Currency Crises and Fiscal Sustainability

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This chapter discusses the relation between fiscal sustainability and the sustainability of a fixed exchange rate regime. At the most general level, fiscal sustainability simply corresponds to the notion that a government's intertemporal budget constraint holds without explicit default on its debt. This requires that the initial real value of a government's debt be equal to the real present value of its future primary surpluses plus the present value of inflation-related revenues (e.g. seigniorage). In contrast, sustainability of a fixed exchange rate regime requires that the government not raise any inflation-related revenues.[1] So under such a regime, fiscal sustainability reduces to the condition that the real value of a government's initial debt must equal the real present value of its primary fiscal surpluses.

The classic example of a fixed exchange rate regime that is not sustainable is analyzed in the seminal papers of Krugman (1979) and Flood and Garber (1984). These authors consider a situation in which a government is running persistent primary deficits. A key implicit assumption of their analyses is that future primary surpluses will not be large enough to balance the government's intertemporal budget constraint. Since it is unfeasible to indefinitely borrow and repay the resources needed to cover the ongoing deficits, the government will eventually have to print money to raise seigniorage revenues. This means that the fixed exchange rate regime is not sustainable. As we will see below, the precise timing of the fixed exchange rate collapse depends on various assumptions about government behavior and the demand for domestic money. But collapse it will. Precise timing aside, in this scenario an analyst would observe large ongoing deficits and rising debt levels prior to the collapse of the fixed exchange rate regime.

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One might be tempted to conclude that large primary deficits are a necessary symptom of fiscal nonsustainability under a fixed exchange rate regime. But that is not the case. Unless the assumptions of the Krugman-Flood-Garber analyses hold, it is very difficult to assess whether a given country is on a fiscally sustainable path using historical data on standard macroeconomic aggregates like deficits. Deciding whether a fixed exchange rate regime is sustainable necessarily involves forecasting the future values of government purchases, transfers and tax revenues. This is particularly difficult in a world where governments incur large contingent liabilities. Such liabilities often arise because governments are committed to bailing out large sectors of the economy (e.g. banks and other large financial institutions) should they fail. A government which was running substantial fiscal surpluses may switch to a fiscally nonsustainable path, once large contingent liabilities are triggered. This happens when the government does not have a credible way of raising future primary surpluses to pay for the activated liabilities. Under these circumstances, activated contingent liabilities translate into prospective deficits which the government must fund via inflation-related revenues. It follows that the fixed exchange rate regime is no longer sustainable. Again, the precise time at which the collapse depends on various assumptions. But as in the Krugman-Flood-Garber case the collapse is inevitable.

Below we display examples, motivated by the recent East Asian currency crises, in which the fixed exchange rate regime collapses before the government begins to pay the activated liabilities, incur primary deficits or print money. In such a situation primary deficits are obviously a very poor indicator of fiscal nonsustainability. Instead the analyst must carefully assess the nature of a government's contingent liabilities, the probability that those liabilities will be activated and the extent to which the government is willing to raise revenues not related to inflation to pay for its prospective expenditures. In practice such an assessment will involve detailed institutional information about the country in question. Statistical analysis of standard macroeconomic data—no matter how well informed by economic theory—will not suffice.

The remainder of this chapter is organized as follows. Section 2 displays a simple version of the government budget constraint. In chapter XX we will discuss a more realistic version of the budget constraint that will be useful for organizing data and analyzing particular episodes. Here the crucial issue will be whether the government needs to raise resources via inflation-related revenues or not. Section 3 considers the classic Krugman-Flood-Garber

experiment. In Section 4 we turn to the case of currency crises triggered by prospective deficits. In addition we argue on empirical grounds that this case provides a good description of the origins of the East Asian currency crises. Section 5 briefly reviews the shortcomings of our analysis and serves as an introduction to chapter XX.

1. The Government's Intertemporal Budget Constraint

In this section we develop a simplified version of the government's intertemporal budget constraint. The key simplification is that the only inflation-related source of revenue available to the government is printing money. In chapter 3 we discuss additional sources of inflation-related revenue: deflating the real value of outstanding nonindexed nominal debt and reducing the real value of government expenditures by an *implicit* fiscal reform. By the latter we mean that the government can deflate the real value of its outlays that are fixed, at least temporarily, in nominal terms (e.g. civil servant wages or social security payments).

To proceed we assume that there is a single good whose domestic currency price is P_t. The foreign currency price of this good is P_t^* and purchasing power parity (PPP) holds:

$$P_t = P_t^* S_t.$$

Here S_t is the exchange rate expressed in units of local currency per unit of foreign currency (so a depreciation means a rise in S_t). For simplicity we assume that $P_t^* = 1$ so that $P_t = S_t$.

The government can borrow and lend in international capital markets at a constant real interest rate r. It also has assets in the form of foreign reserves which earn the real interest rate r_t. We denote the dollar value of the government's debt net of foreign reserves by b_t. Net government debt evolves according to:

$$\dot{b}_t = rb_t - (\tau_t - g_t - v_t) - \dot{M}_t/S_t. \tag{1.1}$$

The variable g_t represents real government spending, v_t represents real transfers and τ_t real tax revenues. The term $\tau_t - g_t - v_t$ represents the real government surplus, while M_t denotes the level of the money supply. We use the notation $\dot{x}$ to represent the derivative of x with respect to time, dx/dt.

Equation (1.1) assumes that the government's real debt and the supply of money evolve smoothly over time.[2] In currency crisis models there are typically points in time at which M

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[2]Technically (1.1) applies when b and M are differentiable functions of time.

and b change discretely (one such point in time is the instance at which the exchange rate regime is abandoned). We denote the set of such points in time by I. At these points the change in government debt is given by:

$$\Delta b_t = -\Delta(M_t/S_t). \tag{1.2}$$

Assuming no default on the government debt, using (1.1), the condition $\lim_{t\to\infty} e^{-rt} b_t = 0$, and allowing for discrete jumps in government debt, yields the government's intertemporal budget constraint:

$$b_0 = \int_0^\infty (\tau_t - g_t - v_t) e^{-rt} dt + \int_0^\infty (\dot{M}_t/S_t) e^{-rt} dt + \sum_{t\in I} \Delta(M_t/S_t) e^{-rt}. \tag{1.3}$$

According to (1.3) the initial level of the government debt must be equal the present value of future surpluses $(\int_0^\infty (\tau_t - g_t - v_t) e^{-rt} dt)$ plus the present value of seigniorage $(\int_0^\infty (\dot{M}_t/S_t) e^{-rt} dt + \sum_{t\in I} \Delta(M_t/S_t) e^{-rt})$. We say that a set of monetary and fiscal policies is fiscally sustainable as long as (1.3) holds.

