
INTERDEPENDENT PREFERENCES AND GROUPS OF AGENTS

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Abstract

An individual's preferences are assumed to be malleable and may be influenced by the preferences of others. Mutual interaction among individuals whose preferences are interdependent powers a dynamic process in which preference profiles evolve over time. Two formulations of the dynamic process are presented. One is an abstract model in which the iteration of a mapping from profiles to profiles defines a discrete time dynamic process; the other is a linear discrete time process specified in more detail. Examples motivate the model and illustrate its application. Conditions are given for the existence of a stable preference profile—a rest point of the dynamic process. A stable profile is naturally associated with a division, not in general unique, of the set of agents into subgroups with the property that preference interdependencies within a subgroup are “stronger” than those across subgroups. The conventional case in which each agent's preference relation is exogenously given is, in this model, the special case where each subgroup consists of just one agent.

1. Introduction

A considerable part of economic activity is carried on in groups, including households as well as other formal and informal organizations. This is perhaps especially so in the area of political-economic activities. Indeed, a society is sometimes viewed as a collection of overlapping groups.

An individual belongs to a family, to a circle of friends; perhaps works for a firm; belongs to religious organizations, to clubs, to political groups, and so on. How are we to understand the group structure of an economy

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or a society? In some cases economic theory treats groups, such as households, as economic agents. This raises the question of whether and how a household or other group should be characterized as an agent (see Samuelson 1956). In some cases an individual's membership in a group may be predetermined (e.g., family); in others it is a matter of choice. In any case, the degree and nature of participation in the group as distinct from membership typically involve choice. This is reflected in the model presented in Section 3.

According to the rational choice approach a group is to be understood in terms of the individuals who make up that group. To the extent that a group acts collectively, it presents a social choice problem: how is the joint action of the group related to the preferences of the individuals who constitute the group? If their preferences are too different, then they may not be able to find a mutually satisfactory collective action. If individuals can influence one another's preferences, then it is possible that the social choice problem of the group is mitigated. For example, if the members of the group come to have the same preferences, over the objects relevant to the choice at hand, then the problem of choosing a group action is trivial.

In economic theory preferences are taken as given. For good reasons they are usually taken as primitives. Yet from a descriptive standpoint it is generally understood that preference relations are in some sense endogenous—socially influenced, if not determined. It seems plausible that individuals are not born with a preference for the music of Beethoven over that of the Beatles or for rice over wheat. Yet persons from Japan reveal a preference for rice, while those from the United States may reveal a preference for wheat. Moreover preferences seem to change over time. Some who at an earlier stage of life preferred the music of the Beatles later come to prefer the music of Beethoven. The idea that an individual's preferences are shaped by others, perhaps through some process of socialization, is certainly not new. Relationships with others that persist for some time and involve interactions in activities important to the participants gives them opportunities to “get to know one another.” In more intimate relationships, such as those in families, or in some work situations, people come to know a good deal about one another's preferences and to influence those preferences. One way, among others, in which a person can influence the preferences of another is to provide a new experience for the other that leads to a change in her preferences. This is perhaps most germane when her preference relation was originally incomplete. Preferences can also be influenced by public forms of communication and interaction, such as exposure to advertising or to entertainment media.

Beyond unwittingly influencing another's preferences, a person can intend to influence the preference ordering of another. The desire to change the preference orderings of others can derive from a desire to change their actions in a given situation, or in a class of situations, or

from a direct concern with their preference orderings. In a situation in which an outcome depends on the actions of other agents, and in which the relation between their actions and the consequences cannot be changed, then, in the rational choice framework, the only hope of changing the actions of those agents is to change their preferences.¹ If preferences are malleable, a potential influencer may have means to push them in a direction more to her liking.

An individual who is contemplating joining a particular group and who is aware of the potential interaction of the preferences of others in the group with her own preferences should take this into account in deciding whether or not to participate in that group. All this can result in behavior that is not easily explained in a framework in which preferences are assumed to be immutable. The common notion of an opinion leader or tastemaker suggests that a group may be influenced by a member and vice versa. In economic theory the behavior of a group is to be understood in terms of the behavior of agents, and the behavior of agents is understood according to the rational choice paradigm. According to this paradigm a *situation* is specified by a set of alternative actions and their consequences. An agent is assumed to have a preference ordering over the set of consequences; this induces a preference ordering over actions.² But, there are situations that cannot be understood satisfactorily in this framework when preferences are assumed to be fixed.³ A situation of this kind involving groups of agents is presented in Example 2.2 below.

Malleability of individual preferences and the desire of one agent to influence the preferences of another can play a role in the formation of groups. This is particularly interesting in connection with political action groups. Groups of that kind usually comprise many agents, in contrast to more intimate groups such as families. The media of mass communication are often used by those who seek to influence a large number of people. Section 2 contains an example (Example 2.2) in which political-economic action turns on the ability of a small group to change the preferences of a sufficient number of agents in a large group.

¹ It is sometimes possible to influence the action of an agent with given preferences over a given set from which an action must be chosen by invoking a constraint not already expressed by preferences or by the set of available actions and their consequences. The mother who whispers to her child, "You must not take the last cookie," is an example.

² If the situation is one in which the consequences of an action are uncertain, or incomplete, the description of the situation becomes more complicated, involving information and beliefs. But it remains the case that once the situation has been completely specified, the agent's action is determined by his preference relation over the set of consequences. If the situation is one in which several agents interact strategically in a game, the notion of "best action" depends on the solution concept of the game. But it remains true that given a full specification of the situation, including the game with its solution concept, the behavior of a player depends on his preferences.

³ It is often possible in such a situation to introduce by ad hoc assumption new elements that allow the situation to be rationalized with fixed preferences, but this should not necessarily be regarded as resulting in a satisfactory understanding of the given situation.

It seems clear that close and persistent interactions with others can influence one's own patterns of behavior. Whether this phenomenon is best analyzed as interchange of information altering beliefs when preferences are fixed, or as interactions that change preferences, no doubt depends on the particular problem setting. Nevertheless, the two approaches are not equivalent. For example, Alfredo and Bettina are thinking about whether or not to marry. They are strongly attracted to each other—they are in love. They have been dating for some time and have come to know one another's likes and dislikes quite well. Alfredo prefers high style modern surroundings and Italian food and clothing; is gregarious and likes to party; is not interested in children. Bettina prefers traditional surroundings, antique furniture, and oriental food; likes to spend time at home pursuing her private activities; and wants to have a large family. They each know these things. While neither can predict the future, there is no reason for either of them to anticipate that the other dimensions of life will change in any particular way.

The standard model would lead each of them to calculate expected payoff in a game in which each knows the payoff structure or has beliefs about it. In the case described, we may suppose that beliefs about the other's payoff are very close to certainty (probability one). In that framework, the decision depends on whether the contribution to payoff from being together and in love outweighs the deductions from payoff that result from mismatched preferences. One possible consequence of the mismatch of preferences is that it will be very difficult for them to arrive at mutually acceptable joint decisions.

In an alternative model, each of them recognizes the possibility that the preferences they now have may be changed as a result of continuing interaction with the other. Alfredo could think that if he married Bettina, his preferences would come much closer to hers than they do now. He could contemplate this with horror or equanimity (i.e., he has preferences over his and her preferences), and his decision would turn on which of those is the case. Similarly for Bettina. The outcome could well be that the standard model would predict that they do not marry, while the alternative model predicts that they do, or vice versa.

More generally, a decision to join a certain group carries with it among its consequences the effect of the group on the individual who joins and in addition the effect of the joiner on the rest of the group and perhaps also on its future history.

This paper formalizes a notion of interdependent preferences and relates the group structure of the set of agents to interdependencies among their preferences. In this formulation preference relations, usually taken as a primitive, become endogenous. If preferences are endogenous something else must be the exogenous. The new primitive formal concept can be interpreted as expressing an agent's susceptibility to influence by others. With this we model the way in which the preferences of one agent interact with and are influenced by the preferences of others. The process

of mutual interaction of preferences is presented as a dynamical system. This system is presented in two related ways: first, in terms of a specific linearized dynamic process of mutual evolution of preferences over time; second, in terms of a mapping between preference profiles and individual preference relations, (therefore also preference profiles) whose iteration determines a dynamic process of the evolution of preferences. The first dynamic formulation is a specific example of the more general and abstract second formulation. It is used to make the dynamic process more specific and to analyze some examples of interdependent preferences. The second formulation is used to analyze limiting or equilibrium profiles and to show how the structure of interrelations of preferences is related to the group structure of the set of agents.

In this formulation an agent's knowledge of the preference relation of another may be partial, but it is knowledge (i.e., there is no uncertainty about what is known).

Endogeneity of preferences raises at least two analytical questions. The first is about the existence of a consistent preference profile. The second is normative: does a concept of optimality, for example, Pareto optimality, make sense when present action may lead to a situation in which the present actors have new preferences? The problem raised by these questions is illustrated in Example 2.3 of Section 2. Conditions under which consistent profiles of interdependent preferences exist are given in the Appendix.

We note that interdependence of preferences is a kind of externality, one that is different from more familiar preference externalities in which the preference of one agent depends on the consequences accruing to another or even the utility of another.

There is a substantial literature in which externalities in preferences play a role. An important motivation for this literature was the need to understand the family or household as an economic unit (Samuelson 1956; Becker 1981). A part of this literature is aimed at bringing inside the framework of rational choice theory the phenomenon of uncompensated unilateral transfers of resources from one individual to others within families, within or across generations. These models are referred to as models of "altruism." (See, e.g., Bergstrom 1996; and the references given therein.) The models in this area are in terms of utility functions (or functionals) that depend on the "level of happiness" of other agents, in addition to the usual arguments of utility functions. The level of happiness of another is represented by the value of a variable that enters the utility function of the "caring" agent in the same way that the measure of any ordinary commodity does. In models of this kind it is generally assumed that the happiness of an agent is represented by the value of her utility function. This carries us back into the world of the utilitarians; it assumes that each agent has a cardinal utility scale, invariant under positive linear transformation, that measures "happiness."

Bergstrom (1991) presents a careful formulation of this model. In Bergstrom's model a utility function that has the happiness of another as an argument is assumed to be weakly separable in the sense that comparison by agent i of two allocations that differ only in the consumption of agent i is independent of the happiness measure of other agents. Even with this restriction the utility to agent i depends on the utility measures of other agents in the sense that changing the measure of utility (e.g., by a positive monotone transformation) of an agent whose utility value enters agent i 's utility function changes the utility function of agent i . The utility measures that serve as measures of happiness cannot be ordinal. In dealing with this problem, Bergstrom defines two systems of utility functions that have the values of others' utility functions as arguments to be equivalent if they induce the same preferences over allocations. He further restricts attention to additively separable utility functions and shows that, even in that restricted framework, two utility functions are equivalent (induce the same preferences) if and only if one of them is a monotone increasing affine transformation of the other.⁴ Thus, it is no accident that models of altruism typically work with additively separable utility functions.

The idea of interdependence of preferences considered in this paper is different from that expressed in the altruism model. In the latter model the preference relation of each party in an altruistic relationship is fixed. A change in the preferences of a target of altruism does not produce a change in the preferences of the altruist, any more than a change in the quantity of an ordinary commodity consumed by the altruist results in a change in his preference relation. Neither does a change in the utility function of the altruist produce a change in the utility function of his beneficiary.

The interdependency of preferences considered in this paper is not aimed at rationalizing uncompensated unilateral transfers among agents, though it does present a formulation using only ordinal preferences in which an agent can care about the welfare of another, and in which uncompensated transfers from one agent to another can be rational. This is part of what is shown in Example 2.1 in Section 2.

The idea that people know one another's preferences is not new to economic theory; it is part of the common knowledge assumption in the theory of games of complete information. However, there preferences are fixed—immutable. The focus is on rational action in the strategic setting defined by those preferences and the possibilities of action. There are other ideas that have the flavor of making room for interdependent subjective or psychological elements in formal models of economic or political behavior. Geanakoplos, Pearce, and Stacchetti (1989) introduced a class of psychological games in which payoffs are allowed to depend on

⁴See Bergstrom (1991).

beliefs of the players in an incomplete information game. The interpretation of these games includes modeling emotions, such as anger and surprise, as influencing payoffs like preferences do. (See also Gilboa and Schmeidler 1988.) Because beliefs are incorporated into payoffs, interactions among the players take place at a psychological level as well as at the level of action. Emotions may be responses to the perceived intentions of others as well as to their actions. This phenomenon is studied further by Rabin (1993) in situations where a player's perception that another player is trying to hurt him, or to be kind to him, has an effect on the actions of the perceiving player.

