Hedging and Financial Fragility in Fixed Exchange Rate Regimes*

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Abstract

Currency crises that coincide with banking crises tend to share at least three elements. First, banks have a currency mismatch between their assets and liabilities. Second, banks do not completely hedge the associated exchange rate risk. Third, there are implicit government guarantees to banks and their foreign creditors. This paper argues that the first two features arise from banks' optimal response to government guarantees. We show that guarantees completely eliminate banks' incentives to hedge the risk of a devaluation. Our model also articulates one reason why governments might be tempted to provide guarantees to bank creditors. Guarantees lower the domestic interest rate and lead to a boom in economic activity. But this boom comes at the cost of a more fragile banking system. In the event of a devaluation, banks renege on foreign debts and declare bankruptcy.

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1. Introduction

In the post Bretton Woods era, currency crises have often coincided with banking crises. Prominent examples include Southeast Asia in 1997, Chile in 1982, Mexico in 1994, and Sweden and Finland in 1992.[1] Three key features of such 'twin' crises are (i) banks have a currency mismatch between their assets and liabilities, (ii) banks do not completely hedge the associated exchange rate risk, and (iii) the government makes implicit guarantees to banks and their foreign creditors.[2] This paper argues that the first two features arise from banks' optimal response to government guarantees.

The fact that banks are exposed to unhedged currency risks lies at the core of competing theories of currency crises. In some cases, it is simply a maintained assumption of the analysis. See for example, Aghion, Bacchetta and Banerjee (2000), Chang and Velasco (1999), and Krugman (1999). In other cases it is assumed that, to a first approximation, banks in emerging markets must borrow in units of foreign currency and simply cannot hedge the resulting exchange rate risk (see Eichengreen and Hausmann (2000)).

This paper argues that banks choose to expose themselves to exchange rate risk. In a world with government guarantees, it is optimal for banks to have an unhedged currency mismatch between their assets and liabilities. When a devaluation occurs, banks simply renege on foreign debt and go bankrupt. To minimize the value of the assets that they surrender in bankruptcy, banks may actually find it optimal to magnify their exchange rate exposure by selling dollars forward and lose money in the forward market when there is a devaluation.

Given the central role that government guarantees play in our model, it is useful to provide intuition for how they affect the optimal hedging strategies of banks. The government guarantees that foreign creditors will be repaid in full if there is a devaluation and banks default on their debt. Suppose a bank contemplates hedging foreign exchange rate risk via forward contracts. The hedging profits that are realized when a devaluation occurs and the bank declares bankruptcy are seized by the government. So the bank assigns zero value to them. But these contracts generate losses when there is no devaluation and the bank does not go bankrupt. It follows that banks have no incentive to enter forward contracts that generate positive payoffs when there is a devaluation.

[1] Kaminsky and Reinhart (1999) study empirically the link between banking and currency crises. See Diaz-Alejandro (1985) and Nyberg and Vihriälä (1993) for analyses of the 1982 Chilean and 1992 Finnish crises, respectively. Garber and Lall (1998) and Krueger and Tornell (1999) discuss the 1994 Mexican crisis.

[2] Mishkin (1996) and Obstfeld (1998) argue that a government's promise to maintain a fixed exchange rate is often seen as providing an implicit guarantee to banks' creditors against the effects of a possible devaluation. Corsetti, Pesenti and Roubini (1999) and Dooley (2000) also emphasize the role of government guarantees to the banking sector in 'twin crises' episodes.

Banks will therefore not be perfectly hedged, and will go bankrupt when a devaluation occurs. To the extent that they have any assets in that state of the world, these will be seized by the government. Hence banks ought to minimize their asset holdings in bankruptcy states of the world. A simple strategy for doing this is to sell dollars forward, which generates profits in the no devaluation state and losses in the devaluation state.

In addition to explaining the mismatch between banks' assets and liabilities, our analysis has an important policy implication: absent regulation of banks' hedging behavior, capital requirements have no effect on the frequency of bank bankruptcies after a devaluation. Banks undo the effects of these requirements by enhancing their exposure to exchange rate risk via forward markets.

If government guarantees lead to increased risk in the banking system, why are they so prevalent? One possibility is that guarantees lower domestic interest rates and lead to a boom in economic activity, a temptation government officials may find impossible to resist. To formalize the idea that guarantees lead to increased economic activity we embed our model of banking into a general equilibrium model. The key feature of our model is that firms in the output sector must borrow working capital from banks to pay labor. Government guarantees to banks' foreign creditors act like a subsidy to the banking industry, thereby lowering the domestic interest rate and the cost of working capital. This reduction in costs leads to higher aggregate employment, output and real wages.

Our paper is organized as follows. In section 2 we provide empirical motivation for our analysis. Section 3 lays out a competitive banking model. Section 4 studies banks' hedging decisions in a setting where the level of bank loans is pre-determined. In section 5 we characterize the equilibrium of the banking industry when lending and hedging decisions are made simultaneously. Section 6 discusses the effects of introducing capital requirements. In section 7 we study a version of the model where banks lend to domestic firms in dollars but face uncertainty with respect to loan repayment rates in the event of a devaluation. In section 8 we embed our banking model in a general equilibrium environment and study the macroeconomic effects of government guarantees and devaluations. Section 9 contains concluding remarks.

2. Empirical Motivation

In the introduction we noted that there are three important common elements in episodes where banking crises are associated with currency crises. The first is the existence of implicit government guarantees to domestic and foreign bank creditors prior to the currency crises. These guarantees have been extensively discussed in the literature.[3] The size of these government guarantees is of sufficient interest to the private sector that, at the end of 1997, Standard and Poor's began to publish estimates of governments' total contingent liabilities to the banking sector.[4]

The other two common elements in banking/currency crises are that (i) firms and financial intermediaries borrow extensively from abroad, and (ii) they do not completely hedge exchange rate risk. Unfortunately, due to data limitations, there is little empirical work measuring private agents' exchange rate exposure. One way to assess the potential magnitude of this exposure is to consider banks', firms' and financial intermediaries' net foreign assets prior to the onset of a crisis. Two sources of data are: the Bank of International Settlements (BIS) and the International Financial Statistics (IFS). An important advantage of the BIS data is that they are based on reports from major OECD banks. To the extent that accounting standards for banks in OECD countries are more uniform and carefully enforced than in non-OECD countries, the BIS data may be more reliable than the IFS data. The IFS data are based, in part, on reports from agents in non-OECD countries. In addition, there are subtle ambiguities about how a given transaction might be reported in the IFS data.[5] The advantage of the IFS data is that they are more comprehensive in coverage, since the BIS data are based solely on reports from banks that are part of its system. With these limitations in mind, we now report results based on both the BIS and IFS data.

Table 1 presents information on the net foreign assets of banks and firms for various countries prior to the onset of their currency/banking crises. A number of interesting features emerge from the table. First, in all cases, firms and banks in these countries had significant net foreign debt at the time of their crises. For the Nordic countries, the IFS data indicate a much larger net negative position than the BIS data. This may reflect the fact that banks in the Nordic countries were borrowing substantial amounts of funds from entities other than BIS banks. For the East Asian countries, the BIS data indicate a much larger negative net position than the IFS data. This may reflect accounting problems with the data reported to

[3] See the references in footnote 2. In addition, see IMF (1998, page 35), IMF (1999, page 21) and Delhaise (1998) for a discussion of guarantees in Thailand, Malaysia, Indonesia and Korea. IMF (1998, page 39) details the extent to which implicit guarantees became explicit after the crises occurred. Drees and Pazarbaşioğlu (1998) discuss government guarantees in the Nordic country banking crises, while Calomiris (1998) discusses foreign bank creditor bailouts after the 1994 Mexican currency crisis.

[4] See Burnside, Eichenbaum and Rebelo (2000b) for a discussion of these estimates.

[5] Suppose, for example, that Bank A in Country X borrows dollars from abroad and uses the proceeds to make dollar denominated loans to local firms. Bank A may take the position that this transaction does not cause a decline in its net foreign assets and would not report it as such. However, the transaction does expose the bank to another type of exchange rate risk–the increase in loan default rates that often occurs after a devaluation. Since the BIS data are based on reports from the foreign creditors of Bank A (assuming that they are banks in OECD countries) the transaction would show up as a decrease in the net foreign assets of banks in Country X.

TABLE 1

Summary Indicators of Net Foreign Assets

(percent of GDP)

Source:	BIS		IFS		Standard & Poor's
	Banks	Firms	DMB	OFI	Financial System
Nordic Countries (1992)					
Finland	-8.5	-8.1	-23.2	n/a	-17.5
Sweden	-18.4	-6.2	-27.0	0.3	-19.7
Mexico (1994)	-3.8	-8.6	-1.2	-7.5	-9.7
East Asia (1997)					
Indonesia	-6.9	-16.1	-1.7	n/a	-2.4
Korea	-12.8	-5.2	-2.2	-3.2	-3.2
Malaysia	-11.5	-4.4	-7.0	0.1	-4.1
Philippines	-8.5	-2.1	-7.5	n/a	-8.0
Thailand	-48.9	-7.0	-22.8	-2.7	-19.7

Notes: BIS measures of NFA are end-of-quarter. For the Nordic countries, 1991Q3, for Mexico, 1994Q3, for East Asia, 1997Q2. The IFS measures of NFA are end-of-year. For the Nordic countries, 1990, for Mexico, 1993, for East Asia 1996. DMB: Deposit Money Banks. OFI: Other Financial Institutions.

the IFS.

The table pertains to levels of net foreign assets immediately prior to the crises. In most cases, these levels were the result of significant declines of net foreign asset holdings in the immediately preceding years. For example, in discussing Asian crisis countries, Jackson (1999) writes: "banks in each country rapidly increased their net foreign liabilities by large percentages during the four years prior to the crisis." The IMF World Economic Outlook and International Capital Markets (various years) document similar behavior in other crisis countries.

While highly suggestive, Table 1 can only establish that financial institutions had large exchange rate exposure. This exposure could, in principle, have been hedged. Data limitations prevent us from precisely measuring the extent to which large net foreign asset positions were hedged in different crisis countries. Still, many qualitative analyses suggest that these positions were unhedged. Lane et. al. (1999, p. 17) conclude that Indonesia, Korea and Thailand had "large unhedged private short-term foreign currency debt (...); in Korea and Thailand, this debt was mainly intermediated through the banking system, while in Indonesia the corporations had heavier direct exposures to such debt." Folkerts-Landau et. al.

(1997, p. 46) write, with reference to Thailand, "While banks are believed to have hedged most of their net foreign liabilities, the opposite is believed to be true for the corporate sector. The combination of a stable exchange rate and a wide differential between foreign and (much higher) domestic interest rates provided a strong incentive for firms to take on foreign currency liabilities. Hence, in addition to their own foreign exchange exposure, banks may have a large indirect exposure in the form of credit risk to firms that have borrowed in foreign currencies."[6]

The previous discussion leaves open the issue of whether private agents could have hedged exchange rate risk in twin crisis countries. Below we discuss evidence on (i) the agents' ability to borrow externally in units of local currency, (ii) the availability of forward contracts and other derivatives, and (iii) hedging by nonfinancial firms.

External Debt Denominated in Local Currency

One obvious way for banks to hedge exchange rate risk is to denominate their foreign loans in local currency. This strategy was certainly available for at least two twin-crises countries, Sweden and Finland. Table 2, based on BIS data, shows that between 5 and 14 percent of banks' external liabilities were denominated in local currency.

TABLE 2

PERCENTAGE OF BANKS' EXTERNAL LIABILITIES IN LOCAL CURRENCY

Year	1990	1991	1992	1993	1994
Finland	9.4	10.1	9.3	13.4	14.3
Sweden	8.9	5.0	6.7	12.5	11.1

Forwards, Swaps and Other Currency Derivatives

Table 3, which is based on BIS data, provides some evidence on the overall size of foreign exchange derivative markets in various twin crises countries. At least by April 1998, these countries had active derivative markets. While suggestive, Table 3 pertains to a period after the countries' twin crises. To address this concern we refer to the following further sources of information.

[6] See also Eichengreen and Fishlow (1998).