Fiscal sustainability is much more stringent in an economy operating under a fixed exchange rate regime. Abstracting from foreign inflation the price level must be constant in a fixed exchange rate regime, because otherwise PPP would not hold. Abstracting from growth in the demand for real balances (due to growth in output or consumption) this last condition requires that the money supply be constant, so that seigniorage revenues are zero. It follows that (1.3) reduces to:

$$b_0 = \int_0^\infty (\tau_t - g_t - v_t) e^{-rt} dt. \tag{1.4}$$

The key point here is that sustainability of a fixed exchange rate regime requires that the government balance its intertemporal budget constraint without resorting to inflation-based revenues. The forward looking nature of (1.4) makes clear why it is so difficult to determine from real time data whether a country is on a fiscally sustainable path. There is no way to evaluate fiscal sustainability without forecasting the future paths of expenditures and taxes. For example a country could be running a sustained deficit for a period of time yet (1.4) could still hold because the government will credibly run future surpluses that will offset the deficits. In contrast, a country could be running a surplus but have future deficits that are so large that (1.4) does not hold.

This last possibility is more than a theoretical curiosity. Table 1 presents data on the fiscal surpluses for the United States and several Asian countries. Notice that the countries

involved in the Asian currency crises of 1997 (Indonesia, Korea, Malaysia, Philippines and Thailand) were running either surpluses or modest deficits. At the same time the U.S., which did not suffer large adverse movements in its exchange rate, was running a fiscal deficit. We will return to this example later when we discuss the importance of contingent liabilities and their impact on government budget constraints.

2. Fiscal Sustainability and Speculative Attacks

When sustainability condition (1.4) does not hold it is inevitable that a fixed exchange rate regime will be abandoned. The only questions are: when and what will the aftermath look like? The answers to these questions depends on three elements: (i) the nature of money demand; (ii) the rule for abandoning fixed exchange rates; and (iii) the post-crisis monetary policy. We will now add these elements to our analysis and discuss two experiments. In the first case there is an immediate increase in government transfers which induces the government to begin running a deficit. In the second case agents find out that there will be an increase in future government transfers, say because contingent liabilities to a failing banking system have been activated. This will induces prospective, but not current, deficits.

Money Demand   We adopt a standard specification for the demand for domestic money, due to Cagan (1956):

$$M_t = \theta Y P_t e^{-\eta R_t}, \tag{2.1}$$

where θ is a positive constant. According to (2.1), the demand for domestic money depends positively on Y, the domestic real income of the economy, and negatively on the opportunity cost of holding money, the nominal interest rate, R_t. The parameter η represents the semi-elasticity of money demand with respect to the interest rate. For the sake of simplicity we assume that domestic real output is constant over time.

In the absence of uncertainty the nominal interest rate is equal to the real rate of interest (r_t) plus the rate of inflation, $\pi_t = \dot{P}_t/P_t$:

$$R_t = r_t + \pi_t. \tag{2.2}$$

Combining (2.1) and (2.2) we obtain a differential equation in P_t:

$$\log(M_t) = \log(\theta Y) + \log(P_t) - \eta(r + \dot{P}_t/P_t),$$

The solution to this equation is:

$$\ln P_t = \eta r - \ln(\theta Y) + \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} \ln(M_s) ds. \tag{2.3}$$

Consistent with classic results in Sargent and Wallace (1973), equation (2.3) implies that the current price level is an increasing function of current and future money supplies. To see the intuition behind this result suppose that at time t the economy is under a floating exchange rate regime. Higher growth rates of money in the future translate into higher rates of inflation and a higher nominal interest rate. This in turn lowers the demand for real balances at time t. Under floating exchange rates M_t is exogenously determined by the central bank. So the only way for real balances to fall in equilibrium is for the price level to rise.

Equation (2.3) also holds while the exchange rate is fixed. Suppose first that the fixed exchange rate regime is sustainable. Then inflation is zero and $P_t = S$. The money supply, which is endogenous, must equal the quantity demanded given S:

$$M = S\theta Y \exp(-\eta r). \tag{2.4}$$

When the value of M is constant equation (2.3) reduces to:

$$\ln P_t = \eta r - \ln(\theta Y) + \ln M \tag{2.5}$$

So if the level of the money supply is given by (2.4) then $P_t = S$.

If the government tried to print more money than the level M given by (2.4), private agents would simply trade it in at the fixed exchange rate for foreign reserves or government debt. Thus, as long as the country is in a fixed exchange rate regime, the government cannot generate seigniorage revenues.[3]

The interpretation of (2.3) is more complicated when the exchange rate is fixed at time t but agents know that at some future date t^* the economy will let the exchange rate float. After t^* the path of the money supply is determined by the central bank, so the intuition for (2.3) is as described above. Before t^* the money supply is endogenously determined by the behavior of private agents. To understand the role played by equation (2.3) before t^* we must discuss the determinants of t^*. We now turn to this task.

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[3]If there were growth in P^* or in Y, the government would collect some seignorage revenue in a fixed exchange rate regime.

Rule for Abandoning Fixed Exchange Rates   It is standard in the literature to assume that the government follows a threshold rule for abandoning the fixed exchange rate regime: the fixed exchange rate regime is abandoned in the first period, t^*, in which the government's debt reaches some finite upper bound, $\bar{b}$. This rule turns out to be equivalent to another rule that we will use in our analysis: the fixed exchange rate is abandoned when the amount of domestic money sold by private agents in exchange for foreign reserves exceeds some percentage of the initial money supply, i.e. when the money demand falls to $e^{-\chi}M$, for some $\chi > 0$.

To see why the two rules are equivalent, it is important to recognize that in any equilibrium where the fixed exchange rate regime is abandoned, the inflation rate rises discretely at the time this occurs. Agents, anticipating this, will discretely reduce their domestic money balances an instant before the exchange rate regime is abandoned. Under the fixed exchange rate regime they go to the government and exchange domestic money for dollars at rate S. This reduces the government's reserves, thus raising its net debt. So the rise in debt that sets off the government's threshold rule and the fall in money demand occur simultaneously.

In addition to being a good description of what happens in actual crises, the threshold rule can be interpreted as a short-run borrowing constraint on the government: it limits the amount of reserves that the government can borrow to defend the fixed exchange rate.[4] Rebelo and Végh (2001) discuss the circumstances in which it is optimal for a social planner to follow a threshold rule.[5] While they use a general equilibrium model, their framework is similar in spirit to the model used here.

Post-crisis Monetary Policy   Finally, we adopt the following specification of post-crisis monetary policy: in period T the government engineers a one time increase in the money supply relative to its pre-crisis level, i.e. $M_T = e^{\gamma}M$. Thereafter the money supply grows at the rate μ. This formulation for post-crisis money supply decouples the time of the speculative attack (which will be computed later) from the time at which the government

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[4]Drazen and Helpman (1987), as well as others, have proposed a different rule for the government's behavior: fix future monetary policy and allow the central bank to borrow as much as possible provided the present value budget constraint of the government is not violated. This rule ends up being equivalent to a threshold rule. See Wijnbergen (1991) and Burnside, Eichenbaum and Rebelo (2001) for a discussion.

[5]Rebelo and Végh (2001) show that this rule for abandoning the peg is optimal when: (i) the fiscal shock that makes the fixed exchange rate regime unsustainable is of moderate size; and either (ii) there are significant real social costs associated with a devaluation, such as loss of output or firm bankruptcy; or (iii) while the exchange rate is fixed a fiscal reform may arrive according to a Poisson process that restores the sustainability of the fixed exchange rate regime.

starts to print money. It also nests as a special case the specification used by Krugman (1979) and Flood and Garber (1984) according to which money starts growing at a constant rate μ as soon as fixed exchange rate are abandoned. Finally, this specification is simple enough so that we can provide intuition about the timing of the speculative attack.