In this paper interdependence of preference relations is studied directly. A model in which the preference relation of one agent can change as a result of interactions with others makes possible explanations of certain kinds of behavior that are difficult to understand in other ways. (The first two motivating examples with which the next section begins are instances.) A second, but central, focus of the analysis is on the relationship between the structure of groups and the pattern of interdependence of preferences. This comes out concretely in Example 2.2 and generally in the analysis of rest points of the dynamic process that models the change in preference relations.

The focus of attention in this paper is on the mutual interaction of preferences and the consequences for the group structure of society. The rest of the rational choice model, namely, the determination of actions chosen, is virtually ignored here. When needed in the examples, the simplest model of choice of action given preferences is used. The set of objects over which preferences are defined is not given any particular properties other than that it is a set, except in the Appendix, where a fixed point theorem is used to demonstrate existence of a stable preference profile. It seems possible to combine this approach with any theory of choice that involves preference relations for individuals, for example, those discussed in Sen (1997).

The rest of the paper is organized as follows. Three examples presented in Section 2 motivate and subsequently illustrate the use of the formal structures presented in the following sections. Section 3 contains a formal model whose features include provision for incomplete preference relations. The idea of preferences over preference relations arises as motivation of agents in some of the examples. That idea is not treated formally in this paper. Interpersonal influences operating on preferences are, to begin with, modeled by a discrete time adjustment process whose states are preference profiles. The process is first illustrated by applying it to a familiar example from social choice theory—the example involving three objects of choice and three agents whose initial preference orderings form the familiar cyclic preference profile. In this example, with the specified parameters, the process comes to rest at a state in which all agents have the same preference ordering.

Considered more abstractly the adjustment process defines a mapping, denoted U , from the set of preference profiles to itself, whose iteration gives the steps of the process.

In Section 4 the adjustment process is applied to the three motivating examples presented in Section 2. The first example involves a small number of agents, among them two agents who are already in a close relationship. It shows in detail how the more concrete model of interdependent preferences operates.⁵

The second example involves a large number of agents in a situation in which actions have both an economic and a political aspect. In this example the malleability of preferences is indispensable to a satisfactory understanding of the situation.

The third example illustrates some of the complexities that can arise from interdependence of preferences and illustrates the relevance of preference interdependence to the formation of groups.

In Section 5 we discuss the fixed points of the mapping U in terms of the structure of subgroups of interacting agents. Propositions A1 and A2 show that, under reasonable and weak conditions, the fixed points of U consist of fixed points of related mappings whose iteration describes the interacting influences within each of the subgroups of agents that together make up the subgroup structure of the society. Proposition A3 shows that a group structure of the set of agents that expresses the pattern of interdependence embodied in the mapping U must reflect indirect dependencies as well as immediate ones. This leads to a partial ordering among group structures that may be useful in determining the group structures that are most likely to form.

Section 6 contains a short summary and comments about future lines of research.

The Appendix contains the formal presentation of Propositions A1, A2, and A3 and their proofs. It also presents a theorem giving sufficient conditions for the existence of fixed points of the mapping U when the set of objects of choice is a complete metric space and the mapping involved is a contraction. This covers the case where the choice set is Euclidean and also the case where it is a finite set.

2. Three Examples

We begin with informal presentations of the three examples intended to motivate the model presented in this paper, and subsequently to illustrate its use.

⁵In this example similar conclusions might be reached in a model with fixed preferences but with additional assumptions about complementarities among objects of choice and about beliefs. It is not evident why the required complementarities should exist.

2.1 Father and Son

There are a father, Benjamin, and his son, Franklin, who is about to go to college. He has been admitted to two universities, P and E. P is a large state university with excellent academic programs and a lively popular culture scene. There is also a small group of students interested in high culture activities. E is a large private university with excellent academic programs, a pervasive high culture environment, and a small pop culture scene. E costs significantly more than P. Benjamin is in the process of deciding what to do about his son's education. Benjamin knows that his son is a good student and also that he is much taken with popular culture. The father cares about his son. He cares about his son's lifestyle. The father thinks that many of the finer things in life are associated with high culture and would like to enjoy them with his son. Benjamin could transfer income to his son directly or by paying for his education. He could also transfer income in kind, thereby controlling his son's lifestyle to some extent. But none of these actions are in his opinion likely to produce what the father wants, namely, that the son would himself freely choose a lifestyle with a significant high culture component and enjoy it.

Benjamin thinks that if he offers to pay for his son's education and allows Franklin to choose which university to attend, then Franklin, given his current preferences, would choose P. At P he would be as well prepared for his economic future as he would at E. However, in the father's view, life at P would reinforce his son's preference for pop culture; the son would graduate with pop culture preferences and choose a pop culture lifestyle. The father thinks that if Franklin were to attend E, he would be exposed to influences pushing in the direction of high culture, and as a result Franklin might experience a change in preferences, emerging from E with a preference for high culture. This is the result the father wants. On the other hand, Franklin could join the pop culture minority at E and retain his present preferences and lifestyle. Unless there is a good chance that his son's preferences will change, Benjamin could end up paying the higher cost of E, with the same result as could be achieved at lower cost at P.

Note that the father's concern for the son is not directly about the son's "happiness." The father's concern would not be modeled adequately by assuming that the father's utility depends on the value of the son's utility. Nor would his wishes be satisfied by making gifts in kind to the son, for example, subscriptions to high culture activities. After all, if Franklin does not enjoy them, they are a waste of money and effort. The father's concern is expressed more accurately by assuming that he has preferences over the possible preference orderings of the son. If the son's preference ordering is one that the father prefers, then the son's choice in every situation that might arise would be one that the father also prefers for the

son. In the extreme case that their preferences agreed they would make the same choices in every situation they might face, and the father would know this without knowing the specific situations that might arise.

Benjamin's preference ordering over the possible preferences of Franklin has at least two possible interpretations. One is that it arises from concern for Franklin's experience of life. The other is that Benjamin would like them to enjoy each other's company over the rest of his own life and considers this to be more likely if their interests and tastes are compatible.

The father's preferences over the preferences of the son would be merely wishful unless the son's preferences were malleable. If the son's preference relation is given and not open to change, then it is what it is; the father's wishes are without consequence.

Furthermore, Benjamin is aware that Franklin has been exposed to high culture in the past; it is not information that Franklin lacks and that, if he had access to it, would lead him to choose a life with a substantial component of high culture. For Benjamin to justify paying the high cost of E he must believe that Franklin's experience at E will change his preferences. Such a change must, under the assumptions of the example, come about through interactions with the students at E, especially those who prefer a high culture lifestyle. The father supposes that preferences can be altered as a result of a process of mutual interaction in which the preferences of those involved influence one another. In addition, Benjamin must believe that his son is open to such influences—that he will associate with those with whom interaction might lead to changing his preferences and that he will be an interested and willing participant in such associations.

2.2 Forest and Birds

There is a mature forest located in a national forest in the Pacific Northwest. The trees are suitable for harvest, which would yield valuable lumber. The forest is also the only remaining habitat of a rare bird. This bird would become extinct if the forest were cut down. Harvesting the trees requires the permission of an agency of the federal government.

Those who work in the lumber industry, and those whose incomes depend on the prosperity of that industry, want to harvest the trees. They are willing to give effort and resources to attain that goal. We refer to this group as "loggers." There is also a relatively small group of people, "preservationists," who want to preserve the old forest and the rare birds. They too are ready to give effort and money to attain their goal. The rest of the population is fully informed about the situation but consists of people who are indifferent to whether or not the trees are harvested. Harvesting (or not harvesting) the trees would have a negligible effect on their indi-

vidual economic situations.⁶ The loggers have hired lobbyists and have contributed to the campaign funds of their representatives in Congress to enlist their support for cutting down the forest. If nothing changes, permission to do so will be given.

In order for the preservationists to prevent cutting of the forest they must gain the active political support of a substantial number of those who are presently indifferent. The preservationists do not have the resources to pay off a sufficient number of the indifferent to change the outcome. Even though a tiny payment to an individual with conventional preference would tip her indifference to preference, this would be inadequate, because substantial action in support of the preservationist position is required for a sufficient political effect. Even if the preservationists could find the resources to pay off a sufficient number of the indifferent, it would almost certainly be cheaper to try to change their preferences. In this situation individual preservationists would try to change the preferences of those they know among the indifferent. The group of preservationists would try to influence the preferences of a wider group of fully informed but indifferent agents. A change in preferences from indifference to the preservationists' preferences would mean that cutting the forest would have an external effect on the agents whose preferences are changed, reducing their well-being substantially. Those whose preferences have changed would then be willing to devote some effort and resources to political activities aimed at preserving the forest and the birds—having the same preferences as the preservationists, they would behave the same way in the same situation.

In the attempt to change preferences of the indifferent, preservationists must try to reach a group of the indifferent larger than those whom they might already know personally. They could try to use news media free of charge by arranging newsworthy events, such as demonstrations and other forms of street theater, as well as public meetings, where they could put their message before the general public, including a large number of the indifferent. They could also use their own limited resources to buy time on the mass media to broadcast their construction of the situation and their preferences to the indifferent.

If these efforts were to result merely in giving truthful information to the indifferent without changing their preferences, their efforts would have no effect on the outcome. In order to change the outcome, the information provided by the preservationists would have to change the preferences of originally indifferent fully informed agents, thereby making them willing to support or actively engage in political activity to save

⁶Harvesting the trees would (*ceteris paribus*) increase the supply of a particular kind of lumber, which would (*ceteris paribus*) lead to some decrease in the prices of things made from that kind of lumber. We assume that these general equilibrium effects are small and taking account of them would only complicate the analysis without any compensating benefit to understanding. We therefore ignore them from here on.

the forest and the birds. This must happen to a sufficient number of those agents to produce the necessary political effect. We emphasize that in this example changing the information of indifferent agents without changing their preferences is not sufficient to change their behavior.

The next example demonstrates that when preferences are interdependent there may not be a consistent preference profile. It exhibits a type of preference externality that is different from the more usual concept of preference externality in economic theory. The example also shows how interdependence of preferences can affect the structure of groups of agents.

2.3 Cain and Abel⁷

There are two brothers, Cain and Abel, who must share an inheritance. The inheritance consists of a watch and a ring. The will rules out any allocation in which one brother gets nothing. Therefore, there are two feasible allocations, denoted (r, w) and (w, r) , where the first component indicates what Abel gets and the second what Cain gets. Each brother has preferences. We distinguish several cases. First, each brother might have “selfish” preferences. In that case each brother ranks the allocations according to what that brother gets in each allocation. In a second case, each brother cares about what the other gets, and therefore his preference over allocations depends on the entire allocation. In the present example the two cases give the same strict orderings (ruling out indifference), because there are only two possible allocations. These orderings are

$$(r, w) > (w, r)$$

or

$$(w, r) > (r, w).$$

They form four possible environments. These are shown in Table 1, whose entries are the Pareto optimal allocations corresponding to the four possible combinations of preferences of the two brothers.⁸

Table 1:

Prefs: <i>Cain</i> → <i>Abel</i> ↓	$(r, w) >_C (w, r)$	$(w, r) >_C (r, w)$
$(r, w) >_A (w, r)$	(r, w)	$(r, w), (w, r)$
$(w, r) >_A (r, w)$	$(r, w), (w, r)$	(w, r)

⁷Example 2.3 is taken from Reiter (1970).

⁸In the two cases where the Pareto optimal set is not a singleton, several ways in which the brothers could agree on one of the Pareto optima suggest themselves, for example, toss a coin or, if money is available, have one brother make a side payment to the other.

The four environments shown in Table 1 exhaust the possibilities allowed in conventional preference models.⁹ However, another possibility remains. One or the other brother might be motivated by other considerations. For instance, Abel might want Cain to have the allocation that Cain prefers. In that case Abel's preference ordering over the allocations would depend on Cain's preference ordering. Thus, Abel's preference ordering, denoted $>_A$, would be

$$(r, w) >_A (w, r) \Leftrightarrow (r, w) >_C (w, r)$$

and

$$(w, r) >_A (r, w) \Leftrightarrow (w, r) >_C (r, w).$$

We assume that this is the way Abel's preference relation depends on Cain's, and we consider three different alternative preference conditions for Cain.

