Monetary Policy and the Predictability of Nominal Exchange Rates

Martin Eichenbaum   Benjamin K. Johannsen*   Sergio Rebelo

November 2017

Abstract

This paper studies how the monetary policy regime affects the relative importance of nominal exchange rates and inflation rates in shaping the response of real exchange rates to shocks. We document two facts about countries with floating exchange rates where monetary policy controls inflation using a short-term interest rate. First, the current real exchange rate predicts future changes in the nominal exchange rate at horizons greater than two years both in sample and out of sample. Second, the real exchange rate is virtually uncorrelated with future inflation rates both in the short run and in the long run. These facts do not hold in countries under quasi-fixed exchange rate regimes, crawling-peg regimes or heavily managed floating exchange rate regimes. We construct an open-economy model that accounts quantitatively for the facts that we document.

*The views expressed here are those of the authors and do not necessarily reflect the views of the Board of Governors, the Federal Open Market Committee, or anyone else associated with the Federal Reserve System. We thank Adrien Auclert, Charles Engel, Gaetano Gaballo, Zvi Hercovitz, Oleg Itskhoki, Dmitry Mukhin, Paulo Rodrigues, Christopher Sims, and Oreste Tristani for their comments and Martin Bodenstein for helpful discussions.

1 Introduction

This paper studies how the monetary policy regime affects the relative importance of nominal exchange rates (NERs) and inflation rates in shaping the response of real exchange rates (RERs) to shocks. To describe our findings, we define the RER as the price of the foreign-consumption basket in units of the home-consumption basket and the NER as the price of the foreign currency in units of the home currency.

We begin by documenting two facts about real and nominal exchange rates for a set of benchmark countries. These countries have two characteristics in common over our sample period: they have flexible exchange rates and the central bank uses short-term interest rates to keep inflation near its target level. Our first fact, is that the current RER is highly negatively correlated with future changes in the NER at horizons greater than two years. This correlation is stronger the longer is the horizon. Our second fact, is that the RER is virtually uncorrelated with future inflation rates at all horizons.

Taken together, these facts imply that the RER adjusts in the medium and long runs overwhelmingly through changes in the NER, not through differential inflation rates. When a country's consumption basket is relatively expensive, its NER eventually depreciates by enough to move the RER back to its long-run level. These conclusions are consistent with those of Cheung, Lai, and Bergman (2004).[1]

Critically, we argue that these facts depend on the monetary policy regime in effect. To show this dependency, we re-do our analysis for China which is on a quasi-fixed exchange rate regime versus the U.S. dollar; for Hong Kong which has a fixed exchange rate versus the U.S. dollar; and for the euro-area countries, which have fixed exchange rates with each other. In all of these cases, the current RER is highly negatively correlated with future relative inflation rates. In contrast to the flexible exchange rate countries, the RER adjusts overwhelmingly through predictable inflation differentials.

Additional evidence on the importance of the monetary policy regime comes from a set of countries that had crawling pegs or heavily managed floating exchange rates and then moved to floating exchange rates and inflation targeting. This set of countries consists of Brazil, Chile, Colombia, Indonesia, Israel, Mexico, South Korea, and Thailand. We show that when these countries adopted floating exchange rates and inflation targeting, the dynamic co-movements of the NER, the RER, and inflation became qualitatively similar to those in our benchmark countries. This type of sensitivity to the monetary policy regime is precisely what we would expect given the Lucas (1976) critique.

Before discussing a class of models that accounts for our findings, we confront the concern that these findings are spurious in the sense that they primarily reflect small sample sizes and persistent RERs.[2] We address these concerns in two ways. First, we argue on statistical grounds that it is

[1] These authors use an alternative statistical methodology to study the behavior of the exchange rates for four European countries and Japan. Their sample spans the period between the collapse of the Bretton Woods system and the establishment of the euro.

[2] Similar concerns lie at the heart of ongoing debates about the predictability of the equity premium based on variables

very unlikely that our empirical findings could be produced by a data generating process (DGP) with a very persistent RER that is uncorrelated with future changes in the NER. Second, we show that *out-of-sample* forecasts of the NER based on the RER beat a random walk forecast at medium and long horizons. We argue that this finding is extremely unlikely if the NER is not predictable, regardless of whether the RER is stationary or not. Viewed overall, these results are strongly supportive of the view that our key empirical findings for the benchmark countries are not spurious.

Having established our key facts, we turn to the underlying economics. We show that there is a wide class of models consistent with the fact that, for our benchmark countries, the current RER predicts future movements in the NER. This consistency holds in models both with and without nominal rigidities. The key elements of these models are that monetary policy is governed by a Taylor rule and there is home bias in consumption.

We analyze versions of the same class of models in which the foreign central bank follows a managed float. We show that these models are consistent with the fact that, under a managed float, the RER is useful for predicting future movements in differential inflation rates. While the previous findings hold for all versions of the model that we consider, a dynamic stochastic general equilibrium (DSGE) model with nominal rigidities does the best job quantitatively.

We begin our theoretical analysis with a simple flexible-price model where labor is the only factor in the production of intermediate goods. The intuition for why this simple model accounts for our empirical findings about Taylor-rule regimes is as follows. Consider a persistent fall in domestic productivity or an increase in domestic government spending. Both shocks lead to a rise in the real cost of producing home goods that dissipates smoothly over time. Home bias means that domestically produced goods have a high weight in the domestic consumer basket. So, after the shock, the price of the foreign consumption basket in units of the home consumption basket falls, i.e. the RER falls. The Taylor rule followed by both central banks keeps inflation relatively stable in the two countries. As a consequence, most of the adjustment in the RER occurs through changes in the NER. In our model, the NER behaves in a way that is reminiscent of the overshooting phenomenon emphasized by Dornbusch (1976). After a technology shock, the foreign currency depreciates on impact and then slowly appreciates to a level consistent with the return of the RER to its steady-state value. The longer the horizon, the higher is the cumulative appreciation of the foreign currency. So in this simple model, the current RER is highly negatively correlated with the value of the NER at future horizons, and this correlation is stronger the longer is the horizon. These predictable movements in the NER can occur in equilibrium because they are offset by the interest rate differential, i.e., uncovered interest parity (UIP) holds.

An obvious shortcoming of the flexible-price model is that purchasing power parity (PPP) holds at every point in time. To remedy this shortcoming, we modify the model so that monopolist producers set the nominal prices of domestic and exported goods in the local currency where they are sold. They do so subject to Calvo-style pricing frictions. For simplicity, suppose for now that there is a complete set of domestic and international asset markets. Consider a persistent fall in

like the price–dividend ratio (see Stambaugh (1999); Boudoukh, Richardson and Whitelaw (2006); and Cochrane (2008)).

domestic productivity or an increase in domestic government spending. Both shocks lead to a rise in domestic marginal cost. The domestic firms that can reoptimize their prices increase them at home and abroad, so inflation rises. Because of home bias, domestic inflation rises by more than foreign inflation. The Taylor principle implies that the domestic real interest rate rises by more than the foreign real interest rate. So, domestic consumption falls by more than foreign consumption.

With complete asset markets, the RER is proportional to the ratio of foreign to domestic marginal utilities of consumption. So, the fall in the ratio of domestic to foreign consumption implies a fall in the RER. As in the flexible price model, the Taylor rule keeps inflation relatively low in both countries so that most of the adjustment in the RER is attributable to movements in the NER. Again, the implied predictable movements in the NER can occur in equilibrium because they are offset by the interest rate differential, i.e. UIP holds.

While the intuition is less straightforward, our results are not substantively affected if we replace complete markets with incomplete markets or assume producer-currency pricing instead of local-currency pricing. Risk premiums aside, UIP holds conditional on the realization of many types of shocks to the model economy. We introduce shocks to the demand for bonds, for which UIP does not hold. So, when the variance of these shocks is sufficiently large, traditional tests of UIP applied to data from our model would reject that hypothesis.

We show that an empirically-plausible medium-sized DSGE model can quantitatively account for the facts that we document. By "empirically plausible," we mean that the model is consistent with the persistence of $RERs$, the short-run failure of UIP and PPP, and the high correlation between the RER and the NER. Among other features, the model allows for Calvo-style nominal wage and price frictions and habit formation in consumption of the type considered in Christiano, Eichenbaum, and Evans (2005).

In addition, we show that our DSGE model can quantitatively account for the extent to which RER-based medium- and long-run forecasts of the NER outperform random-walk forecasts. Specifically, the model is consistent with the fact that the comparative advantage of RER-based forecasts increases with the forecasting horizon. In addition, the model accounts quantitatively for the average ratio of the root mean squared prediction error (RMSPE) of the RER-based and random walk-based forecasts at all horizons.

Our work is related to four important strands of literature. The first strand demonstrates the existence of long-run predictability in NERs (e.g. Mark (1995) and Engel, Mark, and West (2007)). Our contribution is to show that the ability of the RER to predict the NER at medium and long-run horizons depends critically on the monetary policy regime in effect. The second strand of the literature, which goes back to Meese and Rogoff (1983), studies the out-of-sample predictability of the NER. Authors like Engel and West (2004, 2005) and Molodtsova and Papell (2009) have proposed using variables that might enter into a Taylor rule to improve out-of-sample forecasting. Such variables includes output gaps, inflation, and possibly RERs. Rossi (2013) provides a thorough review of this literature. Recently, Cheung, Chinn, Pascual, and Zhang (2017) highlight the potential role of the RER in helping forecast the NER. Ca'Zorzi, Muck, and Rubaszek (2016) study the forecasting performance of the Justiniani and Preston (2010) DSGE model. Citing an earlier version of this paper, these authors note the potential usefulness of the RER in forecasting the NER. Our contribution relative to these two papers is to thoroughly document that role and show how it depends on the monetary policy regime.

The third strand of the literature seeks to explain the persistence of RERs. See, for example, Rogoff (1996); Kollmann (2001); Benigno (2004); Engel, Mark, and West (2007); and Steinsson (2008). Our contribution relative to that literature is to show that we can account for the relationship between the RER and future changes in inflation and the NER in a way that is consistent with the observed inertia in the RER.

The fourth strand of the literature emphasizes the importance of the monetary regime for the behavior of the RER. See, for example, Baxter and Stockman (1989); Henderson and McKibbin (1993); Engel, Mark, and West (2007); and Engel (2012). Our contribution relative to this literature is to document the importance of the monetary regime in determining the relative roles of inflation and the NER in the adjustment of the RER to its long-run levels.

Our paper is organized as follows. Section 2 contains our empirical results. Section 3 describes a sequence of models consistent with these results. We start with a model that has flexible prices and complete asset markets and where labor is the only factor in the production of intermediate goods. We then replace complete markets with a version of incomplete markets where only one-period bonds can be traded. Next, we introduce Calvo-style frictions in price setting. In Section 4, we consider an estimated medium-scale DSGE model. Section 5 concludes.

2 Some empirical properties of exchange rates

In this section, we present our empirical results regarding NERs, RERs, and relative inflation rates. We use consumer price indexes for all items and average quarterly NERs versus the U.S. dollar.

2.1 Data

We initially focus on a benchmark group of advanced economies—Australia, Canada, Norway, Sweden, and Switzerland—that had floating exchange rates in the period from 1973 to 2007.[3] In choosing the sample period, we face the following trade off. On the one hand, we would like as long a time series as possible. On the other hand, we would like the monetary regime to be reasonably stable in our sample. To balance these considerations, we exclude from our sample data from 2008 to the present because short-term nominal interest rates in the United States were at or near their effective lower bound. We include data since 2008 as a part of our robustness analysis. We exclude Japan from our set of benchmark countries because its short-term interest rates have been at or close to the effective lower bound since 1995. We exclude the United Kingdom, which left the European Exchange Rate Mechanism of the European Monetary System in 1992 after a large

[3]Unless indicated otherwise, a year means that the entire year's worth of data was used.

devaluation. We include data from both Japan and the United Kingdom in our robustness analysis, where we also consider countries that eventually adopted the euro.[4]

We compare results for the benchmark flexible exchange rate economies with those for China (from 1994 through 2007), which has been on a quasi-fixed exchange rate vis-à-vis the U.S. dollar, and for Hong Kong (from 1985 through 2007), which has a fixed exchange rate vis-à-vis the U.S. dollar. We also analyze data starting in 1999 for France, Ireland, Italy, Portugal, and Spain where the RER and relative inflation rates are defined relative to Germany. In addition, we consider a group of countries that had crawling pegs or heavily managed floating exchange rates and then moved to floating exchange rate regimes along with a form of inflation targeting. This set of countries consists of Brazil, Chile, Colombia, Indonesia, Israel, Mexico, South Korea, Thailand, and Turkey.

