CUT-THROAT FRINGE COMPETITION IN AN EMERGING COUNTRY MARKET: TAX EVASION OR THE ABSENCE OF MARKET POWER?

ONLINE APPENDIX: DEMAND DERIVATION AND FURTHER FIGURES AND TABLES

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Derivation of the AIDS Demand Function

This section derives the AIDS demand function in budget share form, following Deaton and Muellbauer [1980a]. Consider a class of preferences represented by a cost function of the following type

$$\log c(u,p) = (1-u)\log\{a(p)\} + u\log\{b(p)\}$$

where u denotes utility, p denotes prices, and $a(p)$ and $b(p)$ are homogeneous of degree one in prices, defined as follows[1]. $\log\{a(p)\}$ is quadratic in log prices

$$\log\{a(p)\} = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k\sum_j \gamma^*_{kj}\log p_k \log p_j$$

and $\log\{b(p)\} - \log\{a(p)\}$ is given by

$$\log\{b(p)\} - \log\{a(p)\} = \beta_0 \prod_k p_k^{\beta_k}$$

where α_i, β_i and γ^*_{ij} are parameters, such that $\sum_i \alpha_i = 1$ and $\sum_j \gamma^*_{kj} = \sum_k \gamma^*_{kj} = \sum_j \beta_j = 0$. (These restrictions are required for the cost function to be linearly homoge-

[1] Note that $c(u,p) = \{a(p)\}^{(1-u)}\{b(p)\}^u$. With some exceptions (see the Appendix in Deaton and Muellbauer [1980a]), u lies between 0 and 1 so that $a(p)$ and $b(p)$ can be regarded as the costs of subsistence ($u = 0$) and bliss ($u = 1$), respectively.

neous in prices.) The (log of the) cost function can then be written

$$\log c(u,p) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_0 \prod_k p_k^{\beta_k} \tag{1}$$

By Shephard's Lemma, $\frac{\partial c(u,p)}{\partial p_i} = q_i(u,p)$, the compensated demand for good i, and hence

$$\frac{\partial \log c(u,p)}{\partial \log p_i} = \frac{p_i}{c(u,p)}\frac{\partial c(u,p)}{\partial p_i} = \frac{p_i q_i(u,p)}{c(u,p)} = s_i(u,p) \tag{2}$$

where s_i denotes the budget share of good i. This budget share is then derived by differentiation of (1):

$$s_i(u,p) = \frac{\partial \log c(u,p)}{\partial \log p_i} = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u\beta_0 \prod_k p_k^{\beta_k} \tag{3}$$

where γ_{ij} denotes the arithmetic mean of γ^*_{ij} and γ^*_{ji}

$$\gamma_{ij} = \frac{1}{2}(\gamma^*_{ij} + \gamma^*_{ji}) \tag{4}$$

and therefore $\gamma_{ij} = \gamma_{ji}$ (symmetry). Utility maximization implies that $c(u,p)$ equal total expenditure Y. The cost function defined by (1) can then be inverted, yielding:

$$\begin{aligned} u\beta_0 \prod_k p_k^{\beta_k} &= \log Y - \left(\alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j\right) \\ &= \log Y - \left(\alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma_{kj} \log p_k \log p_j\right) \end{aligned} \tag{5}$$

where (4) is used in the latter step.

Define a price index P by

$$\log P := \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma_{kj} \log p_k \log p_j \tag{6}$$

The budget share for good i given in (3) can then be written, using (5) and (6), as a function of prices and expenditure:

$$s_i(Y,p) = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(\frac{Y}{P}) \tag{7}$$

The restrictions on the parameters of (1) and the symmetry restriction imply

$$\begin{aligned} \sum_i \alpha_i &= 1 \qquad \sum_i \gamma_{ij} = \sum_i \beta_i = 0 \qquad \text{("adding-up")} \\ \sum_j \gamma_{ij} &= 0 \qquad \text{(homogeneity)} \\ \gamma_{ij} &= \gamma_{ji} \qquad \text{(symmetry)} \end{aligned}$$

Provided these restrictions hold, (7) characterizes a system of demand functions which add up to total expenditure ($\sum_i s_i = 1$), are homogeneous of degree zero in prices and expenditure, and satisfy Slutsky symmetry. Changes in relative prices work through the γ_{ij} parameters: a 1 percent change in the price of good j has an effect γ_{ij} on good i's budget share, with real expenditure $\frac{Y}{P}$ held constant. Changes in real expenditure work through the β_i parameters. These add to zero and are positive for "luxuries" and negative for "necessities".