TABLE 3

REPORTED OVER-THE-COUNTER FOREIGN EXCHANGE DERIVATIVES TURNOVER

(Daily Averages, April 1998, Millions of US Dollars)

Finland	2,334	Philippines	278
Indonesia	759	South Korea	316
Malaysia	541	Sweden	5,235
Mexico	2,378	Thailand	1,925

1. Euromoney (1996) reports what products were offered by the Hong Kong Shanghai Bank Corporation (HSBC) for various Asian countries. The relevant products are: (1) forward markets, (2) local currency swaps, and (3) currency options. HSBC offered all three products in Indonesia, Hong Kong, Singapore and Thailand and the first two products in Korea, Malaysia, and Taiwan. In addition, forward contracts were available in India, Sri Lanka, Pakistan, the Philippines, Mauritius, Macau, and Brunei.

2. Asiamoney (1997) reported, prior to the crisis in Thailand, that there had been "an explosion in both spot and forward trading not only in Bangkok, but also in Singapore and Hong Kong. Recent daily turnover, including swaps, is estimated to be more than US$2 billion per day, over three times the average daily trading amount in 1993. US dollar/baht interbank spot and forward foreign exchange markets are liquid and readily available, with maturities up to six months. US dollar/baht currency swaps and over-the-counter currency options are also popular and active."

3. Table 4, constructed from the BANKSCOPE database, summarizes major Thai banks' purchases and sales of forward market contracts for 1996. The large gross purchases and sales are evidence that the Thai banks were involved extensively in the forward market. However, the relatively small size of their net purchases is consistent with the notion that the banks were acting as agents for their clients, and not hedging their own exchange rate exposure.

4. Asiamoney (1996) reported that liquid forward markets existed in Indonesia for contracts up to six months. In addition, other hedging instruments were becoming more active, including cross-currency and interest rate swaps up to five years as well as currency options with maturities up to two years.[7]

[7] These markets were subject to a variety of regulations designed to discourage 'speculation' in derivative contracts. In particular net open and forward positions of commercial banks were limited to 25 percent of their capital.

TABLE 4

SUMMARY OF FORWARD MARKET ACTIVITIES BY THAI BANKS

(Millions of U.S. Dollars)

	Forward Contracts			Assets	Rank
	Purchased	Sold	Net		(1996)
Bangkok Bank	9394	10345	-952	45097	1
Thai Farmers Bank	1049	1109	-59	25135	2
Krung Thai Bank	2148	n/a	n/a	25012	3
Siam Commercial Bank	3631	3465	166	21026	5
Bank of Ayudhya	239	1477	-1238	14547	6
Thai Military Bank	885	180	705	12921	7
First Bangkok City Bank	526	807	-281	9575	8
Siam City Bank	701	1221	-520	9039	10
Bangkok Metropolitan Bank	172	273	-101	7298	13
Bangkok Bank of Commerce	308	644	-336	6356	14
Bank of Asia	3656	3661	-4	4893	17
DBS Thai Danu Bank	801	770	30	4645	18
Std. Chartered Nakornthon Bank	399	518	-120	2492	26
Bankthai	24	72	-48	2474	27
UOB Radanasin Bank	44	n/a	n/a	n/a	35
Export-Import Bank of Thailand	14	66	-52	1350	38

5. Prior to the crisis in Korea, six-month, one-year and three-year forward contracts were available and traded in domestic markets. However these markets were not very liquid, because of regulation.[8] According to Asiamoney (1997), these liquidity problems led to the development of a large offshore market for nondeliverable forwards. Park and Rhee (2000) report that average daily trading in these contracts was 250 million US dollars. In addition Kregel (1998) reports that Korean banks traded over $550 billion worth of financial derivatives in 1997. We infer that, imperfections aside, markets for hedging the Won existed.

6. Euromoney (1991) reported that most companies in Sweden dealt actively in: (i) forward contracts which were very liquid for maturities up to one year and fairly liquid for maturities up to two years; (ii) currency options which were liquid going out to 18 months in maturity; and (iii) currency swaps. According to the BIS, the notional principal of currency swaps involving the Swedish Krona, valued in billions of US dollars, was 17.7, 29.9, 34.3, and 37.3, as of December 1991, 1992, 1993 and 1994, respectively.

[8] For example, banks were not permitted to carry a spot short position above limits determined by the Bank of Korea.

7. Euromoney (1991) reports that there was an active forward market in U.S. Dollar-Finnish Marka with maturities up to one year.

Hedging by Nonfinancial Firms

Table 5, extracted from Allayannis, Brown and Klapper (2000), presents additional evidence on the ability of private agents to hedge exchange rate risk in the Asian countries prior to the crisis of 1997. Specifically, we display their estimate of the percentage of nonfinancial firms' foreign debt that was hedged in 1996. Notice that with the exception of Taiwan, firms hedged a nontrivial amount of their foreign debt. Presumably, if firms could hedge, so could banks.[9]

TABLE 5

PERCENTAGE OF FOREIGN DEBT HEDGED BY NONFINANCIAL FIRMS, 1996

Hong Kong/China	37.1	Singapore	29.5
Indonesia	28.3	South Korea	n/a
Malaysia	9.1	Taiwan	4.7
Philippines	13.8	Thailand	26.5

Viewed overall, the evidence reviewed in this section provides substantial support for the view that hedging vehicles were available to private agents, including banks, in many of the twin crises countries. Of course the evidence does not tell us what the costs of hedging through these vehicles was or how the cost was affected by macroeconomic developments. This is clearly an important topic for future research.

3. A Model of Banking in an Small Open Economy

In this section we discuss our benchmark model. For the banks in our analysis to face a nontrivial decision, we must make two key assumptions: (i) banks operate in a fixed exchange rate regime that may collapse, and (ii) forward markets exist. With respect to (ii) we assume that forward markets are perfect. There are at least three reasons why this is an interesting benchmark case. First, it allows us to analyze what would happen if all forward market imperfections were eliminated. Our analysis implies that banks would not avail themselves of these markets in the presence of government guarantees. Second, twin

[9]Note that Korea is not included in Table 5. According to Allayannis, Brown and Klapper (2000) Korean law made it difficult for nonfinancial firms to use currency derivatives. Indeed, the IMF (1997) reports that in Korea there were "no specific restrictions on the terms of forward contracts in respect of interbank transactions" but "forward contracts between foreign exchange banks and nonbank customers must be based on a bona fide transaction." Presumably firms could only hedge exchange rate risk associated with export sales or purchases, via an arrangement like a banker's acceptance.

crises do not happen only in emerging markets where forward markets may well be quite imperfect. They also occur in countries such as Sweden where the assumption of well-functioning forward markets seems reasonable. Finally, the benchmark case makes clear one reason why markets for securities to hedge exchange rate risk, such as forward contracts and foreign debt denominated in local currency, may not exist: in fixed exchange rate economies with government guarantees, banks have no incentives to participate in them.

We begin by studying a partial equilibrium model of banks in a small open economy. By assumption there is a single consumption good and no barriers to trade, so that purchasing power parity holds:

$$P_t = S_t P_t^*. \tag{3.1}$$

Here P_t and P_t^* denote the domestic and foreign price levels respectively, while S_t denotes the exchange rate defined as units of domestic currency per unit of foreign currency. For convenience we normalize the foreign price level to one: $P_t^* = 1$ for all t.

The rate of depreciation of the exchange rate is assumed to follow a Markov chain. The economy is initially in a fixed exchange rate regime, $S_t/S_{t-1} = 1$, with exchange rate $S_t = S^I$. The economy can switch to a devaluation regime, $S_t/S_{t-1} = \gamma > 1$, which is an absorbing state. The probability transition matrix is given by:

$$T = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}.$$

Here $p = \Pr(S_{t+1}/S_t = \gamma | S_t/S_{t-1} = 1)$, is the probability of switching from the fixed exchange rate regime to the devaluation regime.[10] The variable $S^D = \gamma S^I$ denotes the level of the exchange rate in the first period of the devaluation regime.[11] Since we focus on banks' hedging strategies prior to the devaluation, we suppress time subscripts throughout much of the rest of the paper.

3.1. The Banking Sector

For now we assume that banks borrow in dollars and lend in local currency, so that they are exposed to a currency mismatch. In section 4 we show that, in the presence of government guarantees, banks are willing to take on this mismatch.

Banks, all of whom are perfectly competitive, finance themselves entirely by borrowing L dollars at the gross interest rate R^b in the international capital market. These funds are

[10] In Burnside, Eichenbaum and Rebelo (2000a) we endogenize the probability of devaluations in a manner consistent with the way banks are modeled in this paper.

[11] The analysis can be generalized to the case where there is more than one possible value of S in the event of a devaluation.

converted into local currency at exchange rate S^I. Banks use these funds to make loans that are repaid in units of the domestic currency at the nonindexed gross interest rate R^a.

To be in the banking business requires an investment of K units of output at the beginning of every period. If the bank does not exit the industry by the end of the period it recoups $(1-\delta)K$ units of its capital investment. Here δ represents the rate of depreciation on capital. If the bank does exit, it cannot retrieve any of its initial capital investment. Since capital can be invested in world capital markets, the per period opportunity cost of entering the banking industry is RK. Here R is the gross interest rate on dollar-denominated loans in the world capital market.

We abstract from the role of banks as producers of information and focus directly on the real costs of making loans.[12] By assumption, the real cost of lending L units of output is $\psi(L)$. The function $\psi(L)$ is positive, twice continuously differentiable, increasing and strictly convex for $L > 0$, so that the marginal cost of making loans is increasing. The bank's end-of-period dollar-denominated profits from lending activities, inclusive of end-of-period capital, are:

$$\pi^L(L, x; S) = \frac{R^a S^I L}{S} + (1-\delta)K - \psi(L) - R^b(L, x)L. \tag{3.2}$$

Here S denotes the exchange rate that prevails at the end of the period. In addition, x denotes the number of units of local currency sold by the bank in the forward market. R^b is written as a function of L and x because banks face a competitively determined schedule of borrowing rates that depends on their choices for L and x. This embodies our assumption that foreign creditors can observe banks' portfolios.

Banks can hedge exchange rate risk by entering into forward contracts.[13] Let F denote the one-period forward exchange rate defined as units of local currency per dollar. To simplify we abstract from transactions costs and assume that these contracts are priced in a risk neutral manner, so that:[14]

$$\frac{1}{F} = (1-p)\frac{1}{S^I} + p\frac{1}{S^D}. \tag{3.3}$$

Dollar-denominated profits from hedging activities are:

$$\pi^H(x; S) = x\left(\frac{1}{F} - \frac{1}{S}\right). \tag{3.4}$$

[12] For a similar model see Chari, Christiano and Eichenbaum (1996) and Edwards and Vegh (1997). For alternative models of the role of banks in currency crises see Akerlof and Romer (1993), Caballero and Krishnamurthy (1998), and Chang and Velasco (1999).

[13] See Albuquerque (1999) for a general discussion of optimal hedging strategies in the presence of transactions costs, including the choice between forwards and options as hedging instruments.

[14] Note that since $P_t^* = 1$, this equation implies that the expected *real* return from a forward contract is zero. This avoids Siegel's (1972) paradox, which arises when the expected *nominal* return to forward contracts is assumed to be zero.

Notice that the expected value of the bank's profits from hedging is $E[\pi^H(\cdot;S)] = 0$. The bank's total dollar profits from loan plus hedging operations are given by

$$\pi(L,x;S) = \pi^L(L,x;S) + \pi^H(x;S). \tag{3.5}$$

By assumption banks can default on loans contracted in the international capital market. It is optimal for banks to default in states of the world where π is negative. The expected profit of a bank that defaults whenever $\pi(\cdot;S) < 0$ is

$$V(L,x) = E[\max\{\pi(L,x;S), 0\}]. \tag{3.6}$$

where E is the expectation with respect to S.

When a bank defaults it has gross assets with a residual value given by

$$\begin{aligned} V^R(L,x;S) &= \frac{R^a S^I L}{S} - \psi(L) + (1-\delta)K + x\left(\frac{1}{F} - \frac{1}{S}\right) \\ &= \pi(L,x;S) + R^b(L,x)L. \end{aligned} \tag{3.7}$$

These assets, net of bankruptcy costs, are distributed to the bank's international creditors. Bankruptcy costs are given by

$$C^B(L,x;S) = \omega + \lambda V^R(L,x;S). \tag{3.8}$$

Here ω represents fixed costs associated with bankruptcy and λ represents the fraction of the bank's gross assets that are dissipated upon default.[15] By including the bank's net profits from hedging in V^R, we have assumed that forward contracts must be settled before the bank's foreign creditors are paid. Condition (3.8) can also be interpreted as a collateral constraint: banks have to pledge part of their revenues to ensure that they can settle their forward positions in the event of a devaluation.

