On endogenous economic regulation

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Abstract

This paper presents a model in which in each of a succession of time periods the State
and the private economy interact to determine rules under which the private agents will
operate in the next period, and rules and resources that constrain interventions of the State
in the next period. The set of State institutions, called regulators, that are the instruments of
State intervention is endogenously determined in each period. The model is a multiperiod
game consisting of two phases. The first is a (noncooperative) game played by private
economic agents in each period, the rules for which are given by the regulators in the
preceding period. The second phase is political. In each period the private agents acting
politic ally determine the legal and budgetary constraints under which the regulators will
operate in the next period, and thereby determine the noncooperative game to be played in
the next period. Formal entities in the model allow a wide variety of regulatory instruments
and transfer payments to be represented.

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1. Introduction

Economic life seems to be dominated by the use of the power of the state to
regulate, restrict and direct economic activity and to redistribute its fruits. Public
opinion and the opinions of economists seem to be that interventions by the state

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in economic affairs are not done very well. There are some who hold the view that state interventions are harmful, and others who hold the view that they are necessary to rectify failures of performance by the private economy. Economic theory recognizes the limits of validity of the classical welfare theorems, which express at a fundamental level the claim that a private market economy free of state regulatory intervention can be expected to produce efficient results. Economic theory provides a justification for regulatory intervention in cases of 'market failure', such as those due to significant indivisibilities, or to increasing returns to scale, or externalities. While some cases of intervention seem to be of this type, such as public utility regulation, it is difficult to believe that most interventions by the state in economic affairs are directed to correcting inefficiencies due to market failures. Indeed, many such interventions seem to achieve their objectives, whatever those may be, at the cost of introducing inefficiencies of their own.

On the one hand, business men, members of the public generally, as well as economists who concern themselves with governmental economic policy, seem to understand very well the nature and sources of state intervention. It is generally understood that economic agents benefit if they can arrange that state power be used to change the rules that govern economic activity in their favor. For example, U.S. sugar growers, who must compete in the world economy with other sugar producing areas with lower costs, can improve their incomes if they can persuade the federal government to subsidize sugar. Real estate developers can benefit from changes in obscure provisions of the federal tax code, and so on. On this view, at least some and perhaps all state intervention in the economic system results from pressure by economic agents in pursuit of their own interests. Such interventions are then a natural development in any private economic system. For brevity we may use the term 'regulation' to refer broadly to state intervention in the economic system. On the view just alluded to, regulation is an endogenous political-economic phenomenon.

There is an extensive literature on regulation in economics. Noll (1989) presents a summary of this literature up to about 1988. As Noll points out, the oldest part of that literature deals with interventions as remedies for market failure. A subsequent component is aimed at assessing the effectiveness of state interventions. More recently since the 1970s, with the publication of Stigler (1971) and Pelzman (1976) analytic attention has been paid to the political causes of regulation. In these papers the focus is on identifying 'interest groups' and trying to assess their relative powers by comparing benefits (positive and negative) to be obtained (or avoided) by a regulatory intervention, to the cost of organizing a group and of its political activity. In the more recent literature on endogenous regulation (surveyed by Hillman (1989)) two important classes of models can be distinguished, those in which there is political competition among opposing candidates or parties (Magee et al. (1989). and Hillman and Ursprung (1988)), and those in which there is an incumbent government that seeks to maximize political
support (Stigler, 1971; Hillman, 1982). More recently, a series of papers by Grossman and Helpman (1994, 1995a,b) and Grossman and Helpman (1996), dealing with international trade policy takes the latter approach. This paper shares with the more recent literature the assumption that regulation arises from the desires of economic agents to use the power of the state to benefit themselves rather than from the postulated existence of some general objective, such as 'suffrage of the common good', or the 'public interest' as the basis for state economic interventions.

The present paper arose out of joint work with the economic historian J.R.T. Hughes (Reiter and Hughes, 1981). Hughes, who had written extensively about economic regulation in the U.S. economy, was trying to understand in historical terms why the U.S. has the kind of regulation that it has. It seemed to us that the existing regulatory institutions and the economy of which they are a part were the results of an evolving process of economic, political and legal interactions which together constituted the economic, legal and political history of the period in which the modern regulated economy developed. It also seemed to us that the existing literature on regulation did not provide an adequate theoretical basis for studying these processes. Moreover, we were not aware of any theoretical model or general framework within which different partial models could be compared with one another. Indeed, in order to distinguish the institutions of regulation that are used from those that might have been used, but were not, it is necessary to specify in some fashion the class of available regulatory instruments: we could not find a specification of such a class in the literature, only descriptions of specific instruments. More recently, the contracting literature, which focuses on the incentive effects of asymmetric information as between regulators and those regulated, or between political authorities and regulators, has improved our understanding of these aspects of regulation. (See Laffont and Tirole, 1993.)

In the present paper, the ideas underlying the formal model used by Reiter and Hughes to organize and interpret U.S. economic history from about 1880 to about 1940 are made the basis for a model that gives more detailed and specific expression to them. The focus in this paper is on the formal model; Reiter and Hughes (1981) and the works cited there can be consulted for connections to the legal and institutional history.

In the model presented here, it is the strategic political interactions of the economic agents that determine the outcomes of the political process, while the politicians are more instrumentalities and so are not modeled explicitly. This has advantages and disadvantages. The main advantage is that it does not commit the model to an excessively simplistic treatment of politics; the main disadvantage is that it forces the model deal with the phenomena of political leadership or political entrepreneurship indirectly. However, it would not be difficult to extend the model to include political agents explicitly, to model their strategic possibilities, and so to include, for example, electoral competition, and financial contributions, phenomena used to define the objectives of political agents in some models.
This paper provides a formal model that combines aspects of implementation theory, political economy and regulation to make a dynamic model of a regulated economy in which the regulators and the regulations are endogenous. The project is incomplete in the sense that there are no theorems in this paper, and, while there are examples relating to elements of the model, there is as yet no application of the model as a whole to the determinination of institutions. Nevertheless it seems worthwhile to present the model at this time, among other reasons because it provides a common framework within which particular models with different assumptions can be compared. A related motivation for presenting the model derives from its relevance to the theory of economic institutions, apart from issues of regulation more narrowly conceived. The model is relevant because it is one in which institutions (rules of the game) are endogenous. They arise in a multistage game played in two phases. In one phase the players interact in a game whose rules are the results of a game played in the second phase. That is, the players interact in a game whose outcome is the set of rules under which they will subsequently play another game whose outcomes are economic payoffs. The two phases form one strategically interrelated multistage game that evolves over time.

The rest of the paper is organized as follows. The model is presented in parts.

We begin in Section 2 with a one period or static model. The unregulated private economy is modeled as a message exchange process and an outcome function. Regulators are added to the model, and the set of regulatory instruments is defined. The outcomes resulting from specific choices by private economic agents and by regulators are introduced, and some examples of regulation are given. Legal and institutional constraints on regulators are modeled, and the evaluation of outcomes is modeled. The sequence of events (a timeline), visualized as taking place within a period of time is explained.

Section 3 contains a multiperiod model with political action in which the moves and events of Section 2 are elements. In this multiperiod model, the set of private agents and regulators can change over time; the set of regulators is made endogenous, i.e., new regulators may be created or existing ones eliminated. Subsection 3.1 models political action and the political process, summarized in a function that represents the net of effect of the political actions taken by economic agents in the current period in determining the regulatory and political framework that will prevail in the next period. Subsection 3.2 presents the economic dynamics.

Section 4 defines a game by specifying the information structures, strategy spaces and payoff functions in the multiperiod structure defined in Section 3. Attention is focused on a class of myopic strategies. The possibility of political cooperation among economic agents is introduced, and modeled in two different ways, one leading to a multistage game with equilibria that are stable against coalitional deviations (strong equilibria) and the other to a characteristic function game.

A brief concluding remark follows.
2. A one-period (static) model

There are two types of agents explicitly in the one-period model. These are: (1) Economic agents, numbered \([1, \ldots, n] = I\); the set \(I\) includes both private economic agents, i.e., producers and consumers, and governmental economic agents, such as the Tennessee Valley Authority or Defense Department agencies; (2) Regulators; the set \(J\) includes existing regulatory authorities. (In the multi-period model described in Section 3 both \(I\) and \(J\) may vary over time.)

2.1 The unregulated economic process

We begin by modeling an unregulated economic process. The adjustment process model of Hurwicz (1969) (see also Mount and Reiter, 1974) in its static equilibrium version is used for this purpose. For expository purposes we focus first on the economic system, including state enterprises, but ignoring the state in its regulatory functions. The latter are introduced in Section 2.2.

We separate those elements that are characteristic of the economic system, such as structure of markets, from those that are given from ‘nature’ or from the past, such as preferences of agents, technology, endowment of natural resources or stock of capital. The latter are called the ‘environment’. A particular environment will be denoted by \(e\); the set of environments by \(E\).

Given an environment \(e\), the characteristics of each agent are determined, i.e., agent \(i\)’s production set, consumption set, and preference relation, being part of the data constituting \(e\), are given when \(e\) is given. Denote by \(e^i\) the characteristic of agent \(i\) when the environment is \(e\) and by \(E^i\) the set of all such characteristics \(e^i\) corresponding to \(e \in E\). It is assumed that agent \(i\) knows his own characteristic \(e^i\).