While such cross-equation restrictions are based on theory at the level of the individual consumer, it is less clear that they should apply at the aggregate representative consumer level, despite their widespread use in applied work (see Deaton and Muellbauer [1980b] for a discussion). Of note, Capps, Church and Love [2003] estimate a multi-stage budgeting system for spaghetti sauce with an AIDS specification at the bottom stage. They conduct likelihood ratio tests and reject both homogeneity and symmetry. Unsurprisingly, restricted estimation yields substantially lower estimated standard errors (in addition to reducing the number of cross-price elasticity estimates that are negative).

In practice (see the empirical application in Deaton and Muellbauer [1980a]), the trans-logarithmic price index P given in (6) may be approximated by a Stone price index P^S (Stone [1954]):

$$\log P^S := \sum_i s_i \log p_i$$

Such linearization of the AIDS demand function that is derived from the consumer's optimization problem may admittedly introduce inconsistency (through, say, the errors-in-variables problem; see, for example, Pashardes [1993], Alston, Foster and Green [1994], and Buse and Chan [2000]) but it buys us a way to deal with non-converging estimation (see Capps, Church and Love [2003]).

Derivation of the Elasticity Expression Corresponding to the Two-Stage Budgeting System

From $s_i = \frac{p_i q_i}{Y}$, one can write

$$\log q_i = \log Y - \log p_i + \log s_i$$

Thus the price elasticities of demand are

$$\eta_{ij} = \frac{\partial \log q_i}{\partial \log p_j} = \frac{\partial\left(\log Y - \log p_i + \log s_i\right)}{\partial \log p_j} = \frac{\partial \log Y}{\partial \log p_j} - 1[i=j] + \frac{\partial \log s_i}{\partial \log p_j} \tag{8}$$

Now, at the bottom stage, s_i is specified by the AIDS demand function (7), such that

$$\frac{\partial \log s_i}{\partial \log p_j} = \frac{1}{s_i}\frac{\partial s_i}{\partial \log p_j} = \frac{1}{s_i}\left(\beta_i \frac{\partial \log Y}{\partial \log p_j} - \beta_i \frac{\partial \log P}{\partial \log p_j} + \gamma_{ij}\right) \tag{9}$$

Using (9) in (8), it follows that

$$\eta_{ij} = \frac{\partial \log Y}{\partial \log p_j}\left(1 + \frac{\beta_i}{s_i}\right) - 1[i=j] + \frac{1}{s_i}\left(\gamma_{ij} - \beta_i \frac{\partial \log P}{\partial \log p_j}\right) \tag{10}$$

From the trans-logarithmic price index (6), one obtains

$$\frac{\partial \log P}{\partial \log p_j} = \alpha_j + \sum_k \gamma_{jk} \log p_k$$

which, in view of (7), can be rewritten as

$$\frac{\partial \log P}{\partial \log p_j} = s_j - \beta_j \log(\frac{Y}{P}) \tag{11}$$

Now consider $\frac{\partial \log Y}{\partial \log p_j}$. One can approximate[2] Y by the product of overall consumption $Q = \sum_i q_i$ and the price index P; that is

$$Y = \sum_i p_i q_i \simeq P \sum_i q_i = PQ$$

which is equivalent to

$$\log Y = \log P + \log Q$$

where $\log Q$ is given by the top-level equation (see equation (2) in the text). Therefore,

$$\frac{\partial \log Y}{\partial \log p_j} = \frac{\partial \log P}{\partial \log p_j} + \frac{\partial \log Q}{\partial \log p_j} = \frac{\partial \log P}{\partial \log p_j}\left(1 + \frac{\partial \log Q}{\partial \log P}\right) = \frac{\partial \log P}{\partial \log p_j}(1+\gamma) \tag{12}$$

Using (11) and (12) in (10), one obtains

$$\eta_{ij} = \left(s_j - \beta_j \log(\frac{Y}{P})\right)(1+\gamma)\left(1 + \frac{\beta_i}{s_i}\right) - 1[i=j] + \frac{1}{s_i}\left(\gamma_{ij} - \beta_i\left(s_j - \beta_j \log(\frac{Y}{P})\right)\right) \tag{13}$$

[2] This approximation can be avoided by defining Q in the top-level equation (equation (2) in the text) as total industry expenditure Y divided by the overall price index P (i.e. $Q := Y/P$), rather than defining Q as overall consumption (i.e. $Q := \sum_i q_i$). (Indeed, this is what I do in the baseline specification of Section III in the text.) In practice, elasticity estimates which follow from either route should be similar.

or simply

$$\eta_{ij} = \left(s_j - \beta_j \log(\frac{Y}{P})\right) \left(1 + \gamma(1 + \frac{\beta_i}{s_i})\right) - 1[i = j] + \frac{\gamma_{ij}}{s_i}$$

Alston, Foster and Green [1994] discuss the several different elasticity expressions that have been employed in the literature based on (slightly) different approximations.