Throughout we assume that bank loans and forward operations are publicly observable. Banks choose L and x subject to the constraint that forward contracts must be honored in all states of the world. This implies that

$$\text{if } \pi(\cdot;S) < 0 \text{ then } V^R(\cdot;S) \geq C^B(\cdot;S), \tag{3.9}$$

so that the bank's residual value net of bankruptcy costs is nonnegative after the settlement of forward contracts whether or not the currency is devalued. We will refer to (3.9) as the 'no default on forward contracts' condition. This is a natural condition to impose. If there were substantial defaults associated with forward contracts, prices would vary significantly across firms. In reality, this does not seem to be the case.

[15] When ω is zero, agents are indifferent between various hedging strategies. As discussed in the Appendix, a positive ω serves the role of a 'tie-breaker'.

Banks' Borrowing Rate Absent Loan Guarantees

By assumption, foreign lenders are risk neutral. Hence they will set a schedule of interest rates, $R^b(L,x)$, with the property that the expected return to their investment equals R for all (L,x). Recall that banks default whenever their total profits, $\pi(L,x;S)$, are negative. It is convenient to define an indicator function $I[\pi(L,x;S)]$ which takes the value 1 when $\pi(L,x;S) < 0$ and the value zero otherwise. The $R^b(L,x)$ schedule is determined by the condition:

$$\begin{aligned} RL &= E\{\{(1 - I[\pi(\cdot;S)])R^b(\cdot)L + I[\pi(\cdot;S)]\left[V^R(\cdot;S) - C^B(\cdot;S)\right]\} \\ &= R^b(\cdot)L + E\{\{I[\pi(\cdot;S)]\left[V^R(\cdot;S) - C^B(\cdot;S) - R^b(\cdot)L\right]\}, \end{aligned} \tag{3.10}$$

where E denotes the expectation with respect to S. To understand the previous equation note that whenever the bank does not default $(I[\pi(\cdot;S)] = 1)$ it repays foreign creditors $R^b(\cdot)L$. If the bank does default $(I[\pi(\cdot;S)] = 0)$, then foreign creditors receive the bank's residual value net of bankruptcy costs, $V^R(\cdot;S) - C^B(\cdot;S)$.

To illustrate how this condition works, suppose that (L,x) is such that a bank is fully hedged, i.e. its profits are always non-negative and $I[\pi(L,x;S)] = 0$ for all S. Then (3.10) implies that $R^b(L,x) = R$. In contrast, suppose that (L,x) is such that the bank's profits are negative when $S = S^D$ and non-negative when $S = S^I$. In this case the bank defaults when a devaluation occurs, so that $R^b(L,x)$ is given by:

$$RL = (1-p)R^b(L,x)L + p\left[V^R(L,x;S^D) - C^B(L,x;S^D)\right]. \tag{3.11}$$

Banks' Borrowing Rate With Loan Guarantees

Suppose that there are government guarantees which apply only to the devaluation state. In this case the $R^b(L,x)$ schedule is determined by the condition:

$$RL = R^b(\cdot)L + (1-p)I[\pi(\cdot;S^I)]\left[V^R(\cdot;S^I) - C^B(\cdot;S^I) - R^b(\cdot)L\right] \tag{3.12}$$

This condition implies that $R^b(L,x) = R$ for all (L,x) such that profits in state $S = S^I$ are non-negative. It does not matter if the bank defaults in state $S = S^D$, because in this case the government pays foreign creditors $R^b(L,x)L$.

We conclude this section with a proposition that provides us with a convenient representation of the firm's objective function.

Proposition 3.1. *The objective function of the representative bank, (3.6), can be rewritten as*

$$V(L,x) = V_e^R(L) - C(L,x), \tag{3.13}$$

where

$$V_e^R(L) = R^a \frac{S^I}{F} L - \psi(L) + (1-\delta)K, \tag{3.14}$$

and $C(L,x)$ is the bank's expected cost of borrowing. It takes the form

$$C(L,x) = E\{\min[R^b(L,x)L, V^R(L,x;S)\}. \tag{3.15}$$

Here E denotes the expectation with respect to S.

Proof: See Appendix.

According to (3.13), the expected value of a bank's profits can be decomposed into two parts, $V_e^R(L)$ and $C(L,x)$. To understand the first part, note that since forward markets are actuarially fair, $E(1/S) = 1/F$. So $V_e^R(L)$ is the bank's expected revenue from loans minus the costs of producing those loans, inclusive of end of period capital. $V_e^R(L)$ does not depend on x since the expected profit from hedging is zero. The term $C(L,x)$ equals the bank's expected cost of borrowing. To see this, note that the bank will either repay its creditors $R^b(\cdot)L$ or $V^R(\cdot;S)$ depending on whether or not it declares bankruptcy. So $C(\cdot)$ is just the probability of not defaulting times $R^b(\cdot)L$ plus the probability of defaulting times the residual assets of the bank, $V^R(\cdot;S)$.

Equation (3.13) is useful because $V_e^R(L)$ does not depend on x. This means we can gain intuition about banks' optimal hedging strategies by considering the value of x that minimizes the expected cost of borrowing, $C(L,x)$, for a given loan size L.

4. Optimal Bank Hedging when Loans are Predetermined

To hone our intuition about the effects of government guarantees we first consider the case where the amount of real lending, L, is exogenously fixed. Given this assumption, a bank's only decision is how much exchange rate risk to hedge. In this section we prove two propositions: (i) it is optimal for a bank to fully hedge exchange rate risk when there are no government guarantees; and (ii) it is not optimal for a bank to hedge exchange rate risk in the presence of government guarantees. These propositions mirror the classic results in Kareken and Wallace (1978) on the impact of deposit insurance on banks' optimal portfolio decisions. While the nature of government guarantees is different, both our results and those of Kareken and Wallace (1978) are driven by: (i) the presence of bankruptcy costs, and (ii) the ability of banks to make portfolio decisions that minimize the value of their assets in bankruptcy states. We conclude this section by showing how our results extend to the case of probabilistic government guarantees.

We begin by characterizing a bank's optimal hedging strategy in an economy without guarantees.

Proposition 4.1. *In an economy with no government guarantees, L fixed, $V_e^R(L) > RL$, and (3.9) satisfied, complete hedging is optimal for $0 < \lambda < 1$. When $\lambda = \omega = 0$ the Modigliani-Miller theorem applies and the bank is indifferent between hedging and not hedging.*

Proof: See Appendix.

To see the intuition behind this result consider a bank that compares defaulting only in the devaluation state with never defaulting. Given Proposition 3.1, and a fixed level of lending, L, the optimal hedging strategy, x, is the one which minimizes the expected cost of borrowing.

If a bank chooses x such that it defaults in the devaluation state. Equations (3.8), (3.15) and (3.11), imply that the bank's expected cost of borrowing is:

$$\begin{aligned} C(L,x) &= RL + p[V^R(L,x;S^D) - C^B(L,x;S^D)] \\ &= RL + p[\omega + (1-\lambda)V^R(L,x;S^D)]. \end{aligned}$$

Conditional on a strategy of defaulting in the devaluation state, $C(x, L)$ is minimized by choosing x to minimize the bank's residual value in that state. This involves choosing the lowest value of x consistent with not defaulting on forward contracts (see equation [3.9]). The minimum value of $C(L, x)$ that can be achieved under this strategy is:

$$C^I(L) \equiv \min_{\{x|\pi(\cdot;S^I)\geq 0, \pi(\cdot;S^D)<0\}} C(L,x) = RL + p\frac{\omega}{1-\lambda}.$$

The term $\omega/(1-\lambda)$ is the residual value that the bank has to leave in bankruptcy states. These resources are not received by the foreign creditors since they are dissipated in bankruptcy costs.

If the bank sets x equal to any value such that the bank is fully hedged and never defaults, then it can borrow at the risk free rate. The expected cost of borrowing is:

$$C^H(L) = RL.$$

As long as there are fixed costs of bankruptcy (i.e. $\omega > 0$), then $C^I(L) > C^H(L)$ and it is optimal for the bank to fully hedge.

We now characterize the optimal hedging strategy in an economy with government guarantees.

Proposition 4.2. *Consider an economy with government guarantees and L fixed. Suppose that: $V_e^R(L) > RL$, $0 < \lambda < 1$, $0 < \omega < (1-\lambda)RL$ and equation (3.9) is satisfied. Then full hedging is not optimal. The bank's optimal strategy is to set x to its lowest permissible bound, given by (3.9), and default on its debt when a devaluation occurs.*

Proof: See Appendix.

To see the intuition behind this result, we again consider the case in which a bank compares defaulting in the devaluation state to being fully hedged. If a bank defaults only in the devaluation state, then equations (3.8), (3.15) and the fact that $R^b(L,x) = R$ imply that the bank's expected cost of borrowing is:

$$\begin{aligned} C(L,x) &= (1-p)RL + pC^B(L,x;S^D) \\ &= (1-p)RL + p[\omega + \lambda V^R(L,x;S^D)]. \end{aligned}$$

To minimize $C(L,x)$ conditional on this strategy, a bank will choose x to minimize $V^R(L,x;S^D)$ subject to condition (3.9). This may involve setting x to be a negative number, i.e. the bank magnifies its exchange rate exposure by selling dollars forward. If the bank is going to go broke it wants to minimize the value of assets that its creditors can seize. Forward contracts that lose money in the devaluation state help the bank do this. The lowest value of $C(L,x)$ that can be achieved under this strategy is:

$$C^I(L) \equiv \min_{\{x|\pi(\cdot;S^I)\geq 0, \pi(\cdot;S^D)<0\}} C(L,x) = (1-p)RL + p\frac{\omega}{1-\lambda}.$$

As before the expected cost of borrowing if the bank chooses x so that it never defaults is:

$$C^H(L) = RL.$$

Suppose that the minimum costs of bankruptcy are lower than the costs of repaying the loans, i.e.

$$\frac{\omega}{1-\lambda} < RL. \tag{4.1}$$

Then $C^I(L)$ is less than $C^H(L)$ and it is optimal to default when a devaluation occurs.

We conclude that with government guarantees, banks go bankrupt in the devaluation state. The lowest permissible value of x will, in some cases, be negative, so that banks adopt a hedging strategy that involves selling dollars forward. While this might be characterized as reckless speculation, it is simply the optimal response of banks to government guarantees.[16]

[16] Interestingly, Kregel (1998) reports that US banks made a profit of roughly \$4.6 billion profits on Won derivative contracts in 1997.

We can modify our analysis to accommodate the case in which agents expect that, with probability α, the government will repay bank's foreign creditors in the event of a devaluation.[17] To conserve on space we only consider the case where banks contemplate defaulting when $S = S^D$. As in the case of certain government guarantees, the expected cost of borrowing for a fully hedged bank is RL. If the bank is not fully hedged and defaults when $S = S^D$, its expected cost of borrowing is: $(1-p)R^b(\cdot)L + pV^R(\cdot; S^D)$. The condition ensuring that international creditors receive expected returns of RL is:

$$RL = (1-p)R^b(\cdot)L + p\alpha R^b(\cdot)L. \tag{4.2}$$

Here we are assuming that the expected value of the government's guarantee conditional on a devaluation, $\alpha R^b(\cdot)L$, is at least as large as what the creditor could otherwise recover in bankruptcy proceedings, $V^R(\cdot; S^D) - C^B(\cdot; S^D)$. Notice that when $\alpha = 1$ we obtain the case of full guarantees underlying Proposition 4.2. When $\alpha = 0$, we obtain the case of no guarantees of Proposition 4.1.

Equation (4.2) implies that the expected cost of borrowing can be rewritten as: $(1-p)RL/[(1-p)+p\alpha] + pV^R(\cdot; S^D)$. The minimum expected cost of borrowing is obtained by choosing x to minimize the residual value of the bank $V^R(\cdot; S^D)$. Condition (3.9) implies that the minimum value of $V^R(\cdot; S^D)$ is equal to $\omega/(1-\lambda)$. Thus the expected cost of borrowing for a bank contemplating default in the devaluation state is: $(1-p)RL/[(1-p)+p\alpha]+p\omega/(1-\lambda)$. Comparing this to RL, the expected cost of borrowing of a fully hedged bank, we conclude that full hedging is not optimal as long as:

$$0 < \omega < \frac{\alpha}{(1-p)+p\alpha}(1-\lambda)RL. \tag{4.3}$$

When $\alpha = 0$, corresponding to the no-guarantee case, the previous condition cannot be fulfilled. Since the right hand side of (4.3) is increasing in α, so too is the class of economies for which full hedging is not optimal. When $\alpha = 1$, so there is no uncertainty about the government bailout, (4.3) reduces to the regularity condition in Proposition 4.2.