The unregulated economic process is modeled (very abstractly) as follows: There is a communication process using some formal language of messages or signals by means of which agents communicate (e.g., the competitive process uses prices and quantities as messages and agents communicate their excess demands.) This process results in stationary or equilibrium messages, which may be interpreted as encoding the economic plans or decisions of the agents. These are translated into actions (productions and trades), which, in turn, determine an allocation. We shall focus attention on the actions, evaluating them by the value of the allocations to which they lead. This is represented formally as follows:

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1 Formally, \(e^i\) is a projection of \(e\) on subspaces corresponding to agent \(i\). If the environment is decomposable (absence of externalities), then \(e = (e^1, \ldots, e^n)\) and \(e^i\) is just the \(i\)th component of the data constituting \(e\).

2 For more detailed accounts of this model, see Hurwicz (1969) and Mount and Reiter (1974). This model does not capture the economic organization the competitive mechanism. A broad class of alternative economic processes can be represented in this way. Therefore, in modeling the unregulated economy in this way, we do not commit ourselves to the position that the economy is competitive, or even that markets and prices are the allocation mechanism in use.
Let \( M^* \) denote the space of possible messages of the economic agents. We may, for the moment, suppose that \( M^* = M_{1}^* \times \ldots \times M_{n}^* \) where \( M_{i}^* \) is the space of messages of economic agent \( i \), \( i = 1, \ldots, n \).

Agent \( i \) is assumed to know only his own component \( e_{i}^* \) of the environment \( e^* \). There is an iterative interchange of information among the agents - a message exchange process - in which each agent has a response function, \( f^* \), that models the way agent \( i \) responds to information received from others in the light of what he knows about the environment. Thus,

\[
f^* (m_{1}^*, \ldots, m_{i}^*, \ldots, m_{n}^*) = m_{i}^* \quad i = 1, \ldots, n.
\]  

(1)

Here the message exchange process is modeled as a first-order difference equation. However, agents responses could depend on the history of past messages sent and received by them. As long as the order of the difference equation is finite - finite memory of past messages - there is no loss of generality in assuming that it is first order, since no assumption has been made about the message space. Each finite history of messages can be regarded as a single message in a higher dimensional space.

Thus, at any stage \( t \) of the communication process, agent \( i \), knowing \( e_{i}^* \), receives message \( m_{i}^* = (m_{i1}^*, \ldots, m_{in}^*) \) from all agents and revises his message \( m_{i}^{t+1} \) at the next stage on the basis of what he then knows.

A message \( n \)-tuple

\[
\overline{m} = (\overline{m}_{1}^*, \ldots, \overline{m}_{n}^*)
\]

such that

\[
f^* (\overline{m}, e_{i}^*) = \overline{m}_{i} \quad i = 1, \ldots, n,
\]

is called an equilibrium message.

Equilibrium messages are given by

\[
f^* (m, e_{i}^*) = m_{i}^* = 0, \quad i = 1, \ldots, n.
\]

We define the individual equilibrium message correspondence

\[
\mu^* (e_{i}^*) = \{ m_{i} \in M_{i}^* | f^* (m, e_{i}^*) = m_{i}^* = 0 \}
\]

and hence the equilibrium message correspondence of the process

\[
\mu^* (e) = \bigcap_{i = 1}^{n} \mu^* (e_{i}^*) = \{ m_{i} \in M_{i}^* | f^* (m, e_{i}^*) = m_{i}^* = 0, i = 1, \ldots, n \}.
\]

In equilibrium analysis the message correspondences can be taken as primitive, rather than the response functions from which they were derived above. The message-correspondence formulation is in the static case more general than that using response functions, but the response-function model is a bit easier to interpret. Thus, let \( M^* \) be a topological space, not necessarily a product space. In many cases \( M^* \) is a Euclidean space. Let

\[
\mu^*: E \rightarrow M^*, \quad i \in I.
\]
be the (equilibrium) message correspondence of economic agent \( i \).

Thus,

\[ \mu^i(e') \subseteq \mathcal{M}^i \]

is the set of messages agent \( i \) "sends" when his environmental component \( e' \in \mathcal{E} \).

For example, in a pure exchange environment \( e, e' \) would denote agent \( i \)'s consumption set, preference relation, and initial endowment. If \( \mu^i \) is to represent \( i \)'s behavior according to the competitive mechanism, then \( \mu^{i*} \) is the graph of agent \( i \)'s excess demand function, viewed as a set in the message space, \( \mathcal{M}^i \subseteq \mathcal{E}^i \), whose points \( (p, q) \) represent prices and allocations (i.e., quantities for all agents), or (multi-lateral) trades.\(^1\)

The correspondence

\[ \mu^i = \bigcap_{e} \mu^i(e') \]

maps environments into equilibrium messages. Thus, in the pure-exchange competitive case,

\[ \mu^i(e) = \bigcap_{e} \mu^i(e') \]

consists of the market-clearing price-quantity pairs, i.e., those that make aggregate excess demands equal to zero.

Returning to the general formulation, the outcome function

\[ h^i : \mathcal{M}^i \rightarrow A \]

translates the equilibrium messages into actions of outcomes. (The set \( A \) is called the outcome space.)

In the pure-exchange competitive case, the outcome function is

\[ h^i(M^i) = h^i(p, q) = q. \]

i.e., \( h^i \) is the projection operator onto the allocation space.

Returning again to the general formulation, in this model of the economic process, the message space \( \mathcal{M}^i \) and the outcome function \( h^i \) may be interpreted as representing institutional arrangements, while the message correspondences \( \mu^i \), \( i \in \mathcal{E} \), represent the behavior of the agents.

For example, consider an organized exchange, such as the Chicago Board of Trade. Bids and offers may be submitted by traders according to the rules of the Exchange: the message space represents conceivable bids and offers. The bids and

\(^1\) Moon and Reiter (1974) give an explicit formulation of the competitive mechanism as a formal resource allocation mechanism of this type.
offers made by traders, i.e., their behavior in choosing their messages, are described by their individual message correspondences, and the trades actually consummated are described by the outcome function applied to the final stage of matching bids and offers according to the rules of trading.

To sum up, the unregulated economic process is represented (in the one-period (static) case) as a triple,

\[ (M^*, \hat{\mu}^*, h^*) \]

where

\[ \hat{\mu}^* = (\mu_1^*, \ldots, \mu_j^*) = \langle \mu_j^* \rangle_{j \in J} \]

(We use the notation \( \langle x_j \rangle_{j \in J} = (x_1, \ldots, x_j) \), \( S = \{x_1, \ldots, x_j\} \); and similarly \( \langle x^D \rangle_{j \in J} = (x^D_1, \ldots, x^D_j) \), \( S^D = \{x^D_1, \ldots, x^D_j\} \). Where there is no ambiguity, we also write \( \langle x \rangle_{j \in J} \) and, more explicitly, \( x_j = \langle x_j \rangle_{j \in J} \).)

2.2. Regulated economy

We next model the regulated economy. Here, in addition to the economic agents we include the set \( J \) of regulators. In the present (static) formulation we suppose temporarily that the number of regulators is a fixed, and write

\( J = \{n + 1, \ldots, n + N\} \).

In Section 3 the set \( J \) is endogenous, and we set of regulators may change over time.

Regulatory instruments are of two kinds. We call them respectively, incentives and direct constraints on behavior. Each consists of an \( n \times N \) matrix denoted \( \theta \) and \( \rho \) respectively, where

\[ \theta = \{ \theta_j \} \text{ and } \rho = \{ \rho_j \}, \quad i \in I, j \in J. \]

(In what follows we keep the principle exemplified by these matrix entries that a label referring to an economic agent appears as a superscript attached to the symbol it labels, and as a subscript if it refers to a regulator.)

We begin by considering direct constraints on behavior of the economic agents.

Regulators may require economic agents to submit certain reports. These may be, indeed typically are, supplementary messages not used by the economic part of the mechanism. We may suppose that the message space has the form

\[ \mathcal{M} = M^* \times M^{**}, \]

where \( M^{**} \) is the component that contains the supplementary messages not used in the operation of the economic mechanism. Correspondingly, the message correspondence of economic agent \( i \) has the form (\( \mu^i, \nu^i \)), where

\[ \mu^i: \mathcal{E} \to M^*, \quad \nu^i: \mathcal{E} \to M^{**}, \]
Here $\mu^{-1}(e')$ is the message $i$ puts into the economic mechanism and $\nu(e')$ is the message consisting of reports $i$ sends to the regulators when $i$’s environmental characteristic is $e'$.

