

The two models of social change presented in this paper are intended for rather different purposes. Roughly speaking, there are three issues: (1) the mechanisms of social mobility; (2) the amount of social interaction of persons in a given system; and (3) the increasing or decreasing rigidity of class boundaries over time. The first model considers the first two issues, and is designed to predict the future status distribution of a population under fixed social mobility conditions. But these conditions do change, and they change at least partly as a result of transitions between social classes. Therefore, a second model is proposed which takes into account the ability of people in a social class to influence the processes which govern transitions to or from that class.

## SOCIAL STATUS AND SOCIAL CHANGE

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### INTRODUCTION<sup>1</sup>

**I**N this paper we present two related models of social change. The first of these deals with social mobility. Assuming that the complex of conditions governing mobility remains unchanged, the model generates the future history of the status distribution of the population. This enables us to trace out the ultimate consequences of given mobility conditions in terms of the status distribution of the population. It is also possible to determine such quantities as the average number of generations a family will spend in a given class, or the average number of generations required to go from one class to another.

The second model drops the assumption of unchanging mobility conditions. The conditions governing mobility are made to change in response to the movements that actually occur. Efforts of people to facilitate some movements and to resist others conflict and interact with one another so as to change the conditions of mobility as movement takes place. In this model there is a simultaneous evolution of the status distribution of the population and of the mobility conditions themselves.

More specifically, we define social position

<sup>1</sup> A version of this paper was presented at the 1961 meeting of the American Sociological Association. Robert Perrucci, Phillip Marcus, and James S. Coleman critically evaluated an early draft of the paper.

in terms of a set of "status symbols"; we choose the generation as our time interval, and we consider the movement of individuals from an initial set of status symbols attached to the family of their parents to a terminal set of status symbols attached to their own families (the ones in which they are the parents). In the first model we make no assumption as to the existence of hierarchy, or "dimensionality," of these positions.

By a status symbol we mean an attribute or quantity that influences the social status of an individual and all of which taken together determine his social status (position). Status symbols may "measure" biological, physical, and economic, as well as more traditionally sociological, phenomena. Thus, they may specify skin color, height, value of automobile owned, lineage. For present purposes a person is characterized by a complete specification of his status symbol values.

The first process we consider is as follows. At any time, the existing population is characterized by a distribution of status symbols among its members. Actions and other events occurring in the course of time lead to modification of the distribution of status symbols. For example, marriages, births, choice of occupation, and the like, result in the course of time in a new distribution of status symbols. We first formulate a process capable of describing this evolution of the distribution of status symbols as time passes,

under the assumption that the social rules, mores, and patterns of behavior remain constant. Relevant patterns of behavior include access to education, occupation, and marriage, among other things. Such a process represents the net effect of social interactions on the status of people, and depends on the particular social mechanisms or rules which connect actions and events with status, e.g., in one society, marriage into a "better family" may improve one's status, while in another, such a marriage may result in loss of status for all concerned.

We shall attempt to state our models in such a way that they may be used in conjunction with the most varied empirical data and with a wide variety of substantive theoretical systems. In order to achieve such generality we will have to drop many of the simplifications common in analyzing Western societies. We shall resort to formal terminology which may, at first, seem rather remote from the usual substantive discourse on such topics. But it is our intent and hope that we shall thereby achieve a theoretical system that does not depend wholly upon current data and viewpoints and may therefore have increasing relevance with increasing knowledge. As it turns out, since we assume as little as we can, the user of the model will have to specify many characteristics of a society before he can generate predictions about it. The fact that such characteristics must be explicit can be viewed as a desirable feature of our models.

#### PART I. STATUS DISTRIBUTION: MOBILITY CONDITIONS FIXED

We begin with a set of persons, a set of status symbols, a set of actions, and a set of consequences of these actions. Each person has a particular combination of status symbols assigned to him. The actions available to persons and taken by them are conditional upon their social positions. The consequences of actions are new or terminal social positions resulting from the configuration of actions taken and initial social position. We wish to compute the resulting distribution of

persons by status symbols one generation later.

The combination of status symbols assigned to a person must include a particular attribute or numerical value on each of the possible types of status symbols which have been selected as relevant for the particular analysis. Thus, if skin color, occupation, and income have been selected, each person must have a designated value or attribute on each of these three symbols, say "white," "painter," "under \$6,000." It is necessary that the combination should be the unit of investigation. Study of each symbol alone will not yield statements about the combination unless we add assumptions about the combination, e.g., is a Negro doctor more similar in status to a Negro ditch digger, a white ditch digger, or a white doctor? If the combination itself is the unit, we need not assume rules of combination, e.g., independence.

*Definition 1.* Let  $I_t = \{1, 2, \dots, R_t\}$  be the set of persons in generation  $t$ .

*Definition 2a.* Let  $S_j$  ( $j = 1, 2, \dots, J$ ) be the set of possible values for the  $j$ th status symbol.

$S_j$  may be a discrete set, as would be the case if the  $j$ th status symbol referred to some qualitative attribute, e.g.,  $j = 1$  refers to skin color,  $S_1 = [\text{Black, Brown, Yellow} \dots]$

*Definition 2b.* Let  $S = S_1 \times S_2 \times \dots \times S_J$ . An element  $s \in S$  is called a *status symbol array* or a *status description*. Thus any element  $s$  is a combination of values on every status symbol.<sup>2</sup> In the special case in which  $S_j$  is a finite set for each  $j = 1, \dots, J$ , then  $S$  is also a finite set and can be enumerated; i.e.,  $S = [s_1, \dots, s_M]$ .