In sum, allowing for probabilistic government guarantees does not change our basic message: guarantees reduce banks' incentives to hedge exchange rate risk. The more likely those guarantees are, the larger is the class of economies for which banks will find it optimal not too fully hedge exchange rate risk.[18]

[17] We thank Pablo Neumeyer for these results.

[18] Given our assumptions, a probabilistic guarantee of all bank loans and a sure guarantee of a fraction of bank loans have identical implications. In particular, suppose that the probability of a bailout when $S = S^D$ is 1 but the government only guarantees that foreigners recover at least $\alpha R^b(\cdot)L$. Then it is straightforward to show that, as above, full hedging is not optimal as long as (4.3) holds.

We conclude this section by asking the question: would the banks in our model prefer to make dollar-denominated loans or borrow in dollars and lend in local currency? Proposition 4.2 implies that they are indifferent between these two alternatives. Banks which make dollar denominated loans can still expose themselves to exchange rate risk by selling dollars in the forward market. In fact, it is optimal for them to choose a value of $x < 0$ which makes their residual value in the devaluation state, net of bankruptcy costs, equal to zero. Then their expected cost of borrowing is: $(1-p)RL + p\omega/(1-\lambda)$. This coincides with the expected cost of borrowing for a bank that borrows in dollars and lends in local currency. It follows that banks are indifferent between denominating their loans in dollars or in local currency. Government guarantees provide banks with an option that has value only if banks have a currency mismatch.[19] The mismatch can arise either from a bank's borrowing and lending operations or from its forward market operations.

5. Equilibrium in the Banking Industry

In this section we study the impact of government guarantees on the equilibrium of the banking industry. For now we assume an exogenous schedule for bank loans (this schedule is rationalized in section 8). In contrast to the previous section we allow banks to simultaneously choose their hedging strategies and how many loans to produce.

Suppose that the total demand for bank loans is given by the function $D(R^a)$ which is assumed to be nonnegative and nonincreasing in R^a. In an economy with no government guarantees the representative bank chooses (L, x) to maximize V, given by (3.6), subject to (3.7), (3.8), (3.9), and (3.10). The equilibrium for the banking industry is defined as follows.

Definition. An *equilibrium for the banking industry* is a value for R^a, a schedule, $R^b(L, x)$, a level of hedging, x, a level of lending L, and a number of banks B such that when banks take R^a and the schedule $R^b(L, x)$ as given, (i) the pair (L, x), maximizes the bank's expected profit, (ii) the bank's expected profit is RK, and (iii) the market for bank loans clears, $LB = D(R^a)$.

We now characterize the equilibrium of an economy with no government guarantees.

Proposition 5.1. *Suppose that $(1-\delta)K - \psi(0) > 0$. The equilibrium of an economy with no government guarantees is unique and has the following properties. First, it is optimal for banks to fully hedge their exchange rate risk. The equilibrium level of hedging is any*

[19] As discussed by Merton (1987) in the context of deposit insurance, guarantees can be interpreted as a free put option provided by the government to banks.

$x_n \in [\underline{X}, \overline{X}]$, where

$$\underline{X} = F\left[R + \psi'\left(L_n\right)\right] L_n - \frac{RK}{\frac{1}{F} - \frac{1}{S^D}}, \tag{5.1}$$

and

$$\overline{X} = F\left[R + \psi'\left(L_n\right)\right] L_n + \frac{RK}{\frac{1}{S^I} - \frac{1}{F}}. \tag{5.2}$$

Second, the equilibrium value of R^a is given by:

$$R_n^a = \frac{F}{S^I}\left[R + \psi'\left(L_n\right)\right]. \tag{5.3}$$

Third, the equilibrium level of lending, L_n, is given by the unique solution to:

$$\psi'(L_n)L_n - \psi\left(L_n\right) + (1-\delta)K = RK. \tag{5.4}$$

Fourth, the equilibrium value of $R^b(L_n, x_n) = R$. Fifth, the number of banks, B, is determined by $B = D(R^a)/L$.

Proof: See Appendix.

The optimal hedging decisions that we characterized in section 4, where L was predetermined, continue to hold when banks choose L and x simultaneously: in the absence of government guarantees it is optimal for banks to fully hedge their exchange rate risk. Equations (5.1) and (5.2) characterize the range of x values consistent with full hedging.

Equation (5.3) follows from the bank's first order condition for L. Since banks never default, their borrowing costs are equal to R. Equation (5.3) implies that the expected return to a bank's loans, measured in dollars, $R_n^a S^I/F$, equals borrowing costs (R) plus the marginal cost of making loans, $(\psi'(L))$. Equation (5.4) is as implication of free entry into the banking industry. Replacing R^a in (3.6) with R_n^a, we find that the maximal value of a bank is $\psi'(L)L - \psi(L) + (1-\delta)K$. By free entry, this value must equal the cost of operating a bank, RK.

We now characterize the equilibrium of an economy with government guarantees.

Proposition 5.2. *Suppose that $(1-\delta)K - \psi(0) > 0$ and $\omega < (1-\lambda)RL_n$. The equilibrium of an economy with government guarantees is unique and has the following properties. First, the equilibrium level of x, x_g, is the minimum permissible value consistent with (3.9) for $S = S^D$. Second, the equilibrium value of R^a is*

$$R_g^a = \frac{F}{S^I}\left[(1-p)R + \psi'(L_g)\right]. \tag{5.5}$$

Third, the equilibrium level of lending, L_g, is the unique solution to

$$\psi'(L_g)L_g - \psi(L_g) + (1-\delta)\,K - p\frac{\omega}{1-\lambda} = RK. \tag{5.6}$$

Fourth, the equilibrium value of $R^b(L_g, x_g) = R$. Fifth, the number of banks, B, is determined by $B = D(R^a)/L$.

Proof: See Appendix.

The optimal hedging decision characterized in section 4 continues to hold: banks choose the lowest value of x consistent with (3.9) and default when a devaluation occurs. Equation (5.5) results from the bank's first order condition for L. Since banks default in the devaluation state, their borrowing costs are equal to $(1-p)R$. Equation (5.3) implies that the expected return to a bank's loans, measured in dollars, $R_g^a S^I/F$, equals the sum of the their marginal borrowing costs $((1-p)R)$ and the marginal cost of making loans,$\psi'(L)\,R_n^a S^I/F$. Finally, (5.6) follows from free entry into the banking industry.

We conclude this section by asking the question: if government guarantees lead to increased risk in the banking system, why are they so prevalent? The following corollary suggests one possible answer: government guarantees lower R^a and lead to a lending boom. Later, in section 8, where we embed our model of banking into a general equilibrium environment, we show that the fall in R^a generated by the guarantees leads to a boom in economic activity and to a rise in real wages.

Corollary 5.3. *For sufficiently small ω the interest rate that banks charge firms is lower in an economy with guarantees than in one without guarantees, i.e. $R_g^a < R_n^a$. In addition, total bank lending is higher in the presence of guarantees.*

Proof: Equations (5.3) and (5.5) imply that:

$$R_g^a - R_n^a = \frac{F}{S^I}\left[-pR + \psi'(L_g) - \psi'(L_n)\right].$$

Equations (5.6) and (5.4) imply that $\lim_{\omega\to 0} L_g = L_n$. This implies that $\lim_{\omega\to 0} R_g^a < R_n^a$. The total level of bank lending, BL, is higher under guarantees because the demand for loans, $BL = D(R^a)$, is downward sloping.

The previous result does not imply that introducing guarantees increases the expected utility of the representative agent. However, it suggests that guarantees generate short term benefits that may be difficult for a short-sighted politicians to ignore.

6. Capital Requirements

In the previous section we argued that government guarantees make the banking system more fragile: banks adopt loan/hedging strategies that make it optimal for them to go bankrupt after a devaluation. In some other models, imposing capital requirements on banks can reduce the probability of bankruptcy. This is not the case in our model, because banks reoptimize their hedging strategies. Recall that banks must commit K units of capital to operate, of which they retrieve $(1-\delta)K$ at the end of the period if they do not default. Suppose that the government imposes capital requirements of the following form. In the beginning of the period a bank must place K' units of capital with the government, which it retrieves if it does not default. If the bank defaults it forfeits the right to K'. In an economy with guarantees, this policy leads to a rise in R^a and a decline in aggregate lending.[20] These contractionary effects might be worth bearing if capital requirements affected the probability of bank default. But they do not. Banks will respond to the policy by choosing a lower value of x to ensure that their residual value, inclusive of K' but net of bankruptcy costs, will continue to be zero in the devaluation state. Banks will therefore find it optimal to go bankrupt when a devaluation occurs.

In our model a much more effective policy for eliminating the fragility of the banking system under government guarantees is to regulate bank's hedging positions. In particular, the government could insist that banks fully hedge their exchange rate risk. In practice this type of regulation may be difficult to implement.

7. Introducing Real Uncertainty

In this section, we analyze the effects of government guarantees when exchange rate risk takes the form of lower repayment rates on the loans made by banks to firms in the devaluation state. This risk can exist even when banks make loans denominated in dollars. This is because these loans are often used to fund concerns that produce nontraded goods whose relative prices decline after a currency devaluation.[21] To focus our analysis here we abstract from risk that arises when banks borrow dollars but lend domestic currency.

[20] These results are similar to those obtained by Kareken and Wallace (1978) who analyze the effect of imposing a capital requirement on fractional reserve banks in the presence of deposit insurance.

[21] For example Gavin and Hausmann (1996) write that "... during the years leading up to the Chilean banking crisis (1982), banks were permitted to borrow in foreign currency but prohibited from taking the exchange risk, so that lending funded by international borrowing was required to be denominated in foreign currency. This was supposed to transfer the currency risk from banks to the nonfinancial firms to which banks made loans, but after the unexpected devaluation many firms found themselves unable to repay their loans in full or on time. Thus, the exchange rate risk that faced nonfinancial firms was to an extent borne by the banking systems in the form of credit risk."

In any given period, a fraction Φ of the firms repay their bank loans. To simplify we assume that Φ can take on only two values: $\Phi = \{\phi, 1\}$ where $\phi < 1$. To concentrate on the effect of bankruptcies associated with devaluations we suppose that $\Pr(\Phi = \phi | S = S^I) = 0$. To allow for imperfect correlation between exchange rate devaluations and firm bankruptcies we assume that $\Pr(\Phi = \phi | S = S^D) = q$. Since currency risk is correlated with default risk, banks can use forward contracts to cross-hedge their default exposure.

We now define profits from lending activities to be:

$$\pi^L(L, x; \Phi) = R^a L\Phi - \psi(L) + (1-\delta)K - R^b(L, x)L.$$

Profits from hedging activities remain the same, while total profits are: $\pi(L, x; \Phi, S) = \pi^L(L, x; \Phi) + \pi^H(x; S)$. The residual value of the bank is

$$\begin{aligned} V^R(L, x; \Phi, S) &= R^a L\Phi - \psi(L) + (1-\delta)K + x\left(\frac{1}{F} - \frac{1}{S}\right) \\ &= \pi(L, x; \Phi, S) + R^b(L, x)L. \end{aligned} \tag{7.1}$$

Bankruptcy costs remain the same: $C^B(L, x; \Phi, S) = \omega + \lambda V^R(L, x; \Phi, S)$. As before we restrict x to take on values such that the bank will be able to settle its forward position in all states of the world:

$$\text{if } \pi(\cdot; \Phi, S) < 0 \text{ then } V^R(\cdot; \Phi, S) \geq C^B(\cdot; \Phi, S). \tag{7.2}$$

In this setting

$$V(L, x) = E[\max\{\pi(\Phi, S), 0\}]. \tag{7.3}$$

The following two proposition summarize banks' optimal hedging strategies with and without government guarantees.

Proposition 7.1. *Consider an economy in which there are no guarantees. Suppose L is fixed, $0 < \lambda < 1$ and condition (7.2) is satisfied. Then fully hedging loan default risk, whenever feasible, is optimal.*

Proof: See Appendix.

Proposition 7.2. *Consider an economy with guarantees. Suppose L is fixed, full hedging is feasible, $\omega < (1-\lambda) RL$, $0 < \lambda < 1$ and the condition (7.2) is satisfied. Then it is not optimal for banks to fully hedge loan default risk.*

Proof: See Appendix.