Write

$$\mu_i = (\mu_i^{-1} \times \nu).$$

Hence,

$$\mu = \bigcap_{i=1}^{n} \mu_i$$

is the message correspondence of the regulated economic process with message space $M$. It represents agent $i$’s economic behavior in the regulated economy, and his compliance with reporting requirements imposed by the regulators.\(^1\) (Note that the value $\nu(e')$ can be a cylinder set in $M^{**}$.) This means that economic agent $i$ can write his own reports, and in effect agree to any report made by the other economic agents. To write $\mu$ as an intersection does not require either that the regulators see only reports on which all economic agents agree, or that each agent's reports to the regulators are the same as those of every other agent.\(^1\)

To model direct constraints on behavior, let the entry $\rho_j$ in the matrix $\rho$ be a correspondence imposed by regulator $j$ on economic agent $i$. Thus

$$\rho_j: E_i \rightarrow M, \quad i \in I, j \in J.$$\

Writing the correspondence $\mu_j$ (and the full regulatory instrument $r$ introduced below) with the environment $e'$ (resp. $e$) as its argument does not commit us to the assumption that regulator $j$ observes or knows $e'$ (resp. $e$). This point is discussed more fully and made clear below after the full regulatory instrument $r_j$ is defined. Briefly, in anticipation of that explanation, the correspondences $\rho_j$ must be compatible with the structure of regulator $j$’s information: if regulator $j$ cannot distinguish environment $e'$ from $e''$, then the correspondence $\rho_j$ must take the same value at $e'$ and $e''$. (If the information were probabilistic then $\rho_j$ would be required to be measurable with respect to that probability structure.)\(^2\)

When agent $i$ has environmental characteristic $e'$, and the regulator $j$ imposes $\rho_j$ agent $i$ must satisfy

$$\mu_i(e') \subseteq \rho_j(e) \quad \text{for every } j \in J$$

or

$$\mu_i(e') \subseteq \bigcap_{j \in J} \rho_j(e') = \rho_i(e).$$

\(^1\) If the report is one that is used by the economic mechanism, such as the federal income tax return, then $\rho$ constrains a part of agent $i$’s message to be the income tax form filled out according to the instructions provided, and the outcome function $s_{j}$ requires the change of income to $i$ corresponding to the appropriate tax transfer to or from the IRS.
This correspondence may be interpreted as a constraint on agent $i$’s behavior, because the outcome function translates equilibrium messages into outcomes.

(Note that
\[ \rho(v) = M \quad \text{for all} \quad v \in E \]
expresses the condition that $i$’s behavior is not constrained by regulator $j$. Therefore we need not distinguish explicitly those agents $i$ that regulator $j$ has the authority to regulate: he issues regulations for all agents, regulations that are not constraining for those outside of $j$’s authority. Formally, each agent is regulated by all regulators.)

To model incentives manipulated by the regulators, we suppose that each regulator $j \in J$ can choose parameters $\theta_j, i \in I$ determining incentives confronting economic agent $i$. For example, a regulator may impose monetary fines for violations of safety regulations, or may provide tax abatements depending on economic behavior.

The matrix
\[ \theta = \{ \theta_j \}_{j \in J} \]
is the regulatory instrument determined by choices made by all the regulators, the element $\theta^j$ being the parameter (possibly multi-dimensional) that regulator $j$ chooses to apply to agent $i$.

The parameter matrix $\theta$ defines an outcome function
\[ h_{\theta}: M \to A. \]

For each $\theta \in \Theta$ where $\Theta$ is the domain in which $\theta$ may be chosen, $h_{\theta}$ is a function from $M$ to $A$. We may regard the parameters $\theta$ as modifying the outcome function of the unregulated economy (the ‘natural’ outcome function) since the regulators may change the outcomes given by $\theta^*$ even if $M = M^*$.

The outcome function $h_{\theta}$, jointly determined by the regulators, may reflect the payment of subsidies or the levying of taxes or other actions that influence the outcomes resulting from the choices made by the economic agents. $^5$

Let
\[ \rho = \{ \rho^1, \ldots, \rho^J \}, \quad \theta = \{ \theta^1, \ldots, \theta^J \} \]
and write
\[ r_j = (\theta_j, \rho_j) \quad \text{for} \quad j \in J. \]
Then, $r_j$ is the full regulatory instrument chosen by regulator $j$.

$^5$ The entitlements program in force in the 1970s prior to the oil shock, which required oil companies to make payments to one another depending on the use of domestic versus imported oil, is an example. Regulations that require transfers between economic agents are rare.
We have written $r$ as a function of the environment $e$. Regulator $j$ may have only partial information about the environment, including the possibility that he has no information about $e$. He may observe certain "signals" depending on the environment. These signals can be built into direct regulations $\rho$ as follows. Say $\eta_j : E \to Y_j$, where $Y_j$ is a space of signals observed by regulator $j$. Then $\eta_j(e) = \eta_j$ is the signal $j$ observes about the environment when it is $e$. His regulation could then be made conditional on $\eta_j$, say by $\rho_j(\eta_j)$. But then defining $\rho = \rho_j(\eta)$ yields the constraint given above.

The extent to which regulator $j$ can know that his regulations are complied with depends, of course on his information. His information can include information about the behavior of economic agents, about the environment, or about the outcome. For $j$ in $J$ let

$\psi : M \to Z$

be the signal about behavior of the economic agents that regulator $j$ sees. Thus, when the environment is $e$ and economic agent $i$ chooses $\mu^i$, regulator $j$ observes

$\psi_j(\mu(e), \eta_j) = z_j$.

Regulator $j$ may also observe something about the outcome, for example, national income statistics, or the results of surveys of economic activity. Let

$\xi : A \to W$

determine the signal

$\xi_j(e) = w$,

that $j$ gets when the outcome is $a$. Because

$\alpha = h_\alpha(\mu(e))$,

and so

$\xi_j(e) = \xi(\mu(e))$.

We may define $\psi$ so that it includes the information $\psi_j$. Recall that regulator $j$ also observes

$\eta_j(e) = \eta_j$,

when the environment is $e$.

Baron and Myerson (1982) have pointed out that regulator $j$ should choose regulations with the structure of his information in mind.
2.2.1. Choices and the outcomes

If
1. regulator $j$ chooses $r_j = (\theta_j, \rho_j)$,
2. economic agent $i$ chooses $\mu' = (\mu', v')$ and
Then the outcome of the economic process is

$$h_a = \mu(e) = a \in A$$

where $(\mu', v')$ must satisfy

$$\{ \mu'(e'), v'(e') \} \subseteq \rho'(e) \quad \text{for all } i \in I \text{ and } e \in E$$

(2)

Let

$$M'(r) = \{ \mu': E \rightarrow M | \mu'(e') \subseteq \rho'(e) \text{ for all } e \in E \}$$

where $r = (\theta, \rho)$. Then, (2) can be written

$$\omega \in \bigcap_i M'(r)$$

2.2.2. Examples

We give some examples to illustrate how the abstract formalism defined above can capture some familiar forms of regulation.

2.2.2.1. Example 1: Federal income tax. The regulator involved is the Internal Revenue Service (IRS). This regulator requires that all economic agents submit reports, namely, IRS Forms filled out according to instructions. (Agents whose income is below a certain value are not required to file. We shall treat that condition here as filing a blank return.) An IRS Form, for instance Form 1040, can be encoded as a vector. That is, the Form consists of lines containing blanks to be filled by the agent. The instructions for filling each blank serve to define a variable, that is, each blank is a place holder (numbered) together with a specification of the set from which the entry is to be selected. Except for those lines to be filled in with information identifying the agent, such as name, address, social security number, the blanks are to be filled with numbers. Thus, the variables are numerical variables. Because the index $i$ already identifies the agent, we may confine attention to the numerical entries. Listing all the IRS Forms, and within each Form the lines of that Form in order, the array of all IRS Forms existing in a given period can be encoded as a real vector space. A given Form, such as IRS Form 1046, defines a certain subspace.

The instructions for filling out the Forms may require that certain entries be dependent on other entries. For instance the entry in the blank labeled ‘adjusted gross income’ is computed as a function of entries in other blanks. This is readily expressed in terms of the rowspace space $M'$ and the correspondence $\rho$. 
In addition, the tax rates determine the transfer of money between the agent and the IRS for any given Tax Return (consisting of the Forms a given agent must fill out). Thus, the tax table, or rate schedule, is part of the parameter \( \theta \), which determines the outcome function.

What any agent gets after taxes depends on both \( p \) and \( \theta \), as well as on his economic activity (represented by the message correspondence \( \mu \) in the relevant period).

2.2.2.2. Example 2: Emission of pollutants. The message space \( M^* \) of the economic mechanism may provide for the transmission of the input–output vector chosen by a producer. (This is true of the so-called concrete processes defined in Hurweitz (1960).) Alternatively, the regulator, say the Environmental Protection Agency (EPA), may require the producer, say agent \( i \), an electric utility using a coal-fired electric power generating plant, to report the amount of sulfur emitted into the atmosphere in a given period of time. The regulations \( \rho_{EPA} \) thus impose limits on emissions, or, together with \( \theta_{EPA} \), may impose a schedule of monetary transfers depending on the amounts emitted. The latter would include regulatory schemes involving purchase of rights to emit sulfur.