As we will see below, in general, given the threshold rule and our assumptions about monetary policy, the fixed exchange rate regime will be abandoned prior to time T. As we will establish, the fixed exchange rate regime is abandoned when the money supply falls by χ percent, so the post-crisis behavior of the money supply can be summarized as follows:

$$M_t = \begin{cases} e^{-\chi}M, & \text{for } t^* \leq t < T \\ e^{\gamma + \mu(t-T)}M, & \text{for } t \geq T. \end{cases} \tag{2.6}$$

With these elements in place we can now discuss the timing of the speculative attack once agents become aware at time zero that the fixed exchange rate regime is unsustainable.

Determining the Timing of the Crisis Note that just before t^* the exchange rate and the price level are still fixed. This means that instantaneous inflation is zero ($\pi_t = 0$ for $t < t^*$) and equation (2.5) holds: i.e. $P_t = S$ for $t < t^*$. An instant after time t^* the exchange rate is floating and the price level is given by (2.3). In order for P_t to be continuous equations (2.5) and (2.3) must both imply that $P_{t^*} = S$.[6] Given this fact, and given (2.6), it is clear that the demand for real balances falls discontinuously at time t^* from M/S to $e^{-\chi}M/S$. This is accomplished by private agents exchanging domestic currency for dollars at t^* at the exchange rate S. It is precisely this flight from local currency into dollars that activates the government's rule for abandoning fixed exchange rates.

If we take the post-crisis growth rate of money, μ, as given, we can solve for t^* by (i) computing P_{t^*} using (2.3) and the path for the money supply, (2.6), and (ii) combine this with the fact $P_{t^*} = S$. In the appendix we show that this yields the following expression for the time of the speculative attack:

$$t^* = T - \eta \ln \left(\frac{\chi + \gamma + \mu\eta}{\chi} \right). \tag{2.7}$$

If the value of t^* implied by (2.7) is less than 0, the attack happens immediately, i.e. $t^* = 0$. In this case the exchange rate is discontinuous at time zero. It is also possible for the crisis

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[6]Proceeding as in the literature we use the fact that the exchange rate must be a continuous function of time. So, the exchange rate is the same the instant before and after the collapse of the fixed exchange rate regime. Were this not the case agents could take advantage of jumps in the exchange rate to make infinite profits.

to happen at time T, but this is only possible if γ is negative; more specifically it requires that $\gamma = -\chi$ and $\chi = \mu\eta$. The appendix considers these special cases in greater detail.

Other things equal, t^* is larger the longer the government delays implementing its new monetary policy (the larger is T) and the more willing the government is to accumulate debt (we will see that the higher χ is the more debt the government accumulates before the crisis occurs). In addition, the higher is the interest rate elasticity of money demand (the larger is η) and the more money the government prints in the future (the higher are γ and μ), the smaller is t^*. The intuition underlying these results is as follows. Once the fixed exchange rate regime is abandoned, inflation rises in anticipation of the increase in the money supply that occurs from time T on. A higher elasticity of money demand (η) makes it easier for the money supply to fall by χ percent. This means that the threshold rule is activated sooner, thus reducing the value of t^*. Higher values of μ and γ also reduce t^* because they lead to higher rates of inflation making it possible for a drop of χ percent in the money supply to happen sooner.

Some caution is required in interpreting these results because we are not free to vary the parameters on the right-hand side of (2.7) independently of each other. When one parameter is varied either γ or μ must be adjusted to ensure that the government resource constraint continues to be satisfied. To fully characterize t^* we have to solve for the combination of γ and μ such that (1.3) holds.

One natural question is: why doesn't the attack happen at time zero, when people find out that the government will run either ongoing or prospective deficits? To understand why the collapse generally occurs after time zero, two issues must be kept in mind. First, as long as the government has access to foreign reserves and is willing to use them, it can fix the price of its currency. It does so by exchanging domestic money for foreign reserves at the fixed price S. In our model the government is willing to do this until the level of domestic money falls by χ percent. Put differently, a fixed exchange rate regime is a price fixing scheme that will endure as long as the government has the ability and the willingness to exchange domestic currency for dollars. If the government were not willing to endure any increases in its debt, i.e. it wasn't willing to buy back any of the domestic money supply at $S_t = S$, then the exchange rate regime would collapse at $t = 0$. Given the government's willingness to buy back no more than χ percent of the money supply, the key determinant of when the fixed exchange rate regime collapses is when money demand falls by χ percent. Second, as a

result of the discrete increase in money supply at time T, inflation is monotonically increasing between t^* and time T. This reflects the fact that in standard Cagan money demand models, the price level at time t is a function of discounted current and future money supplies. An important feature of this function is that the further out in time is the increase in the money supply, the less impact it has on the initial price level [see (2.3)]. In general, inflation is too low at time zero to produce a fall in money demand large enough to trigger the government's threshold rule. This would be the case if the demand for real balances at time zero fell by at least χ. Such a situation may occur if γ, μ and η are sufficiently large. As we stated above, in this case there may be a discontinuous jump in the exchange rate at time zero.

As the previous discussion makes clear the timing of the devaluation is deterministic–everybody knows the precise time at which the fixed exchange rate regime will collapse. This shortcoming can be remedied by introducing some element of uncertainty into the model, such as money demand shocks.[7] We abstract from uncertainty since it complicates the analysis considerably but does not change the basic message about fiscal sustainability.

We will now turn to our two experiments. First we consider the classic Krugman-Flood-Garber case in which the government begins to run ongoing deficits which make the fixed exchange rate regime unsustainable. The key feature of this example is that the deficits would be a real time indicator of fiscal nonsustainability. We then discuss a version of the analysis in Burnside, Eichenbaum and Rebelo (2001) in which private agents come to expect that the government will run future deficits that will not be offset by future primary surpluses. We will see that this results in a collapse of the fixed exchange rate regime after agents receive information about the higher future deficits but before the government starts to run those deficits or print money. So, here past deficits would be a useless indicator of fiscal sustainability.

3. Ongoing Deficits

Consider an economy that is initially in a sustainable fixed exchange rate regime—i.e. (1.4) holds—with a constant primary surplus, $\tau - g - v$. The level of government debt is constant and equal to:

$$b_0 = (\tau - g - v)/r. \tag{3.1}$$

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[7]See Flood and Garber (1984), Blanco and Garber (1986), Drazen and Helpman (1988), Cumby and Wijnbergen (1989), and Goldberg (1994) for stochastic versions of speculative attack models.

At time zero information arrives that there has been a permanent rise in government transfers to a new level $\bar{v}$. In order for the fixed exchange rate regime to still be sustainable the government must adjust its taxes or government spending so that (1.4) continues to hold. This requires that:

$$(\bar{v} - v)/r = \int_0^\infty [(\bar{\tau}_t - \tau) - (\bar{g}_t - g)]e^{-rt}dt. \tag{3.2}$$

where $(\bar{v} - v)/r$ is the increase in the present value of government transfers, while $\bar{\tau}_t$ and $\bar{g}_t$ denote the new values of taxes and government spending. Notice that since this is a constraint on the present value of transfers and taxes, fiscal sustainability is consistent with a persistent ongoing primary deficit. Of course, that deficit must be offset, at some point, by a persistent ongoing primary surplus in the future.