These results are analogous to those of propositions 4.1 and 4.2. In the absence of guarantees, banks use forward contracts to hedge their default risk. Government guarantees eliminate the incentive to hedge.

8. Macroeconomic Implications

In this section, we embed our banking model into a general equilibrium environment. This allows us to endogenize the demand for bank loans, $D(R^a)$, and derive the equilibrium implications of government guarantees for output, employment, real wages, and interest rates. It also allows us to show how a reduction in R^a induced by government guarantees leads to a boom in economic activity. This makes concrete the benefits that might induce a short-sighted politician to offer government guarantees. Finally, we show that the decline in economic activity induced by a devaluation is larger in an economy with guarantees. Throughout this section, we concentrate on the type of exchange rate risk analyzed in sections 3–5 and abstract from loan default.

8.1. The Model Economy

Before describing the problems of the agents in the economy, we provide an overview of the timing of their interactions. This timing was chosen so that banks face exchange rate risk and there are no wage rigidities. We abstract from labor market imperfections to focus on the role of banking frictions. Each period is divided into three subperiods. In subperiod,1 banks borrow funds from abroad, enter into forward contracts and make loans to firms. In addition, firms hire labor and enter into forward contracts. Finally, the household makes its portfolio decisions. In subperiod 2 the exchange rate is realized, forward contracts are settled and firms pay labor in units of the local currency. In subperiod 3 production and consumption occur. In addition, bankruptcy costs, if any, are incurred and foreign loans are repaid.

We will now describe the maximization problem of the different agents in our economy with the exception of the banks. Their problem is specified in section 4.

Households

There is a continuum of unit measure of identical households who maximize expected utility defined over sequences of consumption, C_t, and labor supply, H_t:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t), \qquad 0 < \beta < 1.$$

Here E_0 denotes the expectation conditional on the representative household's time zero, subperiod one information set. In order to obtain analytical results we make three simplifying assumptions. First, we assume that momentary utility takes the form:

$$u(C_t, H_t) = \log(C_t - \frac{1}{\eta} H_t^{\eta}), \qquad \eta > 1.$$

The advantage of this specification is that labor supply depends only on the real wage rate (see (8.1) below). Second, we assume that the household's real financial wealth, a_t, is invested in a perfectly diversified international portfolio that yields a constant dollar-denominated gross rate of return R. Both domestic firms and banks are part of this portfolio and the risk associated with their returns is perfectly diversifiable. The budget constraint of the representative household is:

$$a_{t+1} = Ra_t + w_t H_t + \tau_t - C_t - \frac{M_{t+1}^d - M_t^d}{P_t}.$$

The variable τ_t represents lump sum transfers from the government, M_t^d denotes money holdings at the beginning of period t, w_t is the real wage rate, and P_t denotes the price level.

The household faces a cash-in-advance constraint on consumption:

$$P_t C_t \leq M_t^d + P_t w_t H_t.$$

We assume that $R > 1$, so that the previous constraint holds with equality.

For future reference we note that the household's first order condition for H_t implies:

$$H_t = w_t^{1/(\eta-1)}, \tag{8.1}$$

so that $1/(\eta - 1)$ is the elasticity of labor supply.

Output Producers

There is a continuum of measure N of perfectly competitive firms, where N is an endogenous variable. Each firm produces the single consumption good in the economy using labor, h, according to the following decreasing returns to scale technology:

$$y = f(h) - \zeta. \tag{8.2}$$

Here $f'(h) > 0$, $f''(h) < 0$, $f(0) = 0$ and $f'(0) = \infty$. The parameter $\zeta > 0$ represents a fixed cost of production.

Before the exchange rate is realized, firms hire labor in a competitive spot market at the real wage rate w. Firms borrow d units of local currency from banks at the gross interest rate R^a, and sell x^f units of the local currency in the forward market. The firm must have in hand a sufficient amount of the local currency to pay its nominal wage bill at the end of the period (Pwh), regardless of the realized value of the exchange rate. This cash-in-advance constraint on the nominal wage bill can be written as:

$$Pwh \leq d + Px^f \left(\frac{1}{F} - \frac{1}{S} \right) \quad \forall S. \tag{8.3}$$

The firm's real profits, $\pi^f(h, d, x^f; S)$, are given by:

$$\pi^f(h, d, x^f; S) = f(h) - \zeta - wh - (R^a - 1)\frac{d}{S} + x^f\left(\frac{1}{F} - \frac{1}{S}\right). \tag{8.4}$$

The representative firm's problem is to maximize its expected profits

$$\pi_e^f(h, d) \equiv E[\pi_f(h, d, x^f; S)] = f(h) - \zeta - wh - (R^a - 1)\frac{d}{F} \tag{8.5}$$

subject to (8.3).

Since borrowing is costly, constraint (8.3) holds with equality for all possible S. Evaluating (8.3) at S equal to S^I and S^D, we obtain a system of two equations in d and x^f. Solving this system yields $d = x^f = Fwh$. Once a devaluation occurs, there is no uncertainty, and from then on the forward rate is the same as the future spot rate, i.e. $F = S$, and we have $d = Fwh = Swh$.

Under either exchange rate regime, substituting $d = Fwh$ into (8.5), we see that the firm chooses h to maximize

$$\pi_e^f(h, Fwh) = f(h) - \zeta - R^a wh.$$

For future reference we note that the first order condition for h implies:

$$f'(h) = R^a w. \tag{8.6}$$

Government

In our model the government does not purchase goods and services. Its only roles are to: (i) provide lump-sum transfers to households, (ii) supply money, (iii) decide whether to issue guarantees to banks' creditors, and (iv) determine exchange rate policy. As described in section 3, initially the government fixes the exchange rate at the value $S_t = S^I$. With probability p the economy switches to a devaluation regime. In this regime the exchange rate is given by $S_t = \gamma^{t-t^*} S^D$, where t^* is the date at which the devaluation regime begins. Under both regimes the government sets the money supply (M_t^S) to be consistent with the exchange rate path and money demand.

The government must supply enough new money to finance its lump-sum transfers to domestic households plus any transfers to foreign creditors generated by its guarantees to bank creditors. Hence, it must adjust lump-sum transfers according to

$$\tau_t = (M_{t+1}^S - M_t^S)/S_t - \Gamma_t, \tag{8.7}$$

where Γ_t is the cost, if any, of honoring government guarantees.

In the absence of government guarantees, $\Gamma_t = 0$ for all t. Since in an economy with guarantees banks default only when the fixed exchange rate regime is abandoned (t^*), $\Gamma_t = 0$ for $t \neq t^*$, while $\Gamma_{t^*} = BRL$. Here B denotes the number of banks and BRL is the total amount owed by domestic banks to foreign creditors in case of bankruptcy. The government must cover the entire obligations of the banking system in period t^*, because banks choose to wipe out their residual value net of bankruptcy costs in the devaluation state.

Equilibrium

Free entry into the goods producing sector implies,

$$E[\pi^f(\cdot; S)] = 0. \tag{8.8}$$

Labor market clearing implies:

$$Nh = H. \tag{8.9}$$

The total supply of goods is

$$Y = yN. \tag{8.10}$$

We now define the equilibrium of our model economy which applies both to economies with and without government guarantees.

Definition *A competitive equilibrium is a set of stochastic processes for quantities* $\{C_t, H_t, a_t, M_t^d, h_t, d_t, x_t^f, L_t, x_t, B_t, N_t\}$, *prices* $\{w_t, P_t, F_t, R_t^a\}$ *and government policy variables,* $\{M_t^S, \tau_t\}$ *such that: (i) given the stochastic process for prices and policy variables,* C_t, H_t, a_{t+1}, *and* M_{t+1}^d *solve the household's problem,* h_t, d_t, *and* x_t^f *solve the firm's problem and* L_t *and* x_t *solve the bank's problem; (ii) the government's budget constraint, (8.7), holds; (iii) the money market clears with* $M_t^S = M_t^d$; *(iv) the loan market clears with* $S_{t-1}B_tL_t = N_tF_tw_th_t$; *(v) the forward market clears and (3.3) holds, (vi) the labor market clears, i.e. (8.9) holds; and (vii) the free entry conditions for the banking and output sectors, (3.6) and (8.8), hold.*

8.2. Characterizing the Equilibrium

In this subsection we characterize the equilibrium of the model economy with and without government guarantees. Given our assumption that households can borrow and lend in world capital markets at rate R, production and consumption decisions can be uncoupled. In particular, we can solve for equilibrium employment, H, output, Y, real wages, w, and the interest rate, R^a, without deriving the equilibrium stochastic processes for consumption and the money supply.[22] As emphasized in the introduction, the focus of this paper is on the effects of government guarantees on the banking industry and on aggregate economic activity. Accordingly, we restrict ourselves to characterizing the behavior of H, Y, w, and R^a.

We use the same procedure to study the versions of the model with and without government guarantees. We first derive an equilibrium demand schedule that relates the demand for loans to R^a. We then use the analysis of section 5 to determine the equilibrium value of R^a. Finally, we show how government guarantees influence the equilibrium values of H, Y, w, and R^a.

To derive the equilibrium relation between the total demand for loans and R^a note that (8.4), (8.6) and (8.8) imply that the equilibrium number of hours employed by each firm is the solution to:

$$f(h) - f'(h)h = \zeta. \tag{8.11}$$

Note that h is entirely determined by technology as summarized by the function $f(\cdot)$ and ζ. It follows that output per firm does not depend on whether or not there are government guarantees.

Given h and R_t^a, equation (8.6) determines the real wage, $w_t = f'(h)/R_t^a$, which in turn determines total labor supply via relation (8.1), $H_t = [f'(h)/R_t^a]^{1/(\eta-1)}$. The number of firms is given by $N_t = H_t/h$.

The demand for loans in units of local currency is $N_t d_t$. Since loans are made prior to the realization of the time t exchange rate, the demand for loans in units of foreign currency is $N_t d_t/S_{t-1}$. Substituting in the fact that $d_t = F_t w_t h$ and the expressions for N_t and w_t above, we find that the demand for loans is:

$$D(R_t^a) = \frac{N_t d_t}{S_{t-1}} = \frac{F_t}{S_{t-1}} \left[\frac{f'(h)}{R_t^a}\right]^{\eta/(\eta-1)}. \tag{8.12}$$

Here h is the solution to (8.11). Since $\eta > 1$, the demand for loans, $D(R_t^a)$, is a decreasing function of R_t^a. This rationalizes the assumptions that we made on the demand for loans in section 5.

The equilibrium value of the forward rate is given by:

$$F_t = \begin{cases} \frac{\gamma}{(1-p)\gamma+p} S_{t-1} & \text{for } t \leq t^* \\ \gamma S_{t-1} & \text{for } t > t^*. \end{cases} \tag{8.13}$$

In section 4 we showed that the loans per bank, L, and R^a are determined by the equilibrium of the banking industry. With no guarantees these are given by equations (5.3) and

[22] See Calvo and Drazen (1998) for an analysis of consumption in an open economy model where government policy follows a regime switching process similar to ours.

(5.4). With guarantees they are given by (5.5) and(5.6). The equilibrium level of loans in the economy is determined by substituting either (5.5) or (5.3) into (8.12), depending on whether or not there are government guarantees.

The equilibrium value of $D(R_t^a)$ and the level of loans per bank, L, imply that the number of banks is:

$$B_t = \frac{D(R_t^a)}{L}. \tag{8.14}$$

Given the previous results, the equilibrium stochastic processes for consumption, the price level, and the money supply can be characterized by solving the household's problem as well as imposing PPP and money market clearing.

8.3. The Effects of Government Guarantees

How does the presence of guarantees affect the behavior of the economy before a devaluation? Recall that the number of hours employed by each firm (h) and its level of output ($f(h)$) are independent of the presence of government guarantees (see (8.11)). From section 5 we know that guarantees lower R^a. Since $w_t = f'(h)/R_t^a$, real wages are higher in an economy with guarantees. This leads to higher aggregate employment, H, (see (8.1)), a greater number of firms, $N = H/h$, and higher aggregate output, $Nf(h)$. Finally, (8.14) implies that the total number of banks is also higher in an economy with guarantees.

In sum guarantees generate a boom in economic activity: the levels of employment, output, and total lending increase as do the number of banks and firms. It would not be surprising if short-sighted policymakers were tempted to introduce guarantees to engineer this type of boom. After all, the benefits of this policy are realized up front. The costs only come later, when a devaluation occurs.