2.2.2.3. Example 3. Approval of new drugs. The right to market a drug requires approval by the Food and Drug Administration (FDA). Approval is contingent on demonstration of safety and effectiveness, usually involving one or more clinical trials, which themselves require approval. Even in a static model this process can be expressed in terms of conditions on \( \rho \). For instance, agent \( i \) seeking approval would have to file reports \( \psi \) that meet the standards (expressed by \( \rho_{FDA} \)) established by the FDA at each stage for which approval is sought. But, from a descriptive point of view, perhaps this sort of example fits more naturally in the multi-period model discussed in Section 3.

2.2.3. Constraints on regulators

Regulators are constrained by legal and institutional constraints as well as by resource constraints.

The legal and institutional constraints operating on regulator \( j \) are expressed by the requirement that \( r_j \) belong to a set \( L_j \) of legally available regulations.\(^6\)

The resource constraint on regulator \( j \) is expressed by his budget \( b_j \in \mathbb{R} \) (the non-negative real numbers) and the function \( K_j \), which attaches to each element \( r_j = (\theta_j, \mu_j) \) a cost (transactions cost) \( K_j(r_j) = K_j(\theta_j, \mu_j) \) in dollars. of administering the regulations \( r_j \). Generally, the cost should depend on the environment \( e \).

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\(^6\)The legal constraint on regulators is here modeled as absolute, i.e., inviolate. This parallels the approach taken in the study of regulators on economic agents. Hence, there is no explicit role for the courts in this model, and no problem of enforcement.
as well. Here, we are implicitly ‘averaging out’ the environment. (Where convenient we shall use the notation
\[ A_j = \{ L_j, b_j \} \]
to refer to the legal and budgetary framework of regulator \( j \), and
\[ 1 = \{ A_j \}_{j \in J} \]
for the full legal and budgetary framework of regulation.)

Regulator \( j \) is constrained to choose \( r_j = (\theta_j, \rho_j) \) in the set
\[ B(1) = B(L_j, b_j) = \{ r_j \in L_j | K_j(t) \leq b_j \} \]

i.e., \( B(L_j, b_j) \) is the class of actions that are legally available and affordable with budget \( b_j \).

We write \( R = \{ L_{x_{-1}}, \ldots, L_{x_{-3}} \} \) and \( b = (b_{-1}, \ldots, b_{-3}) \).

In addition to the legal authority and budget that constrain regulator \( j \), he is also given a regulatory objective. The legal authority, the budget and the regulatory objective together define the official mission of regulator \( j \). This is treated more explicitly below.

If we suppose that regulator \( j \) knows that agent \( i \) knows the value of \( e' \) when the environment is \( e' \), and that economic agent \( i \) also knows that the regulator’s information about \( e = \eta = \eta(e) \) then regulator \( j \) may anticipate that agent \( i \) will choose his behavior ‘optimally’, in his own view, within the constraints of detectability imposed by \( j \)’s information. Therefore, regulator \( j \) can choose his regulatory instrument \( r_j \) so as to bring \( i \)’s choice as near as possible to \( j \)’s own objective.

2.2.4. Evaluation of outcomes

Each agent, whether an economic agent or a regulator, has preferences over actions or outcomes, represented by a utility function. The utility of an economic agent depends on the outcome (e.g., resource allocation) and on the environment. For economic agent \( i \) the utility function is therefore
\[ U_i : A \times E \to \mathbb{R} \]

where \( A \) is the outcome space, \( E \) is the space of environments, and \( \mathbb{R} \) is the real numbers. The utility function is a function of the environment \( e \) in several ways. First, if the outcomes given by \( b_{-1} \) are actions, rather than final states such as allocations, then the evaluation of those actions will be derived from the utility of the final states they lead to. For example, if the actions are trades and the final states are allocations, then the utility of trades will depend on the initial holdings of the agents. But the initial endowment is part of the environment \( e \). So the utility depends on \( e \). Furthermore, \( i \)’s preference relation on \( A \), denoted \( \text{pref}_i \), which is also part of the environment \( e \), determines \( i \)’s utility function (up to monotone or affine transformation, depending on whether or not the outcomes are certain).
Then the utility to agent $i$ when the action is $a$ and the environment is $e$, can be written

$$U(a, e').$$

This can also be written more explicitly, if somewhat redundantly, as

$$U(a, e'; \text{pref}').$$

Regulator $j$’s utility function expresses both his (official) mission and his private preferences. Writers on bureaucracy frequently comment on the existence of bureaucratic self-interest and its effects on the behavior of bureaucrats. Regulator $j$ may be supposed to prefer a larger scope of authority to a smaller one, and a larger organization to a smaller one. Thus, both $L_j$ and $b_j$ are arguments of his utility function.

His official mission also enters. This may be supposed to aim at bringing about certain desired economic results, through influencing the behavior of the economic agents via $j$’s regulatory instruments. Regulator $j$’s utility may be supposed to depend on his (legal authority $L_j$, his budget $b_j$, the degree to which his official mission is achieved, as evaluated in terms of his information about the economy, and his private preferences, $\text{pref}'$. This may be expressed abstractly by writing the utility of regulator $j$ as

$$U(\Lambda_j, z_j, w_j, s_j, \text{pref}(\epsilon)) = U(L_j, b_j, \Psi_j((\mu'(e')), \eta_j(e); \text{pref}(\epsilon)).$$

Thus, regulator $j$ may observe some function of the economic messages and the reports submitted by the agents, for example, he might see the equilibrium message, and all the reports submitted to him, or he might see only some function of the outcome of the economic mechanism, such as statistics that appear in the national income accounts, and the reports submitted by agents. The regulator may also have some information about the environment.

The argument $\text{pref}(\epsilon) = \text{pref}'$ represents regulator $j$’s private preferences over the domain of legal authority, size of his agency and the performance of his mission, relative to his information, when the environment is $e$.

In some cases we will make the simplifying assumption that regulator $j$ sees only the economic action $a$, i.e., that

$$\Psi_j((\mu'(e')), \eta_j(e); \text{pref}(\epsilon)) = a,$$

and that $j$ does not directly observe anything about $e$, so that in this simplification

$$\eta_j(e)$$

is constant.

2.3. Sequence of events

The events and choices just described may be visualized as taking place within a time period of fixed length. Within that period the sequence of events and choices is as follows.
At the beginning of the period the regulator’s objective, legal framework of regulation and budgets are given; preferences of the economic agents and the regulators are also given. Thus, the entities

\((\nu_i)_{i \in I}, \ (\text{pref}_i)_{i \in I}, \ (\text{pref}_j)_{j \in J}\)

are determined.

Next, each regulator \(j \in J\) chooses the regulatory instruments,

\(r_j \in R_j(\cdot),\)

where \(r_j \in L_j\). They incur costs \(K_j(r_j)\) satisfying the budget constraint

\(K_j(r_j) \leq b_j.\)

Following this, the economic agents choose their actions, namely their message correspondences, knowing the choices made by the regulators, and their own preferences. Thus, economic agent \(i\) chooses the message correspondences \((\mu^{i}(•), \nu^{i}(•))\) satisfying the regulatory restrictions. i.e.,

\((\mu^{i}(•), \nu^{i}(•)) \subseteq \rho^{i}(•).\)

Finally, Nature chooses the remaining components of \(e.\) Then,

for all \(e \in E, e = (e^j, \cdot \cdot \cdot, e^{i}, \cdot \cdot \cdot, e^e).\)

\((\mu^{i}(e^j), \nu^{i}(e^j)) \subseteq \rho^{i}(e),\)

for every economic agent \(i\).

The value \(\mu(e)\) is then determined along with the action (outcome) \(a = h_j(\mu(e))\). Then the utilities

\(U_j(a, e^j) = U_j(\hat{h}_j, \mu(e)), e^j, \text{pref}_j),\)

and

\(U_i(\mu(e^i), e^j, \eta_i(e); \text{pref}_j, e^e),\)

are also determined.

(It is implicit in the way the utility function of regulator \(j\) is written that \(j’s\) payoff depends on the cost function \(K_j).\)

2.4 A game

This structure defines a rather simple strategic situation, a game in which the regulators move first, choosing their regulations with knowledge of \(A\) and of their preferences. Then the economic agents choose their behaviors, knowing the economic agents choose their message correspondences knowing their own preferences, but not knowing anything else about the environment. Having chosen their message correspondences, their messages are determined when their environments become known to them.
regulations and their preferences, the environment becomes known and all other relevant variables are determined, leading to the payoffs described.

An equilibrium of this game would be a vector of message correspondences and regulations such that no agent could improve his payoff by changing his behavior $\mu'$ and no regulator could improve his payoff by changing his regulation $r_j$ given that no other agent or regulator changes his choices.

On the other hand, there may well be another equilibrium that agent $i$ would prefer, but cannot reach by his own actions alone. If agent $i$ could persuade others like him to change the rules of the game, they could all benefit. Legal restrictions prevent economic agents from conspiring to coordinate their economic actions, but constitutional guarantees protect their right to coordinate political actions aimed at the same objectives. While they cannot undo the actions and outcomes in period $t$, they can try to change the rules governing period $t+1$.

The availability of higher payoffs to some of the agents from changing the regulations in their favor constitutes a powerful motivation to use political means to affect change in the rules governing the economy. There are several ways in which political action in this context might be modeled. This is most naturally done in a multi-period model.