2.2 Results for flexible exchange rate countries

We define the RER for country i relative to the United States as

$$RER_{i,t} = \frac{NER_{i,t} P_{i,t}}{P_t}, \tag{1}$$

where $NER_{i,t}$ is the nominal exchange rate, defined as U.S. dollars per unit of foreign currency. The variables P_t and $P_{i,t}$ denote the consumer price index in the U.S. and in country i, respectively. We assume that the RER is stationary and offer supporting evidence later in this section. Given this assumption, the RER must adjust back to its mean after a shock via changes in the NER or changes in relative prices.

Figure 1 displays scatter plots for Canada of the $\log(RER_{i,t})$ against $\log(NER_{i,t+h}/NER_{i,t})$ at different horizons, h. The analogue figures for the other benchmark flexible exchange rate countries are displayed in the appendix. Two properties of this figure are worth noting. First, consistent with the notion that exchange rates behave like random walks at high frequencies, there is no obvious relationship between the $\log(RER_{i,t})$ and $\log(NER_{i,t+h}/NER_{i,t})$ at a one-year horizon. However, as the horizon expands, the correlation between $\log(RER_{i,t})$ and $\log(NER_{i,t+h}/NER_{i,t})$ rises. The negative relation is very pronounced at longer horizons. This pattern holds for all of the benchmark flexible exchange rate countries included in the appendix.

[4] For bilateral exchange rate data between the United States and other countries, we use the H.10 exchange rate data published by the Federal Reserve, available at http://www.federalreserve.gov/releases/H10/Hist. We compute quarterly averages of the daily data. When the H.10 data do not include a country, we use exchange rate data from the International Monetary Fund's International Financial Statistics database. For price indexes, we also use the International Monetary Fund's International Financial Statistics database. When consumer price indexes are not available from the International Financial Statistics database, we use data from the Organization for Economic Cooperation and Development (OECD), which were downloaded from FRED, a database maintained by the Federal Reserve Bank of St. Luois ("Main Economic Indicators - complete database," Main Economic Indicators (database)).

2.2.1 Nominal exchange rate regressions

We now discuss results based on the following NER regression

$$\log\left(\frac{NER_{i,t+h}}{NER_{i,t}}\right) = \alpha_{i,h}^{NER} + \beta_{i,h}^{NER}\log(RER_{i,t}) + \varepsilon_{i,t,t+h}^{NER}, \tag{2}$$

for country i at horizon $h = 1, 2, \ldots, H$ years. Panel (a) of Table 1 reports estimates of $\beta_{i,h}^{NER}$, along with standard errors, for the benchmark flexible exchange rate countries.[5] A number of features are worth noting. First, for every country and every horizon, the estimated value of $\beta_{i,h}^{NER}$ is negative. Second, for almost all countries, the estimated value of $\beta_{i,h}^{NER}$ is statistically significant at three-year horizons or longer. Third, in most cases, the estimated value of $\beta_{i,h}^{NER}$ increases in absolute value with the horizon, h. Moreover, $\beta_{i,h}^{NER}$ is more precisely estimated for longer horizons.

Panel (a) of Table 1 also reports the R^2s of the fitted regressions. Consistent with the visual impression from the scatter-plots, the R^2s are relatively low at short horizons but rise with the horizon. Strikingly, for the longest horizons, the R^2 exceeds 50 percent for all of our benchmark countries and is 88 percent for Canada.

Taken together, the results in Table 1 strongly support the conclusion that, for our benchmark countries, the current RER is strongly correlated with changes in future NERs, at horizons greater than roughly two years.

2.2.2 Relative price regressions

We now consider results based on the following relative-price regression

$$\log\left(\frac{P_{i,t+h}/P_{t+h}}{P_{i,t}/P_t}\right) = \alpha_{i,h}^{\pi} + \beta_{i,h}^{\pi}\log(RER_{i,t}) + \varepsilon_{i,t,t+h}^{\pi}. \tag{3}$$

This regression quantifies how much of the adjustment in the RER occurs via changes in relative rates of inflation across countries. Panel (a) of Table 2 reports our estimates and standard errors for the slope coefficient $\beta_{i,h}^{\pi}$. Most of the coefficient are positive but statistically insignificant. Panel (a) of Table 2 also reports the R^2s of the fitted regressions. These R^2s are all much lower than those associated with regression (2). These results as a whole suggest that very little of the adjustment in the RER occurs via differential inflation rates. This conclusion is consistent with the results of Cheung, Lai, and Bergman (2004) based on an earlier sample period for Japan and four European countries.

2.2.3 Robustness: Other countries

We now assess the robustness of the previous results by considering other advanced economies with flexible exchange rates—the euro area, Japan, and the United Kingdom. Because the samples for these countries are relatively short, we only estimate regressions (2) and (3) out to a five-year

[5] We compute standard errors using a Newey-West estimator with the number of lags equal to the forecasting horizon plus two quarters.

horizon.[6] Our results are reported in panel (b) of Table 1 and panel (b) of Table 2. The estimated regression coefficients are similar to those obtained for the benchmark countries. The appendix reports results for both these countries and the benchmark countries when we extend the sample to end in 2016:Q4. This change in sample period has little effect on our results.

2.3 Sensitivity to monetary policy

Our basic hypothesis is that the process by which the RER adjusts to shocks depends critically on the monetary policy regime. We provide two types of evidence in favor of this hypothesis. First, we redo our analysis for countries that are on fixed or quasi-fixed exchange regimes. Second, we consider countries that, initially, heavily managed their exchange rates but later allowed their exchange rates to float.

2.3.1 Fixed and quasi-fixed exchange rates

In this subsection, we report the results of redoing our analysis for countries with fixed or quasi-fixed exchange rates. Results for China and Hong Kong, which have quasi-fixed and fixed exchange rates, respectively, are reported in panel (c) of Table 1 and panel (c) of Table 2. Several features of these results are worth noting. First, the estimated values of $\beta_{i,h}^{NER}$ are small relative to the estimates for our benchmark countries. Second, values of $\beta_{i,h}^{\pi}$ are large relative to the estimates for our benchmark countries and statistically significant at every horizon. Third, the estimated value of $\beta_{i,h}^{\pi}$ rises with the horizon, h. Fourth, the R^2 values associated with regression (3), reported in panel (c) of Table 2, are large and increase with the horizon.

We also consider several euro-area countries—France, Ireland, Italy, Portugal, and Spain—vis-à-vis Germany. For these countries, the NER is fixed. Results for regression (3) are reported in Table 3. As was the case for China and Hong Kong vis-à-vis the United States, the estimated values of $\beta_{i,h}^{\pi}$ are large, rise in magnitude with the horizon, and are statistically significant at long horizons. In addition, the R^2 values are large and increase with the horizon, with regression (3) explaining 94 percent of relative price movements between Germany and Portugal at a five-year horizon.

In sum, for economies with fixed or quasi-fixed exchange rates, the RER adjusts overwhelmingly through predictable inflation differentials, not through changes in the NER.

2.3.2 Countries with changes in exchange rate policy

In this subsection, we redo our analysis for a set of countries—Brazil, Chile, Colombia, Indonesia, Israel, Mexico, South Korea, and Thailand—that had crawling pegs or heavily managed floating exchange rates and then adopted floating exchange rates.

[6] We begin the sample for the euro in 1999, when it was created. We start the sample for Japan in 1973 and end the sample in 1994, because Japan has had nominal interest rates near their effective lower bound since the mid-1990s. We begin the sample for the United Kingdom in 1993 because the United Kingdom exited the European Exchange Rate Mechanism of the European Monetary System after a large devaluation in 1992.

We consider two sample periods. The first sample is from 1984:Q1 to 2016:Q4 and covers periods in which all of the countries moved from a managed exchange rate to a floating exchange rate. The second sample spans the period from 1999:Q1 to 2016:Q4. We include the period in which the zero lower bound (ZLB) is binding in the United States and in some other countries in order to have enough observations to estimate our regressions at a five-year horizon. Our experience with the benchmark countries suggests that including the ZLB period has a mild effect on the coefficients and R^2s of regressions (2) and (3).

Tables (4) and (5) report our estimates of $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$, as well as the R^2s from the regressions. In contrast to our benchmark countries, for the sample starting in 1984, the estimates of $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$, and the R^2 values, do not follow the consistent pattern observed for the benchmark flexible exchange rate countries. In addition, the estimates display no apparent pattern across the countries considered.

Tables (4) and (5) also report results for the sample starting in 1999. Notice that for every country except Turkey and every horizon, the estimates of $\beta_{i,h}^{NER}$ are negative, grow in magnitude with the horizon, and are statistically different from zero at longer horizons. In addition (again with the exception of Turkey), the R^2 values for regression (2) using the sample starting in 1999 are much larger than the analogous R^2s from the full sample. By contrast, the estimates of $\beta_{i,h}^{\pi}$ are relatively small in the sample starting in 1999, as are the R^2 values for regression (3).

Clearly, the post-1999 sample produces results that are more similar to those obtained with our benchmark flexible exchange rate countries. We view these results as being supportive of our hypothesis that the monetary policy regime is a central determinant of the way that the RER adjusts to shocks.

3 Are the empirical correlations spurious?

In the previous section, we argue that for our benchmark countries, changes in the NER at long horizons display a strong negative correlation with the current level of the RER. A potential problem with this result is that if the RER is very persistent, we might statistically find in-sample predictability when none is actually present. Boudoukh et al. (2006) make this point in the context of the literature on equity returns predictability. In our context, their critique translates into the statement that the asymptotic standard errors for the regression coefficients reported in the previous section severely understate the importance of sampling uncertainty.

In this section, we address these concerns in two steps. Following the approach proposed by Boudoukh et al. (2006), we examine the small-sample properties of the Wald statistic for the test that the slope coefficients in regression (2), $\beta_{i,h}^{NER}$, are zero at all horizons. We construct bootstrap p-values under the null hypothesis that the RER is a stationary process. These statistics provide strong evidence against the hypothesis that $\beta_{i,h}^{NER}$ are all zero.

Analogue exercises conducted under the null hypothesis that the RER is a difference-stationary process turn out to have very low power, reflecting the diffuse nature of the small-sample distribution of the slope coefficients. Fortunately, in our case, tests based on *out-of-sample* forecasts of the NER

are much more powerful. We show that over medium- and long-run horizons, our forecasts of the NER outperform random-walk forecasts. As discussed later, this finding is unlikely to reflect sampling uncertainty regardless of whether or not the RER has a unit root.

3.1 Testing whether slope coefficients are zero

In this subsection, we test the joint null hypothesis that the slope coefficients in regression (2) are zero at all horizons up to 40 quarters (10 years), i.e.,

$$\beta_{i,1}^{NER} = \beta_{i,2}^{NER} = \cdots = \beta_{i,40}^{NER} = 0. \tag{4}$$

For each country i, we jointly estimate the slope coefficients $\beta_{i,h}^{NER}$ and compute the Wald statistic under the null hypothesis (4).[7] We focus attention on our benchmark flexible exchange rate countries so that we have enough data to include regressions with a horizon of 10 years.

Because the RER is highly persistent, we find in simulations that tests based on the asymptotic distribution of the Wald statistic have poor size. Accordingly, we test the null hypothesis (4) using the following bootstrap procedure. We assume that the stochastic processes for $NER_{i,t}$ and $RER_{i,t}$ are given by

$$\log\left(\frac{NER_{i,t}}{NER_{i,t-1}}\right) = \varepsilon_{i,t}^{NER}, \tag{5}$$

$$A_i(L)\log\left(RER_{i,t}\right) = \varepsilon_{i,t}^{RER}. \tag{6}$$

Here, $A_i(L)$ is a polynomial in the lag operator with roots inside the unit circle so that the RER is a stationary process. The random variables $\varepsilon_{i,t}^{NER}$ and $\varepsilon_{i,t}^{RER}$ are uncorrelated over time (though potentially correlated within a period). This DGP embeds the assumption that changes in the NER are unpredictable at all horizons.[8] We consider up to 10 lags in $A_i(L)$ and choose the lag length separately for each country using the Akaike information criterion (AIC).[9] Given the estimates of $A_i(L)$, we back out a time series for $\varepsilon_{i,t}^{RER}$ and $\varepsilon_{i,t}^{NER}$ from the observed data. We then jointly bootstrap $\varepsilon_{i,t}^{NER}$ and $\varepsilon_{i,t}^{RER}$ to compute 10,000 synthetic time series, each of length equal to our actual sample period.[10] For each synthetic time series, we estimate regression (2) for $h = 1, 2, \ldots, 40$ and compute the corresponding Wald statistics to produce a bootstrap distribution of that statistic.

Table 6 reports the fraction of the bootstrap Wald statistics that are larger than the corresponding Wald statistic that we computed in the data. With the exception of Norway, we can reject the null hypothesis (4) at the 5 percent significance level. For Norway, we can reject it at the 10 percent

[7] We compute standard errors using a Newey-West estimator with the number of lags equal to the forecasting horizon plus two quarters.

[8] Note that if $\log\left(NER_{i,t}/NER_{i,t-1}\right)$ has a non-zero mean, that property is reflected in the fitted shocks from which we construct the bootstrap samples.