8.4. The Effects of A Devaluation

We will now study the impact of a devaluation in economies with and without government guarantees. In both cases the devaluation is followed by a recession. However, the decline in employment and output is more severe in the economy with government guarantees. We will start by considering a situation with no government guarantees.

No Government Guarantees

Since banks are fully hedged, no bankruptcies occur when the fixed exchange regime is abandoned. After that there is no uncertainty, so after time t^* banks can borrow at the rate R. Their objective function is:

$$V = \frac{S_{t-1}}{S_t}\bar{R}_t^a L - \psi(L) + (1-\delta)K - RL.$$

Here $\bar{R}_t^a$ is the value of R^a after the devaluation. The first order condition for L implies that the post-devaluation borrowing rate is a constant

$$\bar{R}^a = \gamma[R + \psi'(L)]. \tag{8.15}$$

Here we used the fact that $S_t/S_{t-1} = \gamma$. The value of L is determined by the banking sector's free entry condition:

$$\psi'(L)L - \psi(L) + (1-\delta)K = RK.$$

This condition holds both before and after the devaluation. Proceeding as above we can deduce the equilibrium level of output, real wages and employment. Recall that prior to the devaluation R^a is given by (5.3), which can be written as:

$$R_n^a = \frac{\gamma}{(1-p)\gamma + p}\left[R + \psi'(L_n)\right].$$

Since $\gamma > 1$ it follows that $\bar{R}^a > R_n^a$. Equations (8.14), (8.12)and (8.13) imply that the total demand for loans and the number of banks decline after a devaluation because

$$D(\bar{R}^a) = [(1-p)\gamma + p]^{-1/(\eta-1)} D(R_n^a) < D(R_n^a).$$

Equations (8.2) and (8.11) imply that h and y are invariant. Equations (8.1), (8.6), (8.9), and (8.10) imply that real wages, employment, the number of banks, the number of firms, and aggregate output all fall when a devaluation occurs.

Government Guarantees

Now consider the situation in which there are government guarantees. Since banks are not hedged they declare bankruptcy when a devaluation occurs. Next, recall that prior to the devaluation R^a is given by:

$$R_g^a = \frac{\gamma}{(1-p)\gamma + p}\left[(1-p)R + \psi'(L_g)\right].$$

where, for ω arbitrarily close to zero, $L_g \cong L_n$.

After the devaluation there is no uncertainty, so government guarantees play no role. So from that point on, the bank's lending rate (given by (8.15)) is identical in the guarantees and no-guarantees economies. However, before the devaluation, R^a was lower in the guarantees economy. So the post-devaluation rise in R^a is larger in the economy with guarantees. It follows that the severity of the declines in real wages, employment, the number of banks, the number of firms, and aggregate output are also all larger in the guarantee economy.

9. Conclusions

This paper analyzed the connection between exchange rate devaluations and banks' hedging behavior. We argued that the presence of government guarantees to banks' creditors completely eliminates banks' incentives to hedge exchange rate risk. So while the policy lowers the interest rate on bank loans and raises aggregate output, it comes at a cost. The banking system becomes fragile. In the event of a devaluation, banks renege on their debt and go bankrupt.

We conclude by discussing some shortcomings of our analysis. To preserve the analytical tractability of our model we made several important simplifying assumptions. First, we assumed that there is no uncertainty about the exchange rate path once a devaluation occurs: the currency depreciates at a constant rate per unit of time. Aside from the counterfactual nature of this assumption, it implies that economies with and without guarantees look identical once the devaluation occurs, at least with respect to the variables we solved for. This would not be the case if there was ongoing exchange rate uncertainty. Second, we assumed that devaluations are exogenous events. In Burnside, Eichenbaum and Rebelo (2000b) we endogenize the probability of devaluations. In particular we show that the presence of government guarantees and the associated fiscal implications of bank bailouts lead to the possibility of self-fulfilling currency crises.[23] Finally, we did not formally address the question of why governments often provide implicit guarantees to banks. We suspect that the answer is related to our model's prediction that such policies generate booms in aggregate activity. Understanding why policymakers focus on this benefit (as well as others not discussed in this paper), rather than the costs is an important task that will no doubt involve political economy type considerations.

[23] For a related analysis where government guarantees play a role in determining the equilibrium probability of a crisis, see Chinn and Kletzer (2000).

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A. Appendix

A.1. Proof of Proposition 3.1

The result follows immediately from the definitions of $V(\cdot)$ and $V^R(\cdot)$ given in (3.6), (3.7). Note that (3.7) implies that (3.6) can be rewritten as

$$V(L,x) = E[\max\{V^R(L,x;S) - R^b(L,x)L, 0\}].$$

Notice that $\max\{V^R(L,x;S) - R^b(L,x)L, 0\} = V^R(L,x;S) - \min\{R^b(L,x)L, V^R(L,x;S)\}$. So we have

$$\begin{aligned} V &= E[V^R(L,x;S)] - E[\min\{R^b(L,x)L, V^R(L,x;S)\}] \\ &= V_e^R(L) - C(L,x). \end{aligned}$$

A.2. Proof of Proposition 4.1

Given that $\pi(L,x;S) = V^R(L,x;S) - R^b(L,x)L$, a bank can fully hedge if, given L, there exists some x such that $\pi(L,x;S) \geq 0$ for all S. It is convenient to define

$$\tilde{\pi}^L(L;S) \equiv R^a(S^I/S)L + (1-\delta)K - \psi(L) - RL, \tag{A.1}$$

in other words profits from lending if the bank's borrowing rate is the risk free rate. It is clear that hedging is feasible if there exists an x such that $\tilde{\pi}^L(L;S) + \pi^H(x;S) \geq 0$ for all S. I.e. if there exists an x such that

$$\tilde{\pi}^L(L,S^I) + x(1/F - 1/S^I) \geq 0$$

and

$$\tilde{\pi}^L(L;S^D) + x(1/F - 1/S^D) \geq 0.$$

Simple rearrangement of these equations shows that this requires that there exists at least one x such that $\underline{X}(L) \leq x \leq \overline{X}(L)$, where

$$\underline{X}(L) \equiv -\tilde{\pi}^L(L;S^D)/(1/F - 1/S^D) \tag{A.2}$$

and

$$\overline{X}(L) \equiv \tilde{\pi}^L(L;S^I)/(1/S^I - 1/F). \tag{A.3}$$

Clearly such an x exists if $\underline{X}(L) \leq \overline{X}(L)$, or

$$-\tilde{\pi}^L(L;S^D)/(1/F - 1/S^D) \leq \tilde{\pi}^L(L;S^I)/(1/S^I - 1/F).$$

Using (3.3) to replace $1/F$ this is equivalent to

$$-\tilde{\pi}^L(L;S^D)/(1-p) \leq \tilde{\pi}^L(L;S^I)/p$$

or $E[\tilde{\pi}^L(L;S)] \geq 0$. But notice that this is equivalent to $V_e^R(L) \geq RL$.

Given Proposition 3.1, and a fixed level of lending, L, the optimal hedging strategy, x, is the one which minimizes the expected cost of borrowing.

Consider, first, the baseline case where $\omega > 0$ and $\lambda > 0$. From (3.15), for those x such that the bank is fully hedged $C(L,x) = RL$, since $V^R(L,x;S) \geq RL$ for all S.

Using (3.15), for those x such that the bank goes bankrupt when $S = S^D$

$$C(L,x) = (1-p)R^b(L,x)L + pV^R(L,x;S^D).$$

The equation determining $R^b(L,x)$, (3.10), implies that for such x

$$(1-p)R^b(L,x)L + pV^R(L,x;S^D) = RL + pC^B(L,x;S^D)$$

and hence

$$C(L,x) = RL + pC^B(L,x;S^D). \tag{A.4}$$

Notice that (3.9) implies that bankruptcy costs are strictly positive, with $C^B(\cdot;S) \geq \omega/(1-\lambda)$. This implies that full hedging strictly dominates *any* strategy consistent with default in only the devaluation state.

Using (3.15), for those x such that the bank goes bankrupt when $S = S^I$

$$C(L,x) = (1-p)V^R(L,x;S^I) + pR^b(L,x)L.$$

The equation determining $R^b(L,x)$, (3.10), implies that for such x

$$(1-p)V^R(L,x;S^I) + pR^b(L,x)L = RL + (1-p)C^B(L,x;S^I)$$

and hence

$$C(L,x) = RL + (1-p)C^B(L,x;S^I). \tag{A.5}$$

This implies that full hedging strictly dominates *any* strategy consistent with default in only the no-devaluation state.

For those x such that the bank defaults for all S, $C(L,x) = V_e^R(L)$. Since $V_e^R(L) > RL$, Hence this strategy is also strictly dominated by full hedging.

Now consider the case where $\lambda > 0$ but $\omega = 0$. When $\omega = 0$ strategies involving default are feasible because the interval of xs consistent with (3.9) becomes

$$-\left[\tilde{\pi}^L(L;S^D) + RL\right]/(1/F - 1/S^D) \equiv \underline{\chi}(L) \leq x \leq \bar{\chi}(L) \equiv \left[\tilde{\pi}^L(L;S^I) + RL\right]/(1/S^I - 1/F).$$

and $[\underline{X}(L), \overline{X}(L)] \subset [\underline{\chi}(L), \bar{\chi}(L)]$. Once again, $C(L,x) = RL$ for any $x \in [\underline{X}(L), \overline{X}(L)]$ (full hedging). The best x that the bank could choose consistent with defaulting when $S = S^D$, is, as can be seen from (A.4), the x that minimizes $C^B(L,x;S^D)$, i.e. $x = \underline{\chi}(L)$. It implies that $C(L,x) = RL$. The best x that the bank could choose consistent with defaulting when $S = S^I$, is, as can be seen from (A.5), the x that minimizes $C^B(L,x;S^I)$, i.e. $x = \bar{\chi}(L)$. It also implies that $C(L,x) = RL$. Hence, the bank is indifferent between $x \in [\underline{X}(L), \overline{X}(L)]$, $x = \underline{\chi}(L)$, and $x = \bar{\chi}(L)$.

If $\omega = 0$ and $\lambda = 0$, notice that there are no costs associated with bankruptcy. That is, $C^B(\cdot;S) = 0$ for all S, implying that $C(L,x) = RL$ for *any* x such that the bank is solvent in at least one state of the world. Hence a version of the Modigliani-Miller theorem applies: the bank is indifferent between all $x \in [\underline{\chi}(L), \bar{\chi}(L)]$. Because $V_e^R(L) > RL$ and bankruptcy costs are zero, there are no x such that the bank is insolvent in both states.

A.3. Proof of Proposition 4.2

As in the proof of Proposition 4.2, hedging fully is feasible given that $E(V^R) > RL$. For $x \in [\underline{X}(L), \overline{X}(L)]$ (full hedging) the expected cost of borrowing is $C(L,x) = RL$.

Here we establish that under guarantees a strategy of defaulting only when $S = S^D$ is feasible if and only if $\omega < (1-\lambda)RL$. Such a strategy requires that there exists at least one x for which (i) $V^R(L,x;S^I) \geq RL$, (ii) $V^R(L,x;S^D) < RL$, and (iii) $V^R(L,x;S^D) \geq C^B(L,x;S^D)$ are satisfied. In (i) and (ii) we have used the fact that $R^b(L,x) = R$ for any such strategy. Also, the last condition follows from (3.9) and can be rewritten using (3.8) as $(1-\lambda)V^R(L,x;S^D) \geq \omega$.

Using results from the previous section the first condition is equivalent to $x \leq \overline{X}(L)$ while the second condition is equivalent to $x < \underline{X}(L)$. The previous section also established that $\underline{X}(L) \leq \overline{X}(L)$ if $V_e^R - RL \geq 0$, which we have assumed. Hence, the first two conditions can be replaced with one: $x < \underline{X}(L)$. Finally, some algebra shows that the third condition is equivalent to $x \geq \underline{Y}(L)$ where

$$\underline{Y}(L) \equiv \left\{\omega/(1-\lambda) - \left[\tilde{\pi}^L(L;S^D) + RL\right]\right\} / (1/F - 1/S^D). \tag{A.6}$$

Hence, for all three conditions to be satisfied it is necessary and sufficient to have $\underline{Y}(L) < \underline{X}(L)$. But this is easily shown to be equivalent to $\omega < (1-\lambda)\, RL$.