We now wish to consider the actions available to persons that determine the acquisition of terminal status symbols. These actions may include education choice, marriage choice, occupation choice, and so on. Again we will use set notation to indicate that the unit of choice is the combination of actions.

*Definition 3a.* Let  $A_k$  ( $k = 1, \dots, K$ ) be

<sup>2</sup> The operator  $\times$ , the Cartesian product, is equivalent to a cross-classification formed from two or more classifications; each element of the cross-classification is a single cell. The symbol  $\in$  is read, "is an element of."

the set of the actions available with respect to the  $k$ th component of individual choice. E.g., if  $k = 1$  refers to education,  $A_1 =$  [Education through high school, Education through Ivy League College, Education through State University. . .]

*Definition 3b.* Let  $A = A_1 \times A_2 \times \dots \times A_K$ . Then  $a \in A$  is called an *action array* or *action description*. In the special case in which  $A_k$  is a finite set for each  $k = 1, \dots, K$ , then  $A$  is also a finite set and can be enumerated; i.e.,  $A = [a_1, \dots, a_L]$ . An action description  $a \in A$  specifies the choices made or actions undertaken with respect to every component of action deemed relevant, i.e., those named  $1, \dots, K$ . Thus, an element  $a \in A$  might describe a combination of actions with respect to education, marriage, consumption patterns, etc. The components of  $a$  are not in general independent. We might regard action as the means of acquisition of a terminal status symbol description. Moreover, the action choices made by a person can be thought to depend on his initial status description; this reflects different access to actions of people with different initial status descriptions as well as their differing preferences among actions. We formalize this dependence of action on status description probabilistically. Note that more than one person may have the same action description and/or status description; therefore we introduce new symbols  $x_i, y_i,$  and  $z_i$  to refer to characteristics of person  $i$  in our definitions. The probabilities that relate status symbol descriptions to action descriptions must be defined in terms of persons who are characterized by both descriptions.

*Definition 4.* Let  $x_i$  be the initial status description of person  $i$ , and  $z_i$  his action description. Let  $a \subset A$  be an event (i.e., event  $a$  happens whenever an action description in  $a$  occurs) and let  $s \subset S$  be an event.<sup>3</sup> Let  $g(a|s)$  be the conditional probability that

<sup>3</sup> The symbol  $\subset$  is read, "is contained in," and is equivalent to, "is a subset of"; a subset may contain any number of elements. Since  $x_i, y_i,$  and  $z_i$  are defined as elements of  $a$  and  $s$ , then  $a$  and  $s$  are themselves sets, but  $a$  and  $s$  remain subsets of  $A$  and  $S$  respectively. It is necessary to define events as we wish to define the probabilities of these events.

$z_i \in a$  given that  $x_i \in s$ . The dependence of the terminal status description of anyone on his actions is also probabilistic.

*Definition 5.* Let  $y_i$  be the terminal status description of person  $i$  and  $z_i$  his action description. Then for  $s \subset S$  and  $a \subset A$ , let  $f(s|a)$  be the conditional probability that  $y_i \in s$  given that  $z_i \in a$ .

*Definition 6.* If  $x_i$  is the initial status description of person  $i$  and  $y_i$  his terminal status description, let

$$p(s|s') \equiv \int_a f(s|a) dg(a|s')$$

be the conditional probability that  $y_i \in s$  given that  $x_i \in s'$ .

In the special case when  $S$  and  $A$  are both finite sets and can be enumerated, then the functions  $g(a|s')$  and  $f(s|a)$  can be represented as stochastic matrices, say  $G$  and  $F$ . Then  $P(s|s') = GF$ .  $G$  is a  $(M \times L)$  matrix,  $F$  is  $(L \times M)$ .

*Definition 7.* Let  $C = [C_1, C_2 \dots C_N]$  be a partition of  $S$  into the subsets  $C_1 \dots C_N$ .

(By a *partition of a set* is meant a classification of the elements of that set into mutually exclusive subsets, which together include the whole of the original set.) We call  $C$  a *social class system*. For our present purposes the particular social content or meaning of the social class system is irrelevant. We may take  $C$  to be an arbitrary partitioning of  $S$ . The numbers  $1, 2, \dots, N$  need not imply rank. However, to be at all useful in representing some specific society, the class system should be such that people whose status descriptions put them in the same social class should exhibit a certain homogeneity of action. Identity of action is, of course, not required, because all the relationships involved are probabilistic. Lack of sufficient homogeneity means that the social class system  $C$  is not a useful one for this model. Given the social class system  $C$  we may confine our attention to the probabilities

$$P(C_j|C_i) \equiv \int_a f(C_j|a) dg(a|C_i).$$

These can be set forth in a matrix  $P = ((p_{ij}))$ , whose entry

$$p_{ij} \equiv P(C_j|C_i).$$

*Definition 8.* The matrix  $P = ((p_{ij}))$ ,  $p_{ij} \equiv P(C_j|C_i)$   $i, j = 1 \dots N$ , is called the *class transition matrix*. The probability  $p_{ij}$  is the probability that a person whose initial status description is in social class  $C_i$  attains a terminal status description in social class  $C_j$ .<sup>4</sup> In the finite case in which  $f(C_j|a) = F, g(a|C_i) = G$ , then  $P = GF$ .  $G$  is a  $(NxL)$  matrix,  $F$  is  $(LxN)$ .