We assume that the government does not change the path of taxes of government spending, so that $\bar{\tau}_t = \tau$ and $\bar{g}_t = g$. Given these assumptions, the primary surplus declines by $\bar{v} - v$ and the stock of debt is no longer constant. Furthermore, for (3.2) to hold, the government must, at some point, print money. Thus, the fixed exchange rate regime is not sustainable, though the exchange rate will remain fixed until the government's threshold rule for floating the exchange rate is activated by a sufficient rise in its debt (or equivalent drop in money demand).

Instead, while the economy remains under a fixed exchange rate regime, it evolves according to:

$$b_t = b_0 + \frac{\bar{v} - v}{r}(e^{rt} - 1), \text{ for } t < t^*. \tag{3.3}$$

Note that the stock of debt rises at an increasing rate while the economy remains in the fixed exchange rate regime:

$$\dot{b}_t = (\bar{v} - v)e^{rt}, \text{ for } t < t^*.$$

Immediately prior to time t^* the debt stock will have risen to the level $b_0 + (\bar{v} - v)(e^{rt^*} - 1)/r$. As we show in the appendix, at time t^* the inflation rate rises discretely to χ/η. This occurs in anticipation of the higher and faster growing money supply path that will prevail in the future. This discrete rise in inflation causes agents to use the last few seconds of the fixed exchange rate regime to reduce their money balances from M to $Me^{-\chi}$. This is accomplished by swapping domestic money for reserves, which would have earned the real interest rate r.

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So at time t^* the debt stock rises, consistent with (1.2) by $\Delta b_{t^*} = (M - Me^{-\chi})/S$, so that

$$b_{t^*} = b_0 + \frac{\bar{v} - v}{r}(e^{rt^*} - 1) + \frac{M - Me^{-\chi}}{S}. \tag{3.4}$$

It is this final jump in debt that sets off the government's threshold rule, the precise timing of the attack is determined by making $b_{t^*} = \bar{b}$. In the appendix we show that it is equivalent to thinking about the downward jump in money demand as the event that triggers the government's threshold rule.

Since the money supply remains constant between t^* and T seigniorage is zero and debt continues to rise according to:

$$\dot{b}_t = r(b_t - b_0) + \bar{v} - v > 0. \tag{3.5}$$

At date T there is another jump, but this time it is downward. The government increases the supply of money to Me^γ by buying back government debt. So, other things equal the effect of this operation is to reduce the level of debt by $(Me^\gamma - Me^{-\chi})/S_T$. It follows from this and (3.5) that b_T is given by:

$$b_T = b_0 + \frac{\bar{v} - v}{r}(e^{rT} - 1) + \frac{M - Me^{-\chi}}{S}e^{r(T-t^*)} - \frac{Me^\gamma - Me^{-\chi}}{S_T}. \tag{3.6}$$

As equation (3.6) shows, three factors determine the change in the government's debt between period 0 and period T. First, from time 0 forward, the government's primary deficit is larger by the amount $\bar{v} - v$. This increase in the primary deficit causes debt to accumulate. At time t^* there is a jump up in the level of debt due to the decline in money balances during the speculative attack. Finally, at time T the government's debt stock jumps down as engineers a discrete increase in money balances.

From time T on the money supply expands at rate μ. As we show in the appendix, this implies that for $t \geq T$, the exchange rate is given by $S_t = e^{\gamma + \mu(\eta + t - T)}S$, the inflation rate is μ, real balances are constant at the level

$$\frac{M_T}{S_T} = \theta Y e^{-\eta(r+\mu)} = e^{-\eta\mu}\frac{M}{S}$$

and the government receives a constant seigniorage flow of: $\mu e^{-\eta\mu}M/S$.

Recall that the level of μ must ensure that the government's intertemporal budget constraint, (1.4), holds. For $t > T$ the sum of the primary surplus and the flow of seigniorage revenue is constant over time. It is easy to see that (1.4) holds if the sum of the primary

surplus and the flow of seigniorage revenue equals the interest payments on the debt accumulated by date T:

$$\tau - g - \bar{v} + \mu e^{-\eta\mu} \frac{M}{S} = r b_T. \tag{3.7}$$

Given the expression for b_T, (3.6), the government must set μ so that (3.7) is satisfied. Notice however, given our expression for b_T, that (3.7) is equivalent to the lifetime budget constraint for period 0:

$$\frac{\bar{v} - v}{r} = e^{-rt^*} \frac{Me^{-\chi} - M}{S} + e^{-rT} \frac{Me^{\gamma} - Me^{-\chi}}{S_T} + e^{-rT} \frac{\mu e^{-\eta\mu}}{r} \frac{M}{S}. \tag{3.8}$$

In this form, we see clearly that the increase in the present value of transfers must be financed with increased seigniorage revenue.

A Numerical Example   To discuss the properties of the model it is useful to present a numerical example. The parameter values that we use, summarized in Table 2, are loosely based on Korean data. For reasons discussed above we do not think that the Krugman-Flood-Garber analysis applies to Korea. But we do take the Korean example more seriously in the context of the next section where we discuss prospective deficits. So, to conserve on space, we discuss here the parameter values that will be used in both examples. These parameters are taken from the analysis in Burnside, Eichenbaum and Rebelo (2001).

We normalized real income, Y, and the initial exchange rate, S, to 1. We set the semi-elasticity of money demand with respect to the interest rate, η, equal to 0.5. This is consistent with the range of estimates of money demand elasticities in developing countries provided by Easterly, Mauro and Schmidt-Hebbel (1985). We set the constant $\theta = 0.06$ so that in the initial steady state the model is consistent with the ratio of the monetary base to GDP in Korea in the late 1990s. We set the real interest rate, r, to 5 percent. This is roughly consistent with dollar interest rates in Korea in the 1990s. For convenience we set $b_0 = \tau - g - v = 0$.

We assume that $\bar{v} = v + 0.012$, which implies that the present value of the increase in transfer spending is $(\bar{v} - v)/r = 0.012/0.05 = 0.24$, or 24 percent of GDP. We set $\chi = 0.12$, $\gamma = 0.12$ and $T = 1$. Our reasons for choosing these parameter values will become clear in the following section. Given these parameter values, we solve (6.2) and (3.8) simultaneously for t^* and μ, which turn out to be 0.45 and 0.24, respectively.

Figure 1 displays the paths for the exchange rate, nominal and real money, inflation and money growth, as well as real government debt. A number of features emerge. First, as anticipated, government debt starts to rise from time zero due to the increase in transfers and the primary deficit to new, permanently higher, levels. The speculative attack takes place at $t^* = 0.49$, at which time the debt jumps discontinuously as agents trade domestic money for reserves. The debt grows smoothly between time t^* and T, at which point it drops discontinuously as the government increases the money supply and starts to generate seigniorage revenues. Second, the exchange rate rises in a continuous way from t^* to T and then depreciates at a constant rate μ. Finally, note that in this example the money supply does not grow before the speculative attack. The only indicator of the crisis to come is the increase in the deficit and the accumulation of increasing amounts of debt.