If Cain's ordering is independent of Abel's, then the possible combinations of preferences and the resulting Pareto optima are as shown in Table 2.¹⁰ The off-diagonal boxes in Table 2 are blank, because those cases cannot arise from the given preferences.

⁹The case in which one brother's utility depends on the utility level of the other reduces to the case in which the brothers' preferences depend on the entire allocation. Thus, let U^i be the utility function of brother i , $i = 1, 2$, and suppose that brother 1's utility depends on the utility value of brother 2. Then $U^1(x, u^2) = U^1(x, U^2(y)) = \bar{U}^1(x, y)$, where $u^2 = U^2(y)$, and x and y denote what Cain, and Abel get, respectively. Here we ignore issues of ordinality.

¹⁰This case models an example of Ragnar Frisch and is quoted in Sen (1997), as follows.

Assume that my wife and I have had dinner alone as we usually do. For dessert two cakes have been purchased. They are very different, but both are very fine cakes and expensive—according to our standard. My wife hands me the tray and suggests that I help myself. What shall I do? By looking up my own total utility function I find that I very much would like to devour one particular one of the two cakes. I will propound that this introspective observation is *completely irrelevant* for the choice problem I face. The really relevant problem is: which of the two cakes does my wife prefer? If I knew that the case would be easy. I would say "yes please" and take the *other* cake, the one that is her second priority.

Sen's interest in this example is in terms of concepts of menu dependence or chooser dependence of choice and of fiduciary responsibility. It seems more straightforward to treat the example as one in which Frisch's preference relation depends on his wife's, while hers, as presented in the example, does not depend on his. (If her preferences relation depended on his, the Pareto optimal allocations would also be as shown in Table 2.) The formulation in terms of interdependence of preferences seems to capture this story more directly and naturally than the concepts of menu dependence or chooser dependence or fiduciary responsibility, and it is not as dependent on the particular menu from which choice is made. The latter concept would, it seems, encounter more complexities when the fiduciary agent acts for more than one person.

Table II:

Prefs: $Cain \rightarrow Abel \downarrow$	$(w, r) \succ_C (r, w)$	$(w, r) \succ_C (r, w)$
$(r, w) \succ_A (w, r)$	(r, w)	
$(w, r) \succ_A (r, w)$		(w, r)

On the other hand, Cain’s preferences might depend on Abel’s. One case arises if Cain wants Abel to have the allocation that Abel prefers. Then, Cain’s preference ordering would satisfy

$$(r, w) \succ_C (w, r) \Leftrightarrow (r, w) \succ_A (w, r)$$

and

$$(w, r) \succ_C (r, w) \Leftrightarrow (w, r) \succ_A (r, w).$$

In that case both feasible allocations are Pareto optimal, as in Table 2.

In the remaining case Cain resents Abel and prefers to frustrate Abel whenever possible. In that case Cain’s preference ordering depends on Abel’s as follows.

$$(r, w) \succ_C (w, r) \Leftrightarrow (w, r) \succ_A (r, w),$$

$$(w, r) \succ_C (r, w) \Leftrightarrow (r, w) \succ_A (w, r).$$

This combination of interdependent preferences results in a fiasco, in which there is no fixed pair of preference orderings over the objects of choice. Consequently there are no Pareto optimal allocations. To see this, consider first the allocation (r, w) . Suppose for the sake of argument that Abel’s preference ordering is $(r, w) \succ_A (w, r)$. Then Cain’s preference ordering is $(w, r) \succ_C (r, w)$. But then Abel’s preference ordering would be $(w, r) \succ_A (r, w)$. It follows that (r, w) is not Pareto optimal, because both prefer (w, r) . Furthermore, since $(w, r) \succ_A (r, w)$, Cain’s ordering would be $(r, w) \succ_C (w, r)$, and hence Abel’s would then be $(r, w) \succ_A (w, r)$, and (r, w) would be Pareto optimal. But this is where we started.

In this case the interdependence of Cain’s and Abel’s preferences does not admit a well-defined preference profile. Because there is no well-defined preference profile, there is no well-defined Pareto optimal set in this environment.

Example 2.3 bears a certain resemblance to an example given by Sen (1970) and further analyzed by Gibbard (1974) in relation to a libertarian principle in the context of social choice theory. However, in spite of apparent similarities these examples are quite different. Sen’s example involves two neighbors, Mr. 1 and Mrs. 2. Mr. 1 wants the color of his bedroom wall to be the same as that of his neighbor, Mrs. 2; the neighbor

wants the color of her walls to be different from that of his walls. A social state can be represented by an ordered array in which the first component is the color of his wall and the second the color of her wall. Thus, if $x = (x_1, x_2, z)$ and $y = (y_1, y_2, z)$ are states (here z denotes the rest of the social state) Mr. 1 prefers x to y if and only if $x_1 = x_2$ and $y_1 \neq y_2$, while Mrs. 2 prefers x to y if and only if $x_1 \neq x_2$ and $y_1 = y_2$. Suppose the set of attainable states is $\{x, y\}$. Then with these preferences, the set of Pareto optimal states is also $\{x, y\}$. Preferences are well defined and the Pareto set is well defined, whereas in the example of Cain and Abel neither of these is the case when Cain is spiteful.¹¹

In Sen's example each agent's preference relation depends on components of the state or allocation other than those that refer only to that agent. This is a case of a conventional externality in preferences. In the example of Cain and Abel, each agent's preference relation can depend on the preference relation of the other, as well as on the entire state or allocation.

3. Formal Model

There is a finite set of agents, denoted $I = \{1, \dots, N\}$. There is a choice set X , which is the set of objects of choice. All that is assumed about X here is that it is a set.¹² The elements of X might be allocations; or they might be states that are consequences of actions, such as result from actions of the government; or they might be actions, like acts in the sense of Savage. An element of X could consist of a part that identifies an allocation and another part that identifies other kinds of outcomes. In some special cases, $X = X^1 \times \dots \times X^N$, where X^i is the set of conceivable states for agent i .

DEFINITION 1: *A preference structure is a pair (Z, W) , where Z is a set (the choice set) and W is a set of complete preorderings on Z .*

For $x, y \in Z$ and $\nu \in W$, we write, as usual, $x\nu y$ for “ x is weakly preferred to y according to ν ”; we also write $x >_\nu y$ for “ x strictly preferred to y according to ν ” and $x \sim_\nu y$ for “ x indifferent to y according to ν .” Where there is no ambiguity we can write $x \succeq y$ or $x \succeq_{\nu^i} y$ for $x\nu^i y$, and so on.

By taking $Z \subset X$, $Z \neq X$, Definition 1 allows us to consider preference relations on X that are not complete. This allows us to address formally the possibility that the influence of one agent i on the preferences of another j is confined to a subset of the objects of choice. For example, a

¹¹Both Sen and Gibbard are studying properties of social choice functions, and neither questions the fact that preferences are well defined in Sen's example or that there are Pareto optimal states in that example.

¹²More structure is assumed in the second part of the Appendix.

member of an academic department might influence the preference relation of a colleague over recruiting decisions but not over domestic arrangements. It also allows that possibility that an element $x \in X \setminus Z$ can provide an agent with a new (to her) experience that leads to an extension of her preference relation on Z to one on $Z \cup \{x\}$ or a superset of that set.

We write V_0 for the set of complete preference relations on X .¹³ Thus,

$$V_0 = \{\nu \subseteq X \times X: \nu \text{ is a complete preordering on } X\}.$$

We write

$$V = \underbrace{V_0 \times \dots \times V_0}_{N \text{ times}} \equiv V_0^1 \times \dots \times V_0^N.$$

V is the set of preference profiles.

We also write the preference profile of the agents other than i as

$$V^{-1} = V_0^1 \times \dots \times V_0^{i-1} \times V_0^{i+1} \times \dots \times V_0^N.$$

For $i \in I$ define

$$U^i: V \rightarrow V_0^i; U = (U_1, \dots, U^N), U: V \rightarrow V^{14}$$

The mappings U^i represent the dependence of agent i 's preference relation over objects in X on the preference relations of other agents. The "vectorial" mapping U , which maps preference profiles into preference profiles, expresses the pattern of mutual interdependence among preferences. Iteration of U defines a discrete time dynamical system for the formation or evolution of preferences.

In this section we also present a more concrete simpler dynamical system that is used to analyze examples.

The possibility that one person has preferences over the possible preferences of another arises both in the motivating example of Benjamin and Franklin and in the example of the forest and birds in Section 2. While preferences over preference relations motivates agents in these and other examples, we do not formalize that concept here. To do so would introduce complexities that it seems best to avoid for now.

There are several ways in which agents influence one another's preferences. First, when people are in close proximity for an extended period of time they come to know one another's preferences, at least in some restricted domain, and to influence one another's preferences. Second, broadcast communication like advertising can provide information, includ-

¹³In the dynamic model presented in this section, preferences are not always assumed to be complete on X .

¹⁴This mapping, U , referred to in Section 1, summarizes the effect on each agent's preference relation of his interactions with other agents. (See the discussion following Table 4a.)

ing information about preferences, perhaps in encoded forms, simultaneously to groups of agents. If in a group of agents, a few of them respond to the advertiser's message, the internal processes of the group can spread the change in preferences to other agents in the group.

Another way in which people can influence one another's preferences is that one may provide another with experiences that he would not otherwise choose and thereby change the preferences of the agent undergoing the new experience. For example, suppose that an agent knows that a certain restaurant exists in his city but he has never visited this restaurant. Therefore, dining at this restaurant is in his choice set X , but because he has no experience of dining there, he is unable to compare it with alternatives he knows. Should this be modeled as part of a complete preference relation on X ? If so, how should this restaurant appear in the ordering compared to ones that have been experienced? The standard approach requires that the agent have preferences and prior beliefs about this restaurant, which might or might not lead him to try it. An alternative is that the preference relation is incomplete—that the restaurant not yet experienced cannot yet be compared to alternatives that have been experienced. If the agent is taken to dine at this restaurant, he would then revise his former incomplete ranking to include the new (to him) restaurant.

Ways of influencing the preference relation of an agent often operate together. One agent takes another to a restaurant, thereby providing a new experience; the host also shows enough enthusiasm for the experience to make clear his high opinion of the restaurant and his enjoyment of the experience, thereby influencing the guest also to think highly of it. However, the guest has some control over his response.

Formally, let the guest's initial preference structure be $(Y, \nu) \cup (Y, V_{0,Y})$, where $V_{0,Y}$ denotes the set of preference relations on Y consisting of restrictions of the elements of V_0 to Y . Dining at the new-to-him restaurant R is not an element of Y . After he has had the experience of R thrust upon him, he revises his preference relation from ν on Y to ν' on $Y \cup \{R\}$. If it is assumed that preference relations must be transitive, then ν' is in the transitive closure of an extension of the relation ν to $Y \cup \{R\}$. Thus the model presented in this section applies with $(Y, V_{0,Y})$ and $(Y \cup \{R\}, V_{0,Y \cup \{R\}})$ in place of (X, V_0) .

We suppose that the effect of influence over preferences takes place in steps. A simple way to model this is as a dynamic discrete time process in which the preference profile $\nu_t = (\nu_t^1, \dots, \nu_t^N)$ changes to $\nu_{t+1} = (\nu_{t+1}^1, \dots, \nu_{t+1}^N)$ in a deterministic way. We first consider a particular example of such a process and subsequently a more general version.

We begin the construction of the dynamic process by identifying the set of agents that a particular agent i can interact with. These are the agents whose preferences become known to agent i . Let $\Gamma: I \rightarrow I$ denote this correspondence. We sometimes refer to the set $\Gamma(i)$ as agent i 's circle.

We represent a preference relation on a choice set Z by its graph in $Z \times Z$.

(i) An *indicator* for a preference relation ν^i on Z is a function $\rho(\nu^i): Z \times Z \rightarrow \{-1, 0, 1\}$, where

$$\rho_{\nu^i}(x, y) = \rho(\nu^i)(x, y) = \begin{cases} 1 & \text{if } x \succ_{\nu^i} y \\ 0 & \text{if } x \sim_{\nu^i} y \\ -1 & \text{if } y \succ_{\nu^i} x \end{cases}$$

When $Z \subseteq X$, we can extend ρ_{ν} to all of X by defining $\rho_{\nu}(x) = \emptyset$, when $x \in X \setminus Z$.