[9] The AIC selected four lags for Australia, seven lags for Canada, eight lags for Norway, four lags for Sweden, and four lags for Switzerland.

[10] We use 100 periods of initial burn in for our bootstraps.

significance level. Based on these tests, we infer that the negative correlations between the RER and the future changes of the NER that we documented are unlikely to be spurious.

3.2 Out-of-sample forecasts

In this subsection, we use out-of-sample forecasting performance to test the null hypothesis that the NER is not predictable. In practice, quarterly consumer price indexes are available with a one-period lag. To avoid any look-ahead bias, we measure the RER for country i using lagged price indices

$$\overline{RER}_{i,t} \equiv \frac{NER_{i,t}P_{i,t-1}}{P_{t-1}}. \tag{7}$$

Our forecasting equation for the NER is

$$\log\left(\frac{NER_{i,t+h}}{NER_{i,t}}\right) = \alpha_{i,h}^{NER} + \beta_h^{NER}\log(\overline{RER}_{i,t}) + \varepsilon_{i,t,t+h}^{NER}. \tag{8}$$

Notice that the parameter β_h^{NER} is common across countries. This specification corresponds to a balanced panel with country-specific intercepts ($\alpha_{i,h}^{NER}$) and common slopes.[11] We set the training period for the regression to the horizon of the forecast, h, plus 40 quarters.

We assess our ability to forecast the NER relative to a forecast of no change. The latter is the benchmark in the literature and corresponds to the assumption that the NER is a random walk without drift. Define the RMSPE for country i associated with forecasts based on equation (8) as

$$\sigma_{i,B,h} = \left\{\frac{1}{T_{i,h}}\sum_{t=0}^{T_{i,h}}\left[f_{i,t,t+h} - \log\left(\frac{NER_{i,t+h}}{NER_{i,t}}\right)\right]^2\right\}^{1/2}. \tag{9}$$

Here, $T_{i,h}$ denotes the number of forecasts for $\log(NER_{i,t+h}/NER_{i,t})$ in our sample, and $f_{i,t,t+h}$ is the forecast of $\log(NER_{i,t+h}/NER_{i,t})$ based on equation (8). We denote by $\sigma_{i,RW,h}$ the corresponding RMSPE associated with the no-change forecast from a random walk model.

For each country i, we report the ratio of the RMSPE associated with the benchmark and random walk specifications, $\sigma_{i,B,h}/\sigma_{i,RW,h}$. We also compute a pooled RMSPE implied by our forecasting equation for all of the countries in our sample, defined as

$$\sigma_{B,h} = \left\{\frac{1}{\sum_i T_{i,h}}\sum_i\sum_{t=0}^{T_{i,h}}\left[f_{i,t,t+h} - \log\left(\frac{NER_{i,t+h}}{NER_{i,t}}\right)\right]^2\right\}^{1/2}. \tag{10}$$

We denote by $\sigma_{RW,h}$ the pooled RMSPE implied by the random walk forecast and report the ratio of the pooled RMSPEs, $\sigma_{B,h}/\sigma_{RW,h}$.

We initially limit the analysis to our benchmark countries. Panel (a) of Table (7) reports relative RMSPEs for each country and for the pooled sample. Forecasts based on equation (8) outperform

[11] In adopting this approach, we follow Mark and Sul (2001), Groen (2005), and Engel, Mark, and West (2007), who use panel error-correction models to improve the forecasting power of exchange rate models.

the random walk model at horizons greater than two years. Remarkably, at the four- and seven-year horizons, forecasting equation (8) outperforms the random walk by 23 percent and 45 percent, respectively.[12]

We now formally test the hypothesis that the relative RMSPEs reported in panel (a) of Table 7 were generated by a DGP in which the NER is a random walk. Under this hypothesis, changes in the NER should not be predictable. We test this hypothesis using a bootstrap procedure similar to the one described in the previous subsection. In particular, we assume that $NER_{i,t}$ and $\overline{RER}_{i,t}$ are generated by equations (5) and (6), where we replace $RER_{i,t}$ with $\overline{RER}_{i,t}$. The lag length of $A_i(L)$ is chosen using the AIC.[13] We construct $10,000$ synthetic time series, each of length equal to the size of our sample, by randomly selecting a sequence of estimated disturbances. We jointly sample the disturbances so as to preserve contemporaneous correlations across the NER and RER and across countries.[14] For each synthetic time series, we compute forecasts based on equation (8) and the random walk without drift. Using these forecasts, we compute RMSPEs for each country and for the pooled countries.

Panel (b) of Table 7 shows the percentage of bootstrap simulations in which the value of the relative RMSPE is less than or equal to the analogue number reported in panel (a) at different horizons. The column labeled "Years 3–7" reports the percentage of bootstrap simulations where the relative RMSPEs are lower than in the data for all yearly horizons 3 through 7.

Consider the first row that reports for all the countries pooled together. For the three- and four year-horizon-specific tests, we can reject the random walk hypothesis at a 5 percent significance level. At the five-, six-, and seven-year horizons, we can reject the random walk hypothesis at the 1 percent significance level.

For the joint test of yearly horizons 3–7, we can reject the random walk hypothesis at the 1 percent significance level. There is some variability in the results for different countries and horizons. But the joint-horizon test provides very strong evidence against the random walk hypothesis for all of our benchmark countries (see last column of panel (b)).

Panel (c) of Table 7 reports robustness results for $\sigma_{B,h}/\sigma_{RW,h}$. The first row repeats our benchmark results. The second row reports results for the case in which we use $\log(RER_{i,t})$ instead of $\log(\overline{RER}_{i,t})$ in forecasting equation (8). The results we obtain are very similar to the benchmark case. The third row reports the results of extending the sample period until the end of 2016. There is a mild overall deterioration in forecasting performance at long horizons. The fourth row reports results obtained by adding Japan to our benchmark specification with the sample ending in December 2016. There is a further mild deterioration in forecasting performance at long horizons.

The fifth row, reports results based on an unbalanced panel that includes the euro area starting in 1999:Q1, the United Kingdom starting in 1993:Q1, and Japan starting in 1973:Q1 and ending

[12]Additional recent evidence against random-walk-based forecasts for the NER comes from Cheung, Chinn, Pascual, and Zhang (2017). These authors examine the ability of a host of economic models to forecast NERs. They find that, relative to random walk forecasts, relative-purchasing-power-parity-based forecasts outperform other economic models.

[13]The AIC selected four lags for Australia, seven lags for Canada, eight lags for Norway, four lags for Sweden, and four lags for Switzerland.

[14]We again have a burn-in period of 100 quarters so that the initial values of $\log(RER_{i,t})$ are different across bootstrap samples.

in 1994:Q4.[15] The results are about the same as the benchmark results at short horizons and only somewhat worse at long horizons. Still, the model outperforms the random walk for all horizons. At the seven-year horizon, the RMSPE associated with our forecasting equation is 40 percent lower than that associated with a random walk.

The panel structure of our benchmark specification assumes that the slope coefficients are the same across all countries. A natural question is, how sensitive are our results to this assumption? The sixth row of Table 7, labeled "Country-by-country regressions," reports results obtained by estimating separate slope coefficients for each country. There is a slight deterioration in forecasting performance. But, even without imposing the panel structure, the model outperforms the random walk at long horizons (by 41 percent at the seven-year horizon).

To this point, we have maintained the assumption that the RER is stationary. To assess the robustness of our results, we redo the out-of-sample bootstrap exercises assuming that $\log(\overline{RER}_{i,t})$ is difference stationary. In particular, we assume that

$$B_i(L)(1-L)\log\left(\overline{RER}_{i,t}\right)=\varepsilon_{i,t}^{RER}. \tag{11}$$

Here, $B_i(L)$ is a polynomial in the lag operator with roots inside the unit circle. We maintain the assumption that changes in the NER are given by (5). As before, we choose the lag length by the AIC and compute the relative RMSPEs.[16] The implied p-values are reported in panel (d) of Table 7. Note that the results are very similar to those reported in panel (b) of that table. We conclude that our results are *not* sensitive to whether we assume that the RER has a unit root.[17]

In summary, the results reported in this section strongly support the view that changes in the NER are predictable at medium- and long-run horizons. By implication, it is highly statistically unlikely that the correlations documented in the previous section are spurious.

4 Interpreting our empirical results: Economic models

In this section, we use a sequence of economic models to interpret the empirical findings documented earlier. We begin with a flexible-price, two-country, complete-markets model, allowing for different specifications of monetary policy, a Taylor rule, an exogenous money growth rule, and a regime where one country seeks to dampen fluctuations in the NER.

Next, we consider a sticky-price model with an incomplete-markets setting in which the only assets traded internationally are bonds. It turns out that the complete- and incomplete-markets versions of our model have very similar implications.

Finally, we consider a medium-sized DSGE model with Calvo-style nominal price and wage rigidities in which producers set prices in local currencies. We allow for technology shocks in each

[15] The euro did not exist until 1999. The United Kingdom left the European Exchange Rate Mechanism in 1992 after Black Wednesday. Japan's short-term interest rate has been at or near the ZLB since 1995.

[16] The AIC selected one lag for Australia, three lags for Canada, one lag for Norway, three lags for Sweden, and one lag for Switzerland.

[17] The results are not sensitive to assuming that the RER is a random walk.

country and shocks to the demand for domestic bonds. The latter shocks imply that unconditional UIP does not hold in our model.

4.1 Flexible-price, complete-markets model

The model consists of two completely symmetric countries. We first describe the households' problems and then discuss the firms' problems.

4.1.1 Households

The domestic economy is populated by a representative household whose preferences are given by

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log\left(C_{t+j}\right) - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} + \mu \frac{\left(M_{t+j}/P_{t+j}\right)^{1-\sigma_M}}{1-\sigma_M} \right]. \tag{12}$$

Here, C_t denotes consumption, L_t hours worked, M_t end-of-period nominal money balances, P_t the price of consumption goods, and E_t the expectations operator conditional on time-t information. We assume that $0 < \beta < 1$, $\sigma_M > 1$, and χ and μ are positive scalars.

Households can trade in a complete set of domestic and international contingent claims. The domestic household's flow budget constraint is given by

$$B_{H,t} + NER_t B_{F,t} + P_t C_t + M_t = R_{t-1} B_{H,t-1} + NER_t R_{t-1}^* B_{F,t-1} + W_t L_t + T_t + M_{t-1}. \tag{13}$$

Here, $B_{H,t}$ and $B_{F,t}$ are nominal balances of home and foreign bonds; NER_t is the nominal exchange rate, defined as in our empirical section to be the price of the foreign currency unit (units of home currency per unit of foreign currency); R_t is the nominal interest rate on the home bond; R_t^* is the nominal interest rate on the foreign bond; W_t is the nominal wage rate; and T_t denotes nominal lump-sum profits and taxes. For notational ease, we have suppressed the household's purchases and payoffs of contingent claims. With complete markets, the presence of one-period nominal bonds is redundant since these bonds can be synthesized using state-contingent claims.

The first-order conditions with respect to labor supply, money balances, and consumption are

$$\chi L_t^{\phi} C_t = \frac{W_t}{P_t}, \tag{14}$$

$$\mu \left(\frac{M_t}{P_t}\right)^{-\sigma_M} = \left(\frac{R_t - 1}{R_t}\right) \frac{1}{C_t}, \tag{15}$$

$$1 = \beta R_t E_t \frac{C_t}{C_{t+1} \pi_{t+1}}, \tag{16}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the inflation rate. Equation (15) characterizes money demand by domestic agents. Since households derive utility only from their country's money, domestic agents do not hold foreign money balances.

We use stars to denote the prices and quantities in the foreign country. The preferences of the foreign household are given by

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log \left(C^*_{t+j} \right) - \frac{\chi}{1+\phi} \left(L^*_{t+j} \right)^{1+\phi} + \mu \frac{\left(M^*_{t+j}/P^*_{t+j} \right)^{1-\sigma_M}}{1-\sigma_M} \right]. \tag{17}$$

The foreign household's flow budget constraint is given by

$$B^*_{F,t} + NER_t^{-1} B^*_{H,t} + P^*_t C^*_t + M^*_t = R^*_{t-1} B_{F,t-1} + NER_t^{-1} R_{t-1} B^*_{H,t-1} + W^*_t L^*_t + T^*_t + M^*_{t-1}. \tag{18}$$

The first-order conditions for the foreign household with respect to labor supply, money balances, and consumption are

$$\chi \left(L^*_t \right)^{\phi} C^*_t = \frac{W^*_t}{P^*_t}, \tag{19}$$

$$\mu \left(\frac{M^*_t}{P^*_t} \right)^{-\sigma_M} = \left(\frac{R^*_t - 1}{R^*_t} \right) \frac{1}{C^*_t}, \tag{20}$$

$$1 = \beta R^*_t E_t \frac{C^*_t}{C^*_{t+1} \pi^*_{t+1}}. \tag{21}$$

As in our empirical section, we define the real exchange rate, RER_t, as the price of the foreign consumption good in units of the home consumption good

$$RER_t = \frac{NER_t P^*_t}{P_t}. \tag{22}$$

With this definition, an increase in RER_t corresponds to a rise in the relative price of the foreign good.