4. Prospective Deficits

We now consider an example in which ongoing deficits are not a good indicator of currency crises. In this example agents know that there will be future deficits that make the fixed exchange rate regimes unstable. This example is motivated by the 1997 currency crises in Indonesia, Korea, Malaysia, the Philippines, and Thailand. In our view—exposited in Burnside, Eichenbaum and Rebelo (2001)—these governments were faced with large *prospective* deficits associated with implicit bailout guarantees to failing banking systems. The expectation that these future deficits would (at least in part) be financed by seigniorage revenues led to a collapse of the fixed exchange rate regimes in Asia.[8] Of course market participants could have believed that governments would fund their obligations by raising taxes or lower expenditures. But in our view this was not credible. The state of the world in which financial intermediaries would suffer grievous losses is exactly the state of the world in which current and prospective real output and tax revenues would fall. While not modeled in this chapter, raising distortionary taxes or lowering government purchases under those circumstances could well be politically unacceptable or socially undesirable relative to the alternative: monetizing the prospective deficits and receiving aid from international agencies like the International Monetary Fund. But this alternative is incompatible with maintaining a fixed exchange rate.

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[8]Corsetti, Pesenti and Roubini (1998) also discuss the possible role played by expectations of future seignorage revenues in the Asian currency crises.

As above we consider an economy that is initially in a sustainable fixed exchange rate regime with a constant government primary surplus, $\tau - g - v$, and a constant level of government debt given by (3.1).

At time zero information arrives that from time $T' > T$ on there will be a permanent rise in government transfers to a new level $\bar{v}$. In order for the fixed exchange rate regime to still be sustainable taxes or government spending must adjust so that (1.4) continues to hold. This requires that:

$$e^{-rT'}(\bar{v} - v)/r = \int_0^\infty [(\bar{\tau}_t - \tau) - (\bar{g}_t - g)]e^{-rt}dt. \tag{4.1}$$

As before we assume that the government does not change the path of taxes of government spending, so that $\bar{\tau}_t = \tau$ and $\bar{g}_t = g$. This implies that (4.1) does not hold so that the fixed exchange rate regime is not sustainable.

Given our assumptions, $\dot{b}_t = 0$ for $t < t^*$. Government debt remains constant until the time of the speculative attack, because the increase in transfers does not occur immediately: $b_t = b_0$, for $t < t^*$.

As in the Krugman-Flood-Garber case, the level of debt jumps discretely at t^* as agents reduce their money balances from M to $Me^{-\chi}$:

$$b_{t^*} = b_0 - (Me^{-\chi} - M)/S.$$

Since the money supply remains constant between t^* and T seigniorage is zero and the evolution of the debt is given by: $b_t = b_0 - e^{r(t-t^*)}(Me^{-\chi} - M)/S$, for $t^* < t < T$. At time T the government increases the supply of money by γ percent so the level of debt at time T is given by:

$$b_T = b_0 - e^{r(T-t^*)}\frac{Me^{-\chi} - M}{S} - \frac{Me^\gamma - Me^{-\chi}}{S_T}.$$

From time T through time T' there is no increase in the primary deficit, but the money supply expands at the rate μ implying that the government receives a seigniorage flow of $\mu M_T/S_T$. This implies that the stock of debt evolves according to:

$$b_t = b_0 - e^{r(t-t^*)}\frac{Me^{-\chi} - M}{S} - e^{r(t-T)}\frac{Me^\gamma - Me^{-\chi}}{S_T} - \frac{e^{r(t-T)} - 1}{r}\mu\frac{M_T}{S_T}.$$

After time T' transfers increase permanently to the level $\bar{v}$. This implies that after date T' debt the stock of debt will evolve according to:

$$b_t = b_0 - e^{r(t-t^*)}\frac{Me^{-\chi} - M}{S} - e^{r(t-T)}\frac{Me^\gamma - Me^{-\chi}}{S_T} - \frac{e^{r(t-T)} - 1}{r}\mu\frac{M_T}{S_T} + \frac{e^{r(t-T')} - 1}{r}(\bar{v} - v).$$

The lifetime budget constraint is satisfied if $\lim_{t \to \infty} e^{-rt} b_t = 0$. From the previous equation it is clear that the lifetime budget constraint is satisfied if debt is constant at the level $b_{T'}$ and

$$e^{-rT'} \frac{\bar{v} - v}{r} = e^{-rt^*} \frac{Me^{-\chi} - M}{S} + e^{-rT} \frac{Me^{\gamma} - Me^{-\chi}}{S_T} + \frac{e^{-rT}}{r} \mu \frac{M_T}{S_T}$$

This equation is equivalent to (3.8) as long as we hold the present value of the increase in transfers the same across the two examples. In this case, the paths of the exchange rate are identical across the two examples, as is the timing of the speculative attack. Other things equal, all that matters is the present value of the new transfers, which are financed by seigniorage.

In the previous section we assumed that $\bar{v} = v + 0.012$. Notice that this implied that the present value of the increase in transfer spending is $(\bar{v} - v)/r = 0.24$, or 24 percent of GDP. This corresponds to a conservative estimate of the fiscal cost of Korea's banking crisis relative to its GDP.[9] In this section, the present value of the increase in transfer spending is $e^{-rT'}(\bar{v} - v)/r$. We set $T' = 1.5 > T = 1$. In order that the increase in the present value of transfers should still be 0.24, we set $\bar{v} - v = 0.24re^{rT'} = 0.0129$. In the previous section we set $\chi = 0.12$. This corresponds to the fall in Korea's monetary base between December 1996 and December 1997. We also set the value of γ to 0.12. This corresponds to the ratio of the average value of the monetary base in the second half of 1999 versus the first half of 1997.

Figure 2 displays the paths for the exchange rate, nominal and real money, as well as real government debt. The key features of this example are as follows. First, as with the first example, the collapse of the fixed exchange rate regime occurs after the new information about the deficit arrives but before the new monetary policy is implemented at time T. Second, inflation begins to rise at t^*, before the change in monetary policy. So consistent with classic results in Sargent and Wallace (1981), future monetary policy affects current inflation.

Note that, in this example, the currency crisis is preceded by neither a rise in government debt nor an increase in the primary deficit, nor an increase in the money supply. This is consistent with the view that past deficits and past money growth rates are not reliable predictors of currency crises or fiscal sustainability.[10] Our analysis suggests that in many

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[9] See Burnside, Eichenbaum and Rebelo (2003) for a discussion.

[10] See Corsetti, Pesenti and Roubini (1999) and Kaminsky and Reinhart (19XX).

cases we should focus our attention on the magnitude of a government's prospective liabilities.

We conclude by reviewing the evidence in Burnside, Eichenbaum and Rebelo (2001) regarding key assumptions in this example as they pertain to the Asian currency crises.