*Definition 9.* Let  $m = (m_1, \dots, m_n)$  be a vector of nonnegative integers. The number  $m_i$  is the number of people in class  $C_i$ ;  $i = 1, \dots, N$ . The Markov chain defined by

$$m(t+1) = m(t) \cdot P \quad t = 0, 1, \dots$$

is a *model of social class mobility under stationary social conditions*.

Because the probability of ending up in a given social class depends only on the class in which one starts, and because we have a finite number of social classes, the stochastic process we have constructed is a *Markov chain*. In such a process it is possible to trace the social class distribution of a population over many generations, and to analyze the long-run distribution on the basis of knowl-

<sup>4</sup> We note that, for a given action description, say  $a_k$ , the probability of attaining a particular terminal class, say  $C_j$ , is given by  $g(C_j|a_k)$  which is independent of the initial class position, say  $C_i$ . Thus if a person from  $C_i$  has a given action description  $a_k$ , then the chance that he achieves  $C_j$ , say  $\pi$ , is identical with the chance that a person starting from  $C_{i+1}$ , given  $a_k$ , will achieve  $C_j$ . For our present purposes this assumption will suffice, but for certain other purposes a more general formulation may be useful. For example, we may wish to analyze data on careers, comparing various action descriptions, and we would like to test the alternative hypothesis that these two chances are different. This can be done by modifying the functions  $f$  and  $g$ . We take the cross partition of  $A$  and  $C$  and obtain a new set of elements. These elements, say  $d_k \subset D$ , can then replace  $a \subset A$  in the functions  $f$  and  $g$ . Thus  $f = f(C_j|d_k)$ ,  $g = g(d_k|C_i)$ . Then two persons from different initial classes will not necessarily have the same chance of attaining a particular terminal class even though they possess identical action descriptions (Perrucci, 1961).

edge of the transition probabilities. We now present an example of such a chain.

### Example 1

We shall consider a hypothetical society in which there are two status symbols, income and "ancestry." Thus, let

$$S_1 = \{x: x \text{ is a nonnegative real number}\}$$

$$S_2 = \{y: y \text{ is a nonnegative integer}\}$$

The set  $S = S_1 \times S_2$  consists of pairs  $(x, y)$ ,  $x \in S_1$ ,  $y \in S_2$ , where  $x$  specifies an income in dollars, and  $y$  ancestry type measured by number of generations in the society at some given time, say the initial time.

We take a social class system with three classes, defined as follows.

$$C_1 = \{(x, y): x \geq \$5,000, y \geq 4\}$$

$$C_3 = \{(x, y) x < \$10,000, y \leq 2\}$$

$$C_2 = S - (C_1 \cup C_3).$$

Briefly, to be in  $C_1$ , the "top" class, one must have an income of at least \$5,000, and four or more generations of ancestry; to be in  $C_3$ , the "lowest" class, one must have an income of less than \$10,000, and no more than two generations of ancestry; everyone not in class  $C_1$  or in  $C_3$  is in  $C_2$ .

We now consider actions and their consequences. Suppose that there are two possible dimensions of action in this society, namely choice of occupation and choice of a marriage partner. Thus,

$$A = A_1 \times A_2$$

where  $A_1$  is the set of possible occupational choices and  $A_2$  the set of marriage choices.

We shall suppose there are two occupations,  $O_1$  and  $O_2$ , so that

$$A_1 = \{O_1, O_2\}$$

and that the significant fact about marriage choice is the ancestry type associated with the marriage partner, so that an action of marriage choice may be represented by the ancestry type "married into."

$$A_2 = \{x; x \text{ is a nonnegative integer}\}.$$

We shall further suppose, for purposes of simplification, that the two dimensions of action may be considered independently.

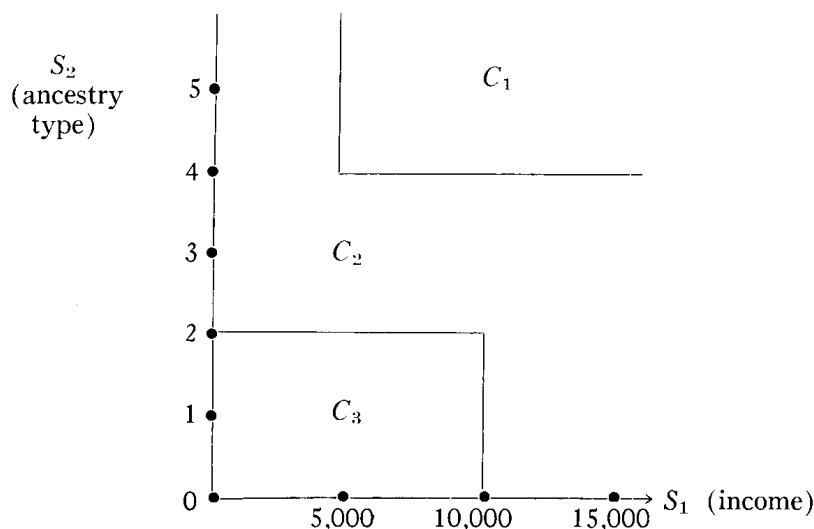


FIG. 1. The Class System and the Status Symbol Set.

The class system and the status symbol set may be pictured as in Figure 1.

We suppose the mores, sanctions, and preferences of this society are such that persons in class  $C_1$  are certain of going into occupation  $O_1$ , persons in class  $C_3$  are certain of entering occupation  $O_2$ , while persons in class  $C_2$  are such that some may enter  $O_1$ , perhaps with difficulty, while the others enter  $O_2$ . We will suppose that the probability of a person in  $C_2$  entering  $O_1$  is  $\frac{1}{2}$ . We may therefore represent that part of the function  $G$ , referring to occupational choice by the matrix,

$$G_1 : \begin{matrix} & \begin{matrix} O_1 & O_2 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix} \end{matrix}$$

whose entries are the relevant probabilities.