Further Figures and Tables

Figures 1 to 3 portray the by-region evolution of prices and value shares for Fanta, Guaraná Antarctica and Pepsi (for family-size bottles sold through self-service outlets). They complement the two panels of Figure 3 in the text, which provide the same information for the aggregation of B brands and the leading A brand Coke.

Table I provides the matrix of price elasticities of demand estimated under restricted OLS (comparable to Table IV—elasticities estimated under restricted 3SLS—provided in the text). Table II reports the "augmented" matrix of elasticities (estimated under restricted 3SLS) for the specification detailed in robustness test 4 of the text's Appendix, where single-size data for selected brands (Coke, Fanta, Guaraná Antarctica and Pepsi) augments the family-size data.

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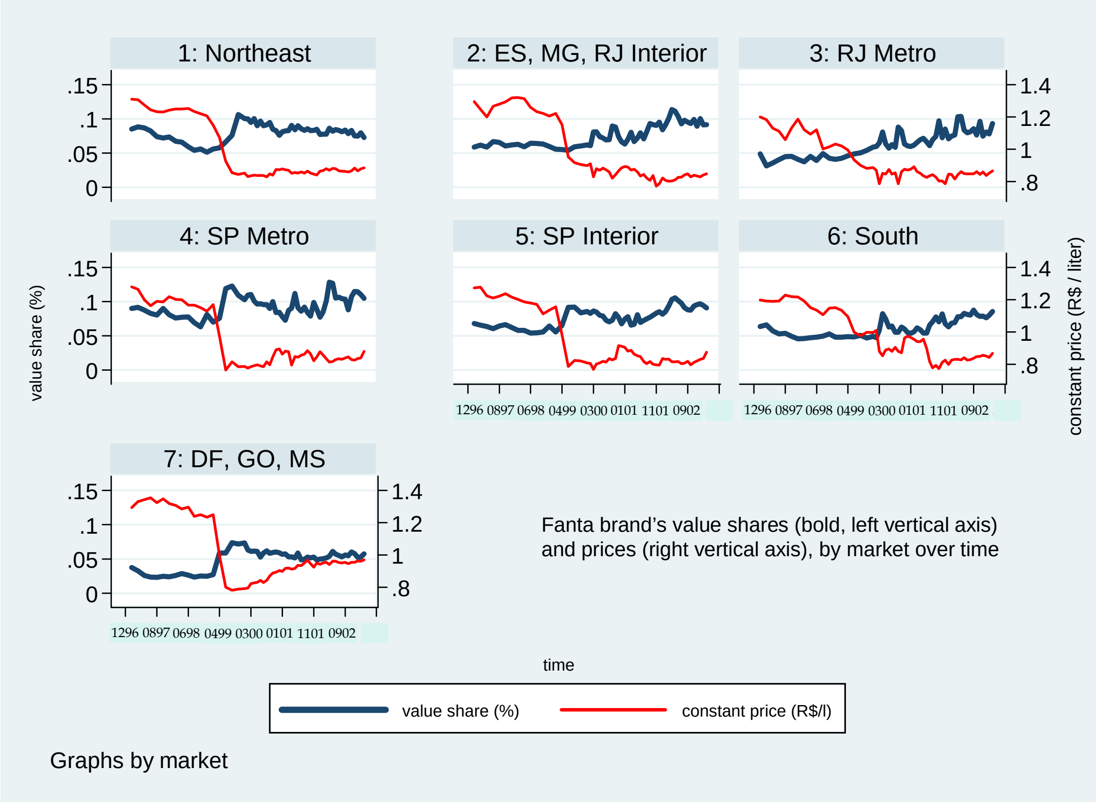

Figure 1

Value shares (bold, left vertical axis) and prices (right vertical axis, in constant March 2003 R$ per liter) for the Fanta brand, by Nielsen regional market (each subpanel). Family-size bottles sold through self-service outlets.