Complete markets and symmetry of initial conditions imply

$$\frac{C_t}{C^*_t} = RER_t. \tag{23}$$

Combining equations (21) and (23) we obtain

$$1 = \beta R^*_t E_t \frac{C_t}{C_{t+1} \pi_{t+1}} \frac{NER_{t+1}}{NER_t}. \tag{24}$$

Equations (16) and (23) imply

$$1 = \beta R_t E_t \frac{C^*_t}{C^*_{t+1} \pi^*_{t+1}} \frac{NER_t}{NER_{t+1}}. \tag{25}$$

4.1.2 Firms

The domestic final good, Y_t, is produced by combining domestic and foreign goods ($Y_{H,t}$ and $Y_{F,t}$, respectively) according to the technology

$$Y_t = \left[\omega^{1-\rho} \left(Y_{H,t} \right)^{\rho} + (1-\omega)^{1-\rho} \left(Y_{F,t} \right)^{\rho} \right]^{\frac{1}{\rho}}. \tag{26}$$

Here, $\omega > 0$ controls the importance of home bias in consumption. The parameter $\rho \leq 1$ controls the elasticity of substitution between home and foreign goods. Similarly, the foreign final good, Y_t^*, is produced by combining domestic and foreign goods ($Y_{H,t}^*$ and $Y_{F,t}^*$, respectively) according to the technology

$$Y_t^* = \left[\omega^{1-\rho}\left(Y_{F,t}^*\right)^{\rho} + (1-\omega)^{1-\rho}\left(Y_{H,t}^*\right)^{\rho}\right]^{\frac{1}{\rho}}. \tag{27}$$

The domestic goods used in the production of the domestic final good ($Y_{H,t}$) and in the production of the foreign final good ($Y_{H,t}^*$) are produced according to the technologies

$$Y_{H,t} = \left(\int_0^1 X_{H,t}(j)^{\frac{\nu-1}{\nu}}\,dj\right)^{\frac{\nu}{\nu-1}} \quad \text{and} \quad Y_{H,t}^* = \left(\int_0^1 X_{H,t}^*(j)^{\frac{\nu-1}{\nu}}\,dj\right)^{\frac{\nu}{\nu-1}}. \tag{28}$$

Here, $X_{H,t}(j)$ and $X_{H,t}^*(j)$ are domestic intermediate goods produced by monopolist j using the linear technology

$$X_{H,t}(j) + X_{H,t}^*(j) = A_t L_t(j). \tag{29}$$

The variable $L_t(j)$ denotes the quantity of labor employed by monopolist j, and A_t denotes the state of time-t technology, which evolves according to

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_{A,t}, \tag{30}$$

where $|\rho_A| < 1$. The parameter $\nu > 1$ controls the degree of substitutability between different intermediate inputs.

The foreign goods used in the production of the domestic final good ($Y_{F,t}$) and in the production of the foreign final good ($Y_{F,t}^*$) are produced according to the technologies

$$Y_{F,t} = \left(\int_0^1 X_{F,t}(j)^{\frac{\nu-1}{\nu}}\,dj\right)^{\frac{\nu}{\nu-1}} \quad \text{and} \quad Y_{F,t}^* = \left(\int_0^1 X_{F,t}^*(j)^{\frac{\nu-1}{\nu}}\,dj\right)^{\frac{\nu}{\nu-1}}. \tag{31}$$

Here, $X_{F,t}(j)$ and $X_{F,t}^*(j)$ are foreign intermediate goods produced by monopolist j using the linear technology

$$X_{F,t}(j) + X_{F,t}^*(j) = A_t^* L_t^*(j), \tag{32}$$

where $L_t^*(j)$ is the labor employed by monopolist j in the foreign country and A_t^* denotes the state of technology in the foreign country at time t, which evolves according to

$$\log(A_t^*) = \rho_A \log(A_{t-1}^*) + \varepsilon_{A,t}^*. \tag{33}$$

Monopolists in the home country choose $\tilde{P}_{H,t}(j)$ and $\tilde{P}_{H,t}^*(j)$ to maximize per-period profits given by

$$\left(\tilde{P}_{H,t}(j) - W_t/A_t\right) X_{H,t}(j) + \left(NER_t\tilde{P}_{H,t}^*(j) - W_t/A_t\right) X_{H,t}^*(j), \tag{34}$$

subject to the demand curves of final good producers

$$X_{H,t}(j) = \left(\frac{\tilde{P}_{H,t}(j)}{P_{H,t}}\right)^{-\nu} Y_{H,t} \qquad \text{and} \qquad X^*_{H,t}(j) = \left(\frac{\tilde{P}^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\nu} Y^*_{H,t}. \tag{35}$$

The aggregate price indexes for $X_{H,t}$ and $X^*_{H,t}$, denoted by $P_{H,t}$ and $P^*_{H,t}$, can be expressed as

$$P_{H,t} \equiv \left(\int_0^1 \left[\tilde{P}_{H,t}(j)\right]^{1-\nu} dj\right)^{\frac{1}{1-\nu}} \qquad \text{and} \qquad P^*_{H,t} \equiv \left(\int_0^1 \left[\tilde{P}^*_{H,t}(j)\right]^{1-\nu} dj\right)^{\frac{1}{1-\nu}}. \tag{36}$$

Monopolists in the foreign country choose $\tilde{P}_{F,t}(j)$ and $\tilde{P}^*_{F,t}(j)$ to maximize profits given by

$$\left(\tilde{P}^*_{F,t}(j) - W^*_t/A^*_t\right) X^*_{F,t}(j) + \left(NER_t^{-1}\tilde{P}_{F,t}(j) - W^*_t/A^*_t\right) X_{F,t}(j), \tag{37}$$

subject to the demand curves of final good producers

$$X_{F,t}(j) = \left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\nu} Y_{F,t} \qquad \text{and} \qquad X^*_{F,t}(j) = \left(\frac{\tilde{P}^*_{F,t}(j)}{P^*_{F,t}}\right)^{-\nu} Y^*_{F,t}. \tag{38}$$

Here, the aggregate price index for $X_{F,t}$ and $X^*_{F,t}$, denoted by $P_{F,t}$ and $P^*_{F,t}$, can be expressed as

$$P_{F,t} \equiv \left(\int_0^1 \left[\tilde{P}_{F,t}(j)\right]^{1-\nu} dj\right)^{\frac{1}{1-\nu}} \qquad \text{and} \qquad P^*_{F,t} \equiv \left(\int_0^1 \left[\tilde{P}^*_{F,t}(j)\right]^{1-\nu} dj\right)^{\frac{1}{1-\nu}}. \tag{39}$$

The first-order conditions for the monopolists imply

$$\tilde{P}_{H,t}(j) = NER_t\tilde{P}^*_{H,t}(j) = \frac{\nu}{\nu-1}\frac{W_t}{A_t}, \tag{40}$$

where $\tilde{P}_{H,t}(j)$ and $\tilde{P}^*_{H,t}(j)$ are prices that the home monopolist charges in the home and foreign markets, respectively. Similarly,

$$NER_t^{-1}\tilde{P}_{F,t}(j) = \tilde{P}^*_{F,t}(j) = \frac{\nu}{\nu-1}\frac{W^*_t}{A^*_t}. \tag{41}$$

Here, $\tilde{P}_{F,t}(j)$ and $\tilde{P}^*_{F,t}(j)$ are the prices that the foreign monopolist charges in the home and foreign markets, respectively. Equations (40) and (41) imply that the law of one price holds for intermediate goods.

4.1.3 Monetary policy, market clearing, and the aggregate resource constraint

In our first specification of monetary policy, the domestic monetary authority sets the nominal interest rate according to the following Taylor rule

$$R_t = (R_{t-1})^{\gamma}\left(R\pi_t^{\theta_\pi}\right)^{1-\gamma}\exp\left(\varepsilon_{R,t}\right). \tag{42}$$

We assume that the Taylor principle holds, so that $\theta_\pi > 1$. In addition, $R = \beta^{-1}$, and ε_t^R is an independently and identically distributed (iid) shock to monetary policy. To simplify, we assume that the inflation target is zero in both countries. The foreign monetary authority follows a similar rule

$$R_t^* = \left(R_{t-1}^*\right)^\gamma \left(R(\pi_t^*)^{\theta_\pi}\right)^{1-\gamma} \exp\left(\varepsilon_{R,t}^*\right). \tag{43}$$

We abstract from the output gap in the Taylor rule to ease the comparison between the flexible-price version of the model, which has a zero output gap, and the sticky-price version of the model. In practice, the output gap coefficients in estimated versions of the Taylor rule are quite small (see, e.g. Clarida, Gali, and Gertler (1998)). Modifying the Taylor rule to include empirically plausible responses to the output gap has a negligible effect on our results.[18]

In our second specification of monetary policy, the domestic monetary authority sets the growth rate of the money supply according to

$$\log\left(\frac{M_t}{M_{t-1}}\right) = x_t^M, \qquad \text{where} \qquad x_t^M = \rho_{X_M} x_{t-1}^M + \varepsilon_t^M. \tag{44}$$

Here, $|\rho_{X_M}| < 1$ and ε_t^M is an iid shock to monetary policy. For convenience, we assume that the unconditional mean growth rate of nominal money balances is zero. The foreign monetary authority follows a similar rule so that

$$\log\left(\frac{M_t^*}{M_{t-1}^*}\right) = x_t^{M*}, \qquad \text{where} \qquad x_t^{M*} = \rho_{X_M} x_{t-1}^{M*} + \varepsilon_t^{M*}. \tag{45}$$

In our third specification of monetary policy, the domestic monetary authority sets the nominal interest rate as in (42), but the foreign monetary authority uses an augmented Taylor rule that includes a term that targets the NER.

$$R_t^* = R\left(NER_t^{-\theta_{NER}}\right) \exp\left(\varepsilon_{R,t}^*\right). \tag{46}$$

We assume that the Taylor principle holds so that $\theta_\pi > 1$, and that $\theta_{NER} > 0$ so that the nominal interest rate in the foreign country rises whenever there is a depreciation of the foreign currency.

We refer to the three specifications of monetary policy as the Taylor rule, the exogenous money growth rule, and the exchange rate targeting rule, respectively.

We assume that government purchases, G_t, evolve according to

$$\log\left(\frac{G_t}{G}\right) = \rho_G \log\left(\frac{G_{t-1}}{G}\right) + \varepsilon_t^G, \tag{47}$$

and that, without loss of generality, the government budget is balanced each period using lump-sum taxes. Here, $|\rho_G| < 1$ and ε_t^G is an iid shock to government purchases. The composition of government expenditures in terms of domestic and foreign intermediate goods ($Y_{H,t}$ and $Y_{F,t}$) is the

[18] Suppose we define the output gap as the percentage deviation of output from its steady-state value and include it in the Taylor rule with a coefficient equal to 0.5. The resulting impulse functions are very similar to those obtained for a version of the model with a Taylor rule that excludes the output gap.

same as that of the domestic household's final consumption good.

Similarly, government purchases in the foreign country, G_t^*, evolve according to

$$\log\left(\frac{G_t^*}{G}\right) = \rho_G \log\left(\frac{G_{t-1}^*}{G}\right) + \varepsilon_t^{G*}, \tag{48}$$

where ε_t^{G*} is an iid shock to government purchases and the government budget is balanced each period using lump-sum taxes. The composition of government expenditures in terms of domestic and foreign intermediate goods ($Y_{F,t}^*$ and $Y_{H,t}^*$) is the same as that of the foreign household's final consumption good.