With regard to marriage choice, we shall in the interests of simplicity assume that marriage mores are such that a person in class  $C_1$  is equally likely to marry in  $C_1$  as in  $C_2$  and never marries in  $C_3$ , a person in  $C_2$  is equally likely to marry in any class as any other, while one in  $C_3$  is equally likely to marry in  $C_2$  or  $C_3$  but never marries in  $C_1$ .

Thus, the marriage probabilities are:

		a person of		
		$C_1$	$C_2$	$C_3$
$G_2 :$	$C_1$	$\frac{1}{2}$	$\frac{1}{2}$	0
	$C_2$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
	$C_3$	0	$\frac{1}{2}$	$\frac{1}{2}$

We come now to the relationship between action descriptions and terminal status descriptions.

We assume that occupation determines income as follows. A person in occupation  $O_1$  earns an income according to the rectangular distribution on the interval  $0 \leq x \leq 15,000$ , while earnings in occupation  $O_2$  are governed by the rectangular distribution on  $0 \leq x \leq 12,000$ . With respect to the transmission of ancestry type, this society allows the maximum ancestry of husband or wife to be the ancestry of the family, i.e., the terminal ancestry type of each parent in that family.

Using the definitions of the various classes in terms of the class origins of families formed, we obtain the probabilities for the attainment of terminal ancestry type conditional upon the class by class marriages (the notation  $C_3 \cdot C_3$  represents marriage between  $C_3$  and  $C_3$ ).

$$\begin{array}{l}
 C_3 \cdot C_3 \\
 C_3 \cdot C_2 \\
 C_3 \cdot C_1 \\
 C_2 \cdot C_2 \\
 C_2 \cdot C_1 \\
 C_1 \cdot C_1
 \end{array}
 \begin{array}{c}
 \left( \begin{array}{ccc}
 y_1 & y_2 & y_3 \\
 1 & 0 & 0 \\
 \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\
 0 & 0 & 1 \\
 \frac{1}{25} & \frac{15}{25} & \frac{9}{25} \\
 0 & 0 & 1 \\
 0 & 0 & 1
 \end{array} \right)
 \end{array}
 .$$

We can now carry out the computations for  $P$ . These are not shown in detail, but the result is:

$$P = \begin{array}{l}
 C_1 \\
 C_2 \\
 C_3
 \end{array}
 \begin{array}{c}
 \left( \begin{array}{ccc}
 C_1 & C_2 & C_3 \\
 \frac{2}{3} & \frac{1}{3} & 0 \\
 \frac{65}{200} & \frac{123}{200} & \frac{12}{200} \\
 \frac{7}{120} & \frac{53}{120} & \frac{60}{120}
 \end{array} \right)
 \end{array}$$

We assume that each family just reproduces the parents. Then, this matrix  $P$  implies that whatever the initial distribution of the population by classes, after a sufficiently long time has elapsed the proportion of the population in the three classes will be very close to

$$\begin{array}{ccc}
 \frac{C_1}{.4707} & \frac{C_2}{.4726} & \frac{C_3}{.0567}
 \end{array}
 .$$

Thus, if left unchanged, this social system would result in a middle-class  $C_2$  equal to an upper-class  $C_1$  and a practically negligible lower-class  $C_3$ .

We may also consider the passage times for this example, the average number of generations a family will remain in a given class, and the average number of generations required to go from one class to another. These times are reported below in a matrix  $M$ .

$$M = \begin{array}{l}
 C_1 \\
 C_2 \\
 C_3
 \end{array}
 \begin{array}{c}
 \left( \begin{array}{ccc}
 C_1 & C_2 & C_3 \\
 2.12 & 3.00 & 35.93 \\
 3.37 & 2.12 & 32.93 \\
 4.98 & 2.35 & 17.64
 \end{array} \right)
 \end{array}
 .$$

The diagonal entries of  $M$  represent the average number of generations spent in each class. Thus the members of  $C_3$  may expect to stay there for seventeen generations, on the average, whereas the members of  $C_1$  and  $C_2$  will remain there for two generations.

The off-diagonal entries of  $M$  represent the average number of generations before a first move to another class. Thus members of  $C_1$  and  $C_2$  will enter  $C_3$  in more than thirty generations, on the average, while they will interchange between themselves in about three generations. Members of  $C_3$  will first enter  $C_1$  in about five generations, on the average, and they will enter  $C_2$  in about two and a half generations.

This example is not very "realistic" so far as any particular society is concerned. But even the representation of this hypothetical society is not a simple matter. The "triviality" of the society should make clear the complexity of any specific application of the model.

Let us review the assumptions that we were required to make before the model could be put to work. First of all, relevant status symbols had to be selected and their values defined. Second, the class system had to be defined as a partition of the set of status symbols. The relevant definition of class for purposes of this model may not be the definition elicited by interviewing the persons in a society; to select a relevant definition of class may in fact be an extremely difficult task. The partition in the example is intended to suggest the relation between income and ancestry in American society, a relation which is undoubtedly more complex than that of our illustration.

It is now necessary to designate the actions that lead to attainment of terminal status symbols as a consequence. We considered the two dimensions of action independently, but in general this simplification *cannot be made*.