3. A multi-period model with political action

We modify the one-period model as follows.

The set of agents may change from period to period. There are countably infinite disjoint sets $I$ and $J$ of names of agents and regulators respectively. The finite subset $I(t)$ of $I$ is the set of economic agents in period $t$. Let $\#(I) = n(t)$. We write $I(t) = \{i_1, \ldots, i_n(t)\}$. This set is exogenously determined. The finite subset $J(t)$ of $J$ denotes the set of regulators in period $t$. It is determined by $I(t)$ as follows. Regulator $j$ is in $J(t)$ if and only if $R_j(t) \neq \emptyset$ and $b_j(t) \neq 0$.

3.1. Political action and political process

Economic agents may engage in political activities in period $t$. There is a space of political actions, and for each economic agent $i$ in $I(t)$ a subset $P(i)$ of political actions legally and institutionally available to agent $i$ in period $t$.

Political action may require resources. Agent $i$ is constrained to choose a political action he can afford. Let $P^*(i, t)$ be a correspondence from $e(i)$ to $P(i)$, so that for $e(i)$ in $e(i)$, $P(e(i), t)$ is the set of political actions that agent

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1Regulators may or may not be legally permitted to engage in political action. For simplicity we assume that direct political activity is not permitted to regulators. (Of course, individuals who are regulators may participate in political action in their roles as citizens.)
i may choose in period t, i.e., that i can afford among the ones legally available. Let

\[ p_i(t) \in P_i^t(e_i(t), \cdot) \]

denote agent i's political action in period t, and let

\[ p(t) = (p_i(t))_{0 < i < n} = (p_1, \ldots, p_n) \]

Thus, in each period economic agents i chooses two actions, a message correspondence, and a political action. Economic agents may communicate and negotiate with one another, or organize formally into groups, to arrive at coordinated choices of their political actions. Having done so, the outcome of the political process in period t is the specification of the legal and institutional framework of regulation, \( L_i(t+1) \), and the budgets of regulators, \( h_i(t+1) \). The political process in period t determines (possibly new) values \( L_i(t+1) \) for each \( j \in (i) \) (Note that by determining \( \cdot \cup \{i\} \) the political process can create or eliminate regulators.)

The political process also determines the rules that govern political activity in the subsequent period.\(^1\)

Elected and appointed officials of governments at the federal, state or local level are not introduced explicitly, nor are the complex and subtle processes by

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\(^1\) The use of group political action to influence regulatory activities by influencing the framework given to the regulators by the government authorities is exemplified by the recent experience of the F.T.C. An old-line agency originally created in 1914 to enforce the antitrust laws, the F.T.C. was over the years given other duties including the power even to stop deceptive advertising. See Wheeler-Lea Act, ch. 490 52 Stat. 111 (1938) (current version at 15 U.S.C. Sec 44, 45, 52-58 (1960)). After some decades of relative inaction by the agency and criticism by consumer groups for being a captive of those it is charged with regulating, the new political strength of the consumer movement led to a more active policy by the agency. Its more aggressive approach to its responsibilities led to alienation of a growing portion of the country's businesses and business groups. By the late 1970s, the F.T.C. was under fire from many directions. The F.T.C. had planned to impose restraints and guidelines on advertisers of products for children, the costs of funeral, information provided by used-car dealers, and standard-setting practices of trade associations. Offended industry groups fought the extension of F.T.C. power through Congress, which controls the F.T.C.'s budget. By the fall of 1976, Congress became the instrument to clip the F.T.C.'s wings. Wall St. J., Oct. 18, 1978, at 48, col. 1. See Reiter and Hughes (1982) for a more extended discussion of this experience.

\(^2\) The legitimacy of the use of state power to regulate economic processes constrained only by political decisions, specifically by legislative decisions, was finally established in the case of Nebbia v. New York. See Reiter and Hughes (1981) for a more complete discussion of the legal history of regulation from Moore v. Illinois, 94 U.S. 113 (1873) to Nebbia v. New York, 291 U.S. 502 (1934).

\(^3\) In the period preceding the Civil War, after a series of political compromises over the extension of slavery to new territories and states, the Supreme Court in the Dred Scott case (Scott v. Sanford, 60 U.S. 1 (1857) (1857) in effect, judged the rules under which political decisions concerning the geographical extension of slavery could be made, by extending Constitutional protection to the right of property in slaves to states in which slavery was prohibited.
which the actions of the economic agents have their effects on the relevant officials. Neither do we introduce explicitly political agents, such as party leaders and political organizers, who may be instrumental in coordinating the actions of the economic agents. Instead the political process is modeled as a black box. The inputs to this box are the political actions of the economic agents, however determined, and the outputs are (i) the legal and budgetary framework of regulation that will govern the regulators in the next period, and (ii) the legal and institutional framework that will govern political action in the next period.

Accordingly, there is a function \( \Pi \) which carries political actions into political outcomes. This

\[
\Pi\left(\{u(t)\}_{t \in H},\{P'(t)\}_{t \in H},\{P(t)\}_{t \in H}\right) = \left(\{u(t+1)\}_{t \in H},\{P'(t+1)\}_{t \in H}\right).
\]

According to this formula, the effect of the political action \( P'(t) \) on the regulatory and budgetary framework of regulators in period \( t \) and on the legal framework for political activity in period \( t \) is obtained by treating these as parameters; we may write the function modeling the effects of the political process as a function of the political actions, as follows:

\[
\Pi(t, \cdot) \equiv \Pi\left(\cdot, u(t), P'(t), \cdot, \cdot, \cdot\right),
\]

where

\[
P(\cdot, t) = u(\cdot, t), \quad u(t) = u(t),
\]

Furthermore, define

\[
\Pi^*(t, \cdot) \equiv \text{proj}_t \Pi(t, \cdot),
\]

\[
\Pi^*_4(t, \cdot) \equiv \text{proj}_t \Pi(t, \cdot),
\]

so that

\[
\Pi^*_4(t, \cdot) \equiv P(t+1),
\]

\[
\Pi^*_4(t, \cdot) \equiv P(t+1).
\]

The function \( \Pi \) represents the net effect of the political actions taken by the economic agents in the current period in determining the regulatory and political framework that will prevail in the next period.

3.2. Economic dynamics

Economic actions, such as production and investment, taken by the agents in period \( t \) (and perhaps also together with independent effects such as weather), determine the environment in period \( t+1 \). This will be modeled here as a deterministic
relation between the environment in $t$, the economic and political actions taken in $t$ and the environment in $t+1$. Thus,

$$e(t+1) = G(e(t), p(t), e(t)) = G(h_{te}, (\mu(e(t), t)), p(t), e(t))$$

Economic agent $i$'s preferences may change as a result of experience. Agent $i$'s experience in period $t$ may be described by the economic outcome in period $t$ and the environment prevailing in period $t$. This is described by the function $G_i$, where

$$\text{pref}(t + 1) = G_{i_{pref}}(at(t), e(t), \text{pref}(t))$$

The functions $G$ and $G_{i_{pref}}$ permit us to complete the recursive step from period $t$ to period $t + 1$, as shown in Fig. 1.

4. A multi-period game

The multi-period model introduced above can be made into a game by defining specific information structures, strategy spaces and payoff functions for the economic agents and the regulators, taking into account the number, possibly infinite, of periods.

An analysis of the multi-period game with 'fully rational' players would assume unlimited abilities of the agents to see into the possible futures, i.e., to know the entire game tree, as well as to remember arbitrarily long histories of play. Since the multiperiod model does not define a repeated game, but rather an infinitely long multistage game in which the utility functions and constraints on choices evolve along the path of play, the demands on information processing capacities of the players that would be imposed by a fully rational game theoretic analysis are very great, exceeding even the demands made by the same analysis of repeated games. This consideration would in itself not be decisive if we could expect a fully rational analysis to produce significant insights that would help in interpreting behavior or outcomes in similar situations with players of limited rationality. This seems unlikely, as well as descriptively wrong.
Furthermore, assumptions about the state of information of the players are problematic. The number of agents is very large. We have already assumed that each economic agent knows only his own environmental characteristic. The now standard approach in this situation, that of games of incomplete information, is to assume that the players have beliefs in the form of probability distributions over the possible types (preferences and other environmental parameters) that affect payoff functions or strategy domains of the players, including themselves where appropriate. See Harary (1967–58) or Mises (1911). It is typically the case in this kind of game that almost any outcome can be rationalized as an equilibrium, if the set of possible beliefs of the players is rich enough. The degree to which a formal analysis of a model fully specified along these lines would be illuminating depends on the questions being asked of the model, in addition to depending on its precise specification.

In order to simplify matters, we first present here a version of the game in which relatively myopic players can see only two periods into the future, and recall only one past period, making the model and the dynamic process into a succession of overlapping two-period games. That is, in the game consisting of periods 1 and 2, the players consider the moves in period 1 looking at the two-period game tree beginning with period 1, and do the same in period 2, looking at the tree of the game for periods 2 and 3. This formulation, though highly simplified, can yield a path of evolution that is not prima facie inconsistent with historical experience.

