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A NOTE ON CONVEXITY OF THE AGGREGATE PRODUCTION SET

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IN RECENT issues of this *Journal*, M. J. Farrell[1] and J. Rothenberg[2] have presented arguments to the effect that the aggregate production possibilities set characterizing technological possibilities for an economy is convex, even if the production possibilities sets of individual firms are not convex, provided only that there are enough firms.

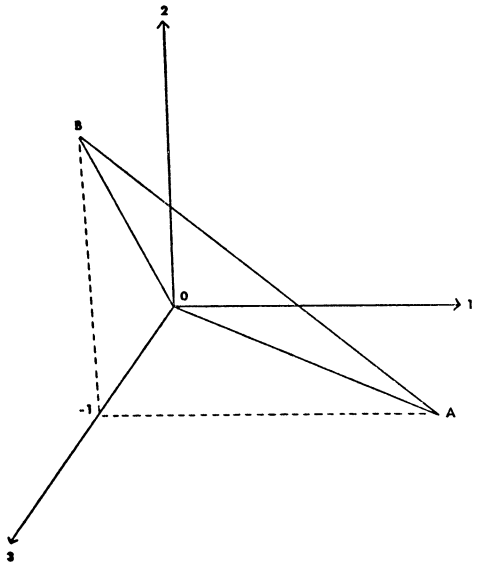

FIG. 1

The motivation for this kind of theorem lies in the fact that convexity of the aggregate production set permits the application of certain theorems identifying competitive equilibriums with Pareto optima. It was previously reasoned that convexity of the aggregate production possibilities set follows from convexity of the individual production sets of firms.[3] Further, the imposition of constraints expressing limited availability of resources results in a convex set of achievable production possibilities; it is the convexity of this set that is needed for the welfare theorems. Unfortunately, while the aggregate production possibilities set may be convex in the limit as the number of firms increases, it is not the case that the set of achievable aggregate production possibilities under constraint of limited resources is necessarily convex, however large the number of firms. The following example establishes this proposition.

Suppose there are three commodities, 1, 2, 3, and two types of firms, Type I, whose technological possibilities are given by the two input-output vectors

$$A = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \qquad O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

and Type II, whose technological possibilities are

$$B = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \qquad O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

We follow the usual conventions, according to which a positive coefficient indicates an output and a negative coefficient an input. Thus, the individual production sets are either $\{O, A\}$ or $\{O, B\}$, which are, of course, not convex. In this case, the limiting aggregate production possibilities set (properly normalized) is the triangle with vertices O, A, B shown in Figure 1.

[1] M. J. Farrell, "The Convexity Assumption in the Theory of Competitive Markets," *Journal of Political Economy*, LXVII (August, 1959), 386 ff.

[2] Jerome Rothenberg, "Non-convexity, Aggregation, and Pareto Optimality," *Journal of Political Economy*, LXVIII (October, 1960), 454 ff.

[3] T. C. Koopmans, "Allocation of Resources and the Price System," in *Three Essays on the State of Economic Science* (New York: McGraw-Hill Book Co., 1957), p. 50.

Suppose further that there is just one unit of commodity 3 available in this economy. Then, there can be just one firm of Type I or one of Type II operating at A or B, while all other firms, of whatever type, must operate at O. Thus, however large the number of firms, the achievable aggregate production possibilities set, under the limitation that there is one unit of commodity 3 available, consists of the three isolated points O, A, B.

This example suggests that the relevant theorem is one asserting the convexity of the limiting achievable aggregate production set (suitably normalized) as both the number of firms and the size of the resource endowment increase. It would follow, then, that, where individual production sets are not convex, competitive organization of production would be more nearly optimal in a rich economy than in a poor one. This assertion differs in an interesting way from the more usual welfare theorems that hold uniformly for all resource endowments.