If the bank chooses $x \in [\underline{Y}(L), \underline{X}(L))$ it will choose to default when $S = S^D$. Because $R^b(L,x) = R$, (3.15) implies

$$C(L,x) = (1-p)RL + pV^R(L,x;S^D)$$

for such x. Notice that this implies

$$C(L,x) = RL + p\left[V^R(L,x;S^D) - RL\right].$$

Since $V^R(L,x;S^D) < RL$ for $x \in [\underline{Y}(L), \underline{X}(L))$ (default when $S = S^D$), it follows immediately that *any* such x strictly dominates any $x \in [\underline{X}(L), \overline{X}(L)]$ (full hedging), for which $C(L,x) = RL$.

Since $V^R(L,x;S^D)$ is strictly increasing in x, it is optimal for a bank that defaults when $S = S^D$ to set x to its lowest permissible value consistent with (3.9), $x = \underline{Y}(L)$. This value of x implies that $V^R(L,x;S^D) = C^B(L,x;S^D) = \omega/(1-\lambda)$. It follows that the minimized value of $C(L,x)$ is $(1-p)RL + p\omega/(1-\lambda)$.

Since government guarantees do not apply to default when $S = S^I$, if the bank chooses a value of x such that it defaults only when $S = S^I$ this strategy is dominated by full hedging for the same reasons given in the proof of Proposition 4.1. Similarly, a bank will not choose a value of x such that it defaults in both states, for the same reasons given in the proof of Proposition 4.1.

A.4. Proof of Proposition 5.1

We first define four sets of (L,x) pairs:

$$\Omega_1 = \{(L,x) | L \geq 0, \pi(L,x|S) \geq 0 \ \forall S\}, \tag{A.7}$$

$$\Omega_2 = \left\{(L,x) | \pi(L,x;S^I) \geq 0, \pi(L,x;S^D) < 0, \text{ (3.9) holds}\right\}, \tag{A.8}$$

$$\Omega_3 = \left\{(L,x)|\pi(L,x;S^I) < 0, \pi(L,x;S^D) \geq 0, \ (3.9) \text{ holds}\right\}, \tag{A.9}$$

$$\Omega_4 = \{(L,x)|\pi(L,x;S) < 0 \ \forall S, \ (3.9) \text{ holds}\}. \tag{A.10}$$

The set Ω_1 consists of those (L, x) pairs consistent with the bank being fully hedged. The set Ω_2 is the set of (L, x) such that the bank will default when $S = S^D$. The set Ω_3 is the set of (L, x) such that the bank will default when $S = S^I$. The set Ω_4 is the set of (L, x) such that the bank will default for all S. Implcit in the four definitions is the definition of $R^b(L, x)$ given by (3.10).

Now define $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4$. The bank's problem is to choose $(L, x) \in \Omega$ to maximize $V(L, x)$. We will frequently use the fact that, bu definition, $V(L, x) \geq 0$ for all $(L, x) \in \Omega$ (see [3.13]).

In the proof of Proposition 4.1 we established that for any $(L, x) \in \Omega_1$ expected profits are given by

$$\begin{aligned} V(L,x) = V^1(L) &\equiv V_e^R(L) - RL \\ &= R^a(S^I/F)L - \psi(L) + (1-\delta)K - RL \end{aligned}$$

and that $V_1(L) \geq 0$ if and only if $\exists (L, x) \in \Omega_1$.

Notice that $V_1(0) = (1 - \delta)K - \psi(0) > 0$, $V_1'(L) = R^a(S^I/F) - R - \psi'(L)$, and that $V_1''(L) = -\psi''(L) < 0$ for $L > 0$. The strict convexity of V_1 implies that V_1' is negative for sufficiently large L. This implies that there exists a finite L_+ such that $V_1(L_+) = 0$. The set $\{L|V_1(L) \geq 0\} = [0, L_+]$. Hence $\Omega_1 = \left\{(L, x)|L \in [0, L_+], \underline{X}(L) \leq x \leq \overline{X}(L)\right\}$.

Consider any $(L, x) \in \Omega_2$. In the proof of Proposition 4.1 we established that for such strategies $C(L, x) = RL + pC^B(L, x; S^D)$. Hence

$$\begin{aligned} V(L,x) = V^2(L,x) &\equiv V_e^R(L) - RL - pC^B(L,x;S^D) \\ &= V_1(L) - pC^B(L,x;S^D). \end{aligned}$$

Since $C^B(L, x; S^D) > 0$ and $V_2(L, x) \geq 0$ it follows that $V_1(L) > 0$ for $(L, x) \in \Omega_2$. Hence $L \in [0, L_+]$ if $(L, x) \in \Omega_2$. It follows immediately that no $(L, x) \in \Omega_2$ is profit maximizing.

Consider any $(L, x) \in \Omega_3$. In the proof of Proposition 4.1 we established that for such strategies $C(L, x) = RL + (1 - p)C^B(L, x; S^I)$. Hence

$$\begin{aligned} V(L,x) = V_3(L,x) &\equiv V_e^R(L) - RL - (1-p)C^B(L,x;S^I) \\ &= V_1(L) - (1-p)C^B(L,x;S^I). \end{aligned}$$

Since $C^B(L, x; S^I) > 0$ and $V_3(L, x) \geq 0$ it follows that $V_1(L) > 0$ for $(L, x) \in \Omega_3$. Hence $L \in [0, L_+]$ if $(L, x) \in \Omega_3$. It follows immediately that no $(L, x) \in \Omega_3$ is profit maximizing.

Within Ω_4 expected profits are $V_4(L, x) = 0$. Since there exists $L \in [0, L_+]$ such that $V_1(L) > 0$, no $(L, x) \in \Omega_4$ can be profit maximizing.

Thus, the profit maximizing choice of (L, x) will always lie in Ω_1, for any R^a, and will be the combination of the L which maximizes $V_1(L)$ and any $\underline{X}(L) \leq x \leq \overline{X}(L)$. Since $V_1(L)$ is strictly concave and $[0, L_+]$ is a compact set we can easily obtain the profit maximizing value of L. It is either the solution to the first order condition

$$V_1'(L) = R^a(S^I/F) - \psi'(L) - R = 0 \tag{A.11}$$

or it is one of the two endpoints, $L = 0$ or $L = L_+$. Notice that the optimum can never be at L_+ since $V_1(0) > 0$ and $V_1(L_+) = 0$.

In equilibrium $V = RK$. The previous subsection showed that in equilibrium $(L, x) \in \Omega_1$ with $L \in [0, L_+)$. We cannot have $L = 0$ in equilibrium since $V_1(0) = (1-\delta)K - \psi(0) < RK$. Hence, the only possibility is that an equilibrium with $(L, x) \in \Omega_1$ exists in which the first-order condition, (A.11), is satisfied. Subsituting this into the expression for $V_1(L)$, in equilibrium expected profits are given by

$$\psi'(L)L - \psi(L) + (1-\delta)K = RK. \tag{A.12}$$

Given the conditions we placed on ψ, there is a unique solution to (A.12). This value of L, which we denote L_n, is the unique equilibrium level of lending in the absence of government guarantees.

The unique equilibrium ask rate, as determined by (A.11), is

$$R_n^a = (F/S^I)\left[R + \psi'(L_n)\right].$$

To complete the proof we verify that $\underline{X}(L_n) \le \overline{X}(L_n)$ so that full hedging is feasible. Notice that

$$\overline{X} = \overline{X}(L_n) = F\left[R + \psi'(L_n)\right]L_n + RK/(1/S^I - 1/F)$$

and

$$\underline{X} = \underline{X}(L_n) = F\left[R + \psi'(L_n)\right]L_n - RK/(1/F - 1/S^D).$$

Clearly $\overline{X} > \underline{X}$.

A.5. Proof of Proposition 5.2

Under government guarantees, the Ω_i, $i = 1, 2, 3, 4$, are still given by (A.7)–(A.10), but implicit in these definitions is the definition of $R^b(L, x)$ under government guarantees, given by (3.12).

We proceed as in the proof of Proposition 5.1. For $(L, x) \in \Omega_1$, expected profits are again given by

$$V(L, x) = V_1(L) \equiv V_e^R(L) - RL.$$

Also $(L, x) \in \Omega_1$ and $V_1(L) \ge 0$ if and only if $L \in [0, L_+]$.

Consider any $(L, x) \in \Omega_2$. In the proof of Proposition 4.2 we established that for such strategies $C(L, x) = (1-p)RL + pV^R(L, x; S^D)$. Hence expected profits are given by

$$\begin{aligned} V(L, x) = V_2(L, x) &\equiv V_e^R(L) - (1-p)RL - pV^R(L, x; S^D) \\ &= V_1(L) - p\left[V^R(L, x; S^D) - RL\right] \\ &= V_1(L) - p\pi(L, x; S^D). \end{aligned}$$

Since, for $(L, x) \in \Omega_2$, $\pi(L, x; S^D) < 0$ this implies that $V_2(L, x) > V_1(L)$ for $(L, x) \in \Omega_2$.

It follows by the same arguments used in the proof of Proposition 5.1 that no $(L, x) \in \Omega_3$ nor any $(L, x) \in \Omega_4$ can be profit maximizing.

We next eliminate the possibility that in equilibrium any banks will be fully hedged. We know from the proof of Proposition 5.1 that the zero profit condition implies that if any banks are fully hedged, in equilibrium they must have $L = L_n$, the solution to (A.12) and the ask rate must be R_n^a given by (5.3). But, using the same argument used in the proof to Proposition 4.2, the fact that $\omega < (1-\lambda)RL_n$, implies that there exists at least one x with $(L_n, x) \in \Omega_2$. And, using any $(L_n, x) \in \Omega_2$, a bank would have expected profits of

$$V_2(L_n, x) = V_1(L_n) - p\pi(L_n, x; S^D).$$

But, for $(L_n, x) \in \Omega_2$, $\pi(L_n, x; S^D) < 0$, so that $V_2(L_n, x) > V_1(L_n)$. Hence banks cannot be fully hedged in equilibrium.

Now we check whether there is an equilibrium with $(L, x) \in \Omega_2$. For a given L, the proof of Proposition 4.2 showed that $(L, x) \in \Omega_2$ if and only if $\underline{Y}(L) \leq x < \underline{X}(L)$. We also have

$$\partial V_2(L, x)/\partial x = -p\partial\pi(L, x; S^D)/\partial x = -p(1/F - 1/S^D) < 0.$$

So the bank will set x equal to its lowest feasible value, i.e. $x = \underline{Y}(L)$ defined in (A.6). Substituting $x = \underline{Y}(L)$ into the expression for $V_2(L, x)$ we get

$$V_2[L, \underline{Y}(L)] = V_1(L) + p[RL - \omega/(1-\lambda)].$$

The lower bound on the values of L which are feasible in Ω_2 is the value of L for which $\underline{X}(L) = \underline{Y}(L)$, which is $\underline{L} = \omega/[R(1-\lambda)] < L_n$. The largest value of L which is feasible in Ω_2 is one for which $V_2[L, \underline{Y}(L)] = 0$ which we denote by $\overline{L}$. We assume, for the moment, that $\underline{L} < \overline{L}$, so that Ω_2 is non-null. We now choose $L \in (\underline{L}, \overline{L}]$ to maximize $V_2[L, \underline{Y}(L)]$.

Since $\overline{L}$ cannot be an equilibrium (it implies zero profits), and $V_2[L, \underline{Y}(L)]$ is strictly concave we look for an interior solution. Differentiating $V_2[L, \underline{Y}(L)]$ with respect to L, we obtain the first order condition

$$(S^I/F)R^a - \psi'(L) = (1-p)R. \tag{A.13}$$

In this case expected profits would be

$$\psi'(L)L - \psi(L) + (1-\delta)K - p\omega/(1-\lambda) = RK. \tag{A.14}$$

There is a unique solution, $L = L_g$, to (A.14) such that $L_g > L_n$. Notice that this implies $L_g > \underline{L}$, which confirms our initial assumption that $\underline{L} < \overline{L}$.

The unique equilibrium ask rate, as determined by (A.13), is

$$R_g^a = (F/S^I)\left[(1-p)R + \psi'(L_g)\right].$$

The equilibrium hedge position of the bank is

$$x_g = \underline{Y}(L_g).$$

A.6. Proof of Proposition 6.1

Under noncontingent government guarantees, the Ω_i, $i = 1, 2, 3, 4$, are still given by (A.7)–(A.10), but implicit in these definitions is the fact that $R^b(L, x) = RL$ for all $(L, x) \in \Omega$.