Next one must provide the probabilities

of particular actions conditional upon the initial classes; the ease of this task depends upon the extent to which existing data conform to the definition of class employed in the model.

We must further specify the relationship between actions and terminal status symbols. We shall determine terminal social class only after the configuration of terminal status symbols is obtained—the class will be defined by the partition of the status symbols. In the case of occupation and income our example is fairly straightforward. In the case of ancestry transmission by marriage complications arise. Our ancestry transmission rule is chosen to represent the blurring of matrilineal and patrilineal descent in American society as exemplified in the criteria for membership in the D.A.R. We assumed that marriages were contracted with an eye to class membership but that the transmission rules are stated in terms of ancestry combinations that are not necessarily simply related to the definitions of class. The problem is quite unpleasant but is very realistic.

### Applications

Some comments on the application of this scheme in contemporary sociological research are in order. First of all, the problem of intergenerational mobility is a classical one. The classical theory and research is summarized by Sorokin (1960) and is brought up to date in Lipset and Bendix (1960). Recent research (Rogoff, 1953; Glass, 1954) on occupational mobility has been analyzed (Matras, 1960; Prais, 1955a, 1955b; Kemeny & Snell, 1960) using Markov chains. However, in these instances the transition matrix has been obtained directly from empirical data on fathers' and sons' occupations, and has not explicitly related actions or rules to the transition matrix.

In contrast, our approach provides a theory in which the transition matrix  $P$  can in principle be computed from knowledge of the social "rules." The functions  $f$  and  $g$ , or, in the finite case,  $F$  and  $G$ , may themselves be estimated from empirical data or they may be specified by the "rules" of the society. In the case of occupation and income in our

example we would expect that empirical data would best supply an estimate. There is considerable data of this type available from census and other surveys. In the case of marriage and ancestry type we could use empirical data, but we could also use marriage rules supplied to us, as we did in the example. In this latter case the empirical data on marriages might be interpreted as measures of the degree of conformity of the members of a society to its rules. In any case, we have two alternative interpretations of our model. We may take empirical data and generate the long-run consequences of these data for our society, or we may take rules, express them as hypothetical data, and generate the long-run consequences for "Utopia." The latter interpretation enables us to study theoretical constructs of society, to spell out the eventual implications of systems of social rules. Both interpretations may yield results pertinent to sociologists. The necessary computations themselves are easily performed on a modern computer.

The approach we take also differs from that of Prais and of Kemeny and Snell in that the unit of analysis is not an element from a single status symbol, for example occupation, but it is the combination of elements from several status symbols. Thus the joint effect of the several status symbols, including their complex interactions, becomes the focus of attention. Unfortunately it is not easy to illustrate this situation with sociological data. Sociologists have often restricted their attention to a single status symbol, or they have used the correlation among status symbols as a criterion for constructing a single index to represent these status symbols, but they have seldom turned their attention to the possibility of complex interactions among status symbols and therefore they have not collected the data relevant to such an interest. The research on status crystallization represents an important departure from tradition.

### PART II. STATUS DISTRIBUTION: MOBILITY CONDITIONS CHANGING

We now wish to consider the possibility that the social rules, traditions, and sanctions

are themselves subject to a process of social change rather than fixed and constant over time. To this end we shall suppose that people in a social class can influence the occurrence of transitions from that class or to that class, whether by personal action or organized action. We can then think of the probabilities of transition from class to class as being determined by the net effect of these efforts and actions. We shall call the ability of a class to influence the probability of transitions to or from it "class power" and we shall consider how class power changes with changing composition of the social classes. The result is an evolutionary process in which the class power configuration determines the probabilities of interclass transitions, while the actual transitions that take place change the class power configuration. Thus, we must specify two things. First, we must specify how class power determines the transition probabilities. Second, we must specify how class power is determined by the social class distribution of the population. Definitions 10a and 10b below accomplish the first job; Definition 11 does the second.

We must now suppose that the classes can meaningfully be ordered, say in a hierarchy from top to bottom. What follows is for a given ordering.

*Assumption 1.* The classes are ordered  $C_1, \dots, C_N$ .

*Definition 10a.* A vector  $v = (v_1, \dots, v_N)$  whose elements are positive numbers is called a *vector of class power indicators*,  $v_i$ , also called a *class power vector*.

*Definition 10b.* If  $v$  is a class power vector let  $Q(v) = ((q_{ij}))$  be the  $N \times N$  matrix defined for the fixed ordering  $1, \dots, N$ , by

$$q_{ij} = \begin{cases} \frac{v_j}{v_i} \cdot \frac{1}{D_i} & i \leq j \\ \frac{v_i}{v_j} \cdot \frac{1}{D_i} & i > j \end{cases}$$

where the numbers  $D_i$  are determined by the conditions

$$\frac{1}{D_i} \left[ \sum_{j=1}^{i-1} \frac{v_i}{v_j} + \sum_{j=i}^N \frac{v_j}{v_i} \right] = 1 \\ i = 1, \dots, N.$$

According to Definition 10b the matrix  $Q(v)$  is one whose row vectors are probability vectors, i.e., the components of each row are nonnegative numbers whose sum is unity. Thus, Definitions 10a and 10b determine transition probabilities in terms of class power.

*Definition 11.* Let  $B = ((b_{ij}))$  be an  $N \times N$  matrix of nonnegative entries having at least one nonzero entry in each row. If  $m = (m_1, \dots, m_N)$  is strictly positive (each co-ordinate is positive), define  $v$  by  $v' = Bm'$ .<sup>5</sup>

According to Definition 11, the vector  $v$  is strictly positive and is therefore a class power vector. However, because of the restriction to strictly positive  $m$ , our models apply only to societies in which there are some people in each social class.

