

Question on Promised vs. Expected Return (Midterm 1998)

You are trying to decide between two investment opportunities. The first opportunity consists of a loan to your friend's start-up venture. It requires an initial investment and promises a single risky cashflow in 30 years. With 50% probability the cashflow in 30 years is very high and with 50% probability it is zero (you lose your entire investment). All of the risk associated with the new business venture is idiosyncratic (firm-specific).

The second opportunity consists of an investment in a zero-coupon U.S. government bond with a 30-year maturity. The current yield on the 30-year bond is 6.5%, the current T-Bill (3-month government bond) rate is 5.3%, and the risk premium associated with 30-year government bonds is 1.4%.

(A) Which of these two investments has the higher expected return? Why? (10)

(B) What is the 30 year risk free rate of return? (5)

(C) What is the *annualized promised* rate of return from investing in the start-up venture?
Show your work. (10)

(D) Does the market expect interest rates to rise or fall? Explain. (5)

Answer

(A) As long as there is a risk premium in the term structure, long-term government bonds have some systematic risk and will therefore command an expected return in excess of the 30-year risk free rate. Conversely, because the risk associated with the new business venture is all idiosyncratic, its expected return will equal the risk-free rate. Therefore, the government bond will have a greater expected return. In this case, the expected return on the 30-year bond is 1.4% greater than the expected return on the risky project.

This is somewhat counter-intuitive—the government bond is “safer” in that it has less total risk and it has a higher return. But remember that only systematic risk, not total risk, is priced. The risky venture will have to pay a higher promised return to entice you to invest. However, because all of its risk is idiosyncratic, it does not pay a higher expected return. A good example of a risky zero-beta investment is gold. Over the 20-year period from 1974 through 1994, the beta on gold was roughly equal to zero. Yet the volatility (total risk) of gold prices was quite high over this period.

(B) The problem states that the risk premium associated with long-term government bonds is 1.4%. From CAPM, we know that:

$$r_{30\text{-year}} = r_{f, 30\text{-year}} + \beta(8.4\%)$$

Therefore,

$$r_{f, 30\text{-year}} = r_{30\text{-year}} - \beta(8.4\%) = 6.5\% - 1.4\% = 5.1\%$$

(C) We can use the expected return on the risky business venture to “back-out” the promised return. Because all of the risk associated with this venture is idiosyncratic, the expected return must equal the 30-year risk free rate (see part (A)). Therefore, the expected cashflow must equal the initial investment multiplied by $(1+r_f)^{30}$. Let I equal the initial investment. Then:

$$\begin{aligned}(1 + r_{f, 30\text{-year}})^{30} &= (0.5)(\text{Cashflow}_{\text{High}}) + (0.5)(\text{Cashflow}_{\text{Low}}) \\ \Rightarrow (1.051)^{30} I &= (0.5)(\text{Cashflow}_{\text{High}}) + (.05)(0) \\ \Rightarrow \frac{(\text{Cashflow}_{\text{High}})}{I} &= 8.894\end{aligned}$$

We also know that the promised return must satisfy the following equation:

$$\begin{aligned}(1 + r_{\text{Promised}})^{30} I &= \text{Cashflow}_{\text{High}} \\ \Rightarrow \frac{(\text{Cashflow}_{\text{High}})}{I} &= (1 + r_{\text{Promised}})^{30} = 8.894 \text{ (from above)} \\ \Rightarrow r_{\text{Promised}} &= 7.56\%\end{aligned}$$

(D) The yield curve can be upward sloping for two reasons. First, interest rates may be expected to increase. Second, long-term bondholders demand compensation for

bearing interest rate (or inflation) risk. One way to determine whether the market expects rates to increase or decrease is to compare the short-term risk-free rate to the long-term risk-free rate. In this case, the long-term risk-free rate (5.1%) is below the short-term risk-free rate (5.3%). Therefore, the market expects rates to fall.