Since bonds are in zero net supply, bond-market clearing implies

$$B_{H,t} + B_{H,t}^* = 0 \qquad \text{and} \qquad B_{F,t} + B_{F,t}^* = 0. \tag{49}$$

Labor market clearing requires that

$$L_t = \int_0^1 L_t(j)\, dj \qquad \text{and} \qquad L_t^* = \int_0^1 L_t^*(j)\, dj. \tag{50}$$

Market clearing in the intermediate input markets requires that

$$X_{H,t}(j) + X_{H,t}^*(j) = A_t L_t \qquad \text{and} \qquad X_{F,t}(j) + X_{F,t}^*(j) = A_t^* L_t^*. \tag{51}$$

Finally, the aggregate resource constraints are given by

$$Y_t = C_t + G_t \qquad \text{and} \qquad Y_t^* = C_t^* + G_t^*. \tag{52}$$

4.1.4 Impulse response functions

In the examples in this section we use the following parameter values. We assume a Frisch elasticity of labor supply equal to 1 ($\phi = 1$) and, as in Christiano, Eichenbaum, and Evans (2005), set $\sigma_M = 10.62$. We set the value of β so that the steady-state real interest rate is 3 percent. As in Backus, Kehoe, and Kydland (1992), we assume that the elasticity of substitution between domestic and foreign goods in the consumption aggregator is 1.5 ($\rho = 1/3$) and the import share is 10 percent ($\omega = 0.9$) so that there is home bias in consumption. We set $\nu = 6$, which implies an average markup of 20 percent. This value falls well within the range considered by Altig et al. (2011). We normalize the value of χ, which affects the marginal disutility of labor, so that hours worked in the steady state is equal to 1. We assume that monetary policy is given by the Taylor rules (42) and (43). We set θ_π to 1.5 so as to satisfy the Taylor principle. For ease of exposition, in this section, we set $\gamma = 0$ so that there is no interest rate smoothing. We set $\rho_A = 0.946$, a value that is very similar to those used in the literature (e.g., Hansen (1985)). In section 5.1.2, we discuss the estimation procedure underlying our choice of this value. We solve the model by log-linearizing the equilibrium conditions.

Figure 2 displays the impulse response to a negative technology shock in the home country.

Home bias in consumption plays a critical role in the impulse response function. First, the RER falls since home goods are more costly to produce and the home consumption basket places a higher weight on these goods. Second, domestic consumption falls by more than foreign consumption because domestic agents consume more of the home good whose relative cost of production has risen. Third, the households' Euler equations imply that the domestic real interest rate must rise by more than the foreign real interest rate. The Taylor rule and the Taylor principle imply that high real interest rates are associated with high nominal interest rates and high inflation rates. It follows that the domestic nominal interest rate and the domestic inflation rate rise by more than their foreign counterparts. This result is inconsistent with the naive intuition that inflation has to be lower in the home country in order for the RER to return to its pre-shock level. In fact, inflation is persistently *higher* in the home country. So the RER reverts to its steady-state value via changes in the NER. The NER has to change by enough to offset both the initial movement in the RER and the difference between the domestic and foreign inflation rates.

From Figure 2, we see that the Taylor rule keeps prices relatively stable and the RER falls at time zero via an appreciation of the home currency. To understand the last result, it is useful to consider the model's log-linearized equilibrium conditions. These conditions imply that the response of the RER to a technology shock is given by

$$\widehat{RER}_t = \kappa \hat{A}_t. \tag{53}$$

Here, $\hat{x}_t$ denotes the log deviation of x from its steady state level and

$$\kappa = \left\{ \left[\frac{2\omega}{\rho - 1} \frac{2(\omega - 1)}{2\omega - 1} + \frac{2\omega - 1}{\sigma} \right] \frac{C}{Y} \phi + \frac{1}{2\omega - 1} \right\}^{-1} (\phi + 1). \tag{54}$$

Equation (53) implies that the RER inherits the first-order autoregressive (AR(1)) nature of the technology shock so that

$$E_t \widehat{RER}_{t+1} = \rho_A \widehat{RER}_t. \tag{55}$$

Combining the linearized home- and foreign-country intertemporal Euler equations (16) and (21), the relation between the two countries' marginal utilities implied by complete markets (23), and the Taylor rules for the two countries (42) and (43), we obtain

$$\hat{\pi}_t - \hat{\pi}_t^* = -\frac{1 - \rho_A}{\theta_\pi - \rho_A} \widehat{RER}_t. \tag{56}$$

Since the Taylor principle holds ($\theta_\pi > 1$), we have $\left| \frac{1-\rho_A}{\theta_\pi - \rho_A} \right| < 1$. Given that the RER is equal to $NER_t P_t^*/P_t$, equation (56) implies that, on impact, the RER_t falls by more than P_t^*/P_t. It follows that NER_t must initially fall, i.e., the home currency *appreciates* on impact.

Recall that in response to the technology shock, both the real and the nominal interest rates rise more at home than abroad. The technology shock is persistent, so there is a persistent gap between the domestic and foreign nominal interest rates. Since UIP holds in the log-linear equilibrium, the domestic currency must depreciate over time to compensate for the nominal interest rate gap. So the home currency appreciates on impact and then depreciates. This pattern is reminiscent of the overshooting phenomenon emphasized by Dornbusch (1976).[19] Domestic inflation is persistently higher than foreign inflation, so the domestic price level rises by more than the foreign price level. This result, along with the law of one price for intermediate goods, implies that the home currency depreciates over time to an asymptotically lower value (the figure displays the price of the foreign currency, which is rising to a higher value).[20]

As the previous discussion makes clear, home bias plays a critical role in our results. Absent that bias, the consumption basket would be the same in both countries, and the RER would be equal to 1 in each period after the technology shock. Equation (56) implies that if the RER is constant, so, too, is the relative inflation and the NER.

4.1.5 Implied regression coefficients

We now assess the model's ability to account for the basic regressions that motivate our analysis (equations (2) and (3)). In a version of our model driven only by shocks to A_t and A_t^*, the probability limits (plims) of the regression coefficients, $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$, are given by

$$\beta_{i,h}^{NER} = -\frac{1-\rho_A^h}{1-\rho_A/\theta_\pi}, \tag{57}$$

and

$$\beta_{i,h}^{\pi} = \frac{1-\rho_A^h}{\theta_\pi/\rho_A - 1}. \tag{58}$$

Equation (57) implies that $\beta_{i,h}^{NER}$ is negative for all h and increases in absolute value with h. The intuition for these results is as follows. In the model, a low current value of the RER predicts a future appreciation of the foreign currency, so the slope of the regression is negative. The slope increases in absolute value with the horizon because the cumulative appreciation of the foreign currency increases over time.

Notice that the more aggressive is monetary policy (i.e., the larger is θ_π), the smaller is the absolute value of $\beta_{i,h}^{NER}$. The intuition for this result is as follows. After a domestic technology shock, π_t is higher than π_t^*. Equation (55) implies that the RER must revert to its steady-state level at a rate ρ_A. The higher is θ_π, the lower is $|\pi_t - \pi_t^*|$, and the less the domestic currency needs to depreciate to bring about the required adjustment in the RER. So the absolute value of $\beta_{i,h}^{NER}$ is decreasing in θ_π. Equation (58) implies that $\beta_{i,h}^{\pi}$ is positive for all h and converges to $\rho_A/\left(\theta_\pi - \rho_A\right)$. Consistent with the previous intuition, the higher is θ_π, the lower is $\beta_{i,h}^{\pi}$ for all h.

[19] In Dornbusch (1976), an unanticipated permanent change in the money supply causes the NER to overshoot relative to its new long-run level.

[20] In this version of the model, temporary technology shocks generate permanent changes in the NER. This property does not generally hold in versions of the model where both countries adopt Taylor rules of the form (46). In our numerical experiments, we find that when we place a small weight on the exchange rate (θ_{NER}), the NER becomes stationary. At the same time, the model's quantitative properties as summarized by the implied values of $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$ are virtually unchanged.

The sum of the two slopes is given by

$$\beta_{i,h}^{NER} + \beta_{i,h}^{\pi} = -(1 - \rho_A^h). \tag{59}$$

This sum converges to negative 1 as h goes to infinity, reflecting the fact that the RER must eventually converge to its pre-shock steady-state level through changes in relative prices or changes in the NER.

Figure 4 displays the small-sample average estimates of $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$. These statistics are calculated as follows. We simulate 10,000 synthetic time series using the model with only technology shocks as the DGP. Each synthetic time series is of length equal to our sample size. For each sample, we estimate $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$. We then compute the average values across the different samples. Consistent with our analytic expressions for the plims of these regressions, the absolute value of each coefficient grows with the horizon.[21]

The ability of the model to rationalize the regression coefficients does not depend on technology shocks per se. Consider a government spending shock that is mean reverting. An increase in government spending causes domestic consumption to fall by more than foreign consumption because the government consumes the domestic consumption bundle. As a result, the RER falls (see equation (23)). Because the shock is mean reverting, domestic consumption and foreign consumption rise over time, but by more in the home country. As a result, real interest rates are higher at home than abroad. Under a Taylor rule, relatively high real interest rates accompany relatively high inflation rates, which, all else being equal, would cause the RER to decline further. As a result, the RER converges to its pre-shock level through the NER. The exact magnitude of the response of inflation and the NER depends on other model features, such as sticky prices and wages.

4.1.6 Economy with money growth rule

We now show that, when monetary policy follows a money growth rule (equation (44)), the flexible-price model is much less successful at accounting for our regression results.

The impulse response functions to a technology shock are displayed in Figure 3. The following features are worth noting. First, prices in both countries move by much more than they did under the Taylor rule. So the movements in the NER required to validate the given equilibrium path of the RER are much smaller than under a Taylor rule. Second, since the growth rate of money does not increase after the shock, the price level eventually reverts to its pre-shock steady-state level. As a result, the NER also reverts to its steady state. Third, not all of the adjustment in the RER occurs via the price level, so there are still predictable movements in the NER. But these movements are much smaller than under a Taylor rule. This property is reflected in the model-implied plim of the small-sample regression slopes for our NER regression. These slopes are much smaller than under a Taylor rule (see Figure 4). The reason that movements in the NER are smaller than under a Taylor rule is that relative inflation rates help move the RER back to the

[21] In practice, we do not find a large difference between the plims of $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$ and the small-sample average estimates.

steady state. Under a Taylor rule, prices move the RER away from the steady state.

4.1.7 Economy with NER targeting rule

We now discuss the response of the economy to a productivity shock when monetary policy is given by equation (46) in the foreign country and by equation (42) in the domestic country. We set $\theta_{NER} = 1$, which reduces the volatility of the per-period change in the NER by 75 percent relative to our benchmark calibration. This setting corresponds to a situation in which foreign monetary policy places a high priority on exchange rate stabilization.

Figure 5 shows the response of this economy to a technology shock. Since prices are flexible, the behavior of the real variables is the same as when both countries follow a Taylor rule. The NER is much more stable in this version of the model. Since UIP holds, this stability requires the nominal interest rates to be similar in the two countries.

After the shock, consumption growth is higher in the domestic economy. As a consequence, the domestic real interest rate exceeds the foreign real interest rate. Since the Fisher equation holds, to make the two nominal interest rates similar, foreign monetary policy must ensure that foreign expected inflation is higher than domestic inflation. As a result, P_t^* rises faster than P_t, so the behavior of prices drives the RER back to its steady state. This property is reflected in the regression coefficients displayed in panel (b) of Figure 4. We see that the bulk of the adjustment of the RER toward the steady state is accomplished by prices, not by the NER. This pattern stands in sharp constrast with the behavior of the economy under a Taylor-rule regime.

4.2 Sticky-price, incomplete-markets model

As it turns out, the implications of the flexible price model discussed previously and those of the sticky price model discussed in this section are very similar when markets are complete or when the only assets that can be traded internationally are one-period nominal bonds. The basic structure is as in the previous subsection, with addition features similar to models used in Kollmann (2001) and in Gali and Monacelli (2005).

4.2.1 Incomplete asset markets

It is well known that with incomplete asset markets, the equilibrium process for the RER in models like ours has a unit root. To avoid this implication, authors like Schmitt-Grohe and Uribe (2003) assume that there is a small quadratic cost to holding bonds. We make a similar assumption in our model. The domestic household's budget constraint is given by

$$\begin{aligned} B_{H,t} + NER_t B_{F,t} + P_t C_t + M_t + \frac{\phi_B}{2}\left(\frac{NER_t B_{F,t}}{P_t}\right)^2 P_t &= \\ R_{t-1}B_{H,t-1} + NER_t R^*_{t-1} B_{F,t-1} + W_t L_t &+ T_t + M_{t-1}. \end{aligned} \tag{60}$$

We assume that the quadratic cost of holding bonds applies to bonds from the other country. The foreign household's budget constraint is given by

$$\begin{aligned} B^*_{F,t} + NER_t^{-1} B^*_{H,t} + P^*_t C^*_t + M^*_t + \frac{\phi_B}{2} \left(\frac{NER_t^{-1} B^*_{H,t}}{P^*_t} \right)^2 P^*_t \quad &= \\ R^*_{t-1} B^*_{F,t-1} + NER_t^{-1} R_{t-1} B^*_{H,t-1} + W^*_t L^*_t \quad &+ \quad T^*_t + M^*_{t-1}. \end{aligned} \tag{61}$$

Equation (23) no longer holds, and the home household's budget constraint is used to close the model.