4.2. Assumptions

In formulating this succession of two-period games we make some further simplifying assumptions:

**Assumption 1.** At each t the economic agents correctly assume that
(i) \( R(t+1) = R(t) \) and
(ii) \( \text{pref}(t) = \text{pref}(t+1) \), i.e., that preferences of economic agents are the same in period \( t+1 \) as in \( t \).

**Assumption 2.** For all \( j \),
(i) \( \psi_j(\mu_j^t(a_j^t)) = a_j(t) \) and
(ii) \( \eta_j(e(t)) = \text{constant} \), i.e., regulator \( j \) observes directly only the action that is the outcome of the economic process. and
(iii) \( \text{pref}_j(t) = \text{pref}(t+1) \).

Whose regulator \( j \) can deduce about the environment from this is only that
\( e(t+1) \in G(\text{E}(t), a(t)) \).
Since regulator \( j \) observes \( p(t) \) at the end of period \( t \), he can deduce
\[
\lambda(t + 1) = \Pi_j(t, p(t))
\]
and hence also,
\[
\lambda(t + 1) = \Pi_{\lambda_j}(t, p(t))
\]

4.2. Histories

A history for a regulator \( j \) at period \( t \) consists of
\[
\gamma_j(t) = \{ \text{pref}(j, t), \alpha(t), \lambda(t - 1), P(t - 1), p(t - 1), r(t - 1) \}.
\]

A history for an economic agent \( i \) at period \( t \) consists of
\[
\gamma_i(t) = \{ \text{pref}(i, t), \lambda(t - 1), \mu(\nu(t - 1), i, t - 1), P(\cdot, t - 1), p(t - 1), r(t) \}.
\]
where
\[
\gamma_i(t) = \{ \gamma(t), \mu(\nu(t), i, t), e(t) \}.
\]

Regulator \( j \) can deduce \( \lambda(t) \) and \( \Pi(\cdot, t) \) from \( \gamma_j(t) \) using the function \( \Pi_j \).
Similarly, economic agent \( i \) can deduce \( \lambda(t) \) and \( \Pi(\cdot, t) \) from \( \gamma_i(t) \), using the function \( \Pi_i \).

Therefore, agent \( i \) can deduce \( \lambda(t) \) and \( \Pi(\cdot, t) \) from \( \gamma_i(t) \), using the outcome function \( h_{\lambda_i}(\cdot, t) \), and the function \( \Pi(\cdot, t) \).

This formulation of histories embodies assumption 3 that each agent, whether economic agent or regulator, looks back only one period.

4.3. Strategies

A myopic strategy for an economic agent \( i \) in the game for the periods \( t \) through \( t + \tau \), denoted \( f_i \), is a sequence of functions, where \( f_i \) consists of \( 2\tau \) functions, which we denote
\[
f_i = (f_{i,1}, \ldots, f_{i,2\tau})
\]
where \( f_{i,1} \) gives the economic action planned by economic agent \( i \) in period \( t \) for period \( t + k \), and \( f_{i,2\tau} \) gives the political action planned by \( i \) at \( t \) for period \( t + k \).

We also write
\[
f_i = (f_{i,1}, \ldots, f_{i,2\tau}).
\]

All functions have histories as arguments. Thus for \( \tau = 1 \),
\[
f_{i,1}(\gamma(t)) = f_{i,2}(r(t)) = \mu(\nu(t)).
\]
where
\[ f_{i,t} : F_t \rightarrow M_r(\tau(1)) \].

On the other hand,
\[ f_{i,t} \rightarrow \gamma'(1) = \tilde{f}_{i,10} \],

where
\[ \tilde{f}_{i,10} : F_t \rightarrow P', (e'(t), t) \].

Hence,
\[ f_{i,t} \rightarrow \gamma'(1) = \tilde{f}_{i,11}(\tau(1), \mu(\sigma(1), t), e'(t)) = \tilde{p}'(t) \],

which determines \(\tilde{p}'(t)\). Notice that the economic action of \(i\) in period \(t\) is taken without knowledge of the environment that prevails in period \(t\), while the political action of economic agent \(i\) in period \(t\) is taken with knowledge of \(e'(1)\).

Next, agent \(i\) at time \(t\) knows that he will learn the value of \(\alpha(t+1)\) (the regulations governing period \(t + 1\)) before he must decide on his message correspondence in period \(t + 1\). Therefore his action in period \(t\) for period \(t + 1\) is to choose a function of \(\tilde{f}_{i,t+1}\) denoted \(f_{i,t} \rightarrow \gamma'(1)\).

Thus,
\[ f_{i,t+1} : \gamma(1) \rightarrow \tilde{f}_{i,t+1} \],

where \(\tilde{f}_{i,t+1}\) the set of functions from \(A(t+1)\) to \(M_r(\tau(t))\). Thus,
\[ f_{i,t} \rightarrow \gamma'(1) = \tilde{f}_{i,t} \],

where
\[ \tilde{f}_{i,t+1} : \gamma(1) \rightarrow \tilde{f}_{i,t} \].

i.e.,
\[ f_{i,t} \rightarrow \gamma'(1) = \tilde{f}_{i,t} \rightarrow \gamma(1) = \tilde{f}_{i,t+1} \rightarrow \gamma(1) = \tilde{f}_{i,t} \rightarrow \gamma(1) = \tilde{f}_{i,t+1} \].

Finally, the last component of \(i\)'s strategy at \(t\) is a function that determines the political action taken by \(i\) in period \(t + 1\) as a function of the information \(i\) will have in period \(t + 1\). Thus,
\[ f_{i,t+1} : \gamma(1) \rightarrow \tilde{f}_{i,t+1} \],

where
\[ \tilde{f}_{i,t+1} : \gamma(1) \rightarrow P'(t(1), t + 1) \].

i.e.,
\[ f_{i,t+1} : \gamma(1) \rightarrow \tilde{f}_{i,t+1} : \gamma(1) \rightarrow \tilde{f}_{i,t+1} : \gamma(t + 1), \mu(\sigma(t + 1), t + 1), e'(t + 1) \rightarrow \tilde{p}'(t + 1) \].
A myopic strategy for regulator \( j \) when the horizon is \( v \), is

\[ g^j_i = (g^j_{i+1})_{i=0}^{\infty}. \]

In the two-period game with periods \( t \) and \( t+1 \), a strategy of regulator \( j \) is a pair of functions \( (g^j_t, g^j_{t+1}) \), such that

\[ g^j_t([\gamma(t)]) = \tilde{g}^j_t. \]

where

\[ \tilde{g}^j_t : B([1,t]) \to [1,(t+1)]. \]

Thus,

\[ g^j_t([\gamma(t)]) = \tilde{g}^j_t([B([1,t]))) = r(t) = (\theta(t), \rho_j(t)). \]

If needed, this can be written more explicitly in terms of the component functions,

\[ g^j_t([\gamma(t)]) = (g^j_{i=0}([\gamma(t)])[\gamma(t)]) \]

\[ \tilde{g}^j_t([\gamma(t)]) = (\tilde{g}^j_{i=0}([\gamma(t)])[\gamma(t)]) = r(t) = (\theta(t), \rho_j(t)). \]

Similarly,

\[ g^j_{t+1}([\gamma(t)]) = \tilde{g}^j_{t+1}. \]

where

\[ \tilde{g}^j_{t+1} : B([1,(t+1)]) \to [1,(t+1)]. \]

Thus,

\[ g^j_{t+1}([\gamma(t)]) = \tilde{g}^j_{t+1}([B([1,(t+1)])]) = r(t+1) - (\theta(t+1), \rho_j(t+1)). \]

and

\[ g^j_t([\gamma(t+1)]) = g^j_{t+1}([\gamma(t)])[\gamma(t+1)]. \]

\[ \tilde{g}^j_{t+1}([\gamma(t)]) = (\tilde{g}^j_{i=0}([\gamma(t)])[\gamma(t)]) = r(t+1) - (\theta(t+1), \rho_j(t+1)). \]

and

\[ \tilde{g}^j_{t+1}([\gamma(t+1)]) = (\tilde{g}^j_{i=0}([\gamma(t+1)]) \tilde{g}^j_{i=0}([\gamma(t+1)]) = (\theta(t+1)). \]

We write

\[ g^j = (g^j_{i=0})_{i=0}^{\infty}. \]
4.4. Utilities and payoffs

The period utility that players \(i\) and \(j\) respectively get when the strategy complex \((f_i, g^i)\) is played are given by period payoff functions \((\tilde{U}_{ij})_{x t_1}, (\tilde{U}_{ij}^g)_{x t_1}\) as follows:

\[
\begin{align*}
\tilde{U}_{ij}(t) = \tilde{U}_{ij}(f_i, g^i; \text{pref}(t)) &= U_{ij}(t; e(t), p(t); \text{pref}(t)) \\
&= U_{ij}\left(h_{i,e_i(t)}, g_{i,t} ; \beta_{i,t}\left(\bigotimes_{i \in I} \gamma_{i,t}\right)\right)
\end{align*}
\]

\[\text{pref}(t) \quad (4.1)\]

Note that the definition of the strategies given above means that the term involving the intersection of the functions \(g^i_{t_1}\) also depends on all the functions \(g^i_{t_2}\). Similarly, in period \(t+1\),

\[
\begin{align*}
\tilde{U}_{ij}(t+1) = \tilde{U}_{ij}(f_i, g^i; \text{pref}(t+1)) &= U_{ij}(t+1; e(t+1), p(t+1); \text{pref}(t+1)) \\
&= U_{ij}\left(h_{i,e_{i+1}(t+1)}, g_{i,t+1} ; \beta_{i,t+1}\left(\bigotimes_{i \in I} \gamma_{i,t+1}\right)\right) \\
&= \left(U_{ij}\left(h_{i,e_{i+1}(t+1)}, g_{i,t+1} ; \beta_{i,t+1}\left(\bigotimes_{i \in I} \gamma_{i,t+1}\right)\right) \right)
\end{align*}
\]

\[\text{pref}(t+1) \quad (4.2)\]

where, environment and preferences evolve according to the functions \(G\) and \(G_{\text{pref}}\) respectively.