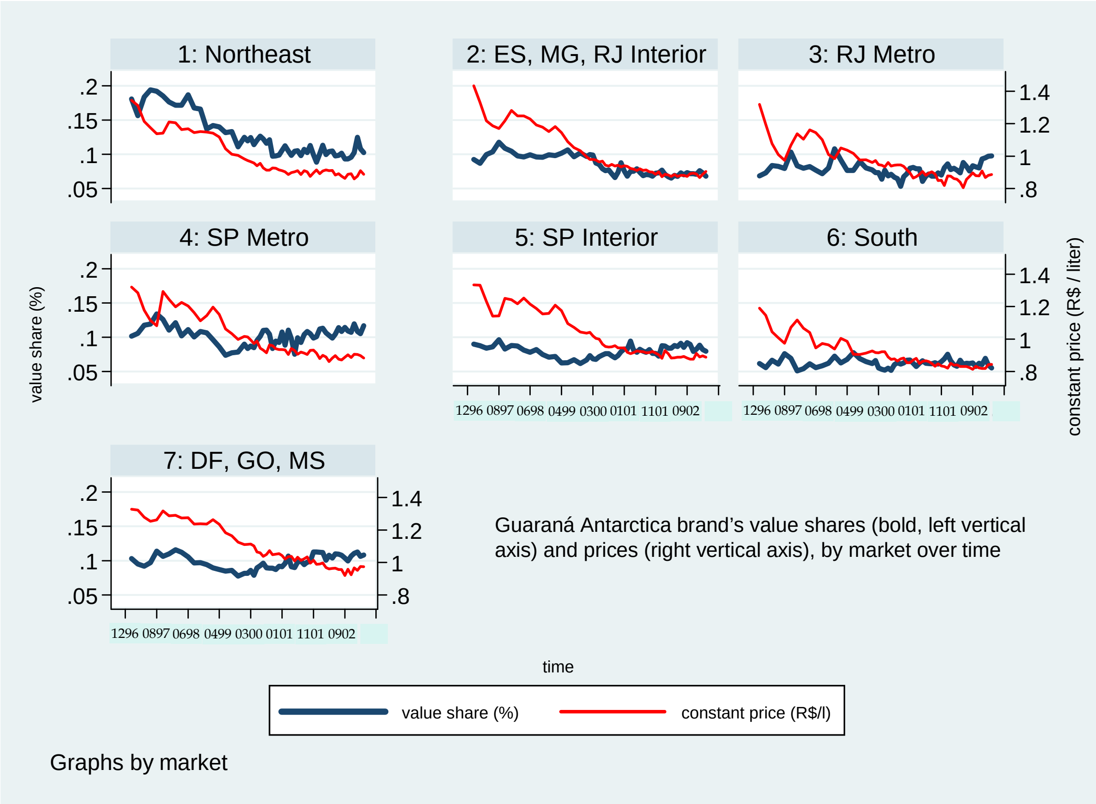

Figure 2

Value shares (bold, left vertical axis) and prices (right vertical axis, in constant March 2003 R$ per liter) for the Guaraná Antarctica brand, by Nielsen regional market (each subpanel). Family-size bottles sold through self-service outlets.