4.2.2 Sticky prices

Monopolist producers set nominal prices in currency units that are local to where the good is sold. The technology for producing final goods is still given by equation (26). Intermediate-goods-producing firms set prices according to a variant of the mechanism spelled out in Calvo (1983). In each period, a firm faces a constant probability, $1 - \xi$, of being able to reoptimize its nominal price. The ability to reoptimize prices is independent across firms and time. Domestic intermediate goods firms choose $\tilde{P}_{H,t}(i)$ and $\tilde{P}^*_{H,t}(i)$ to maximize the objective function

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left\{ \left(\frac{\tilde{P}_{H,t}(i)}{P_{t+j}} - MC_{t+j} \right) X_{H,t+j}(i) + \left(NER_{t+j} \frac{\tilde{P}^*_{H,t}(i)}{P_{t+j}} - MC_{t+j} \right) X^*_{H,t+j}(i) \right\}, \tag{62}$$

subject to the demand equations (35). Here, MC_{t+j} denotes the real marginal cost in period $t+j$, and $\beta^j \Lambda_{t+j}$ is the utility value of profits in period $t+j$ to the household in period t.

Foreign intermediate goods firms choose $\tilde{P}_{H,t}(i)$ and $\tilde{P}^*_{H,t}(i)$ to maximize the objective function

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda^*_{t+j} \left\{ \left(\frac{\tilde{P}^*_{F,t}(i)}{P^*_{t+j}} - MC^*_{t+j} \right) X^*_{F,t+j}(i) + \left(NER^{-1}_{t+j} \frac{\tilde{P}_{F,t}(i)}{P^*_{t+j}} - MC^*_{t+j} \right) X_{F,t+j}(i) \right\}, \tag{63}$$

subject to equations (38). In all other respects, the model is the same as in the previous subsection.

4.2.3 Impulse response to a technology shock

Figure 6 displays the response of the economy to a negative technology shock in the home country. These effects are similar to those in the flexible-price model. The key difference is that in the sticky-price model, the response of $\pi_{H,t}$, $\pi_{F,t}$, $\pi^*_{H,t}$, $\pi^*_{F,t}$ is attenuated relative to the flexible-price model. Interestingly, the effect of sticky prices on overall inflation is ambiguous. When prices are flexible, producers of the foreign good initially reduce the price they charge in the home market. This effect helps reduce the domestic rate of inflation in the flexible-price model. With sticky prices, this effect is attenuated relative to the flexible-price model. So, depending on parameter values, domestic inflation can be higher or lower in the sticky price model than in the flexible-price model.

Because the negative technology shock leads to a decline in RER_t followed by a persistent depreciation of the home currency, the model-implied values for $\beta_{i,h}^{NER}$ in the economy with only technology shocks are negative and grow in absolute value with the horizon. As in the flexible-price model, the basic intuition is that a negative technology shock drives down the RER. Over time, the NER rises to its new steady-state value. So a low value of the RER is associated with future increases in the NER.

4.3 Medium-scale model with nominal rigidities

In this subsection, we investigate whether an empirically plausible version of our model can account for the facts that we document. By "empirically plausible," we mean that the model is consistent with the persistence of $RERs$, the short-run failure of UIP and PPP, and the high correlation between the RER and the NER. In addition, the model should be able to account for our out-of-sample NER forecasting results.

4.3.1 Monopolists

The production of domestic and foreign final goods (Y_t and Y_t^*) and domestic ($Y_{H,t}$ and $Y_{H,t}^*$) and foreign ($Y_{F,t}$ and $Y_{F,t}^*$) intermediate goods is the same as in the benchmark models. The final good is used for consumption (C_t and C_t^*), investment (I_t and I_t^*), and government purchases (G_t and G_t^*) so that

$$Y_t = C_t + I_t + G_t \qquad \text{and} \qquad Y_t^* = C_t^* + I_t^* + G_t^*. \tag{64}$$

Differentiated intermediate goods supplied by monopolist i ($X_{H,t}(i)$ and $X_{H,t}^*(i)$) are produced with capital ($K_t(i)$) and labor ($L_t(i)$)

$$X_{H,t}(i) + X_{H,t}^*(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha}. \tag{65}$$

The variable A_t denotes the time-t level of technology, which again evolves according to equation (30). Marginal cost in the home country is given by

$$MC_t = \frac{(R_{K,t})^{\alpha} (W_t/P_t)^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha} A_t}. \tag{66}$$

As before, domestic monopolist i sets prices in *local* currency subject to Calvo-style frictions. The monopolist maximizes the objective function given by equation (62) subject to the demand curves for its goods. To conserve on space, we do not describe the problem of the foreign monopolist i. That problem is entirely symmetric to that of the domestic monopolist i.

4.3.2 Households

Each household has a continuum of members indexed $j \in (0,1)$. Each member of the household belongs to a union that monopolistically supplies labor of type j. The union sets the wage $W_{j,t}$ subject to constraint (68) and Calvo-style wage frictions, modeled as in Erceg, Henderson, and

Levin (2000). The wage for labor of type j remains constant with probability ξ_w and is updated with probability $1-\xi_w$.

Intermediate producers purchase a homogeneous labor input from a representative labor contractor. The latter produces the homogeneous labor input by combining differentiated labor inputs, $l_{j,t}$, $j \in (0,1)$, using the technology

$$L_t = \left[\int_0^1 l_{j,t}^{\frac{\nu_L - 1}{\nu_L}} dj \right]^{\frac{\nu_L}{\nu_L - 1}}. \tag{67}$$

Let W_t denote the cost of hiring a unit of L_t. Labor contractors are perfectly competitive and take the nominal wage rates, W_t, and $W_{j,t}$, $j \in (0,1)$, as given. Profit maximization on the part of contractors implies

$$l_{j,t} = \left[\frac{W_{j,t}}{W_t} \right]^{-\nu_L} L_t. \tag{68}$$

Perfect competition and equation (67) imply

$$W_t = \left[\int_0^1 {W_{j,t}}^{1-\nu_L} dj \right]^{\frac{1}{1-\nu_L}}. \tag{69}$$

The preferences of the household are given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\log\left(C_{t+i} - h\bar{C}_{t+i-1}\right) - \frac{\chi}{1+\phi} \int_0^1 L_{j,t+i}^{1+\phi} dj + \mu \frac{\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-\sigma_M}}{1-\sigma_M} + \log(\eta_{t+i}) V\left(\frac{B_{H,t+i}}{P_t}\right) \right]. \tag{70}$$

Here, C_t is consumption, $\bar{C}_t$ is average aggregate consumption, and L_t is hours worked. The scalar h denotes the degree of habit formation in preferences.

Recall that the only assets that can be traded internationally are one-period nominal bonds. As in McCallum (1994), we allow for shocks that break UIP in log-linearized versions of the model. Instead of introducing a shock directly into the UIP condition, we assume that households derive utility from domestic bond holdings and that this utility flow varies over time.

The function V that governs the utility flow from the stock of domestic bonds is increasing, is strictly concave, and has both positive and negative support. For convenience, we assume that η_t is 1 in the steady state, so the flow utility from bonds is zero in the steady state. Outside of the steady state, there may be shocks that put a premium on one bond or the other—those arising from flights to safety or liquidity, for example.

The household budget constraint is

$$\begin{aligned} B_{H,t} + NER_t B_{F,t} + P_t C_t + P_{I,t} I_t \quad = \quad & R_{t-1} B_{H,t-1} + NER_t R^*_{t-1} B_{F,t-1} + P_t R_{K,t} K_t \\ & - \frac{\phi_B}{2} \left(\frac{NER_t B_{F,t}}{P_t} \right)^2 P_t + \int_0^1 W_{j,t} L_{j,t} dj + T_t, \end{aligned} \tag{71}$$

where $R_{K,t}$ is the rental rate on capital, K_t is the stock of capital, and I_t is investment. For notational ease, we have suppressed the household's purchases and payoffs of domestic contingent claims. The capital accumulation equation is

$$K_{t+1} = I_t \left[1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] + (1-\delta) K_t. \tag{72}$$

The sequence of events within each period is as follows. First, the technology shocks and spread shocks are realized. Second, the household makes its consumption and asset decisions. Third, wage rates are updated.

The problem of the foreign household is entirely symmetric with one exception. We assume that foreign households derive utility from their holdings of the domestic country's bonds, which we conceptualize as bonds that are internationally desired for special reasons.

4.3.3 Breaking UIP

In a log-linearized version of the model without shocks to the utility flow from real bond holdings, UIP holds. To show this result, consider the log-linearized first-order conditions of the home household with respect to $B_{H,t}$ and $B_{F,t}$

$$\hat{C}_t = CV'(0)\,\hat{\eta}_t + \hat{R}_t + E_t\left(-\hat{C}_{t+1} - \hat{\pi}_{t+1}\right), \tag{73}$$

$$\hat{C}_t + \phi_B b_{F,t} = \hat{R}^*_t + E_t\left(-\hat{C}_{t+1} - \hat{\pi}_{t+1} + \Delta\widehat{NER}_{t+1}\right). \tag{74}$$

Here, $\Delta\widehat{NER}_{t+1} \equiv \log(NER_{t+1}/NER_t)$, and C is the steady-state level of C_t. It is convenient to normalize $V'(0)$ to be equal to $1/C$. Combining equations (73) and (74) and ignoring the small term associated with ϕ_B, we obtain

$$\hat{R}_t - \hat{R}^*_t = E_t\left(\Delta\widehat{NER}_{t+1}\right) - \hat{\eta}_t. \tag{75}$$

This equation is identical to the reduced-form equation assumed by McCallum (1994).[22]

Absent the spread shocks $\hat{\eta}_t$, equation (75) corresponds to the classic UIP condition

$$\hat{R}_t - \hat{R}^*_t = E_t\left(\Delta\widehat{NER}_{t+1}\right). \tag{76}$$

All of the other shocks in our model induce movements in nominal interest rates and exchange rates that are consistent with equation (76). Conditional on these shocks occurring, UIP holds. However, UIP does not hold unconditionally in the presence of spread shocks, and traditional tests would reject the hypothesis of UIP. For example, the classic Fama (1984) test involves running the regression

$$\Delta\widehat{NER}_{t+1} = \alpha_0 + \alpha_1\left(\hat{R}_t - \hat{R}^*_t\right) + \varepsilon_t, \tag{77}$$

and testing the null hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 1$. Our model implies that this null hypothesis

[22] If we do not ignore ϕ_B, equation (75) is replaced by $\hat{R}_t - \hat{R}^*_t = E_t\left(\Delta\widehat{NER}_{t+1}\right) - \hat{\eta}_t - \phi_B b_{F,t}$.

should be rejected because of a negative covariance between the error term and the interest rate differential. To see this result, consider a positive iid shock to $\hat{\eta}_t$. A rise in $\hat{\eta}_t$ is equivalent to a rise in ε_t. Since domestic bonds are in zero net supply, the yield on domestic bonds must fall, leading to a decline in $\hat{R}_t - \hat{R}_t^*$. So ε_t covaries negatively with $\hat{R}_t - \hat{R}_t^*$ which causes the plim of an ordinary least squares estimate of α_1 to be negative in an economy driven only by spread shocks.

4.3.4 Calibration

For the purposes of calibration and estimation, we assume that the domestic and foreign monetary authorities follow Taylor rules (42) and (43), respectively. We divide the parameters into two categories: those that we calibrate and those that we estimate. The calibrated parameters are listed in Table 8. We maintain the parameter values used in the previous sections and set the habit persistence parameter, h; the probability that firms cannot adjust their price, ξ; and the probability that labor suppliers cannot readjust their nominal wage, ξ_W, to the point estimates reported in Christiano, Eichenbaum, and Evans (2005). We set the value of ν_L so as to imply a 5 percent steady state markup. We calibrate the parameters $\rho_\eta = 0.85$. This value is equal to the persistence of the spread shock in the closed-economy version of the new-Keynesian model estimated by Gust et al. (2016).

We estimate the remaining parameters ρ_A, σ_A, and σ_η so that the model is consistent with the following moments of the data. Technology shocks are assumed to be uncorrelated across countries. We require that the first-order autocorrelation of HP-filtered model output and the standard deviation of the innovation to a fitted AR(1) coincide with the analogous statistics estimated using quarterly U.S. data for the period from 1973:Q1 through 2007:Q4.[23] In addition, we require that the model be consistent with the slope coefficient of the Fama regression—defined by equation (77)—being equal to 0.5.[24] We find that the parameters that give us the best fit are $\rho_A = 0.946$, $\sigma_A = 0.010$, and $\sigma_\eta = 0.001$.

4.3.5 Model results

Here, we discuss the model's implications for the statistics that we emphasized in our empirical analysis. Table 9 reports the model's implications for the coefficients in regressions (2) and (3). We report results for two versions of the model (with flexible prices and wages and with sticky prices and wages) and three monetary regimes (Taylor rule, money growth rule, and NER targeting).

Two results are worth noting. First, the model with a Taylor rule does a good job of accounting for the estimated values of $\beta_{i,h}^{NER}$ and their rise in absolute value with the regression horizon. Second, if sampling uncertainty is taken into account, the model with a Taylor rule is also consistent with the positive values of $\beta_{i,h}^{\pi}$ and the fact that they rise with the horizon. With a money growth rule and an NER targeting rule, the model predicts negative values of $\beta_{i,h}^{\pi}$ and values of $\beta_{i,h}^{NER}$ that are small in magnitude relative to the data.