Let $\nu = (\nu^1, \dots, \nu^N)$ and let $\rho_{\nu} = \rho(\nu) = (\rho(\nu^1), \dots, \rho(\nu^N)) = (\rho_{\nu^1}, \dots, \rho_{\nu^N})$. The function ρ encodes a preference relation, or a preference profile, in any preference structure, in terms of the alphabet $\{-1, 0, 1, \emptyset\}$.

To model the influence of the preferences of others on the preference relation of agent i , we begin by introducing the weight vector of agent i , intended to model i 's responsiveness to the preferences of others.

(ii) A *weight vector* for agent i , $i \in I$, is a vector $\alpha^i = (\alpha_1^i, \dots, \alpha_N^i)$, where $\alpha_j^i \in [-1, 1]$ and $-1 \leq \sum_{j=1}^n \alpha_j^i \leq 1$, and $\alpha_j^i = 0$, if $j \notin \Gamma(i)$.¹⁵

In general the weight vector of an agent depends on the subset of objects of choice (i.e., the weight α_j^i is the value of a function whose domain is the collection of subsets of the choice set X), as well as on the agents i and j . However, the examples we analyze here are sufficiently simple that we can specify the choice set so as to avoid having to consider subsets of it.

The weight vectors together can be written as a *weight matrix* $A = [\alpha_j^i]$, $i, j = 1, \dots, N$. The weight matrix is a primitive of the model. Along with initial preference relations, the weight matrix is the exogenous entity that replaces exogenous preferences.

The weights can be interpreted as follows. A person has qualities that are deeper and more enduring than preferences. These qualities together determine the *personality* or *character* of the individual—the kind of person that he or she is. These qualities in agent j perceived by agent i might lead i to include or exclude j in her circle and to give weight to j 's views and opinions, including j 's preferences.

The weight vector of an agent expresses the weight given by that agent to the preferences of each of the other agents. For instance, if $\alpha_j^i(t) > 0$, then agent j 's preferences influence the ordering of agent i in the next stage of the process in the direction of j 's preference relation in the current stage. If $\alpha_j^i(t) < 0$, then, if agent j prefers x to y , agent i is more

¹⁵The weights α_j^i are normalized. Suppose we started from numbers β_j^i ; $i, j = 1, 2, \dots, N$ that measure the influence of j 's preference ordering on i 's preference ordering, for some fixed value of i . Define $\alpha_j^i = \beta_j^i / \sum_{\beta_k^i < 0} |\beta_k^i|$ if $\beta_j^i < 0$; $\alpha_j^i = \beta_j^i / \sum_{\beta_k^i > 0} |\beta_k^i|$ if $\beta_j^i > 0$; $\alpha_j^i = 0$ if $\beta_j^i = 0$.

likely to prefer y to x in the next iteration of his preferences. If agent j is not in i 's circle, then agent i gives j 's preferences no weight.

We augment the model to take account of a more aggressive way of influencing preferences of agents. Each agent k can, at a cost, include herself in the circle of every other agent. The action of agent k that results in her inclusion in the circle of the other agents is to use a mass medium of communication to send a message to every agent. For example, agent k can broadcast a TV commercial aimed at changing the preference relations of other agents over some subset of alternatives. The commercial might show attractive people demonstrating by their actions the preference relation over a subset Y of X that agent k would like the receiving agents to have. In this model we ignore the superficial form of the message and focus on the subtext, which is the preference ordering that the sender of the message would like the receivers to have.

Each agent i whose circle includes k as a new member determines a coefficient β_k^i , as discussed in footnote 15, and therefore after normalization there is a new normalized weight vector for agent i , measuring the influence of all the agents, including agent k , who are in agent i 's circle. However, it is still possible that k 's message can be ignored by a receiving agent. That is, it is possible that $\beta_k^i = 0$, and hence $\alpha_k^i = 0$.

We will assume that the preference being transmitted is the restriction of agent k 's true preference relation to the specified subset of alternatives.¹⁶ We make use of this feature of the model, together with consideration of groups of agents, in analyzing Example 2.2.

The function $\tau_{\alpha,\nu}^i$ defined next represents the influence of all others taken together on agent i 's preference relation. We may call it *an influence indicator function for agent i* .

(iii) For $i \in I$ define $\tau_{\alpha,\nu}^i$ at the point $(x, y) \in Z \times Z$ by

$$\tau_{\alpha,\nu}^i(x, y) = \tau^i(x, y; \alpha, \nu) = \sum_{j \in \Gamma(I)} \alpha_j^i \rho_{\nu^j}(x, y),$$

with the convention that $\alpha_j^i \emptyset = 0$.

Note that for every α and profile ν , and for every $(x, y) \in Z \times Z$, $-1 \leq \tau_{\alpha,\nu}^i(x, y) \leq 1$.

(iv) For each $i \in I$, $\gamma^i \in [0, 1]$ is called a *critical value*, or *threshold*, for agent i .

Note also that $-1 \leq \tau_{\alpha,\nu}^i(x, y) \leq -\gamma^i \Leftrightarrow \gamma^i \leq \tau_{\alpha,\nu}^i(y, x) \leq 1$.

For each agent i , consider the collection of pairs of objects of choice for which the influence indicator function exceeds the threshold. Thus, define

$$T_{\alpha^i, \nu^i}^i(\gamma^i) = \{(x, y) \in X \times X : \tau_{\alpha^i, \nu^i}^i(x, y) \geq \gamma^i\}.$$

¹⁶The possibility of sending a message to other agents that could change their preferences suggests that the sender should or would behave strategically and therefore so would the receiving agents. We postpone consideration of strategic behavior for another paper.

This set is the graph of a relation on X that is an extension of a transitive and incomplete relation on X . The extended relation may or may not be transitive. Such a graph can be modified to make it transitive.¹⁷ For convenience we drop reference to α and ν in the notation. If for some agent i and some γ^i , $T^i(\gamma^i)$ is not the graph of a transitive relation, then it contains an intransitive cycle; that is, there are points $a_0, a_1, \dots, a_q, a_{q+1}$, where $a_{q+1} = a_0$, in X , such that the pairs (a_j, a_{j+1}) , $j = 0, 1, \dots, q$ satisfy $\tau^i(a_j, a_{j+1}) \geq \gamma^i$. In other words, $a_0 \succ_i a_1 \succeq_i \dots \succ_i a_q \succ_i a_0$. To remove the intransitivity, we take the following steps.

If there is a value $\gamma^i < \bar{\gamma}^i \leq 1$ such that $T^i(\bar{\gamma}^i)$ is the graph of a reflexive, transitive relation, then replace γ^i by $\bar{\gamma}^i$.

If no such $\bar{\gamma}^i$ exists, then for each intransitive cycle $a_0 \succ_i a_1 \succeq_i \dots \succeq_i a_q \succ_i a_0$ in $T^i(\gamma^i)$, set $\rho^i(a_j, a_{j+1}) = 0$, for $j = 0, 1, \dots, q$. That is, make all the objects in the cycle indifferent to each other.

We illustrate the process first in a familiar example from social choice theory, the example with three agents, three objects, and a cyclic preference profile. Let the set of alternatives be $X = \{a, b, c\}$; let the set of agents be $I = \{1, 2, 3\}$; let the preference profile be $\nu_1^1 = a > b > c$; $\nu_1^2 = b > c > a$; $\nu_1^3 = c > a > b$. We suppose that the circle of each agent includes the other two. The weights are assumed to be

$$A = \begin{pmatrix} \alpha^1 \\ \alpha^2 \\ \alpha^3 \end{pmatrix} = \begin{bmatrix} .3, .4, .1 \\ .1, .4, .3 \\ .2, .4, .3 \end{bmatrix}$$

and suppose that $\gamma^i = 0.1$ for $i = 1, 2, 3$.

Table 3 shows the quantities $\tau_{\alpha^i, \nu_i^i}^i$, which determine the sets $T_{\alpha^i, \nu_i^i}^i(\gamma^i)$.¹⁸ These sets determine the transitions to the preference profile at $t + 1$.

Then, the sets $T_{\alpha^i, \nu_i^i}^i(\gamma^i)$ are

$$T_{\alpha^1, \nu_1^1}^1(\gamma^1) = T_{\alpha^1, \nu_1^1}^1(.1) = \{b, a), (c, a)\},$$

$$T_{\alpha^2, \nu_2^2}^2(\gamma^2) = T_{\alpha^2, \nu_2^2}^2(.1) = \{(b, c), (c, a)\},$$

$$T_{\alpha^3, \nu_3^3}^3(\gamma^3) = T_{\alpha^3, \nu_3^3}^3(.1) = \{(a, b), (b, c), (c, a)\}.$$

¹⁷The modifications proposed here to correct intransitivities perhaps have stronger justification in the context of individual preferences, as opposed to the context of social choice, where the problem is to construct a transitive social preference ordering. One the other hand, an anonymous referee has remarked that “the problem of removing intransitivities frequently influences public debates. A common criticism of an opposing position is that it has implications that are inconsistent with other widely accepted positions. This criticism can be fatal to a position that, in isolation, might otherwise be popular.” I am indebted to Roger Myerson and to Jim Jordan for discussions of this matter. Neither one is responsible for any mistakes.

¹⁸Table 3 is in the presence of transitivity sufficient to determine the values of $\beta_{\nu_i^i}^j$ and of $\tau_{\alpha^i, \nu_i^i}^j$ $j = 1, 2, 3$ on the remaining elements of $X \times X$. The same practice is followed in subsequent tables as well.

Table III:

$X \times X$	$\rho_{\nu_t^1}(\bullet, \bullet)$	$\rho_{\nu_t^2}(\bullet, \bullet)$	$\rho_{\nu_t^3}(\bullet, \bullet)$	$\tau_{\alpha^1, \nu_t^1}^1$	$\tau_{\alpha^2, \nu_t^2}^2$	$\tau_{\alpha^3, \nu_t^3}^3$
(a, b)	1	-1	1	.6	0	.3
(a, c)	1	-1	-1	-.2	-.6	-.5
(b, c)	1	1	-1	0	.2	.1

The first two sets are graphs of a reflexive, transitive relation, when the pairs of the form (x, x) are added, but $T_{\alpha^3, \nu_t^3}^3(\gamma^3)$ is not. We see that for $.1 < \bar{\gamma}^3 \leq 0.3$, $T^3(\bar{\gamma}^3) = \{(a, b), (c, a)\}$. The complete transitive ordering with this graph is the one shown in Table 4, where the preference profile ν_{t+1} is given by the indicator functions.

Table IV:

$X \times X$	$\rho_{\nu_{t+1}^1}$	$\rho_{\nu_{t+1}^2}$	$\rho_{\nu_{t+1}^3}$
(a, b)	-1	-1	1
(a, c)	-1	-1	-1
(b, c)	1	1	-1

Calculating $\rho_{\nu_{t+2}^1}$, $\rho_{\nu_{t+2}^2}$, and $\rho_{\nu_{t+2}^3}$ in the same way, we see that the profile at $t + 2$ is one in which all three preference relations are the same, namely, $b > c > a$. This is a rest point of the process.

It is instructive to consider the same example with

$$A = \begin{pmatrix} \alpha^1 \\ \alpha^2 \\ \alpha^3 \end{pmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}.$$

In this case Table 3a, the counterpart to Table 3 above, is

Table IIIa:

$X \times X$	ρ_t^1	ρ_t^2	ρ_t^3	τ^1	τ^2	τ^3
(a, b)	1	-1	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
(a, c)	1	-1	-1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
(b, c)	1	1	-1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

This results in the preference profile shown in Table 4a.

Table IVa:

$X \times X$	ρ_{i+1}^1	ρ_{i+1}^2	ρ_{i+1}^3
(a, b)	1	1	1
(a, c)	-1	-1	-1
(b, c)	1	1	1

We see that for any i and any $\gamma^i \leq 1$, $T^i(\gamma^i)$ consists of the pairs $(c, a)(a, b)(b, c)$, which form an intransitive cycle. Therefore, according to the procedure specified above, we set $\rho_{i+1}^i(a, b) = \rho_{i+1}^i(a, c) + \rho_{i+1}^i(b, c) = 0$, for $i = 1, 2, 3$. This seems to be a sensible outcome for this preference profile.

Taking $Z = X$, the adjustment process takes an array of preference relations in V and produces a new array of preference relations in V . Although we have described this as taking place in time, it defines a mapping from V to V that we can identify with the mapping $U: V \rightarrow V$; the process of evolution of the preference profile is the iteration of the mapping U . In the special process defined above, the pattern of dependencies expressed by U is specified by the weight matrix $A = \begin{bmatrix} \alpha^1 \\ \vdots \\ \alpha^N \end{bmatrix}$.