The utilities of regulator \(j\) in the game in periods \(t, t+1\) is

\[
\begin{align*}
\tilde{U}_{ij}(t) = \tilde{U}_{ij}(f_i, g^i; \text{pref}(t)) &= U_{ij}(t; e(t), p(t)\text{pref}(t)) = U_{ij}(t; A(x, \cdot)). \\
h_{i,1}(t; e(t), (\cdot))\text{pref}(t) \quad (4.3)
\end{align*}
\]

and

\[
\begin{align*}
\tilde{U}_{ij}(t+1) = \tilde{U}_{ij}(f_i, g^i; \text{pref}(t+1)) &= U_{ij}(t+1; e(t+1), p(t+1)\text{pref}(t+1)) \\
&= U_{ij}(t+1; A(t+1), e(t+1)\text{pref}(t+1)) \\
&= \left(U_{ij}(t+1; A(t+1), e(t+1)\text{pref}(t+1)) \right)
\end{align*}
\]

\[\text{pref}(t+1) \quad (4.4)\]
where, as above,
\[ \theta^i(t) = z^{i+1}_t(\gamma(1)) \]
\[ \rho^i(t) = \tilde{z}^{i+1}_t(1,A(1)) \]
\[ \mu^i(t) = \tilde{z}^{i+1}_t(1,A(t)) \]
\[ \pi^i(t) = \tilde{z}^{i+1}_t(1,A(t)) \]
\[ \theta^i(t) \in U(\rho^i(t)) \]
\[ \theta^i(t-1) = z^{i,j+1}_t(1,A(t-1)) \]
\[ \rho(t+1) = \tilde{z}^{i,j+1}_t(1,A(t+1)) \]
\[ \mu^i(t+1) = \tilde{z}^{i,j+1}_t(1,A(t+1)) \]
\[ A(t+1) = \Pi^j_t(\rho(t)) = \Pi^j_t(\tilde{z}^{i,j+1}_t(1,A(t+1))) \]
\[ \mu(t+1) \in U(\tilde{z}^{i,j+1}_t(\gamma(t))) \]

And, by Assumption 2, for all \(i\) and \(j\),
\[ \text{pref}(t+1) = \text{pref}(t) \]
\[ \text{pref}(t+1) = \text{pref}(t) \]

The payoff to economic agent \(i\) in the game played in periods \(t, t+1\), denoted \(v^i(t)\), is a function of the one period utilities. For example,
\[ v^i(t) = \delta_{k} u^i(t) + \delta_{k} u(t+1) \]
where \(\delta_{k}, k = 1, 2\) are discount factors. Alternatively,
\[ v^i(t) = \min_{u(t), u(t+1)} [u^i(t), u(t+1)] \]
and the same for the payoff \(v^j(t)\) of regulator \(j\) in the game played in periods \(t, t+1\). For simplicity, we take the discount factors equal to 1 in the two-period game.

The payoffs to both economic agents and to regulators do not depend on \(p(t+1)\), the political action taken in period \(t+1\), because these actions have no effect until period \(t+2\), which is beyond the myopic horizon at \(t\). Therefore, those components of action that do not affect the payoffs in periods \(t\) and \(t+1\) do not affect strategies in the period \(t+2\), and hence may be ignored. Similarly, \(P(t+1)\) does not affect payoffs, so the determination of \(P(t+1)\) can be ignored in the period \(t, t+1\) game.

A myopic equilibrium of the two-period game at \(t\) determines the actions taken in period \(t\) and a plan for the actions to be taken in \(t+1\). The actions taken in
period $t$ determine the rules — the legal and budgetary frameworks — effective in period $t + 1$. But the actions planned in period $t + 1$ need not in fact be carried out, because at $t + 1$ the agents can see ahead to period $t + 2$. Hence their actions in period $t + 1$ are determined by the equilibrium myopic strategies for the game played in periods $t + 1, t + 2$. The period $t + 1$ actions called for by those new strategies need not be the same as those determined for period $t + 1$ by equilibrium strategies in the two-period game with periods $t$ and $t + 1$. Accordingly the history of the economy would unfold as a sequence of the first period actions determined by the successive equilibria of the sequence of two period games.

This structure would not be seriously affected by weakening some of the simplifying assumptions made above, such as replacing the assumption that preferences are the same in the two periods with the assumption that each agent in the first period has some way of predicting his second period preferences. For example, he might have a probability distribution over his second period preferences, perhaps conditioned on his first period preferences and the outcome of the economic process, that describes his second period preferences. In that case payoffs would be expected utilities. Similarly, discount factors less than 1 would not change the essentials.)

4.5. Political cooperation

It was pointed out above that economic agents are legally prohibited from organizing or communicating explicitly about their economic actions, but their rights to organize in order to coordinate their political actions are constitutionally protected. Similarly, the regulators are not prohibited from coordinating their regulations. Indeed, the notion of a coherent economic policy on the part of the state seems to entail some degree of coordination among the state's agents of economic policy, although experience suggests that this is as much honored in the breach as in the observation.

This situation can be modeled at least two ways:

1. We may define a multi-move game in which the solution concept is one of an equilibrium stable under certain coalitional deviations; or.

2. We may derive a characteristic function (the two-period model in which economic agents may choose their political actions cooperatively, but not their economic actions, and regulators may cooperate to choose their regulations.

The extent to which regulators and economic agents may cooperate is murky. The notion of capture of a regulatory agency by those regulated involves an extreme form of such cooperation. The pervasive use by economic agents of representatives who communicate on their behalf directly with regulators suggests that at least some degree of coordination between economic agents and regulators exists, and is in principle permitted.

In the characteristic function game, the laws governing political action deter-
mine whether it is one with or without side payments. Certainly a direct payment by an economic agent or a government employee in order to affect a regulatory action is prohibited. If side payments are not legal, or not feasible because the number of agents in a coalition is too large to make negotiation of side payments possible, then the economic agents can make compensatory payments among themselves only through the economic mechanism. Similarly, payments to regulators may only be made in the future through the economic mechanism. Concern about the so-called revolving door between government agencies and industry testifies to the belief that such payments exist. On the other hand, regulators may legally require economic agents to make side payments to one another directly, as in the case of transfers required among oil importers and domestic producers referred to above, or indirectly via transfers among agents carried out by the government.

We present first a more formal treatment of a multistage game, as above, still in the case of the two-period model in which the players are myopic.

First, using the relations (4.1), ..., (4.4) and Assumptions 1 and 2, and using the fact that no payoff depends on \( f_2 \), because the costs of political action in period \( t + 1 \) result in no benefit within the horizon of agent \( i \), and hence no economic agent \( i \) will undertake political action in period \( t + 1 \), we may write the payoffs of economic agent \( i \) in \( H(t) \) in the non-cooperative game played in periods \( t \) and \( t + 1 \), as in (4.5). Here we use the undiscounted sum of per-period utilities as the payoff function. The histories \( \gamma_i, i \in H(t) \), and \( \gamma_i, j \in H(t) \) are initial conditions for the game played in periods \( t \) and \( t + 1 \).

\[
\begin{align*}
v'_i(t) &= u'_i(t) + u'_i(t + 1) \\
&= \Delta_i(f_i(\gamma_i(t)), g'_i(\gamma_i(t) ; \text{pref}(t)) + \Delta_i(f_i(\gamma_i(t)), g'_i(\gamma_i(t) ; \text{pref}(t)) \\
&= V'_i(f_i(\gamma_i(t)), g'_i(\gamma_i(t) ; \text{pref}(t)) \quad (4.5)
\end{align*}
\]

Similarly for regulator \( j \) in \( H(t) \).

\[
\begin{align*}
v'_j(t) &= u'_j(t) + u'_j(t + 1) \\
&= \Delta_j(f_j(\gamma_j(t)), g'_j(\gamma_j(t) ; \text{pref}(t)) + \Delta_j(f_j(\gamma_j(t)), g'_j(\gamma_j(t) ; \text{pref}(t)) \\
&= V'_j(f_j(\gamma_j(t)), g'_j(\gamma_j(t) ; \text{pref}(t)) \quad (4.6)
\end{align*}
\]

We consider several cases.