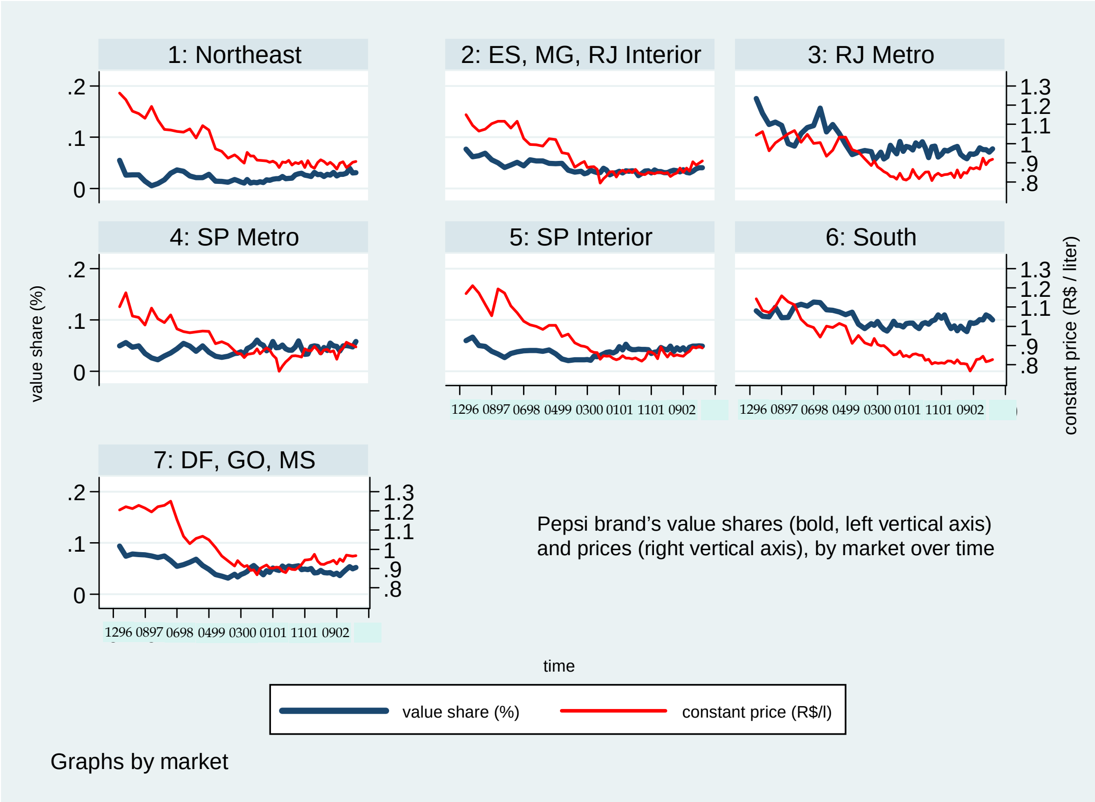

Figure 3

Value shares (bold, left vertical axis) and prices (right vertical axis, in constant March 2003 R$ per liter) for the Pepsi brand, by Nielsen regional market (each subpanel). Family-size bottles sold through self-service outlets.

TABLE I

MATRIX OF ELASTICITIES ESTIMATED UNDER RESTRICTED OLS (SURE)

Elasticity of the demand for	With respect to the price of: Coke	Diet Coke	Fanta	Tai/ Kuat	Other Coke	Guar Antar	Pepsi	Other Ambev	B brands
Coke	-1.929	0.266	0.116	0.132	0.169	0.154	0.283	0.285	0.085
	(0.114)	(0.043)	(0.047)	(0.038)	(0.028)	(0.049)	(0.064)	(0.066)	(0.106)
Diet Coke	1.938	-2.117	-0.217	0.072	0.206	0.130	0.263	0.436	-1.121
	(0.326)	(0.280)	(0.115)	(0.095)	(0.092)	(0.151)	(0.151)	(0.121)	(0.163)
Fanta	0.241	-0.154	-2.208	-0.282	0.098	0.652	0.746	0.536	-0.004
	(0.171)	(0.069)	(0.116)	(0.076)	(0.063)	(0.100)	(0.113)	(0.099)	(0.157)
Tai/ Kuat	1.110	0.106	-0.318	-2.525	0.209	0.082	0.020	0.307	0.533
	(0.207)	(0.084)	(0.111)	(0.133)	(0.076)	(0.125)	(0.147)	(0.127)	(0.192)
Other Coke	1.376	0.238	0.218	0.222	-3.617	0.546	0.314	0.372	-0.072
	(0.216)	(0.107)	(0.126)	(0.101)	(0.120)	(0.141)	(0.157)	(0.125)	(0.184)
Guar Antar	0.220	0.035	0.514	-0.032	0.207	-2.171	0.297	0.484	0.086
	(0.160)	(0.074)	(0.086)	(0.073)	(0.062)	(0.131)	(0.111)	(0.099)	(0.145)
Pepsi	0.508	0.066	0.786	-0.188	0.095	0.250	-1.608	0.149	-0.226
	(0.269)	(0.116)	(0.141)	(0.128)	(0.103)	(0.162)	(0.260)	(0.153)	(0.243)
Other Ambev	0.000	0.161	0.453	-0.001	0.089	0.477	0.046	-3.686	2.465
	(0.269)	(0.103)	(0.138)	(0.124)	(0.094)	(0.163)	(0.182)	(0.202)	(0.260)
B brands	0.570	-0.113	0.160	0.139	0.059	0.245	0.228	0.791	-2.614
	(0.091)	(0.027)	(0.046)	(0.039)	(0.028)	(0.049)	(0.069)	(0.075)	(0.125)

Notes: Family-size bottles sold through self-service outlets. Standard errors in parentheses.