Before continuing with a more abstract formulation using the mapping U , we apply the special model to the motivating examples presented in Section 2. The first is the example of the father and son, Benjamin and Franklin.

4. Application of the Discrete Time Process to Examples 2.1, 2.2, and 2.3

Recall that the father’s choice between offering to pay for Franklin’s education and letting Franklin choose the university, and offering to pay only at E, turns on Benjamin’s analysis of what will happen to Franklin as a consequence of each choice of Benjamin. If Benjamin’s thinks his son’s preference relation is fixed, then, even allowing for the possibility that Franklin’s preference includes concern for Benjamin, he would do best to offer to pay and let the son choose. On the other hand, if the father believes that the son is open to the influences of new experiences and new associates, and if the father’s analysis of what is likely to happen to his

son is expressed by the iterative model of preference evolution presented above, he might prefer to take the other action, in effect forcing the son to go to E. The father's analysis in this example might go as follows.

The agents are $I = \{B, F, 1, 2\}$, where B is the father, Benjamin, F is the son, Franklin, and agents 1 and 2 are representative types of agents.

$$X = \{a, b, e, f\}^{19}$$

$$Y^1 = \{a, b\}, Y^2 = \{e, f\}.$$

The elements of Y^1 are objects of choice typical of popular culture, say, rock concerts, while those in Y^2 are typical of high culture, say, string quartet recitals.

Let \bar{v}', \bar{v}'' be preference preorderings on X such that

$$\bar{v}' = \langle a \succ b \succ z' \succ z'' \rangle \Leftrightarrow \{a\bar{v}^1 b, a\bar{v}^1 z', a\bar{v}^1 z'', b\bar{v}^1 z', b\bar{v}^1 z'', z'\bar{v}^1 z'', z''\bar{v}^1 z'\},$$

$$\bar{v}'' = \langle b \succ a \succ z' \succ z'' \rangle \Leftrightarrow \{b\bar{v}^1 a, b\bar{v}^1 z', b\bar{v}^1 z'', a\bar{v}^1 z', a\bar{v}^1 z'', z'\bar{v}^1 z'', z''\bar{v}^1 z'\},$$

where $z', z'' \in \{e, f\}$, $z' \neq z''$,

and let \bar{v}', \bar{v}'' be preference preorderings on X such that

$$\bar{v}' = \langle e \succ f \succ y' \succ y'' \rangle,$$

$$\bar{v}'' = \langle f \succ e \succ y' \succ y'' \rangle,$$

where

$$y', y'' \in \{a, b\}, y' \neq y''.$$

The preference structure of one who prefers popular culture (and who lacks of experience of high culture) is expressed by the specification that his initial preference structure is $(Y_t^F, \bar{v}_t^F) \in (Y^1, V_{0, Y^1})$, where \bar{v}_t is $\{\bar{v}', \bar{v}''\}$ restricted to V_{0, Y^1} . The preferences of someone who prefers the high culture lifestyle at time t (and has no experience of popular culture) are given by $(Y_t, \bar{v}_t) \in (Y^2, V_{0, Y^2})$, where \bar{v}_t is $\{\bar{v}', \bar{v}''\}$ restricted to V_{0, Y^2} .

To simplify, suppose that all students of the same type have the same preference relation, namely, $\bar{v}' = \bar{v}$ for students of type 1 and $\bar{v}'' = \bar{v}$ for students of type 2.

Benjamin prefers that Franklin have preferences \bar{v}' or \bar{v}'' at time $t + \tau$, $\tau > 0$. In order that Benjamin be willing to pay for his son to attend E, he must think that Franklin is open to the experiences and influences that he will encounter at E and therefore that his initial preferences (at time t) can be changed by those factors. This is expressed in the model by two conditions. The first condition is that the circle $\Gamma(F)$ that Franklin will have at E is one that has a distribution of types representative of the

¹⁹Elements c and d omitted from X are interpreted as choices that determine after-college personal economic capital and therefore income. It is assumed that these are the same at the two schools, and so they are ignored here.

Table V:

$X \times X$	$\rho(\bar{v})$	$\rho(\bar{v} _{Y^1})$	$\rho(\bar{v})$	$\rho(\bar{v} _{Y^2})$
(a, b)	1	1	-1	0
(a, e)	1	0	-1	0
(a, f)	1	0	-1	0
(b, e)	1	0	-1	0
(b, f)	1	0	-1	0
(e, f)	-1	0	1	1

student body at E, and the second condition is that a sufficient number of students at E include Franklin in their circles. To simplify, we assume that for each student i at E, including Franklin, $\Gamma(i)$ is the entire student body. Second, the weight given by Franklin to student i 's preference relation is α_i^F , $i = 1, 2$ if the student is of type i . Thus, Franklin's weight vector is determined by two numbers. Abusing notation, we may write it as

$$\alpha^F = (\alpha_1^F, \alpha_2^F), \quad \text{with } \alpha_2^F > 0.$$

(It may also be the case that $\alpha_1^F > 0$)

We suppose for definiteness that the father thinks that

$$\alpha_1^F = 0.6, \text{ and } \alpha_2^F = 0.4 \tag{1}$$

and

there are N_i^P students of type i at P and N_i^E students of type i at E, $i = 1, 2$.

$$\tag{2}$$

Table 5 shows the encoding of these preferences obtained by applying the operator ρ .

Table 5 also shows the encoding of the restrictions of preferences to the choice sets Y^i , $i = 1, 2$, in the third and fifth columns. The function ρ assigns the value zero to points in $X \setminus Y^i$.

Next we compute the function

$$\tau_{\alpha^F, \bar{v}}^F(x, y) = \sum_{j=1}^N \alpha_j^F \rho_{\bar{v}^j}(x, y) \tag{3}$$

for the case where Franklin attends E, and the preference relations of all agents are defined on all of X .²⁰

²⁰The case in which the preference structures are (Y^1, V_{0, Y^1}) and (Y^2, V_{0, Y^2}) can be analyzed similarly, and can lead to a more complex result, in which v_{i+1}^F agrees with both $\bar{v}|_{Y^1}$ and $\bar{v}|_{Y^2}$, but the ordering of objects in $Y^1 \cup Y^2$ and objects in $X \setminus (Y^1 \cup Y^2)$ depends on the proportions of those with preferences that agree with \bar{v} on $X \setminus (Y^1 \cup Y^2)$ versus those that agree with \bar{v} on $X \setminus (Y^1 \cup Y^2)$.

Because of (2), we may write (3) in the form

$$\tau_{\alpha^F, \bar{v}}^F(x, y) = \hat{\alpha}_1^F \rho_{\bar{v}}^F(x, y) + \hat{\alpha}_2^F \rho_{\bar{v}}^F(x, y),$$

where

$$\hat{\alpha}_i^F = \frac{N_i^E \alpha_i^F}{N_1^E \alpha_1^F + N_2^E \alpha_2^F}, \quad i = 1, 2.$$

If N_2^E is large relative to N_1^E , as is assumed, then the critical value γ^F can be close to zero, while still resulting in the preference relation of the son in the next period changing to one closer to \bar{v} .

The need to calculate T_{α^i, ν^i}^i does not arise, because of the simplicity of the example.

We now take up Example 2.2. The iterative model used to analyze Example 2.1 assumes that the agents are in direct contact with one another. While groups of agents were used (the group of students of type 1 and the group of type 2) they were not essential. Analysis of Example 2.2 involves groups of agents in an essential way. This is to be expected because the number of agents is large and the political aspect is central.

Recall that in Example 2.2 there is a forest that is the sole habitat of a population of rare birds. The choice to be made is whether or not to cut down the forest for lumber. The set of agents may be sorted into three classes: the loggers, the preservationists, and the rest of the population. Thus,

$$I = \{1, \dots, N\}$$

$$= \{1, \dots, N_1, N_1 + 1, \dots, N_1 + N_2, N_1 + N_2 + 1, \dots, N_1 + N_2 + N_3\},$$

where N_3 is the number of loggers, N_2 is the number of preservationists, and N_1 is the number of remaining agents. Here $N_1 + N_2 + N_3 = N$.

The consequences of cutting down the forest can be represented by the allocation that would obtain if the forest were cut down, together with variables that represent the state of the forest and the state of the population of birds. The consequences of not cutting down the forest can be represented in the same way. Choice between the two actions is as usual conceived as choice between their two consequences. Thus, in the model the set X of objects of choice contains at least two objects, denoted x^c, x^n , where x^c is the state that results from cutting the forest and x^n is the state that results from not cutting the forest.

A typical element of X has the form

$$y = \{y_1, \dots, y_{N_1}, y_{N_1+1}, \dots, y_{N_1+N_2+1}, \dots, y_{N_1+N_2+N_3}, z^1, z^2\}$$

where y_i denotes the commodity bundle going to agent i , z^1 is the state of the forest, and z^2 is the state of the bird population.

To simplify we assume that there is a standard commodity bundle; this can be regarded as a single composite consumption good whose quantity

is measured by a real variable. We assume further that the quantity of the consumption good is the same for all agents in the same group. Thus, for each

$$\begin{aligned} 1 \leq i \leq N_1, & & y_i &= y^1, \\ N_1 + 1 \leq i \leq N_1 + N_2, & & y_i &= y^2, \\ N_1 + N_2 + 1 \leq i \leq N_1 + N_2 + N_3, & & y_i &= y^3. \end{aligned}$$

Then a state of the economy can be represented by a vector of the form $(y^1, y^2, y^3, z^1, z^2)$. We also assume that x^c, x^n are such that $y^1 = y^2$, because the only thing that distinguishes preservationists from the other nonloggers is their preferences. Furthermore, the birds cannot exist without the forest, though the forest can exist without the birds; we simplify matters by assuming that one variable can represent the state of the combination of forest and birds.

By choosing units appropriately we can write

$$\begin{aligned} x^c &= (1, 1, 1, 0), \\ x^n &= (1, 1, .5, 1). \end{aligned}$$

Here we are expressing the “fact” that cutting the forest will give every agent one unit of the good, and result in having zero forest and zero population of the birds; not cutting the forest will result in every member of the logging community suffering a reduction by half in the quantity of the good they get, with no measurable change in what the other agents get, and leave the preexisting one unit of forest and bird population as before. Everyone knows these states and knows their association with the actions that bring them about.

The preferences of agents are as follows:

each member of the logging community strictly prefers to cut the forest;

each preservationists strictly prefers not to cut the forest;

every other agent is indifferent between cutting or not cutting the forest.

Using the encoding functions of the iterative model, these preferences can be represented as follows.

$$\begin{aligned} \text{for } 1 \leq j \leq N_1, & & \rho_{\nu^j}(x^c, x^n) &= 0, \\ \text{for } N_1 + 1 \leq j \leq N_1 + N_2, & & \rho_{\nu^j}(x^c, x^n) &= -1. \\ \text{for } N_1 + N_2 + 1 \leq j \leq N_1 + N_2 + N_3, & & \rho_{\nu^j}(x^c, x^n) &= 1. \end{aligned}$$

The remaining parameters of the iterative dynamic model formulated in this section are weight vectors and thresholds.

We recognize that the three groups of agents and the federal agency are players in a game. We do not formalize this game but rather discuss it informally. First we describe the actions available to the players.

The federal agency has two possible actions—to give or to deny permission to cut the forest. We rule out postponing action.

The loggers have available resources amounting to the difference in the amount of the good they would get if the forest were cut down and what they would get if it were not. Under the assumptions we have made this comes to $0.5N_3$ in total. They could use that to make side payments to the preservationists or to the rest of the agents, or it could be used to hire lobbyists and contribute to campaign funds to exert political influence on the federal agency. They could also try to influence the indifferent agents to support cutting down the forest.

The preservationists similarly have resources at most equal to the difference in the amount of the good they could give up without switching their preference between cutting and not cutting the forest. They could use these resources to make side payments to other agents or to lobbyists or campaign funds of officeholders. And they could also use their funds to make their case to the indifferent agents to persuade them to take political action in support of preservation.

The indifferent agents can accept a side payment from one or the other groups and take action accordingly, or they can do nothing.