First the exchange rate crises were preceded by publicly available signs of imminent banking crises. Table 3 displays stock market based measures of the value of financial and nonfinancial sectors in the crisis countries. These data show that in Korea and Thailand, and to a lesser extent in Malaysia and the Philippines, the value of the financial sector had been declining, in both absolute and relative terms, well before their currency crises. For example, by July 1, 1997 and December 31, 1997 the stock market value of the Korea banking sector had declined by roughly 52 percent and 70 percent, respectively, relative to its previous peak value . In contrast, by December 1, 1997, the noncrisis countries' banking sectors had not declined significantly relative to their nonfinancial sectors. This suggests that markets were not particularly concerned about the banks in the noncrisis countries.

Second, failing financial sectors were associated with large prospective government deficits. Table 4 uses information on pre and post currency crisis loan default rates to generate rough estimates of governments' total implicit liabilities to the financial sector. According to these estimates nonperforming loan rates were substantially higher in the crisis countries. Finally, Table 5 depicts estimates of the size of the prospective deficits associated with the need to recapitalize banks in these countries.

5. Conclusion

This chapter discussed the connection between fiscal sustainability and fixed exchange rates. First, we discussed the fact that the sustainability of a fixed exchange rate regime requires that a government satisfy its intertemporal budget constraint without recourse to inflation-related revenues. Second, we argued that ongoing deficits are neither a necessary nor a sufficient condition for the nonsustainability of a fixed exchange rate regime. The 1997 currency crises in Asia are a good illustration of this point.

In the model we used to make these points the only inflation-related revenues available to the government were seigniorage revenues. As a result the model predicts high inflation rates and money growth in the aftermath of a devaluation. In addition, given the PPP assumption the rate of inflation is equal to the rate of devaluation. There are many crises in which these predictions are false. The next chapter we discuss why inflation and money

growth are often low in the aftermath of a currency crisis. In addition, we address a closely related question: how do governments actually pay for the fiscal costs associated with the currency crisis?

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TABLE 1

FISCAL SURPLUS (percent of GDP)

	1995	1996	1997	1998	1999
Indonesia	0.0	0.8	1.2	-0.7	-1.9
Korea	1.0	1.3	1.0	-0.9	-4.0
Malaysia	3.3	2.2	2.1	4.0	-1.0
Philippines	-1.8	-1.4	-0.4	–0.8	–2.7
Thailand	1.9	3.0	2.5	–0.9	-2.5
Hong Kong	-0.3	2.2	6.1	-1.8	0.8
Singapore	13.9	12.3	9.3	9.4	3.6
Taiwan	0.2	0.4	-0.7	–0.6	0.9
Japan	-2.3	-3.6	-4.2	-3.4	-4.3
USA	-3.8	-3.3	-2.4	-1.2	-0.1

SOURCE: Burnside, Eichenbaum and Rebelo (2001).

TABLE 2

PARAMETERS FOR THE NUMERICAL EXAMPLES

$\eta = 0.5$	interest elasticity of money demand
$\chi = 0.12$	threshold rule parameter
$S = 1$	initial exchange rate
$\theta = 0.06$	constant in the money demand function
$r = 0.05$	real interest rate
$Y = 1$	constant level of output
$(\bar{v} - v)/r = 0.24$	present value of new transfers
$b_0 = 0$	initial debt level
$T = 1$	time of switch to new monetary policy
$\gamma = 0.12$	% increase in M at T relative to $t = 0$

TABLE 3

CHANGES IN BANKING SECTOR STOCK MARKET VALUES (7/1/97=100)

| | Pre 7/1/97 Peak | | 7/1/97 | Peak to 7/1/97 % Change | | 12/31/97 | Peak to 12/31/97 % Change | |
	Date	Value	Value	Level	Relative to Nonfinancials	Value	Level	Relative to Nonfinancials
Indonesia	2/28/97	103.2	100.0	-3.1	-3.2	26.3	-74.5	-65.0
Korea	11/7/94	207.3	100.0	-51.8	-34.4	62.5	-69.8	-27.7
Malaysia	2/25/97	121.6	100.0	-17.7	-4.0	36.3	-70.1	-48.0
Philippines	1/31/97	136.8	100.0	-26.9	-13.2	56.4	-58.8	-34.7
Thailand	1/31/96	281.1	100.0	-64.4	-29.6	60.1	-78.6	-48.9

SOURCES.— Burnside, Eichenbaum and Rebelo (2001).

TABLE 4

ESTIMATED NONPERFORMING LOANS (June 1997)

	Domestic Bank Lending[*]	Private Nonbank Foreign Borrowing[†]	Total Lending	Nonperforming Credit (as a percentage of)		
	(percent of GDP[‡])			a) All Loans	b) GDP[‡] c)	Government Revenue[‡]
Indonesia	54.6	14.7	69.3	14	9.7	65.8
Korea	129.9	5.1	135.0	19	25.7	128.0
Malaysia	143.0	6.7	149.7	12.5	18.7	79.9
Philippines	56.4	5.5	61.9	20.5	6.5	33.8
Thailand	135.9	7.3	143.1	24.5	35.1	194.7
Hong Kong	166.1	14.8	180.9	2	3.6	18.3
Singapore	113.9	8.5	122.4	4	4.9	12.5
Taiwan	149.5	1.1	150.5	4	6.0	50.4

SOURCE.—Burnside, Eichenbaum and Rebelo (2001).

TABLE 5

Costs of Restructuring and Recapitalizing the Banking System

	(percent of GDP)	Date of Estimate*
Indonesia	65	Nov. 99
Korea	24	Dec. 99
Malaysia	22	Dec. 99
Thailand	35	Jun. 99

Source.—Burnside, Eichenbaum and Rebelo (2001).