TABLE II

MATRIX OF ELASTICITIES ESTIMATED UNDER RESTRICTED 3SLS FOR FAMILY-SIZE DATA AUGMENTED WITH SINGLE-SIZE DATA FOR SELECTED BRANDS (COKE, FANTA, GUARANÁ ANTARCTICA AND PEPSI)

Elasticity of the demand for	With respect to the price of										
	Coke w/Diet "Family"	Coke w/Diet "Single"	Fanta "Family"	Fanta "Single"	Other Coke "Family"	Guar Antar "Family"	Guar Antar "Single"	Pepsi "Family"	Pepsi "Single"	Other Ambev "Family"	B brands "Family"
Coke w/Diet "Family"	-1.752	0.636	0.081	0.018	0.306	-0.036	0.096	0.499	0.094	0.314	-0.361
	(0.156)	(0.124)	(0.065)	(0.034)	(0.069)	(0.059)	(0.055)	(0.091)	(0.053)	(0.075)	(0.153)
Coke w/Diet "Single"	0.569	-1.346	0.094	-0.075	0.031	-0.114	0.355	0.485	0.251	0.079	-0.414
	(0.136)	(0.141)	(0.061)	(0.032)	(0.066)	(0.055)	(0.058)	(0.082)	(0.050)	(0.072)	(0.150)
Fanta "Family"	0.043	0.383	-2.287	0.150	-0.225	0.315	-0.028	0.860	0.360	0.378	-0.041
	(0.250)	(0.149)	(0.161)	(0.080)	(0.138)	(0.134)	(0.066)	(0.161)	(0.105)	(0.127)	(0.210)
Fanta "Single"	0.323	-0.207	0.463	-1.592	0.211	-0.181	0.170	-0.069	0.889	0.165	-0.282
	(0.326)	(0.181)	(0.199)	(0.179)	(0.208)	(0.227)	(0.080)	(0.241)	(0.166)	(0.185)	(0.266)
Other Coke "Family"	1.226	0.299	-0.150	0.053	-2.682	-0.094	0.246	0.550	0.126	-0.185	0.510
	(0.219)	(0.146)	(0.120)	(0.073)	(0.174)	(0.128)	(0.064)	(0.154)	(0.100)	(0.123)	(0.216)
Guar Antar "Family"	0.118	0.194	0.377	-0.044	0.009	-2.481	0.052	0.635	0.333	0.749	-0.057
	(0.202)	(0.138)	(0.113)	(0.077)	(0.127)	(0.177)	(0.062)	(0.153)	(0.102)	(0.127)	(0.190)
Guar Antar "Single"	-0.139	1.095	-0.088	0.017	0.202	-0.153	-1.370	0.046	0.111	-0.109	0.307
	(0.141)	(0.131)	(0.062)	(0.034)	(0.070)	(0.058)	(0.064)	(0.082)	(0.051)	(0.074)	(0.155)
Pepsi "Family"	0.771	0.695	0.683	-0.242	0.286	0.332	-0.288	-2.243	0.573	0.182	-0.759
	(0.329)	(0.180)	(0.184)	(0.119)	(0.206)	(0.209)	(0.082)	(0.324)	(0.167)	(0.192)	(0.288)
Pepsi "Single"	-1.077	0.997	0.589	0.768	-0.095	0.489	-0.020	1.229	-2.272	-0.069	-0.559
	(0.450)	(0.226)	(0.259)	(0.173)	(0.289)	(0.306)	(0.100)	(0.349)	(0.320)	(0.267)	(0.382)
Other Ambev "Family"	0.509	-0.490	0.264	-0.059	-0.757	0.882	-0.390	0.305	-0.016	-2.982	2.703
	(0.338)	(0.191)	(0.177)	(0.110)	(0.200)	(0.213)	(0.090)	(0.234)	(0.156)	(0.260)	(0.304)
B brands "Family"	0.250	0.414	0.207	0.021	0.376	0.098	0.330	0.340	0.165	0.831	-3.166
	(0.140)	(0.130)	(0.064)	(0.034)	(0.075)	(0.060)	(0.057)	(0.102)	(0.059)	(0.082)	(0.181)

Notes: Matrix corresponds to Robustness test 4 of the printed Appendix. Sales through self-service outlets. Standard errors in parentheses.