We proceed as in the proof to Proposition 5.2. For $(L, x) \in \Omega_1$ expected profits are again given by

$$V(L, x) = V_1(L) \equiv V_e^R(L) - RL.$$

For $(L, x) \in \Omega_2$ expected profits are again given by

$$V(L, x) = V_2(L, x) \equiv V_1(L) - p\pi(L, x; S^D)$$

Again, these are maximized by setting $x = \underline{Y}(L)$, which implies that

$$V_2[L, \underline{Y}(L)] = V_1(L) + p[RL - \omega/(1 - \lambda)].$$

For $(L, x) \in \Omega_3$ expected profits are given by

$$V(L, x) = V_3(L, x) \equiv V_1(L) - (1 - p)\pi(L, x; S^I).$$

Analagously to the previous case, these are maximized by setting x equal to the highest feasible value consistent with (3.9), i.e.

$$x = \overline{Y}(L) \equiv \left[\tilde{\pi}^L(L; S^I) + RL - \omega/(1 - \lambda)\right]/(1/S^I - 1/F). \tag{A.15}$$

Substituting $x = \overline{Y}(L)$ into the expression for $V_3(L, x)$ we get

$$V_3[L, \overline{Y}(L)] = V_1(L) + (1 - p)[RL - \omega/(1 - \lambda)].$$

For $(L, x) \in \Omega_4$ expected profits are again equal to 0.

In equilibrium $V = RK$. It follows by the same arguments used in the previous proofs that no $(L, x) \in \Omega_1$ nor any $(L, x) \in \Omega_4$ can be equilibria. There are two possibilities that we must consider.

If there was an equilibrium with $(L, x) \in \Omega_2$ then it would be at L_g and

$$V_3[L_g, \bar{Y}(L_g)] = V_2[L_g, \underline{Y}(L_g)] + (1 - 2p)[RL_g - \omega/(1 - \lambda)].$$

Since $p < 1/2$ and $L_g > L_n$, we have $V_3[L_g, \bar{Y}(L_g)] > V_2[L_g, \underline{Y}(L_g)] \geq 0$. The fact that $V_3(L_g) > 0$ implies that L_g is feasible within Ω_3. Hence $(L, x) \in \Omega_2$ cannot be an equilibrium.

To check whether there is an equilibrium with $(L, x) \in \Omega_3$, we differentiate $V_3[L, \bar{Y}(L)]$ with respect to L, to obtain the first order condition

$$(S^I/F)R^a - pR = \psi'(L). \tag{A.16}$$

In this case expected profits would be

$$\psi'(L)L - \psi(L) + (1 - \delta)K - (1 - p)\omega/(1 - \lambda) = RK. \tag{A.17}$$

There is a unique solution, $L = L_u$, to (A.17) such that $L_u > L_g > L_n$.

The unique equilibrium ask rate, as determined by (A.16), is

$$R_u^a = (F/S^I)\left[pR + \psi'(L_u)\right].$$

The equilibrium hedge position of the bank is

$$x_u = \overline{Y}(L_u).$$

A.7. Proof of Proposition 7.1

Given the definitions of V and V^R given in (7.3), (7.1) It follows that

$$V(L,x) = E[\max\{V^R(L,x;\Phi,S) - R^b(L,x)L, 0\}].$$

Notice that $\max\{V^R(\cdot;\Phi,S) - R^b(\cdot)L, 0\} = V^R(\cdot;\Phi,S) - \min\{R^b(\cdot)L, V^R(\cdot;\Phi,S)\}$. So we have

$$\begin{aligned} V(L,x) &= E[V^R(L,x;\Phi,S)] - E[\min\{R^b(L,x)L, V^R(L,x;\Phi,S)\}] \\ &= V_e^R(L) - C(L,x). \end{aligned}$$

Hence, the bank's problem of choosing x to maximize $V(L,x)$ is equivalent to it choosing x to minimize $C(L,x)$.

Given L, if the bank chooses x so that it is fully hedged then $C(L,x) = RL$. There are three alternatives to this strategy: the bank could choose x so that (i) it defaults when $S = S^D$ regardless of the value of Φ, (ii) it defaults when $S = S^D$ and $\Phi = \phi$, or (iii) it defaults when $S = S^I$.

For x consistent with strategy (i)

$$C(\cdot) = (1-p)R^b(\cdot)L + p(1-q)V^R(\cdot;1,S^D) + pqV^R(\cdot;\phi,S^D)$$

and its borrowing rate is determined by

$$\begin{aligned} RL &= (1-p)\,R^b(\cdot)L + p(1-q)\left[V^R(\cdot;1,S^D) - C^B(\cdot;1,S^D)\right] \\ &\quad pq\left[V^R(\cdot;\phi,S^D) - C^B(\cdot;\phi,S^D)\right]. \end{aligned}$$

Hence we can rewrite

$$C(\cdot) = RL + p\left[(1-q)C^B(\cdot;1,S^D) + qC^B(\cdot;\phi,S^D)\right].$$

So strategy (i) is dominated by full hedging.

For x consistent with strategy (ii)

$$C(\cdot) = (1-pq)R^b(\cdot)L + pqV^R(\cdot;\phi,S^D)$$

and its borrowing rate is determined by

$$RL = (1-pq)\,R^b(\cdot)L + pq\left[V^R(\cdot;\phi,S^D) - C^B(\cdot;\phi,S^D)\right].$$

Hence we can rewrite

$$C(\cdot) = RL + pqC^B(\cdot;\phi,S^D).$$

So strategy (ii) is dominated by full hedging.

For x consistent with strategy (iii)

$$C(\cdot) = (1-p)V^R(\cdot;1,S^I) + pR^b(\cdot)L$$

and its borrowing rate is determined by

$$RL = pR^b(\cdot)L + (1-p)\left[V^R(\cdot;1,S^I) - C^B(\cdot;1,S^I)\right].$$

Hence we can rewrite

$$C(\cdot) = RL + (1-p)\,C^B(\cdot;1,S^I).$$

So strategy (iii) is dominated by full hedging.

A.8. Proof of Proposition 7.2

As in the proof of Proposition 7.2, the bank's problem of maximizing $V(L,x)$ is equivalent to it minimizing $C(L,x)$. Given L, if the bank chooses x so that it is fully hedged then $C(L,x) = RL$. There are three alternatives to this strategy: the bank could choose x so that (i) it defaults when $S = S^D$ regardless of the value of Φ, (ii) it defaults when $S = S^D$ and $\Phi = \phi$, or (iii) it defaults when $S = S^I$.

First, we show that if full hedging is feasible and $\omega < (1-\lambda)\,RL$ either strategy (i) or strategy (ii) is also feasible. Full hedging being feasible requires that there exists at least one x for which $V^R(L,x;\Phi,S) \geq RL$ for all Φ, S. This, in turn, would imply that $V_e^R(L) \geq RL$. It is useful to define $\tilde{\pi}^L(L;\Phi) = R^a L\Phi - \psi(L) + (1-\delta)K - RL$. Notice that $V_e^R(L) = E[\tilde{\pi}^L(L,\Phi) + RL]$, so the fact that $V_e^R(L) \geq RL$ implies $E[\tilde{\pi}^L(L,\Phi)] \geq 0$. Since $\tilde{\pi}^L(L;1) > \pi^L(L;\phi)$, this further implies that $\tilde{\pi}^L(L;1) > 0$.

Using the expressions for $V^R(\cdot;\Phi,S)$ we see that $V^R(\cdot;\Phi,S) \geq RL$ for all Φ, S is equivalent to

$$x \leq \overline{x}_H = \frac{\tilde{\pi}^L(L;1)}{1/S^I - 1/F} \quad x \geq \underline{x}_{H1} = -\frac{\tilde{\pi}^L(L;1)}{1/F - 1/S^D} \quad x \geq \underline{x}_{H\phi} = -\frac{\tilde{\pi}^L(L;\phi)}{1/F - 1/S^D}.$$

Since $\tilde{\pi}^L(L;\phi) < \tilde{\pi}^L(L;1)$, $\underline{x}_{H1} < \underline{x}_{H\phi}$. So the three conditions reduce to $\underline{x}_{H\phi} \leq x \leq \overline{x}_H$. An x consistent with full hedging will exist if $\underline{x}_{H\phi} \leq \overline{x}_H$ or, equivalently, $E[\tilde{\pi}^L(L,\Phi)] \geq 0$.

Strategy (i) requires that there exist at least one x for which (a) $V^R(L,x;1,S^I) \geq RL$, (b) $V^R(L,x;\Phi,S^D) < RL$, $\forall\Phi$, and (c) $V^R(L,x;\Phi,S^D) \geq C^B(L,x;\Phi,S^D)$, $\forall\Phi$ are satisfied. The last condition can be rewritten as $(1-\lambda)V^R(L,x;\Phi,S^D) \geq \omega$ for all Φ.

Using the same logic as above, conditions (a) and (b) are equivalent to

$$x \leq \overline{x}_H \quad x < \underline{x}_{H1} \quad x < \underline{x}_{H\phi}.$$

Notice that the first and third of these conditions are redundant since $\underline{x}_{H1} < \underline{x}_{H\phi} \leq \overline{x}_H$. So we are left with $x < \underline{x}_{H1}$. Since $\tilde{\pi}^L(L;\phi) < \tilde{\pi}^L(L;1)$ condition c) reduces to one condition

$$x \geq \underline{x}_I = \left\{\omega/(1-\lambda) - \left[\tilde{\pi}^L(L;\phi) + RL\right]\right\}/(1/F - 1/S^D).$$

Thus, strategy (i) is feasible if $\underline{x}_I < \underline{x}_{H1}$. Some algebra shows that this requires

$$\omega < (1-\lambda)\left[RL - (1-\phi)R^a L\right].$$

Strategy (ii) requires that there exist at least one x for which (a) $V^R(L,x;1,S^I) \geq RL$, (b) $V^R(L,x;1,S^D) \geq RL$, (c) $V^R(L,x;\phi,S^D) < RL$ and (d) $V^R(L,x;\phi,S^D) \geq C^B(L,x;\phi,S^D)$ are satisfied. The last condition can be rewritten as $(1-\lambda)V^R(L,x;\phi,S^D) \geq \omega$.

Using the same logic as above, conditions (a), (b) and (c) are equivalent to

$$x \leq \overline{x}_H \quad x \geq \underline{x}_{H1} \quad x < \underline{x}_{H\phi}.$$

Since full hedging is feasible $\underline{x}_{H\phi} \leq \overline{x}_H$, making the first condition redundant. So we are left with $\underline{x}_{H1} \leq x < \underline{x}_{H\phi}$. Condition (d) requires $x \geq \underline{x}_I$. Thus, in order for an x consistent with strategy (ii) to exist, we need $\underline{x}_I < \underline{x}_{H\phi}$. Some algebra shows that this is equivalent to

$$\omega < (1-\lambda)RL.$$

So strategy (ii) is always feasible.

Having established that strategy (ii) is always feasible and that strategy (i) might also be feasible, we turn, now to showing what the expected cost of borrowing is under the three alternative strategies.

For x consistent with strategy (i)

$$\begin{aligned} C(\cdot) &= (1-p)RL + p(1-q)V^R(\cdot;1,S^D) + pqV^R(\cdot;\phi,S^D) \\ &= RL + p(1-q)\left[V^R(\cdot;1,S^D) - RL\right] + pq\left[V^R(\cdot;\phi,S^D) - RL\right] \end{aligned}$$

By definition, for strategy (i), $V^R(\cdot;\phi,S^D) < V^R(\cdot;1,S^D) < RL$. Hence strategy (i), when feasible, will dominate full hedging.

For x consistent with strategy (ii)

$$\begin{aligned} C(\cdot) &= (1-pq)RL + pqV^R(\cdot;\phi,S^D) \\ &= RL + pq\left[V^R(\cdot;\phi,S^D) - RL\right]. \end{aligned}$$

By definition, for strategy (ii), $V^R(\cdot;\phi,S^D) < RL$. Hence strategy (ii), when feasible, will dominate full hedging.

As before, for x consistent with strategy (iii)

$$C(\cdot) = RL + (1-p)\,C^B(\cdot;1,S^I),$$

so that strategy (iii) is dominated by full hedging.

Since we have shown that strategy (ii) and/or strategy (i) is feasible if full hedging is feasible, the bank will not fully hedge.