The interpretation of the matrix  $B$  is as follows. The coefficient  $b_{ij}$  "measures" the contribution to the class power of class  $i$  made per person of class  $j$ . It reflects the extent to which "typical class  $j$  behavior" contributes to the class power of class  $i$ . Thus, the matrix  $B$  represents the entire complex of interclass influences per person of each class. Through these coefficients we may express the notion that the members of class  $k$  may in some ways support the efforts of people in class  $j$  to move to class  $i$ , and in other ways hinder those efforts. To see this, note that the transition probability  $q_{ij}$  is, aside from  $D_i$ , either  $v_i/v_j$  or  $v_j/v_i$ . Suppose for definiteness,

$$q_{ij} = \frac{v_i}{v_j} = \frac{b_{i1}m_1 + b_{i2}m_2 + \dots + b_{iN}m_N}{b_{j1}m_1 + b_{j2}m_2 + \dots + b_{jN}m_N}.$$

The contribution  $b_{i2}$  per person of class 2 to the power of class  $i$  will increase the probability of transition from  $i$  to  $j$ , while the contribution  $b_{j2}$  to the power of class  $j$  will decrease that same transition probability. In the end, the transition probabilities depend on the fixed pattern of interclass influences, and on the variable distribution of people among classes.

This function  $Q$  is adapted from Guttman's simplex hierarchy theory for correlation matrices (1954). The function has the

<sup>5</sup> If  $x$  is a vector  $x'$  is its transpose.

property that, when classes are ordered according to the magnitude of  $v$ , that is, according to their power, with  $C_1$  having the greatest power, then the resulting transition matrix  $Q$  has entries such that in each row the maximum entry is on the main diagonal and the other entries monotonically decrease away from the main diagonal. Thus the greater the difference in power between two classes, the less the mobility between the two classes. For example, if  $v = (9,3,1)$  then

$$Q = \begin{pmatrix} \frac{9}{13} & \frac{3}{13} & \frac{1}{13} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{1}{13} & \frac{3}{13} & \frac{9}{13} \end{pmatrix}$$

Since power is indexed by a single number,  $v$ , the resulting  $Q$  necessarily represents a single

hierarchy of power. However, changes of  $v$  over time may yield shifting patterns of social mobility. Such changes over time may be represented in several ways.

We now define two enlarged stochastic processes.

*Definition 12a.* The process  $\mathbf{S} = [(m(t), Q(t))]$  determined by

- (i)  $m(t + 1) = m(t) Q(t)$
- (ii)  $Q(t + 1) = Q(m(t) \cdot B') = Q(v(t))$

is called a *process of social change*.

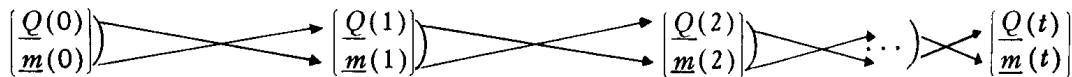
*Definition 12b.* Alternatively let the process  $\mathbf{R} = [(m(t), Q(t))]$  be defined by:

- (i')  $m(t + 1) = m(t) Q(t)$
- (ii')  $Q(t) = Q(m(t) \cdot B') = Q(v(t))$

which together imply

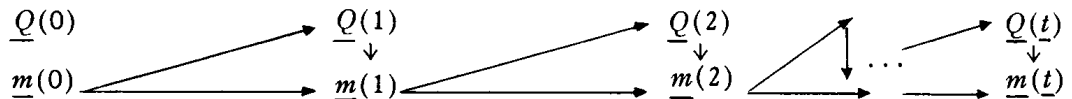
- (iii')  $m(t + 1) = m(t) \cdot Q(m(t) \cdot B')$ .

We can picture these two processes schematically. For the process  $\mathbf{S}$  we have



I.e., at any time ( $t$ ), the class distribution of the population  $m(t)$  determines the transition probabilities which prevail in the next generation,  $Q(t + 1)$ , via  $B$ , and  $m(t)$  and

the transition probabilities governing the  $t^{\text{th}}$  generation together determine  $m(t + 1)$ . For the process  $\mathbf{R}$  we have:



I.e., for the  $\mathbf{R}$  process the class distribution of the population in any generation determines both the transition probability matrix of the subsequent generation and through that the class distribution of the population prevailing in the next generation. This process requires only one initial condition, namely  $m(0)$ , while the  $\mathbf{S}$  process requires two initial conditions,  $Q(0)$  and  $m(0)$ . The  $\mathbf{S}$  process can be related to the Markov chain of Part I by taking  $Q(0) = P$ . Both the  $\mathbf{S}$  process and the  $\mathbf{R}$  process are Markovian, but not Markov chains, for though the probability that the process is in a given state ( $m(t)$ ,

$Q(t)$ ) at times  $t$  depends only on the state of the process at  $t - 1$ , there is no longer a finite number of states because  $q_{ij}(t)$  can take either any rational or any real values between 0 and 1, depending on whether  $B$  consists only of integers or not. Further, these processes are not stationary, because the transition probabilities  $q_{ij}(t)$  depend on  $t$ . It is therefore interesting to ask under what conditions the "trend" of the evolution of the process tends to a statistical equilibrium.