If resources were such that a side payment could settle the decision, then the need to consider actions to change the preferences of the indifferent reduces to comparison of the costs of the two types of action. While there are indeed situations where side payments in some form do solve the problem, there are also situations in which they do not. Our example is one of the latter kind.

We suppose that the group of loggers is too small relative to the other groups to make a side payment sufficient to induce a nonnegligible number of either of the two other groups to support cutting the forest. The best action for the loggers is to lobby congresspersons to support their position with the federal agency. We suppose that they have done this, with the result that if nothing else changes, the agency will give permission to cut the forest.

The preservationists are also too small a group to be able to make effective side payments to either of the other groups or to buy the forest from the government, if that were possible. The only action that has any chance of preventing cutting of the forest is to convince a sufficient number of the indifferent to support the preservationists' position. Though it would take an arbitrarily small addition to what the indifferent agents get in either of the two state vectors to tip the choice of an indifferent agent, more is required here, namely, that the agent undertake political action in support of the action he prefers. This entails substantial effort, say, a donation to support political action by the preservationists or the loggers. This amounts to a nonnegligible reduction in the quantity of the

consumption good the agent will get. Such a reduction would overwhelm the effect of a tiny side payment intended to tip indifference to preference. This leaves the preservationists with the option of using news media and other means of mass communication as well as public and private meetings to make their case to the indifferent.

Mass communication of messages that only give information about the forest and the birds, perhaps also about the loggers, and about the consequences of cutting the forest would be redundant, because all agents know the allocation that results from each action; the indifferent are indifferent between cutting and not cutting having full knowledge of the state that would result from each action. Broadcast messages and meetings will be effective only if they succeed in *changing the preferences* of a sufficient number of the initially indifferent agents. The “case” to be made to the indifferent agents may be described as showing them, perhaps in attractive or dramatic ways, the preferences of the preservationists and inviting them to have the same preferences. The specific actions taken to change the preferences of the indifferent could include use of the free media, such as news organizations, by staging newsworthy events, such as street theater, demonstrations, meetings, and the like, as well as the use of commercially produced messages. All this can be represented formally in the model by the preservationists choosing a representative agent, k , and providing the resources required for her to take the action denoted $b = 1$, that is, to communicate to every agent the part of her preference relation that is defined on the set consisting of the two state vectors that describe the consequences of cutting and not cutting the forest, respectively. A response of the loggers to this action of the preservationists might be to mount their own campaign of persuasion.

The analysis using the iterative adjustment model in Example 2.2 is exactly parallel to that shown in Example 2.1. The outcome predicted by the model depends on the parameter values. With some values, the preservationists prevail; with others the loggers prevail. One implication of this example is that if the agents have sufficient information about the consequences of each action, and if the preferences of agents are immutable, and if the agents are rational in the sense of the paradigm, then further activities by loggers or preservationists to generate support would make no sense. Activities like “making the case” to the indifferent rational agents would be undertaken by rational agents only if they think that changing the preferences of those in the target population is possible.

We frequently see the use of mass media to persuade agents to support a particular course of action. Sometimes the message represents, or misrepresents, the consequences to be expected from a particular course of action. In that case the message is transmitting information, or misinformation, that might lead the target agents to change their behavior without changing their preferences. But a message supplying false or misleading information can be and often is refuted by opposing groups.

In that situation it is often the case that the messages of all sides continue to be sent. It is not clear in that situation whether the message carries information other than that the preferences of one of the opposed groups carry more weight than those of the other group. In a commercial context, the message sometimes consists of images of attractive people or celebrities demonstrating a preference.

We turn now to Example 2.3, which involves Cain and Abel. In terms of the formal dynamic model, Abel (agent 1) is in the circle $\Gamma(2)$ of Cain (agent 2) and vice versa. The preferences are that Abel wants Cain to have the allocation that Cain prefers. Thus $\alpha^1 = (\alpha_1^1, \alpha_2^1) = (0, 1)$. There are two possible Cains: the “generous” Cain is characterized by $\alpha^2 = (\alpha_1^2, \alpha_2^2) = (0, 1)$, and the “spiteful” Cain has $\bar{\alpha}^2 = (\bar{\alpha}_1^2, \bar{\alpha}_2^2) = (-1, 0)$. Recall that the set X consists of the two allocations $x = (r, w)$, $y = (w, r)$. Ignoring indifferences, the set of possible preference orderings V_0 consists of $\bar{\nu}$ such that $x\bar{\nu}y$, and $\bar{\bar{\nu}}$ such that $y\bar{\bar{\nu}}x$. Applying the function ρ yields $\rho(\bar{\nu})(x, y) = 1$, $\rho(\bar{\nu})(y, x) = -1$, and $\rho(\bar{\bar{\nu}})(x, y) = -1$, $\rho(\bar{\bar{\nu}})(y, x) = 1$. Therefore, if the preference profile is $\nu = (\bar{\nu}, \bar{\bar{\nu}})$, the rule (3) results in

$$\begin{aligned} \tau_{\alpha^1, \bar{\nu}}^1(x, y) &= (0, 1)(\rho_{\bar{\nu}}(x, y), \rho_{\bar{\nu}}(x, y)) = 1, \\ \tau_{\alpha^1, \bar{\nu}}^1(y, x) &= (0, 1)(\rho_{\bar{\nu}}(y, x), \rho_{\bar{\nu}}(y, x)) = -1. \end{aligned}$$

Hence, for every admissible value of γ^i , $\nu_{i+1}^1 = \bar{\nu}$. The generous Cain, having the same weight vector as Abel, makes the same transition in preferences.

However, the spiteful Cain has the transition

$$\begin{aligned} \tau_{\bar{\alpha}^2, \bar{\nu}}^2(x, y) &= (-1, 0)(\rho_{\bar{\nu}}(x, y), \rho_{\bar{\nu}}(x, y)) = -1, \\ \tau_{\bar{\alpha}^2, \bar{\nu}}^2(y, x) &= (-1, 0)(\rho_{\bar{\nu}}(y, x), \rho_{\bar{\nu}}(y, x)) = 1. \end{aligned}$$

Hence $\nu_{i+1}^2 = \bar{\bar{\nu}}$.

It is evident that any other preference structure could be substituted for (X, V_0) , with a similar result, because $\bar{\alpha}^2 = (-1, 0)$. The preference relation of Abel, say ν , would be converted to ν' for Cain in which all points $x\nu y$ are converted into $y\nu'x$.

When an individual is considering whether or not to enter into a relationship that involves close and enduring contact with others, such as joining an academic department, or a firm, or a marriage, the standard decision model confronts a rather complex modeling problem in order to arrive at a “rational” decision. The decision to join or not to join may be conceived as a decision to play a sequence of games, where the games are unknown at the time of decision, or perhaps only known to be in a large class of games. The players in some initial segment of the sequence—the present players—are known at the present time, but all that is known of the players in future games is that they will be selected by the present

generation of players or by their successors. The individual is forced to decide whether he would prefer to play such a sequence of unknown games with this group or not.

In Example 2.3, Abel might well not want to interact with the spiteful Cain for two reasons. First, independent of the preference structure, Abel, knowing $\bar{\alpha}^2$, would know that he is in for a lot of frustration. He might also be concerned about the possibility that long association with Cain might influence him to become more like Cain, which Abel might well regard as a dismal prospect. This sort of reasoning is not far from ordinary experience. Indeed, it has happened that a faculty member leaves an otherwise satisfactory position in order to avoid continuing interaction with certain colleagues. People faced with a decision whether or not to enter a long-term relationship with a certain group sometimes deal with it by meeting personally with the members of the group, trying to “size them up” as people, (which in the present model means trying to learn their characters or personalities, i.e., their α 's), and base decisions to some extent on whether they think they will be able to work together effectively in the presently unknown situations in which they must later interact. While it seems possible to model the phenomena involved here in terms of beliefs in the standard model of decision theory or game theory, it also seems possible to do so in the present framework in perhaps a more parsimonious if less detailed way. This way does not require people to anticipate specific situations, including events and probabilities that they do not seem capable of anticipating.

Returning to Example 2.3, a decision by Abel to avoid Cain is expressed by excluding Cain from his circle. Formally, $2 \notin \Gamma(1)$. It might still be the case that Abel is in Cain's circle, that is, $1 \in \Gamma(2)$. Whether Abel is or is not included in Cain's circle, $\alpha_2^1 = 0$. Then, independent of Cain's preferences, there is a stable profile.

Anticipating the next section, we examine Example 2.3 using the mapping U , rather than the special iterative model. Expressed in terms of U , we have already established that when Cain and Abel are in each other's circles, and interact, $U(\nu^1, \nu^2) = (U^1(\nu^1, \nu^2), U^2(\nu^1, \nu^2))$ does not have a fixed point.²¹ However, we have seen that if Cain is not in Abel's circle, then Abel's preference is independent of Cain's. Then $U^1(\nu^1, \nu^2)$ is independent of ν^2 . Hence, ν^1 is constant. Thus, $U^1(\nu^1, \nu^2) = \nu^1$ identically in ν^2 . It follows that $U^2(\nu^1, \nu^2) = \bar{U}^2(\nu^2) = \nu^2$, for the fixed value of ν^1 . Therefore $U(\nu^1, \nu^2) = (U^1(\nu^1, \nu^2), U^2(\nu^1, \nu^2)) = (\nu^1, \nu^2)$; that is, U has a fixed point.

Furthermore, the fixed point of U consists of fixed points of the component mappings U^1 and U^2 . These fixed points are fixed points of

²¹It can be seen from Example 2.3 that this mapping is not a contraction mapping and so does not satisfy the sufficient condition for the existence of a fixed point given in the Appendix.

two groups of agents. One of these groups consists of Abel, and the other can be thought of as consisting either of Cain and Abel or of Cain alone. Thus, the structure of fixed points depends on the pattern of interdependencies of preferences and is related to the structure of subgroups of agents that reflects the pattern of interdependencies.

In the next section we explore these relationships in a more general formulation of the problem. This formulation is not concerned with the process of evolution of preference profiles but instead is focused on the group structures that can result from preference profiles that are the stable or rest points of a dynamic process. We take that process to be the one generated by iterating a function U . Thus, in Section 5 we study the structure of groups that are consistent with and therefore might arise from stable patterns of interdependent individual preference relations, namely, the fixed points of U .

5. Preference Dependencies and Fixed Points

We turn now to the more abstract formulation of our model. The main purpose of this model is to enable us to relate decompositions of the set of agents into subgroups, possibly overlapping, to patterns of interdependence of individual preferences.

As we have seen, the interdependence of Cain's and Abel's preferences when Cain is spiteful does not admit a stable preference profile. As was noted above, in the case of interdependent preferences, the situations in which it is plausible to suppose that agents might know one another's preference orderings, at least over some part of the domain of choice, are those where the agents are in close contact for an extended period of time, for example, members of a household; or partners in a business or professional firm; or members of an academic department.²² But it is an aspect of reality that if in such situations people differ so fundamentally that they cannot "get along together," then the relationships break up. On the other hand, if preferences are interdependent, socializing influences can bring about sufficient commonality of preferences to ensure that the institutions, whether family or other joint enterprises, can function. Indeed if preferences are sufficiently similar (e.g., to mention an extreme case, if preferences are identical), the problem of existence of a satisfactory social ordering is trivial. If patterns of interdependence are such as to permit socializing influences to operate within subgroups of agents, interaction might bring about a degree of consistent interdependencies sufficient to produce coherent preference profiles. The existence of pervasive media of communication, like television, could operate to the same effect, at least with respect to some restricted set of objects of choice.

²²Use of mass media of communication can extend interdependence to larger groups as well, as seen in Example 2.2.

In the Appendix we present a formal model of what is discussed informally in this section, and we also present sufficient conditions for the existence of fixed points of U , that is, coherent patterns of preferences in subgroups.²³ We have so far operated in a set theoretic framework. However, the theorem (stated and proved in the Appendix) establishing existence of fixed points uses a more structured setting. The conditions used include the specification that the choice set be a subset of a Euclidean space (or a complete metric space) and the condition that certain mappings associated with U be contraction mappings. In this section the discussion proceeds in the set theoretic framework, assuming that fixed points exist. The focus is on the relationship between patterns of interdependencies and fixed points of the whole society and of its subgroups.

