project next year in each of the six states. The project's value is the discounted value of current and future cashflows. You should include the cashflow you will received next year. You can assume that next year's cashflow provides you no information about whether Chicago looks like Berkeley or Cleveland. Put your answers in cells C17-H17.

B) Calculate the realized return in each state. You should find that the return is higher when the economy and stock market are doing well. You should also find that the return is higher when Chicago turns out to be like Berkeley than when Chicago turns out to be like Cleveland. It should make sense why each of these two statements are correct.

C) Calculate the correct project discount rate. Is the discount rate you calculated the same one I used in lecture? To do this you must first estimate the asset β. Remember β is estimated as the covariance of the return on the asset minus the risk free with the return on the market minus the riskfree divided by the variance of the return on the market minus the risk free. This means you will have to calculate the excess return on the asset. Your answer from B) is the return on the asset so you need to subtract off the riskfree rate. The formula from Finance I is:

[1] To calculate the NPV of the project, you can use the formula for a finite annuity from the Review of Financial Concepts.

$$\beta_A = \frac{\text{Cov}\left(r_{market} - r_{free} , r_{asset} - r_{free} \right)}{\text{Var}\left(r_{market} - r_{free} \right)}$$

If you prefer pictures, a graph of the excess return on the assets versus the excess return excess return on the market can be found on the third tab of spreadsheet.

D) Value the project. In lecture we took the expected cashflow and discounted them at the risk adjusted rate. This time take the expected asset value next year (from row 17 of the spreadsheet) and discount this as the risk adjusted rate.[2] The risk free rate is 2.0 percent. The market price of risk is 8.5 percent.

E) Extra question for the quantitatively inclined. You calculated the returns and the β conditional on an initial valuation of 120.3 (see cell B18). You should have noticed that to get this number, we had to assume the asset β was equal to one and the correct discount rate was 10.5 percent -- even though our objective was to estimate the β and the discount rate. To convince yourself that this approach yields a uniquely correct answer, use a different estimate of the project's value (i.e. replace the 120.3 in cell B18). You may want to save your worksheet first. The correct estimate of the project's value (cell B18) is the one which produces a final estimate (cell B29) which is equal to your initial estimate. An easy way to do this is to use Excel's Goal Seek. Tell Goal Seek to adjust your initial guess (cell B18) until the difference between your initial guess (B18) and the final value (cell B28) is zero. This difference is contained in cell B29.

[2] You can use the sumproduct( ) function in Excel to calculate expected values or cashflows. For example, sumproduct(A1:A3,C1:C3) equals A1 * C1 + A2 * C2 + A3 * C3. If the project value in each state is in row C and the probability weights are in row A, this gives you the expected project value. You used this formula to calculate the covariance and variance in cells B24 and B25

2) Discount rates and systematic risk (version 2). When we hired the market research firm we deferred the project a year. This changed the expected value of the project in one year. It also changed the correct discount rate which we used to discount values from year one to today. This question calculates this discount rate.

A) Since the 100 investment is not made today — we are deferring our decision to invest for one year — the value of the project today is just the NPV we calculated in lecture (25.3). Calculate the value of the project next year in each of the six states. Professor Gupta will tell us next year whether Chicago looks like Berkeley or Cleveland. Based on this information we will either build the coffee cafes or not. When you calculate next year's value of the project, do not include the 100 investment.

B) Calculate the realized return in each state.

C) Estimate the asset β. β is estimated as the covariance of the return on the asset minus the risk free with the return on the market minus the riskfree divided by the variance of the return on the market minus the risk free. The formula from Finance I is:

$$\beta_A = \frac{\text{Cov}\left(r_{market} - r_{free} , r_{asset} - r_{free} \right)}{\text{Var}\left(r_{market} - r_{free} \right)}$$

If you prefer pictures, a graph of the excess return on the assets versus the excess return excess return on the market can be found on the fourth tab of spreadsheet.

D) Value the project. Discount next year's expected asset value using the risk adjusted discount rate. Is the discount rate you calculated the same one I used in lecture?

Lecture 4: Supplementary exercises answers

1) Project valuation and discount rate when the market research firm is not hired

A) The value of the project next year depends upon both the cashflow which is received next year as well as the cashflow which is expected in years 2 through 9. Since we assumed that we learn nothing from next year's cashflow about whether Chicago looks like Cleveland or Berkeley, then the expected cashflow in years 2 thorough 9 is still 20. The only reason that the project value differs across the 6 states is the cashflow in year 1 differs. As an example, the value of the project in state 1 (Berkeley, High) is:

$$V = 32 + \sum_{t=1}^{9} \frac{20}{(1 + .105)^t} = 144.9$$

The values for the other states are reported in row 19.

B) The realized returns are the value in year 1 divided by the value in year 0 minus 1. Thus the realized return in state 1 (Berkeley, High) is 20.5%. The realized returns are reported in row 20.

C) The covariance of the (excess) asset return and the (excess) market return is calculated (cell B24). It is equal to 1.07%. The variance of the market return is reported in cell B25. It is also equal to 1.07%. Thus the asset β (cell B26) is one. Given an asset β of one, the correct discount rate is 10.5 percent.

$$r_A = 2.0\% + 1.0\ 8.5\% = 10.5\%$$

D) To value the project take the expected value of the project next year. This is the probability weighted average of row 17. Discount this number (132.0) by 1.105 and you get the value of the project is 120.3 — the same as we calculated in lecture.

2) Project valuation and discount rate when the market research firm is hired

A) The value of the project next year depends upon what the market research firm tells us. If they report that Chicago looks like Cleveland, then we will not build the coffee cafe and the value of the project will be zero. If the market research firm reports that Chicago looks like Berkeley, then we will build the coffee cafe. In this case the value of the project is 32.3. Notice that the state of the economy does not affect the value of the project. This is where we used the assumption that the probability of a boom (high) or recession (low) is the same next year independent of whether there was a boom or a recession this year. Thus the current state of the economy has no effect on the expected cashflow to the project next year (it is always 22 once we know Chicago looks like Berkeley) or the value of the project (it is always 32.3 next year one we know Chicago looks like Berkeley).

B) The realized asset returns are -100 percent if we are told Chicago looks like Cleveland and 27.8 percent if we are told Chicago looks like Berkeley.

C) The covariance between the return on the market and the return on the project is zero (cell B24). This implies that the asset β is zero (cell B26). Look at the graph of the asset returns against the market return. Notice that the correlation is zero.

D) Since the asset β is zero, the correct discount rate is the riskfree rate — 2 percent. The value of the project is therefore 25.3.