We consider now necessary conditions for the long-run convergence of the  $\mathbf{S}$  process. Convergence to statistical equilibrium means

that we can drop the  $t$ s from all variables, giving us that

$$m = mQ(mB').$$

But this implies that, in equilibrium,  $Q$  has the form

$$Q = \begin{pmatrix} \frac{m_1}{m} & \frac{m_2}{m} & \dots & \frac{m_N}{m} \\ \frac{m_1}{m} & \frac{m_2}{m} & \dots & \frac{m_N}{m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{m_1}{m} & \frac{m_2}{m} & \dots & \frac{m_N}{m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{m_1}{m} & \frac{m_2}{m} & \dots & \frac{m_N}{m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{m_1}{m} & \frac{m_2}{m} & \dots & \frac{m_N}{m} \end{pmatrix}$$

where  $m. = \sum_{i=1}^N m_i$ .

But,

$$Q(m. B') = \begin{pmatrix} \frac{1}{D_1} & \frac{b_2 m'}{b_1 m' D_1} & \dots & \frac{b_N m'}{b_1 m' D_1} \\ \frac{b_2 m'}{b_1 m' D_2} & \frac{1}{D_2} & \dots & \frac{b_N m'}{b_2 m_2 D_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b_N m'}{b_1 m' D_N} & \frac{b_N m'}{b_2 m' D_N} & \dots & \frac{1}{D_N} \end{pmatrix}$$

where  $b_i = (b_{i1}, \dots, b_{iN})$  is the  $i^{\text{th}}$  row vector of  $B$ . Thus we have two expressions for  $Q$ , which must be the same.

$$\begin{aligned} \frac{m_1}{m} &= \frac{1}{D_1}; \frac{m_2}{m} = \frac{b_2 m'}{b_1 m' D_1}; \dots; \frac{m_N}{m} = \frac{b_N m'}{b_1 m' D_1} \\ \frac{m_1}{m} &= \frac{b_2 m'}{b_1 m' D_2}; \frac{m_2}{m} = \frac{1}{D_2}; \dots; \frac{m_N}{m} = \frac{b_N m'}{b_2 m' D_2} \\ \frac{m_1}{m} &= \frac{b_N m'}{b_1 m' D_N}; \frac{m_2}{m} = \frac{b_N m'}{b_2 m' D_N}; \dots; \frac{m_N}{m} = \frac{1}{D_N} \end{aligned}$$

Note that  $D_i = \frac{m.}{m_i}$ .

$$\frac{-m_2}{m_3} < 0.$$

For clarity consider these equations for  $N = 3$ . They can then be written:

- (i)  $m_2(b_1 m') = m_1(b_2 m')$
- (ii)  $m_3(b_1 m') = m_1(b_3 m')$
- (iii) (iv)  $m_1(b_1 m') = m_2(b_2 m') = m_3(b_3 m')$
- (v)  $m_2(b_3 m') = m_3(b_2 m')$

giving us five equations in the nine variables  $b_{ij}$   $i, j = 1, 2, 3 \dots$ . Equations (iv) and (v) together imply

$$\frac{-m_2}{m_3} (b_2 - b_3) m' = (b_2 - b_3) m'$$

which implies either

$$(b_2 - b_3) m' = 0 \text{ or } \frac{-m_2}{m_3} = 1.$$

But  $m_2$  and  $m_3$  are positive and hence

We may conclude  $(b_2 - b_3) m' = 0$ . Subtracting (ii) from (i) yields:

$$(m_2 - m_3)(b_1 m') = m_1(b_2 - b_3) m' = 0$$

which implies either  $m_2 = m_3$  or  $b_1 m' = 0$ . But  $b_1 m' \neq 0$  because  $b_1$  is a nonnegative nonzero vector and  $m$  is positive. Hence  $m_2 = m_3$  and it follows from (i) and (iii) that  $m_1 = m_2 = m_3$ . It follows that

$$Q = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

and that  $\sum_{j=1}^3 b_{ij} = 1$  for  $i = 1 \dots 3$ .

The argument for an arbitrary value of  $N$  is similar.

Thus we see that a necessary condition that the **S** process be in statistical equilibrium is that the transitions from one class to another be equally probable for all classes and that the distribution of population among classes be uniform.

Moreover, there is a  $B$  matrix consistent with this solution, e.g.,

$$B = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix}$$

The necessary conditions for long-run equilibrium of the **S** process may be interpreted to mean that the only possible equilibrium is one in which movements from class to class are random. All transitions are equally likely. In this circumstance there is no particular use in distinguishing one social class from another; indeed, we may interpret this situation as one way of representing a 1-class society. If  $B$  is restricted to be a diagonal matrix so that  $v_i$  depends on  $m_i$

only, then the  $b$  vectors each contain a single nonzero element, say  $b_1, b_2, \dots, b_N$  and  $b_1 = b_2 = \dots = b_N$ . Thus  $v_1 = b_1 m_1, v_2 = b_2 m_2, \dots, v_N = b_N m_N$  and the process can only reach equilibrium if  $b_1 = b_2 = \dots = b_N$ . In other words, if the relative power of a class depends only on the size of that class, then the system reaches equilibrium when all classes have equal size.

We next present an illustration of the computations for the **S** process.

**Example 2**

We shall present two cases. In each case  $B$  is a diagonal matrix with nonzero elements  $b_1, b_2, \dots, b_n$ . In our first example the initial sizes of classes are equal while the coefficients  $b$  differ by a constant factor  $k$ . Thus, the initial differences among power indicators depend only on the choice of  $k$ . In our second illustration the  $b$ s are chosen equal but the initial class sizes differ by a constant factor. Thus the initial difference among power indicators is determined solely by differing initial class size. We shall continue to ignore fertility and mortality differentials in this example.