[23]We measure output using per capita real gross domestic product. We calculate model moments as small-sample average estimates using 140 periods—the same length as in our data.

[24]This value is well within standard errors of the slope coefficients reported in the literature.

Table 11 reports the standard deviations of ΔRER and ΔNER for the countries in our sample and the Taylor-rule version of our model. In addition, we report estimates for the autocorrelation of the RER. Four features of Table 11 are worth noting. First, our model is consistent with the well-known fact that real and nominal exchange rates are equally volatile (Mussa (1986), Rogoff (1996), and Burstein and Gopinath (2015)). Second, the model understates the volatility of ΔRER and ΔNER. This understatement reflects in part the small number of shocks included in the model (two technology shocks and a spread shock). Third, the model produces persistent estimates of the cyclical component of HP-filtered RERs that are within sampling uncertainty of the data. Fourth, the model with nominal rigidities accounts for the classic Mussa observations that changes in real and nominal exchange rates are highly correlated.

Table 11 also reports the key properties of a version of our model without nominal rigidities. This version of the model captures many qualitative features of the data. However, the model does not account for the high correlation between the NER and RER. For every country in our sample, that correlation is above 0.95. In the model without nominal rigidities, this correlation is 0.65.

4.3.6 Forecasting implications

Here, we assess whether the model can account for the key characteristics of our out-of-sample forecasting results. The results from our panel regressions in the data are repeated in the first row of Table 10. We use our economic model to generate 10,000 synthetic samples, each of length equal to the number of quarters in our sample, for the same number of countries as our benchmark flexible-exchange rate countries. We redo our panel forecasting exercise on each of the synthetic data sets and compute the expected value of the small-sample RMSPEs, i.e., we compute the average of the forecasts at different horizons across the 10,000 synthetic samples. In addition, we compute the standard deviation of the small-sample RMSPEs across the synthetic data sets. The second row of Table 10 reports the average value of the ratio of the RMSPE of our forecasts to the RMSPE of the random walk forecasts across the synthetic data sets when we assume that monetary policy follows a Taylor rule. Standard deviations are reported in brackets. Two key results emerge. First, the model reproduces the fact that the relative performance of our benchmark specification improves with the horizon. Second, the RMSPEs are well within standard errors at longer horizons.

To provide intuition for the performance of the model, it is useful to consider a simplified version of the model in which prices and wages are flexible, there are complete markets across the two countries, and there is no capital or habit formation in utility. The only shocks in this simple model are country-specific technology shocks that follow an AR(1) process with autocorrelation ρ_A. The standard deviation of the innovation to the shock is equal to σ_A. Using equations (53) and (56), along with the definition of the RER, we have that

$$\begin{aligned}\log(NER_{t+h}) - \log(NER_t) \quad = \quad & \log(NER_{t+h}) - E_t \log(NER_{t+h}) \\ & + \left(\rho_A^h - 1\right)\kappa\left(\hat{A}_t - \hat{A}_t^*\right) + \sum_{k=1}^{h} \frac{\rho_A - 1}{\theta_\pi - \rho_A}\rho_A^k \kappa\left(\hat{A}_t - \hat{A}_t^*\right).\end{aligned} \tag{78}$$

The left-hand side of equation (78) is the forecast error associated with a random walk forecast. The first term on the right-hand side is the forecast error associated with the rational expectations forecast assuming that the structural model is the DGP. Squaring both sides of equation (78) using the fact that the rational expectations forecast error is orthogonal to variables in the time-t information set, we obtain

$$\begin{aligned} E\left(\log(NER_{t+h}) - \log(NER_t)\right)^2 &= E\left(\log(NER_{t+h}) - E_t \log(NER_{t+h})\right)^2 \\ &+\kappa^2 \left[\left(1-\rho_A^h\right) + \sum_{k=1}^{h} \frac{1-\rho_A}{\theta_\pi - \rho_A}\rho_A^k\right]^2 \frac{\sigma_A^2}{1-\rho_A^2}. \end{aligned} \tag{79}$$

Two important conclusions follow from equation (79). First, as ρ_A approaches 1, the difference between the two mean-squared forecast errors converges to zero. This convergence is due to the fact that when ρ_A approaches 1, the RER and the NER become increasingly like random walks. The closer ρ_A is to 1, the more difficult it is in small samples, to reject the hypothesis that the NER is a random walk. This observation is reminiscent of a key result in Engel and West (2005). That paper works with a reduced form class of models in which the NER depends on current and future expected 'fundamentals' like relative money and output growth rates. Engel and West show that as the rate of time discounting (β) approaches 1, the NER becomes increasingly difficult to distinguish from a random walk. Note that β plays no role in our analytic results. Second, in equation (79), the difference between the two mean-squared forecast errors increases with the horizon, h, similar to what we find in the data.

Monetary policy plays a key role in the model's implications for the average squared predictions of a rational forecast relative to a forecast of no change in the NER. For example, equation (79) implies that a larger value of θ_π reduces the difference between the average squared prediction error of the rational expectations forecast and that of the random walk forecast. For intuition, consider the extreme case in which θ_π is equal to infinity so that domestic and foreign prices are fixed. Then movements in the NER and RER are equal to each other. Since the RER is close to a random walk, it is difficult to distinguish both the NER and the RER from a random walk in small samples. When θ_π is very close to 1, prices respond by a relatively large amount to shocks, which causes the movements in the NER to be much larger than movements in the RER. The magnitude of the movements in the NER after a shock can then be exploited in small samples to predict future values of the NER.

Monetary policies other than Taylor rules also affect average squared prediction errors. Suppose that monetary policy follows a constant money growth rule or an NER targeting rule. We generate 10,000 synthetic time series from versions of the model under these policies and generate the model's implications for our forecasting exercises. Table 10 reports the average RMSPE from our forecasting equation relative to the average RMSPE from a random walk forecast. Notice that our forecasting specification no longer outperforms the random walk in the simulated data. The RER does not outperform the random walk forecast because the money growth and the NER targeting rule generate weaker correlations between the current RER and future changes in the NER than those that arise under the Taylor rule (see Table 9). These weaker correlations are more difficult to estimate and exploit for forecasting in small samples. The basic lesson from these results is that we should not expect forecasting performance to be robust across different monetary policy regimes.

5 Conclusion

This paper shows that in countries with floating exchange rates where monetary policy uses a short-term interest rate to control inflation, $RERs$ adjust toward parity in the medium and long runs through changes in NERs not via differences in inflation rates. We base this conclusion on two facts. First, the current value of the RER is highly correlated with changes in the NER at horizons longer than two years. Second, the current value of the RER is uncorrelated with differential inflation rates at all horizons.

In our theoretical analysis, we show that there is a large class of open-economy models consistent with these facts: models with and without nominal rigidities as well as complete- and incomplete-markets models. But to account for our empirical findings, models must feature home bias in consumption and monetary policy guided by Taylor rules.

References

[1] Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Linde, “Firm-specific capital, nominal rigidities and the business cycle.” *Review of Economic Dynamics* 14.2 (2011): 225-247.

[2] Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. “International real business cycles.” *Journal of Political Economy* (1992): 745-775.

[3] Baxter, Marianne, and Alan C. Stockman. “Business cycles and the exchange-rate regime: some international evidence.” *Journal of Monetary Economics* 23.3 (1989): 377-400.

[4] Benigno, Gianluca. “Real exchange rate persistence and monetary policy rules.” *Journal of Monetary Economics* 51.3 (2004): 473-502.

[5] Burstein, Ariel, and Gita Gopinath. “International prices and exchange rates.” *Handbook of International Economics* 4 (2014): 391-451.

[6] Calvo, Guillermo A. “Staggered prices in a utility-maximizing framework.” *Journal of Monetary Economics* 12.3 (1983): 383-398.

[7] Ca’Zorzi, Michele, Marcin Kolasa, and Michał Rubaszek. ”Exchange Rate Forecasting with DSGE models.” *Journal of International Economics* 107 (2017): 127-146.

[8] Ca’Zorzi, Michele, Jakub Muck, and Michal Rubaszek. “Real exchange rate forecasting and PPP: This time the random walk loses.” *Open Economies Review* 27.3 (2016): 585-609.

[9] Cheung, Yin-Wong, Kon S. Lai, and Michael Bergman. “Dissecting the PPP puzzle: the unconventional roles of nominal exchange rate and price adjustments.” *Journal of International Economics* 64.1 (2004): 135-150.

[10] Cheung, Y. W., Chinn, M. D., Pascual, A. G., & Zhang, Y. (2017). Exchange rate prediction redux: new models, new data, new currencies (No. w23267). National Bureau of Economic Research.

[11] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. “Nominal rigidities and the dynamic effects of a shock to monetary policy.” *Journal of Political Economy* 113.1 (2005): 1-45.

[12] Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. “Understanding the great recession.” *American Economic Journal: Macroeconomics* 7.1 (2015): 110-167.

[13] Clarida, Richard, Jordi Gali, and Mark Gertler. “Monetary policy rules in practice: some international evidence.” *European Economic Review* 42.6 (1998): 1033-1067.

[14] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc “Optimal monetary policy in open economies” in Ben Friedman and Michael Woodford *Handbook of Monetary Economics, vol. III*, Elsevier, 2010.

[15] Dornbusch, Rudiger. “Expectations and exchange rate dynamics.” *Journal of Political Economy* (1976): 1161-1176.

[16] Engel, Charles, and Kenneth D. West. “Accounting for Exchange-Rate Variability in Present-Value Models When the Discount Factor Is Near 1.” *American Economic Review* 94.2 (2004): 119-125.

[17] Engel, Charles and West, Kenneth, (2005), “Exchange Rates and Fundamentals,” *Journal of Political Economy*, 113, issue 3, p. 485-517.

[18] Engel, Charles, Nelson C. Mark, and Kenneth D. West. “Exchange Rate Models Are Not As Bad As You Think.” *NBER Macroeconomics Annual* 2007, Volume 22. University of Chicago Press, 2008. 381-441.

[19] Engel, Charles. “Real Exchange Rate Convergence: The Roles of Price Stickiness and Monetary Policy.” Manuscript (2012).

[20] Engel, Charles, “Exchange Rates and Interest Parity,” Volume 4, 2014, Pages 453–522, *Handbook of International Economics*.

[21] Erceg, Christopher J., Luca Guerrieri, and Christopher J. Gust. “SIGMA: a new open economy model for policy analysis.” Board of Governors of the Federal Reserve System Research Paper Series (2005).

[22] Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. “Optimal monetary policy with staggered wage and price contracts.” *Journal of Monetary Economics* 46.2 (2000): 281-313.

[23] Fama, Eugene F. “Forward and spot exchange rates.” *Journal of Monetary Economics* 14.3 (1984): 319-338.

[24] Fisher, Jonas DM. “On the Structural Interpretation of the Smets-Wouters ‘Risk Premium’ Shock.” *Journal of Money, Credit and Banking* 47.2-3 (2015): 511-516.

[25] Elliott, Graham, Thomas J. Rothenberg, and James H. Stock. ”Efficient Tests for an Autoregressive Unit Root.” *Econometrica* 64.4 (1996): 813-836.

[26] Gali, Jordi, and Tommaso Monacelli. ”Monetary policy and exchange rate volatility in a small open economy.” *The Review of Economic Studies* 72.3 (2005): 707-734.

[27] Gust, Christopher J., Edward Herbst, David López-Salido, and Matthew E. Smith. “The empirical implications of the interest-rate lower bound.” Board of Governors of the Federal Reserve System Research Paper Series (2016).

[28] Hansen, Gary D. “Indivisible labor and the business cycle.” *Journal of Monetary Economics* 16.3 (1985): 309-327.

[29] Henderson, Dale W., and Warwick J. McKibbin. ”A comparison of some basic monetary policy regimes for open economies: implications of different degrees of instrument adjustment and wage persistence.” *Carnegie-Rochester Conference Series on Public Policy*. Vol. 39. North-Holland, 1993..

[30] Justiniano, Alejandro, and Bruce Preston. “Monetary policy and uncertainty in an empirical small open-economy model.” *Journal of Applied Econometrics* 25.1 (2010): 93-128.

[31] Kollmann, Robert. “The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation.” *Journal of International Economics* 55.2 (2001): 243-262.

[32] Lucas, Robert Jr, 1976, ”Econometric policy evaluation: A critique,” Carnegie-Rochester Conference Series on Public Policy, Elsevier, vol. 1(1), pages 19-46, January.

[33] Mark, Nelson C. “Exchange rates and fundamentals: Evidence on long-horizon predictability.” *American Economic Review* (1995): 201-218.

[34] McCallum, Bennett T. “A reconsideration of the uncovered interest parity relationship.” *Journal of Monetary Economics* 33.1 (1994): 105-132.