FIGURE 1

Equilibrium Paths for Crisis Models

Nominal and Real Balances

Nominal

Real

Inflation

Money Growth

Exchange Rate

Government Debt

Ongoing Case

Prospective Case

Primary Deficit

Ongoing Case

Prospective Case
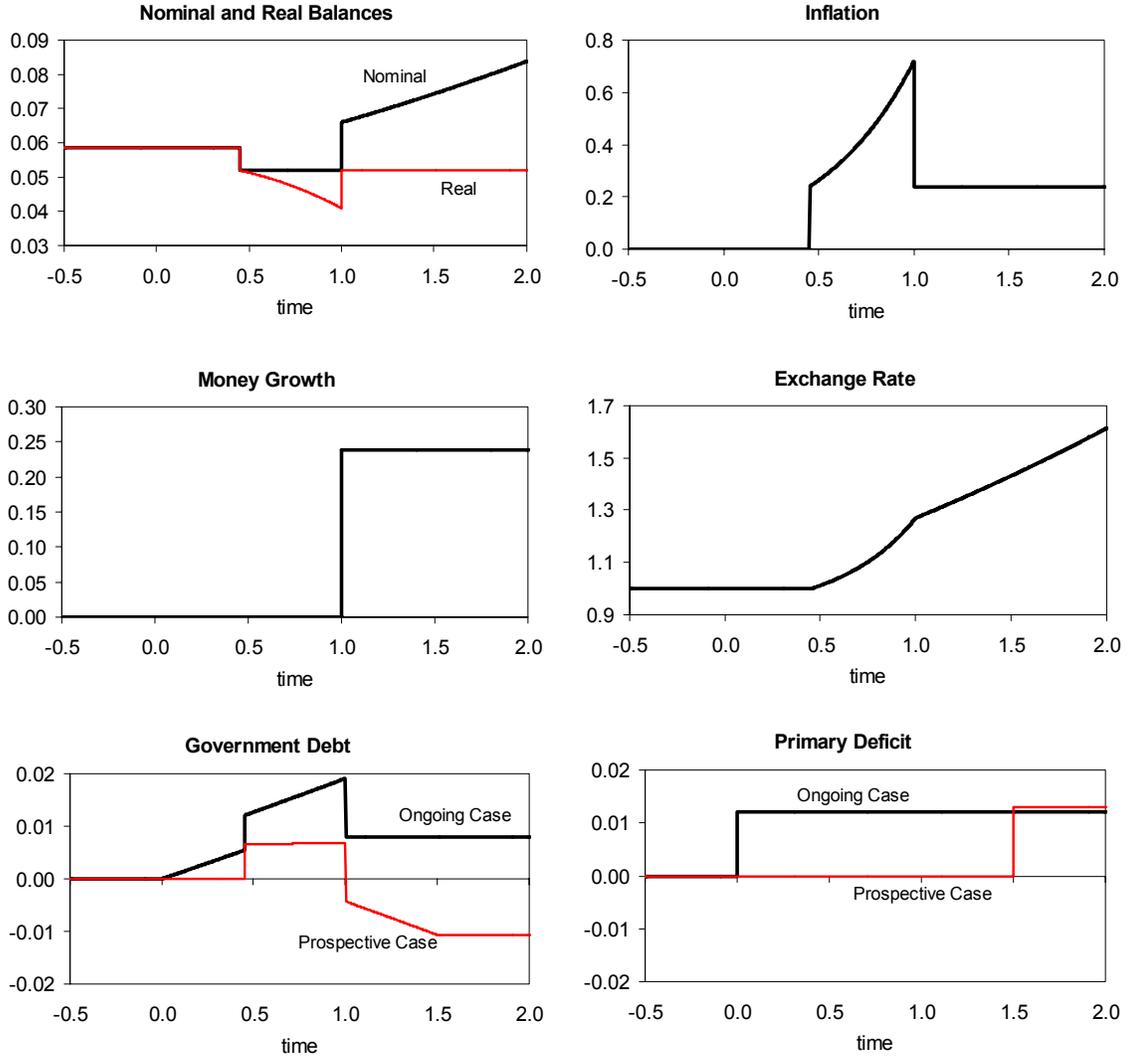
6. Technical Appendix

The Timing of the Crisis

A Crisis Between 0 and T We begin by solving for the time, t^*, at which the speculative attack occurs and the exchange rate is floated. Notice that (2.3) implies that

$$
\begin{aligned}
\ln P_{t^*} &= \eta r - \ln(\theta Y) + \frac{1}{\eta}\int_{t^*}^{T} e^{-(s-t^*)/\eta}\ln(e^{-\chi}M)ds + \frac{1}{\eta}\int_{T}^{\infty} e^{-(s-t^*)/\eta}\ln(e^{\gamma+\mu(s-T)}M)ds \\
&= \eta r - \ln(\theta Y) + \ln M + (\chi + \gamma + \mu\eta)e^{(t^*-T)/\eta} - \chi \\
&= \ln S + (\chi + \gamma + \mu\eta)e^{(t^*-T)/\eta} - \chi
\end{aligned}
\tag{6.1}
$$

Since we know that $P_{t^*} = S$, this implies that

$$
\chi = (\chi + \gamma + \mu\eta)e^{(t^*-T)/\eta},
$$

or, equivalently, that

$$
t^* = T + \eta \ln[\chi/(\chi + \gamma + \mu\eta)].
\tag{6.2}
$$

A Crisis at Time 0 When $\chi < e^{-T/\eta}(\chi + \gamma + \mu\eta)$ the expression in (6.2) implies $t^* < 0$. In this case, the crisis must happen at $t^* = 0$. and the price level jumps at time 0 to the level implied by (6.1):

$$
P_0 = S\exp\left[(\chi + \gamma + \mu\eta)e^{-T/\eta} - \chi\right] > S.
$$

A Crisis at Time T It is also possible that $t^* = T$, i.e. the crisis and the switch in monetary policy have the same timing. This occurs if $\gamma = -\chi$. Notice that in this case, (2.3) implies that

$$
\begin{aligned}
\ln P_{t^*} &= \eta r - \ln(\theta Y) + \frac{1}{\eta}\int_{T}^{\infty} e^{-(s-T)/\eta}\ln(e^{-\chi+\mu(s-T)}M)ds \\
&= \eta r - \ln(\theta Y) + \ln(M) - \chi + \mu\eta \\
&= \ln S - \chi + \mu\eta
\end{aligned}
$$

Since $P_{t^*} = S$ when $t^* > 0$ we have $\chi = \mu\eta$. I.e. the crisis can only happen at time T if the government's threshold rule parameter $\chi = \mu\eta$ is determined by the speed of post-crisis money growth and the interest elasticity parameter.

Ongoing Deficits

Crisis Happens at $0 < t^* < T$ To see the equivalence between a threshold rule based on money demand, and one based on the government's debt stock, suppose we assumed that for $t^* \le t < T$ the stock of money remained constant at the level M_{t^*}, its level immediately

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after the government floats the exchange rate. For $t^* \leq t < T$, the price level, given by (2.3), would be given by

$$\ln P_t = \eta r - \ln(\theta Y) + \left(1 - e^{(t-T)/\eta}\right) \ln M_{t^*} + e^{(t-T)/\eta} \left(\gamma + \ln M + \mu \eta\right).$$

Notice that this implies that the money supply (and demand) must fall to some level less than M at the time the fixed exchange rate regime is abandoned. If it did not, notice that we would have

$$\ln P_{t^*} \geq \ln S + e^{(t^*-T)/\eta} \left(\gamma + \mu \eta\right),$$

which would imply a jump in the exchange rate at time t^*. We denote the lower level of money demand at time t^* as $Me^{-\chi}$ with $\chi > 0$.

We have $\dot{b}_t = r b_t - (\tau - g - \bar{v})$ for $0 < t < t^*$. Since $\tau - g - v = r b_0$ we can rewrite this as $\dot{b}_t = r(b_t - b_0) + (\bar{v} - v)$ for $0 < t < t^*$. Hence

$$b_t = b_0 + \frac{\bar{v} - v}{r}(e^{rt} - 1) \text{ for } 0 < t < t^*.$$

So

$$\lim_{t \uparrow t^*} b_t = b_0 + \frac{\bar{v} - v}{r}(e^{rt^*} - 1).$$

We have seen that there must be a jump in nominal balances, to some lower level $Me^{-\chi}$, at time t^* implying that

$$b_{t^*} = \lim_{t \uparrow t^*} b_t + \frac{M - Me^{-\chi}}{S} = b_0 + \frac{\bar{v} - v}{r}(e^{rt^*} - 1) + \frac{M - Me^{-\chi}}{S}.$$

If the fixed exchange rate regime is abandoned at time t^* this means that b_{t^*} must be equal to the threshold level of debt, $\bar{b}$. So we have

$$\bar{b} = b_0 + \frac{\bar{v} - v}{r}(e^{rt^*} - 1) + \frac{M - Me^{-\chi}}{S}.$$

But we also know that if money demand falls by a factor $e^{-\chi}$ at the time of the attack, then t^* is given by (6.2). Hence,

$$\bar{b} = b_0 + \frac{\bar{v} - v}{r}(e^{r\{T + \eta \ln[\chi/(\chi + \gamma + \mu\eta)]\}} - 1) + \frac{M - Me^{-\chi}}{S}.$$

This shows that there is a one-to-one mapping between $\bar{b}$ and χ. Therefore we can parameterize the threshold rule in terms of debt or in terms of money demand.