**Case I.** The regulators do not cooperate strategically.
For each value of the strategy complex \( f_i = (f_{i,t}, i \in I_t) \) the sets of players, \( R(t) \) and \( A(t) \), the strategy domains of \( f_{i,t} \), \( f_{i,t+1} \), \( s_i \), \( g_{i,t} \), \( j \in R(t), j \in A(t) \), in the original two-period non-cooperative game and the payoff functions \( V' \) and \( V'_{i,t} \), define a non-cooperative game, which we denote \( \text{NC}(f_i) \). (This notation suppresses the fact that the two-period game at time \( t \) depends on the initial conditions.)

When the economic agents in period \( t \) make the choice of \( f_i \), they are together specifying the non-cooperative game \( \text{NC}(f_i) \). But, as we have already pointed out, the economic agents are permitted to make that choice cooperatively. Though the decision facing the economic agents can be modeled in different ways, the essence of the matter is that the economic agents can decide cooperatively by choice of \( f_i \), which NC game to play in periods \( t, t+1 \). While they might want to incorporate into this decision a further decision about how to play the game \( \text{NC}(f_i, t) \), such as agreeing on which equilibrium to play, or signaling so as to coordinate their strategies in \( \text{NC}(f_i, t) \), they are legally prohibited from doing so. Economic agent \( i \), however, may make her political action contingent on the economic outcome, and therefore to some extent on her first-period economic action.

We model this as a multistage game in which the one move is made cooperatively, and the rest non-cooperatively. Recall that the strategy domain of player \( i \) is the set of functions from the set of histories for \( i \) up to time \( t \), denoted \( T_i \), to \( P(T) \). We denote this by \( \Phi_i = P(T) \).

This game is shown in Fig. 2. To simplify notation, we write \( f_i = f, V' = V' \), and denote the equilibria of \( \text{NC}(f) \) by \( E(f) \). When \( E(f) \) is finite, it can be written \( \{ (x_1, \ldots, x_{|I|}) \} \).

Fig. 2 shows that at the first move the economic agents cooperatively choose a strategy complex \( f = (f_1, \ldots, f_{|I|}) \). The choice of \( f \), say \( f = f' \), determines the non-cooperative game \( \text{NC}(f') \). In the diagram, \( \text{NC}(f') \) has \( k \) equilibria, \( \xi_1(f'), \ldots, \xi_k(f') \), where \( k = |R(f')| \) and \( \xi_j(f') \) denotes the strategy com-
plex \( \{(f'_1, f'_2, \ldots, f'_{n-1}, g^{*}, f_n) \in \mathbb{F} \} \) that constitutes the \( q \)th equilibrium of NCE \( f \). This equilibrium leads to the payoff vector

\[ V(\xi(f)) = \{V(\xi_1(f)), \ldots, V(\xi_n(f))\}. \]

Similarly for the strategy complex \( f' \). The regulators, who in this case are not strategic players in the cooperative phase of the multistage game, receive payoffs according to their payoff functions and strategic choices in the game NCE \( f' \).

Suppose for simplicity that the game NCE \( f \) has a unique equilibrium for each

\[ f = (f_1, f_2, \ldots, f_n) \in \mathbb{F}. \]

In that case, the payoff functions of the multistage game are well-defined functions. An equilibrium of the game consists of a strategy complex

\[ \xi(f) = \{(f_1, f_2, \ldots, f_n, g^*)\} \in \mathbb{F}, \]

such that \( \langle f_1, f_2, \ldots, f_n \rangle \) is stable with respect to coalitional deviations, while the remaining components of \( \xi(f) \) are stable with respect to individual deviations.

Case II.a. The regulators cooperate strategically with each other but not with economic agents.

Case II.b. The regulators and economic agents may cooperate.

In Case II.a the regulators choose \( g^* \) cooperatively while the economic agents choose \( f_n \) cooperatively. This amounts to the requirement that, in the notation of Case I, \( \xi(f) \) is stable with respect to deviations from \( f \) by coalitions of economic agents, deviations from \( g^* \) by coalitions of regulators, and the remaining components of \( \xi(f) \) are stable with respect to individual deviations.

Case II.b differs from II.a only in that \( f \) and \( g^* \) must be stable with respect to deviations by any (possibly mixed) coalition of economic agents or regulators.

A solution in the multiperiod game implicitly determines a coalition structure in that game. That coalition structure may well be a covering of the set of players, rather than a partition. It may be expected that a relatively small group of agents will have some important interests in common, say to obtain protection for the products they make. There may be a larger group including the same agents who also have in common other interests that are not sufficiently important to warrant paying the costs of coordinated political action. This appears to be the typical case with consumers who usually do not organize to oppose policies that raise the cost of imported goods that they buy. And, of course, the same agents who cooperate to advance certain common interests may have other interests that are in conflict, so that a given agent may participate in several, even many, different political groups in pursuit of her various interests. This can be described by a covering of the set of agents, perhaps in some cases a partition, where each subset in the
covering is a set of agents pursuing some political agenda aimed at promoting a limited interest they have in common. 12

The second modeling approach is to use the game defined above to derive a characteristic function game. Let the set of players be \( \{ I \} \), i.e., we consider the case in which economic agents may choose their political actions cooperatively.

12 The concept of coalitions of agents is natural and useful in interpreting economic and political history. In Reuter and Hughes (1981) U.S. economic history from about 1880 to about 1940 is summarized using the following coalitions of economic agents:

1. Big Business. Business were the large firms growing in connection with a powerful wave of industrialization that ended in the first great merger movement of 1887–1904. The policy objectives of this group were:
   1. High tariffs.
   2. A strict speculative monetary standard.
   3. High (relatively) interest rates.
   4. Easy access to natural resources on public lands.
   5. Unlimited immigration.

2. Farmers. In different political coalitions – Greenbackers, Grangers, Populists – the farmers began to press the Federal Government systematically after 1865. Basically they wanted:
   1. Free or easy access to public lands.
   2. Low tariffs.
   3. A “soft money” monetary standard (paper money, free silver, etc.) and cheap credit.
   4. Restrictions on immigration.
   5. Control by government of big-business monopoly powers.
   6. Restrictions on land ownership by aliens and corporations.
   7. A progressive income tax.

3. Organized Labor. Following earlier abortive efforts to create permanent organization, the American Federation of Labor, after 1886, followed the political policy of “reward our friends, punish our enemies”. The Federation attempted to gain advantages for the rising class of urban artisans, and there was continuous effort to form unions among shop workers and factory employees. Organized labor wanted help from government. It favored:
   1. The eight-hour day and legal restrictions on child labor.
   2. Government support for the right to organize and strike.
   3. Restrictions on immigration.
   4. Cheap money.
   5. A progressive income tax and redistribution of income and wealth.
   6. High tariffs.

Urban Consumers, Small Businesses, and Professionals. These small businesses, shopkeepers, white-collar workers, and professionals – the most rapidly increasing interest group formed the 13th of the urban population, the most rapidly growing part of the total, which by 1920 would be population in members. These urban populations would ultimately form the basis labor–consumer–Democratic coalitions. Their interest were best served by:

1. Low tariffs.
2. Suppression of big-business monopoly power.
3. A Federal income tax to redistribute in favor of urban overhead investment.
4. Cheap mass transportation.
5. Conservation protection.
6. Cheap-money policies and a soft-money standard.

See Reuter and Hughes (1981) for references and further discussion.
and regulators may not cooperate. The characteristic function $\xi$ may be defined as follows.

Let $C$ be a coalition, and let $C \subset C$ denote the set of players in $(C \setminus C$. Let the players in $C$ agree to play $f^i_C$, $i \in C$ and let the players not in $C$ play $f^i_C$, $i \in C$, and denote the resulting complex $f^C_C$ by $f_C, f_{C-C}$, a permutation of $f^C_C$. When the players in $(C \setminus C$ play $f_C, f_{C-C}$, the game $\text{NOR}(f_C, f_{C-C})$ is determined. Under the assumption that this game has a unique equilibrium $(f_C, f_{C-C})$, the corresponding payoff vector $V(\xi(f_C, f_{C-C})) = (V(\xi(f_C, f_{C-C}), \ldots, V^*(\xi(f_C, f_{C-C})))$ is determined. Assuming transferable utility, we define the characteristic function $\chi_C$ by

$$\chi_C(\xi) = \max_{f_C, f_{C-C} \in \text{NOR}} \sum_{i \in C} V^*(\xi(f_C, f_{C-C}))$$

The Aumann–Maschler bargaining set (Maschler, 1992) may be used to analyze this game. One of the attractions of cooperative game formulation is that it makes explicit the role of coalitions. As remarked above, this accords well with our observation of what goes on in the economy around us.

5. Concluding Remark

The objective of this paper is to provide a general framework for the study of interactions between the state and the private economy in which regulatory interventions are endogenous. The model has been applied to the task of interpreting a period of the economic history of the U.S. (see Reiter and Hughes, 1981). While it appears to be useful in providing a structure that is helpful in organizing our understanding of experience, and in giving formal representation to a variety of different regulatory instruments, it remains to apply the model directly to examples in which specific strategic interactions of economic agents determine specific institutional arrangements as solution outcomes.

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References