Recall that for $i \in I$ $U^i: V \rightarrow V_0^i$ and $U = U^1, \dots, U^N$. For each $i \in I$, and $\nu \in V$, the value $U^i(\nu)$ is a possible preference relation of agent i . That relation might depend on one or more of the preference relations of other agents. For example, $U^1(\nu^1, \nu^2, \nu^3)$ might depend on ν^2 but might not depend on ν^3 . Thus, the mapping U incorporates or expresses certain interdependencies among the preferences of agents. Starting from a given preference profile, either iterations of U continue until a fixed point of U is reached, or iterations go on indefinitely. In the latter case the environment does not admit a mutually consistent set of preferences over objects of choice. Assuming that there are fixed points, we may ask, "What information do the fixed points of U give us about the group structure of the society that U describes?" We address that question by looking at coverings of the set of agents by subsets of agents, expressing the idea that an agent may belong to several groups at the same time. We consider all coverings that are compatible with the mapping U in the following sense. A covering expresses a certain pattern of dependencies among the preferences of agents. We look at coverings that express the same pattern of dependencies that U does. As we noted, a covering represents a decomposition of the set of agents into possibly overlapping groups. A mapping U of preference profiles to preference profiles determines mappings of subprofiles, corresponding to subsets of agents that are elements of a given covering of the set of agents. These mappings may be regarded as essentially the restrictions of U to the sets in the covering. We call these *local mappings*. We show that a fixed point of U determines fixed points of the local mappings. Thus, the society can be seen as a collection of possibly overlapping groups such that a stable preference profile for the

²³The formal presentation of the propositions is relegated to the Appendix because technical difficulties make the definitions and proofs of the propositions difficult to follow and tend to obscure the content of the propositions. The basic intuitive idea is that if the interdependencies of a group are expressed by the mapping U then interdependencies within a subgroup of that group should be expressed by the restriction of U to that subgroup. The problem becomes complicated because agents can belong to more than one subgroup.

society is made up of corresponding subprofiles within each group. This is the content of Proposition A1 in the Appendix.

It is also the case (under the mild assumption that an agent has the same preference relation in every subgroup he belongs to) that the combination of a fixed point of each of the local mappings associated with the subgroups in a covering makes up a fixed point of the mapping U for the entire society. This is the content of Proposition A2 in the Appendix.

An additional consequence of the hypotheses of Proposition A2 is that the group structure must capture indirect influences of the preferences of one agent on another via a third agent. This is part of the content of Proposition A3.

In general the mapping U determines a *class* of coverings that express the same pattern of interdependencies, rather than a unique covering. We want to know which of these coverings is most likely to form. It is likely to be a covering that gives the most information about the pattern of interdependencies. This is a covering that consists of the “smallest” sets. This is discussed following Proposition A3 in the Appendix.

Because the formal definitions and notation required for the propositions are unavoidably technical, they are relegated to the Appendix.

6. Conclusion

Beginning from the ideas that an individual’s preferences are malleable and that they may be influenced by the preferences of others, we have given two formulations of the dynamic process by which preference profiles evolve as preference relations change in a mutual interaction. One is an abstract model in which the iteration of a mapping from profiles to profiles defines a discrete time dynamic process; the other is a linear version of that process specified in more detail that is a special case of the more abstract process. Examples motivate the model and illustrate its application.

Conditions are given for the existence of a stable preference profile—a rest point of the dynamic process. A stable profile is naturally associated with a division, not in general unique, of the set of agents into subgroups with the property that preference interdependencies within a subgroup are “stronger” than those across subgroups. The conventional case in which each agent’s preference relation is given is in this model the special case in which each subgroup consists of just one agent.

The definition of an acceptable welfare criterion is problematic when preferences are endogenous. The model presented here suggests that Pareto optimality can be a consistent welfare criterion when based on stationary or “final” preferences.

The model suggests research in several directions. One is to apply the model to economic and political or social phenomena in which interdependence of preferences appears to offer natural and convincing expla-

nations. Phenomena like fashion, advertising, and political persuasion seem to be likely possibilities.

In another direction, the present model can provide a foundation for social norms. A group of agents whose preferences come to agree, or are sufficiently compatible, on a certain subset of alternatives will tend to take the same action when choosing from that subset. Convergence of preferences would in such cases provide a dynamic basis for rationalizing a “social norm.”

In some of the examples, preferences over preferences motivate one or more of the agents, but this concept is not given formal expression in the model. Another direction of theoretical research is to explore the implications of introducing “preferences over preferences” formally.

Finally, strategic issues have not been treated in this paper. These seem to fall into two classes. First, the present model assumes that people come to know one another’s preferences. It is plausible that their knowledge is incomplete or uncertain. This would create intermediate cases between those in which preferences are private information and those in which preferences are completely known. With no special features these cases might fall into games that are already analyzed; it might also be the case that some examples have special features that call for new analysis. Second, payoff functions in games depend on preferences. In repeated games with interdependent preferences, while the action sets and the consequences of actions remain the same as the stage game is repeated, the preferences of the players evolve. As preferences evolve, the strategic structure of the stage game also evolves. Analysis of the repeated game with evolving payoff functions in the stage game is likely to be quite different from the analysis of the repeated game with a constant stage game.

Appendix

A collection C of subsets $K \subseteq I$ is a covering of I if $\cup_{K \in C} K = I$.

DEFINITION A1: For $i, j \in I$ the function U^i does not depend on the variable v^j if and only if for all $v, \bar{v} \in V$, $v^l = \bar{v}^l$, $l \neq j$ implies $U^i(v) = U^i(\bar{v})$. U^i does not depend on $K \subseteq I$ if and only if for every $j \in K$ U^i does not depend on v^j . In other words U^i does not depend on $K \subseteq I$ if and only if for all $\bar{v}, \bar{\bar{v}} \in V$, $\forall l \in I \setminus K, \bar{v}^l = \bar{\bar{v}}^l$ implies $U^i(\bar{v}) = U^i(\bar{\bar{v}})$.

DEFINITION A2: A covering C of I expresses the pattern of interdependencies, abbreviated EPI, of U if for each $i \in I$, for each $K \in C$, and for each $j \in I \setminus K$, U^i does not depend on v^j .

We would like to show that for a given mapping $U: V \rightarrow V$ and a covering C of I that expresses the interdependencies of U , each fixed point of U consists, in an appropriate sense, of fixed points of the restric-

tions of U to the sets K in C . In order to state and prove a proposition to this effect we need a few new concepts and additional notation.

The mapping U^i depends on the order of its arguments, unless U^i is invariant under permutations of the array of its arguments, and therefore U depends on the order of its arguments. However, when a covering C of I that expresses the interdependencies of U is introduced, the sets K in C might scramble the order of the arguments. To deal with the problems this can cause we first define $I = \langle 1, 2, \dots, N \rangle$ to be an ordered set with the ordering $1 < 2 < \dots < N$. Subsets $K \subseteq I$ inherit the ordering of I .

Let X be an arbitrary set, let $K \in C$, and let $K = \langle j_1, j_2, \dots, j_{q_K} \rangle$.²⁴ Let $x_K = \langle x^{j_1}, x^{j_2}, \dots, x^{j_{q_K}} \rangle$ denote an ordered array of elements of X indexed by K . We define a binary operation \otimes on the set of ordered arrays of elements of a set X indexed by K . This operation, or something like it, is needed in order to carry out the equivalents of set operations with ordered sets and arrays.

DEFINITION A3: *Let K and K' be ordered subsets of I and let x_K and $y_{K'}$ be ordered arrays indexed by K and K' , respectively. Let $z_{K \cup K'}$ be an ordered array indexed by $K \cup K'$, where the value of z^j at a point $j \in K \cup K'$ is the set of elements of the arrays x_K and $y_{K'}$ corresponding to j . Thus,*

$$z^j = \begin{cases} \{x^j\} & \text{if } j \in K \setminus K' \\ \{y^j\} & \text{if } j \in K' \setminus K \\ \{x^j, y^j\} & \text{if } j \in K \cap K' \end{cases} .$$

We see that if $j \notin K \cap K'$, then z^j is a singleton; if $j \in K \cap K'$ then in general z^j is not a singleton. This foreshadows a difficulty that appears in connection with Proposition A2. The difficulty is dealt with by assuming that an agent has the same preferences in every subset he is a member of. Here we define the binary operation denoted \otimes only for cases in which z^j is a singleton for every $j \in K \cup K'$. In that case we can identify the set z^j with its only element. Then the binary operation \otimes on ordered arrays x_K and $y_{K'}$ given by

$$x_K \otimes y_{K'} = z_{K \cup K'}$$

is well-defined. The operation \otimes is associative and commutative.

We can now introduce the notations needed to state and prove the propositions.

²⁴A formally correct notation for the elements of K requires that the indices j_p also be labeled so as to distinguish j_p when the set is K from j_p when the set is K' . What is needed is for each set $K \in C$ is a function $f_K: \langle 1, 2, \dots, q_K \rangle \rightarrow I$ that generates the elements of K . We can write this function $j_p^K = f_K(p)$ and omit the superscript K when there is no chance of confusion or use a letter different from j to name the function.

For $K \in C$, where $K = \langle j_1, j_2, \dots, j_{q_K} \rangle$, our notation gives

$$V_K = V^{j_1} \times V^{j_2} \times \dots \times V^{j_{q_K}}, \nu_K = \langle \nu^{j_1}, \nu^{j_2}, \dots, \nu^{j_{q_K}} \rangle,$$

$$U^{j_i}: V \rightarrow V^{j_i} \quad i \in \langle 1, 2, \dots, q_K \rangle, \quad \text{and} \quad U_K: V \rightarrow V_K,$$

where

$$U_K = \langle U^{j_1}, U^{j_2}, \dots, U^{j_{q_K}} \rangle.$$

Note also that for $K, K' \subseteq i$, $U_K \otimes U_{K'} = U_{K \cup K'}$, because, if $j \in K \cup K'$, then U^j is the unique element in the ordered arrays $U_K, U_{K'}$ that has the index j .

DEFINITION A4: Suppose $L \subseteq I$ and for $i \in I \setminus L$, U^i does not depend on L . Let $L^c = I \setminus L$, and define $\hat{U}_{L^c}^i: V_{L^c} \rightarrow V_{L^c}^i$ by the condition $\hat{U}_{L^c}^i(\nu_{L^c}) = U^i(\nu)$. Note that $U^i(\nu) = U^i(\nu_L \otimes \nu_{L^c})$.

Propositions

Proposition A1 tells us that a stable preference profile for the entire society is made up of fixed points of subsocieties or subgroups, provided that the subgroups together form a covering of the set of agents that expresses the interdependencies of the mapping U .

PROPOSITION A1: Let C be a covering of I of agents that EPI of U ; that is, for each $K \in C$ the function U_K does not depend on the preferences ν^l of agent l for all $l \in I \setminus K$.

If ν^* is a fixed point of $U: V \rightarrow V$, then, for each $K \in C$, ν_K^* is a fixed point of \hat{U}_K .

Proof of Proposition A1: The proof is given for the case of three agents, with a covering C' that is assumed to express the pattern of interdependencies of U . The proof is the same when there are more agents, or more sets, except for notational complexities.