For $t^* \leq t < T$ notice that $\dot{b}_t = r b_t - (\tau - g - \bar{v})$. Hence

$$b_T = e^{r(T-t^*)} b_{t^*} + \frac{e^{r(T-t^*)} - 1}{r}(\bar{v} + g - \tau) - \frac{Me^{\gamma} - Me^{-\chi}}{S_T}$$

If we substitute in the expression for b_{t^*} we have

$$b_T = b_0 + (e^{rT} - 1)(\frac{\bar{v} - v}{r}) + e^{r(T-t^*)}\frac{M - Me^{-\chi}}{S} - \frac{Me^{\gamma} - Me^{-\chi}}{S_T}.$$

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Given values of T, χ, γ, and μ, we can see that t^*, $S_T = Se^{\gamma+\mu\eta}$ and b_T are determined.

From date T forward the government prints money according to $M_t = e^{\gamma+\mu(t-T)}M$ so that $\dot{M}_t = \mu M_t$. From (2.3) its is straightforward to show that $S_t = Se^{\gamma+\mu(\eta+t-T)}$ for $t \geq T$. Hence $\dot{M}_t/S_t = \mu e^{-\mu\eta}M/S$ for $t \geq T$, where $M/S = \theta Y e^{-\eta r}$. This implies that if $\lim_{t\to\infty} e^{-rt}b_t = 0$ then

$$b_T = \int_T^\infty (\tau - g - \bar{v})e^{-r(t-T)}dt + \int_T^\infty \mu e^{-\mu\eta}\frac{M}{S}e^{-r(t-T)}dt.$$

This can be rewritten as

$$rb_T = \tau - g - \bar{v} + \mu e^{-\mu\eta}M/S. \tag{6.3}$$

Given T, χ and γ, this is an implicit equation in μ.

Crisis Happens at $t^* = 0$ As we saw above, if the crisis happens at time 0, then S_0 jumps to the level

$$S_0 = S\exp\left[(\chi + \gamma + \mu\eta)e^{-T/\eta} - \chi\right]$$

During the crisis, the government's debt rises as it exchanges money for debt at the exchange rate S. Hence, immediately after the crisis, the government's debt stock is $b_0 + (M - Me^{-\chi})/S$. Similar to what we saw in the previous section we will have

$$b_T = b_0 + (e^{rT} - 1)(\frac{\bar{v} - v}{r}) + e^{rT}\frac{M - Me^{-\chi}}{S} - \frac{Me^{\gamma} - Me^{-\chi}}{S_T}.$$

Given T, χ, γ and μ, $S_T = Se^{\gamma+\mu\eta}$ and b_T are determined. Given T, χ and γ, (6.3) becomes an implicit equation in μ. Once this equation is solved for μ one would need to check whether, in fact, $t^* = 0$.

Crisis Happens at $t^* = T$ Given the same logic as in the previous subsection, but imposing $t^* = T$ and $\gamma = -\chi$ we have

$$b_T = b_0 + \frac{\bar{v} - v}{r}(e^{rT} - 1) + \frac{M - Me^{-\chi}}{S}.$$

Given values of T and χ, b_T is determined.

From date T forward the government prints money according to $M_t = e^{-\chi+\mu(t-T)}M$ so that $\dot{M}_t = \mu M_t$. From (2.3) its is straightforward to show that $S_t = Se^{-\chi+\mu(\eta+t-T)}$ for $t \geq T$. As we saw above, since there can be no jump in the exchange rate at time $t^* = T$ this means $\mu = \chi/\eta$. Since μ is pinned down by χ, this implies that the threshold rule parameter, χ, is not a free parameter. It must adjust to satisfy the government's lifetime budget constraint, (6.3).

Prospective Deficits We have $\dot{b}_t = rb_t - (\tau - g - v) = 0$ for $0 < t < t^*$. Hence $b_t = b_0$ for $0 < t < t^*$. There is a jump in nominal balances, to some lower level $Me^{-\chi}$, at time t^* implying that

$$b_{t^*} = b_0 + \frac{M - Me^{-\chi}}{S}.$$

For $t^* \leq t < T$ notice that $\dot{b}_t = rb_t - (\tau - g - v)$. Hence

$$b_T = e^{r(T-t^*)}b_{t^*} + \frac{e^{r(T-t^*)} - 1}{r}(v + g - \tau) - \frac{Me^{\gamma} - Me^{-\chi}}{S_T}$$

If we substitute in the expression for b_{t^*} we have

$$b_T = b_0 + e^{r(T-t^*)}\frac{M - Me^{-\chi}}{S} - \frac{Me^{\gamma} - Me^{-\chi}}{S_T}. \tag{6.4}$$

Given values of T, χ, γ, and μ, we can see that t^*, $S_T = Se^{\gamma + \mu\eta}$ and b_T are determined.

As above, for $t \geq T$, $M_t = e^{\gamma + \mu(t-T)}M$, $\dot{M}_t = \mu M_t$, $S_t = Se^{\gamma + \mu(\eta + t - T)}$ and $\dot{M}_t/S_t = \mu e^{-\mu\eta}M/S$. This implies that if $\lim_{t \to \infty} e^{-rt}b_t = 0$ then

$$b_T = \int_T^{\infty} (\tau - g)e^{-r(t-T)}dt - \int_T^{\infty} v_t e^{-r(t-T)}dt + \int_T^{\infty} \mu e^{-\mu\eta}\frac{M}{S}e^{-r(t-T)}dt.$$

Given that $v_t = v$ for $t < T'$, and $v_t = \bar{v}$ for $t > T'$ we can rewrite this as

$$b_T = \int_T^{\infty} (\tau - g)e^{-r(t-T)}dt - \int_T^{T'} ve^{-r(t-T)}dt - \int_{T'}^{\infty} \bar{v}e^{-r(t-T)}dt + \int_T^{\infty} \mu e^{-\mu\eta}\frac{M}{S}e^{-r(t-T)}dt$$

or

$$b_T = \frac{\tau - g - v}{r} + e^{r(T-T')}\frac{v - \bar{v}}{r} + \frac{1}{r}\mu e^{-\mu\eta}M/S \tag{6.5}$$

Given T, χ and γ, this is an implicit equation in μ.

As above, given T, χ and γ, (6.5) is an implicit equation in μ which can be solved while noting that b_T is given by (6.4).