*Case 1.* The set of status symbol arrays  $S$  and the class structure  $C$  are the same as in the first example. For this case the  $b$ s differ by a constant factor but the  $m$ s are equal. The results are:

$Q(0) = \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \begin{bmatrix} C_1 & C_2 & C_3 \\ .6667 & .3333 & 0 \\ .2500 & .7500 & 0 \\ 0 & .5833 & .4167 \end{bmatrix}$ <p> <math>m_1(0) = m_2(0) = m_3(0) = 100</math>  <math>b_1 = kb_2, b_2 = kb_3, b_3 = 1</math>  <math>k = 3.</math> </p>	$Q(1) = \begin{bmatrix} .6923 & .2308 & .0769 \\ .2000 & .6000 & .2000 \\ .0769 & .2308 & .6923 \end{bmatrix}$ <p> <math>m(t) = (91.67 \quad 166.66 \quad 41.67)</math> </p>
$t = 0$	$t = 1$
$Q(5) = \begin{bmatrix} .6973 & .2507 & .0520 \\ .2295 & .6382 & .1323 \\ .0582 & .1617 & .7801 \end{bmatrix}$ <p> <math>m(5) = (107.54 \quad 114.42 \quad 78.04)</math> </p>	$Q(12) = \begin{bmatrix} .6805 & .2510 & .0685 \\ .2247 & .6090 & .1663 \\ .0733 & .1988 & .7279 \end{bmatrix}$ <p> <math>m(12) = (99.16 \quad 110.02 \quad 90.92)</math> </p>
$t = 5$	$t = 12$

Case 2. Everything the same as Case 1, except that  $b_1 = b_2 = b_3 = 1$  and

$$m_1(0) = \frac{1}{k} m_2(0)$$

$$m_2(0) = \frac{1}{k} m_3(0)$$

$$k = 3. m_3(0) = 900.$$

Then:

$Q(0) =$	.6667	.3333	0	$Q(1) =$	.0769	.2308	.6923
	.2500	.7500	0		.4286	.1428	.4286
	0	.5833	.4167		.6923	.2308	.0769
				$m(1) =$	(141.67	783.33	375)
$Q(5) =$	.2820	.3012	.4168	$Q(8) =$	.3395	.3317	.3288
	.3094	.2897	.4009		.3292	.3369	.3339
	.3828	.3583	.2589		.3272	.3349	.3379
$m(5) =$	(418.86	447.19	433.95)	$m(8) =$	(424.21	428.14	447.65)
	$t = 5$				$t = 8$		

In this case it is clear from the computation that the process is converging to:

$$Q(\infty) = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

and

$$m_1 = m_2 = m_3.$$

Social structure has sometimes been regarded as the end result of the distribution of power in society. In our models power is expressed in terms of the matrix  $B$ , whose entries were interpreted as measuring the specific interclass influences available per person of each class. This formulation allows "power" to affect social structure in an explicit and computable way. However, this notion of power suffers from some obvious defects. Although the dependence of the power indicator of one class on coefficients

associated with others allows the model to express the possibility that power is shared in the sense that the power of one class depends upon characteristics of the other classes, the assumed constancy of  $B$  requires that certain features of that distribution be unchanged during the time span covered by the model. Clearly more sophisticated notions of power are possible. However, to be of use it is necessary to be able to relate such a notion of power to the remainder of the model in an explicit way.

Measurement of power is notoriously difficult; one may wonder whether the matrix  $B$  has any empirical significance, whether it is a mere imaginative construction with a name suggesting applicability, but without the possibility of application. We offer one very simple illustration to meet this point.

In a democratic society voting strength is an important aspect of the power of a class. We may use this to obtain an interpretation of  $B$  as follows. Suppose for simplicity that we take  $B$  diagonal. Then let  $b_i$  be the fraction of the  $i^{\text{th}}$  class that typically votes. Then,  $v_i = b_i m_i$  is the total vote cast by class  $i$ .

These numbers are converted by  $Q$  into relative voting strength. Thus the transition probabilities are computed from voting strength one generation previously. For example, if  $m_1 = m_2 = m_3$  and if the fractions voting are  $b_1 = \frac{3}{4}$ ,  $b_2 = \frac{2}{4}$ ,  $b_3 = \frac{1}{4}$ , then we obtain

$$Q = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{4}{13} & \frac{6}{13} & \frac{3}{13} \\ \frac{2}{11} & \frac{3}{11} & \frac{6}{11} \end{pmatrix}.$$

The value of the models presented here depend on their performance in prediction or "explanation" of observed sociological phenomena in concrete cases. Nothing can be said about this until they have been tried out. However, we may expect that willingness to spend the time and effort necessary to make such an application may depend in part on the extent to which the models are judged adequate to express sociological concepts and data deemed especially relevant to social mobility and structure. In this connection the rather abstract and formal presentation of the model may mislead us about its adequacy to express the rich variety of concepts and data suggested by observation of society. However, the question whether something can or cannot be included in the models is in each case a definite question capable of being

decided by checking against the numbered definitions. The abstract and formal presentation of our models lays bare their structure and intellectual simplicity, for good or bad. Some suggestion of empirical application is given by Beshers (1962, in press).

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Intellect is to emotion as our clothes are to our bodies: we could not very well have civilized life without clothes, but we would be in a poor way if we had only clothes without bodies.

ALFRED NORTH WHITEHEAD, *Dialogues*