Suppose $I = \langle 1, 2, 3 \rangle$,

$$C' = \{K_1, K_2\} = \{\langle 1, 2 \rangle \langle 2, 3 \rangle\},$$

$$V = V_0 \times V_0 \times V_0 = V^1 \times V^2 \times V^3,$$

$$U: V \rightarrow V, \text{ where } U = \langle U^1, U^2, U^3 \rangle,$$

$$\text{and } U^i: V \rightarrow V^i, \quad i = 1, 2, 3.$$

Thus,

$$U_{K_1} = U_{\langle 1, 2 \rangle} = \langle U^1, U^2 \rangle,$$

$$U_{K_2} = U_{\langle 2, 3 \rangle} = \langle U^2, U^3 \rangle.$$

Under the hypothesis of Proposition A1, $U_{\langle 1,2 \rangle} = \langle U^1, U^2 \rangle$ does not depend on ν^3 , and $U_{\langle 2,3 \rangle} = \langle U^2, U^3 \rangle$ does not depend on ν^1 . Therefore, we may define

$$\hat{U}_{\langle 1,2 \rangle} : V_{\langle 1,2 \rangle} \rightarrow V_{\langle 1,2 \rangle} \quad \text{and} \quad \hat{U}_{\langle 2,3 \rangle} : V_{\langle 2,3 \rangle} \rightarrow V_{\langle 2,3 \rangle}$$

by

$$\hat{U}_{\langle 1,2 \rangle}(\nu^1, \nu^2) = U_{\langle 1,2 \rangle}(\nu^1, \nu^2, \nu^3) = \langle U^1 \langle \nu^1, \nu^2, \nu^3 \rangle, U^2 \langle \nu^1, \nu^2, \nu^3 \rangle \rangle \quad (\text{A.1})$$

and

$$\hat{U}_{\langle 2,3 \rangle}(\nu^1, \nu^2) = U_{\langle 2,3 \rangle}(\nu^1, \nu^2, \nu^3) = \langle U^2 \langle \nu^1, \nu^2, \nu^3 \rangle, U^3 \langle \nu^1, \nu^2, \nu^3 \rangle \rangle. \quad (\text{A.2})$$

Suppose that $\nu^* = (\nu^{*1}, \nu^{*2}, \nu^{*3})$ is a fixed point of U . Thus, $U^i \langle \nu^{*1}, \nu^{*2}, \nu^{*3} \rangle = \nu^{*i}$, $i = 1, 2, 3$. It follows from the formulas (A.1), and (A.2) that

$$\hat{U}_{\langle 1,2 \rangle} \langle \nu_{\langle 1,2 \rangle}^* \rangle = \hat{U}_{\langle 1,2 \rangle} \langle \nu^{*1}, \nu^{*2} \rangle = \langle \nu^{*1}, \nu^{*2} \rangle = \nu_{\langle 1,2 \rangle}^* \quad (\text{A.3})$$

and

$$\hat{U}_{\langle 2,3 \rangle} \langle \nu_{\langle 2,3 \rangle}^* \rangle = \hat{U}_{\langle 2,3 \rangle} \langle \nu^{*2}, \nu^{*3} \rangle = \langle \nu^{*2}, \nu^{*3} \rangle = \nu_{\langle 2,3 \rangle}^*. \quad (\text{A.4})$$

Therefore $\nu_{K_i}^*$ is a fixed point of \hat{U}_{K_i} for $i = 1, 2$. ■

The converse of Proposition A1 is not valid without an additional condition. That condition (hypothesis (a) in Proposition A2) is that each agent has the same preference relation in every group that agent belongs to. This condition assures that the binary operation \otimes is well defined.

PROPOSITION A2: *Let C be a covering of I that is EPI for $U: V \rightarrow V$ and suppose that*

(a) *for every $K, K' \in C$, if $j \in K \cap K'$, then $\bar{\nu}^j$ is the preference relation of agent $j \in K$, and $\bar{\nu}^j$ is the preference relation of agent $j \in K'$. Then $\bar{\nu}^j = \bar{\nu}^j$.*

If for each $K \in C$, ν_K^ is a fixed point of \hat{U}_K , then $\langle \nu_{K_1}^* \otimes \nu_{K_2}^* \otimes \dots \otimes \nu_{K_N}^* \rangle$ is a fixed point of $\langle \hat{U}_{K_1} \otimes \hat{U}_{K_2} \otimes \dots \otimes \hat{U}_{K_N} \rangle = U$. (Because I is finite we can write $C = \{K_1, K_2, \dots, K_N\}$).*

Proof of Proposition A2: As in Proposition A1, we give the proof for the case $I = \langle 1, 2, 3 \rangle$ and $C = \{K_1, K_2\} = \{\langle 1, 2 \rangle, \langle 2, 3 \rangle\}$.

Suppose that $\bar{\nu}_{K_1} = \bar{\nu}_{\langle 1,2 \rangle}$ is a fixed point of $\hat{U}_{\langle 1,2 \rangle} = \hat{U}_{K_1}$, and that $\bar{\nu}_{K_2} = \bar{\nu}_{\langle 2,3 \rangle}$ is a fixed point of $\hat{U}_{\langle 2,3 \rangle} = \hat{U}_{K_2}$.

By assumption (a) of the proposition it must be the case that $\bar{\nu}^2 = \bar{\nu}^2$. Then substituting, say, $\bar{\nu}^3$ in (A.3) and (A.4), and using (A.1) and (A.2), we see that

$$(U^1(\bar{\nu}^1, \bar{\nu}^2, \bar{\nu}^3), U^2(\bar{\nu}^1, \bar{\nu}^2, \bar{\nu}^3)) = (\bar{\nu}^1, \bar{\nu}^2)$$

and

$$(U^2(\bar{v}^1, \bar{v}^2 \bar{v}^3), U^3(\bar{v}^1, \bar{v}^2, \bar{v}^3)) = (\bar{v}^2, \bar{v}^3).$$

It follows that

$$U(\bar{v}^1, \bar{v}^2, \bar{v}^3) = \langle U_{(1,2)} \otimes U_{(2,3)} \rangle \langle \langle \bar{v}^1, \bar{v}^2 \rangle \otimes \langle \bar{v}^2, \bar{v}^3 \rangle \rangle = (\bar{v}^1 \bar{v}^2, \bar{v}^3).$$

Thus, $\langle \bar{v}^1, \bar{v}^2 \rangle \otimes \langle \bar{v}^2, \bar{v}^3 \rangle$ is a fixed point of U . ■

The hypotheses of Proposition A2 have a further implication. They restrict the coverings that reflect the pattern of interdependencies among agents' preferences so that the group structure of agents must capture explicitly any indirect influences of the preferences of an agent on another via a third agent. This is the subject of Proposition A3.

PROPOSITION A3: *Suppose that C is an EPI covering of I satisfying (a) and that there are two elements K' and K'' of C and $j \in I$ such that $j \in K' \cap K''$. Then there cannot be elements $i, k \in I$, where $i \in K'$, $i \notin K''$ and $k \in K''$, $k \notin K'$, such that U^i and U^k depend on v^j and either U^j depends on v^j or U^j depends on v^k .*

Proof of Proposition A3: To simplify notation, suppose as in the proof of Proposition A1 that $i = 1$, $j = 2$, $k = 3$ and that $K' = \{1, 2\}$ and $K'' = \{2, 3\}$. Because C satisfies (a), U^1 does not depend on v^3 and U^3 does not depend on v^1 . Suppose further that U^1 depends on v^2 and U^3 depends on v^2 . We must show that U^2 does not depend on either v^1 or v^3 . Suppose U^2 depends on v^3 . Stated informally, v^2 is a function of v^3 ; because of (a) the value of v^3 affects the value of v^2 and therefore that of v^1 . That is, $v^2 = U^2(v^1, v^2, v^3)$. But, because U^1 depends on v^1 , and because $v^1 = \hat{U}_{(1,2)}^1 \langle v^1, v^2 \rangle$, it follows from (a) that $v^1 = \hat{U}_{(1,2)}^1(v^1, U^2 \langle v^1, v^2, v^3 \rangle)$. But this contradicts EPI of C . The same argument applies with 1 and 3 interchanged. ■

The mapping U determines a class of EPI coverings of I that satisfy assumption (a), in general not a unique covering. We interpret a covering of I as describing subgroups of agents whose preferences have a degree of independence across subgroups and have a degree of interdependence within subgroups. In general we would want to work with the covering that in a sense gives the most information about the pattern of interdependencies. This is the covering that has the "smallest" sets.

Consider the example where $I = \{1, 2, 3\}$, and $C = \{K_1, K_2, K_3\}$, where $K_1 = \{1, 2\}$, $K_2 = \{2, 3\}$, $K_3 = \{2\}$. Here it is assumed that U is such that v^1 depends on v^2 but not on v^3 ; and v^3 depends on v^2 but not on v^1 . There are three EPI coverings that can also satisfy assumption (a). These are

$$C' = \{1, 2, 3\},$$

$$C'' = \{\{1, 2\}, \{2, 3\}\},$$

and

$$C''' = \{\{1, 2\}, \{2, 3\}, \{2\}\}.$$

C' is automatically EPI because its complement in I is empty. C'' is EPI for three patterns of interdependence:

1. U^1 depends on ν^2 , and U^2 depends on ν^1 ; neither U^1 nor U^2 depends on ν^3 ; U^3 depends on ν^2 but does not depend on ν^1 ;
2. the symmetric case with the roles of agents 1 and 3 interchanged;
3. U^1 and U^3 depend on ν^2 ; U^2 does not depend on ν^1 , and neither U^1 nor U^2 depends on ν^3 ; U^3 does not depend on ν^1 .

Proposition A3 rules out the case in which U^1 and U^3 depend on ν^2 , U^1 does not depend on ν^3 , and U^2 depends on ν^3 and also rules out the symmetric case with the roles of 1 and 3 interchanged. Only C' is EPI for these cases.

The covering C''' is EPI for the pattern in (3).

Let the set of coverings of I that are EPI for U be partially ordered by refinement.²⁵ A minimal EPI covering for U is one that has no proper refinement. These are the coverings that are most informatively EPI. Because they imply the smallest subgroups, they may be the ones most easily formed.

In the example we are considering Proposition A3 tells us that C'' cannot be a minimal refinement of any EPI covering that satisfies assumption (a), because if both U^1 and U^3 depend on ν^2 , but U^1 does not depend on ν^3 , and if U^3 does not depend on ν^1 , then C''' represents the pattern of dependencies in U , while if U^1 is independent of ν^2 , then $[\nu]C = \{\{1\}, \{\{2, 3\}\}$ is a refinement of C'' , or if U^3 is independent of ν^2 , the covering $\bar{C} = \{\{1, 2\}, \{3\}\}$ both is a refinement of U and represents the pattern of dependencies in U . If both conditions hold, the covering $\{\{1\}, \{2\}, \{3\}\}$ is the refinement.

Existence of Fixed Points

Up to this point we have not imposed any structure on the set X of objects of choice beyond requiring that it be a set. We have given examples in which the mapping U of preference profiles to preference profiles has a fixed point and an example in which it does not. The discrete time adjustment process presented in Section 3 does not impose more structure on preferences than that they be transitive, reflexive relations over the set of objects of choice or some subset of it. We have left open the question of conditions sufficient to ensure existence of fixed points of U . Now we take up that question. We present sufficient conditions that the mapping U has a fixed point. These conditions impose more structure on the set X , namely, that it be a complete metric space.

²⁵Let A and B be coverings of I . A is a refinement of B if and only if for every set b in the covering B there is a set a in A such that b is a subset of A , and for some b in B , the containment is proper.

If X is a subset of a Euclidean space, that is, $X \subseteq R^l$, where R^l is equipped with Euclidean distance d , a preference relation V on X is represented by its graph $G(\nu) \subset X \times X$. Consequently the set V_0 of preference relations over X consists of subsets of $X \times X$. Because X is a complete metric space with distance function d , the metric space consisting of nonempty, d -bounded closed subsets of $(X \times X, d)$ equipped with the Hausdorff metric H_d induced by d is also a complete metric space. The Hausdorff metric induced by d is

$$H_d(S, T) = \inf \left\{ \begin{array}{l} \epsilon > 0 \mid S \text{ lies in an } \epsilon \text{ - neighborhood of } T \\ \text{and } T \text{ lies in an } \epsilon \text{ - neighborhood of } S \end{array} \right\},$$

where S and T are d -bounded, closed subsets of $X \times X$.

Next we introduce the concept of a contraction correspondence.

Let (Y, δ) be a complete metric space with distance function δ . A correspondence $\varphi: Y \rightarrow Y$ is a *contraction correspondence* if and only if it has nonempty, closed, δ -bounded values; there is a constant $0 < c < 1$ satisfying

$$H_\delta(\varphi(x), \varphi(y)) \leq c\delta(x, y)$$

for all x, y in Y . The constant c is called the *modulus of contraction*.

THE NADLER FIXED POINT THEOREM:²⁶ *Every contraction correspondence on a complete metric space has a fixed point.*

We apply this theorem to our problem as follows. The space (Y, δ) is the space (V, H_d) , which is a complete metric space. The correspondence φ is the function U considered as a correspondence, that is, $\varphi(\nu) = \{U(\nu)\}$. Because a fixed point $\bar{\nu}$ of the correspondence φ satisfies $\bar{\nu} \in \varphi(\bar{\nu})$, it follows that $\bar{\nu} = U(\bar{\nu})$.

This theorem can be applied directly to the function U or equivalently to the functions U_K , where K is a set in a covering C of I that expresses the interdependencies of U and satisfies assumption (a). It may be more natural to apply the theorem to the functions U_K for each K in the covering C , because the mappings U_K have as arguments only the preferences of the agents in K , and thus U_K does not have to be a contraction with respect to the preferences of those not in K .

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²⁶See Aliprantis and Border (1996, p. 486).

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