[35] Meese, Richard A., and Kenneth Rogoff. ”Empirical exchange rate models of the seventies: Do they fit out of sample?.” *Journal of International Economics* 14.1-2 (1983): 3-24.

[36] Molodtsova, Tanya, and David H. Papell. “Out-of-sample exchange rate predictability with Taylor rule fundamentals.” *Journal of International Economics* 77.2 (2009): 167-180.

[37] Mussa, Michael. “Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications.” Carnegie-Rochester Conference series on public policy. Vol. 25. North-Holland, 1986.

[38] Obstfeld, Maurice, and Kenneth Rogoff. “Exchange Rate Dynamics Redux.” *Journal of Political Economy* 103.3 (1995): 624-60.

[39] Obstfeld and Rogoff, “The six major puzzles in international macroeconomics: is there a common cause?” *NBER Macroeconomics Annual 2000*, Volume 15, 2001

[40] Rogoff, Kenneth. “The purchasing power parity puzzle.” *Journal of Economic Literature* 34.2 (1996): 647-668.

[41] Rossi, Barbara. “Exchange rate predictability.” *Journal of Economic Literature* 51.4 (2013): 1063-1119.

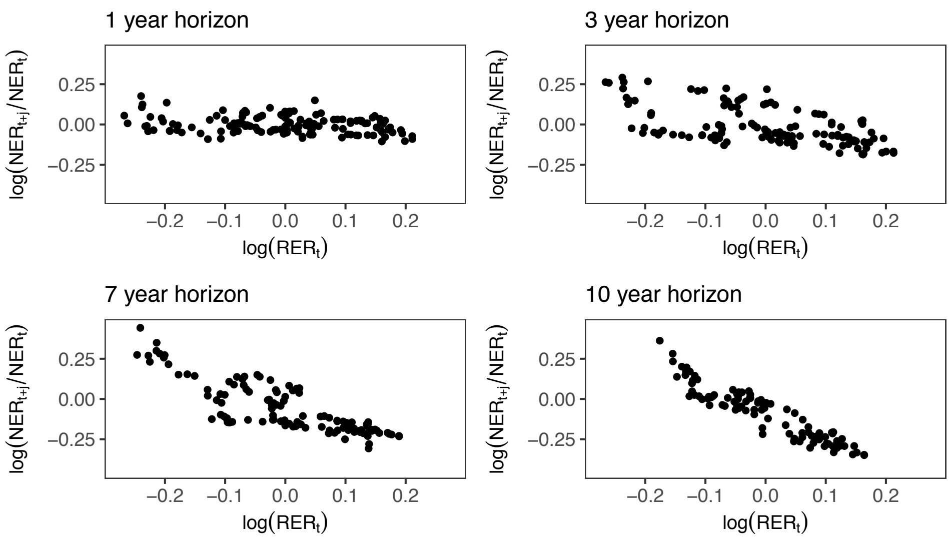
[42] Schmitt-Grohé, Stephanie, and Martın Uribe. “Closing small open economy models.” *Journal of international Economics* 61.1 (2003): 163-185.

[43] Smets, Frank, and Rafael Wouters. “Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach.” *American Economic Review* 97.3 (2007): 586-606.

[44] Steinsson, Jón. “The dynamic behavior of the real exchange rate in sticky price models.” *American Economic Review* 98.1 (2008): 519-533.

6 Tables and Figures

Figure 1: Canada: NER and RER data



Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations.

Table 1: NER regression results

	$\beta_{i,h}^{NER}$ Horizon (in years)					R^2 Horizon (in years)				
	1	3	5	7	10	1	3	5	7	10
(a) Flexible, benchmark										
Australia	-0.20	-0.70	-1.06	-1.13	-1.59	0.10	0.39	0.59	0.60	0.75
	(0.10)	(0.19)	(0.21)	(0.22)	(0.14)					
Canada	-0.12	-0.55	-0.94	-1.16	-1.66	0.08	0.35	0.59	0.69	0.88
	(0.07)	(0.18)	(0.18)	(0.14)	(0.12)					
Norway	-0.21	-0.76	-1.29	-1.47	-1.25	0.07	0.29	0.55	0.65	0.51
	(0.12)	(0.15)	(0.25)	(0.29)	(0.05)					
Sweden	-0.20	-0.75	-1.14	-1.37	-1.28	0.11	0.41	0.65	0.76	0.67
	(0.10)	(0.16)	(0.19)	(0.13)	(0.21)					
Switzerland	-0.31	-0.91	-1.37	-1.30	-1.13	0.15	0.45	0.71	0.79	0.71
	(0.12)	(0.14)	(0.19)	(0.12)	(0.13)					
(b) Flexible, other										
Euro Area	-0.13	-0.86	-0.89			0.03	0.46	0.67		
	(0.17)	(0.29)	(0.13)							
Japan	-0.06	-0.45	-0.68			0.01	0.16	0.33		
	(0.11)	(0.25)	(0.29)							
United Kingdom	-0.29	-1.31	-1.66			0.10	0.58	0.65		
	(0.16)	(0.35)	(0.17)							
(c) Fixed										
Hong Kong	0.00	-0.01	-0.02	-0.03	-0.03	0.04	0.32	0.62	0.76	0.77
	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)					
China	-0.12	-0.21	-0.26			0.26	0.29	0.45		
	(0.03)	(0.06)	(0.10)							

Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations.

Table 2: Relative price regression results

	$\beta^{\pi}_{i,h}$ Horizon (in years)					R^2 Horizon (in years)				
	1	3	5	7	10	1	3	5	7	10
(a) Flexible, benchmark										
Australia	0.01	0.05	0.10	0.20	0.48	0.00	0.01	0.04	0.09	0.24
	(0.04)	(0.09)	(0.08)	(0.08)	(0.18)					
Canada	0.01	0.03	0.04	0.08	0.26	0.01	0.02	0.01	0.02	0.10
	(0.01)	(0.04)	(0.06)	(0.11)	(0.18)					
Norway	-0.07	-0.15	-0.11	-0.06	-0.06	0.11	0.11	0.04	0.01	0.00
	(0.02)	(0.11)	(0.17)	(0.19)	(0.20)					
Sweden	0.01	0.08	0.11	0.05	-0.02	0.01	0.06	0.06	0.01	0.00
	(0.02)	(0.06)	(0.10)	(0.19)	(0.21)					
Switzerland	-0.02	0.00	0.08	0.10	0.01	0.03	0.00	0.02	0.03	0.00
	(0.02)	(0.06)	(0.09)	(0.16)	(0.18)					
(b) Flexible, other										
Euro Area	-0.04	-0.08	0.03			0.51	0.63	0.07		
	(0.01)	(0.01)	(0.01)							
Japan	-0.08	-0.15	-0.11			0.22	0.15	0.06		
	(0.04)	(0.11)	(0.12)							
United Kingdom	-0.02	-0.03	-0.04			0.02	0.03	0.03		
	(0.01)	(0.05)	(0.03)							
(c) Fixed										
Hong Kong	-0.09	-0.45	-0.93	-1.32	-1.63	0.13	0.37	0.66	0.88	0.99
	(0.05)	(0.14)	(0.16)	(0.14)	(0.03)					
China	-0.43	-0.93	-1.05			0.37	0.67	0.91		
	(0.19)	(0.20)	(0.07)							

Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations.

Table 3: Euro area relative price regression results

	$\beta^{\pi}_{i,h}$ Horizon (in years)			R^2 Horizon (in years)		
	1	3	5	1	3	5
France	-0.22	-1.00	-1.31	0.12	0.59	0.78
	(0.12)	(0.17)	(0.19)			
Italy	-0.17	-0.47	-0.61	0.40	0.71	0.80
	(0.05)	(0.08)	(0.06)			
Ireland	-0.30	-0.83	-1.10	0.40	0.73	0.83
	(0.09)	(0.08)	(0.09)			
Portugal	-0.22	-0.66	-0.83	0.48	0.85	0.94
	(0.060)	(0.06)	(0.04)			
Spain	-0.16	-0.44	-0.65	0.49	0.75	0.88
	(0.03)	(0.08)	(0.07)			

Sources: International Monetary Fund, International Financial Statistics; OECD Main Economic Indicators; authors' calculations.

Table 4: NER regression results for other countries

	Sample Starting in 1984						Sample Starting in 1999					
	$\beta_{i,h}^{NER}$ Horizon (in years)			R^2 Horizon (in years)			$\beta_{i,h}^{NER}$ Horizon (in years)			R^2 Horizon (in years)		
	1	3	5	1	3	5	1	3	5	1	3	5
Brazil	-0.31	0.38	1.75	0.02	0.01	0.05	-0.17	-0.59	-1.11	0.07	0.25	0.59
	(0.39)	(0.57)	(0.71)				(0.12)	(0.24)	(0.13)			
Chile	-0.22	-0.59	-0.89	0.05	0.12	0.14	-0.32	-0.98	-1.24	0.16	0.50	0.74
	(0.15)	(0.39)	(0.57)				(0.14)	(0.17)	(0.16)			
Colombia	-0.07	-0.29	-0.76	0.01	0.03	0.07	-0.15	-0.50	-0.93	0.05	0.19	0.43
	(0.14)	(0.42)	(0.69)				(0.12)	(0.27)	(0.16)			
Indonesia	-0.25	-0.59	-0.89	0.08	0.20	0.29	-0.14	-0.46	-0.55	0.06	0.36	0.43
	(0.09)	(0.16)	(0.29)				(0.12)	(0.14)	(0.16)			
Israel	0.70	0.57	0.45	0.07	0.03	0.01	-0.39	-0.64	-1.13	0.20	0.37	0.63
	(0.71)	(1.00)	(1.33)				(0.14)	(0.21)	(0.19)			
Mexico	0.78	1.33	1.48	0.23	0.13	0.12	-0.32	-0.96	-0.54	0.06	0.23	0.10
	(0.32)	(0.75)	(0.90)				(0.38)	(0.13)	(0.34)			
South Korea	-0.42	-1.04	-1.35	0.21	0.55	0.70	-0.49	-1.07	-1.35	0.22	0.54	0.78
	(0.12)	(0.22)	(0.15)				(0.23)	(0.22)	(0.11)			
Thailand	-0.26	-0.59	-1.02	0.16	0.33	0.52	-0.13	-0.42	-0.73	0.09	0.34	0.56
	(0.12)	(0.20)	(0.25)				(0.06)	(0.13)	(0.10)			
Turkey	0.40	1.22	1.97	0.15	0.22	0.20	0.16	0.13	-0.21	0.03	0.01	0.02
	(0.11)	(0.38)	(0.59)				(0.20)	(0.42)	(0.30)			

Sources: International Monetary Fund, International Financial Statistics; OECD Main Economic Indicators; authors' calculations.

Table 5: Relative price regression results for other countries

	Sample Starting in 1984						Sample Starting in 1999					
	$\beta^{\pi}_{i,h}$			R^2			$\beta^{\pi}_{i,h}$			R^2		
	Horizon (in years)			Horizon (in years)			Horizon (in years)			Horizon (in years)		
	1	3	5	1	3	5	1	3	5	1	3	5
Brazil	-0.28	-1.24	-2.80	0.02	0.06	0.12	-0.02	0.00	0.07	0.05	0.00	0.10
	(0.22)	(0.56)	(0.74)				(0.02)	(0.05)	(0.03)			
Chile	-0.05	-0.23	-0.46	0.01	0.03	0.06	0.06	0.16	0.14	0.17	0.42	0.24
	(0.10)	(0.35)	(0.56)				(0.03)	(0.03)	(0.04)			
Colombia	-0.14	-0.39	-0.52	0.12	0.11	0.07	-0.04	-0.06	-0.08	0.12	0.10	0.11
	(0.08)	(0.31)	(0.57)				(0.02)	(0.05)	(0.03)			
Indonesia	-0.07	-0.02	0.01	0.06	0.00	0.00	-0.06	-0.12	-0.23	0.12	0.33	0.61
	(0.03)	(0.07)	(0.14)				(0.03)	(0.03)	(0.03)			
Israel	-0.99	-1.32	-1.56	0.16	0.13	0.13	0.07	0.09	0.00	0.08	0.05	0.00
	(0.67)	(1.07)	(1.35)				(0.04)	(0.11)	(0.09)			
Mexico	-1.10	-2.15	-2.55	0.62	0.36	0.32	0.04	0.07	-0.06	0.03	0.02	0.02
	(0.24)	(0.61)	(0.83)				(0.03)	(0.06)	(0.13)			
South Korea	0.06	0.17	0.21	0.19	0.42	0.34	-0.02	0.02	0.12	0.02	0.01	0.34
	(0.02)	(0.03)	(0.05)				(0.02)	(0.06)	(0.03)			
Thailand	0.04	0.11	0.14	0.12	0.21	0.18	0.01	0.01	-0.01	0.02	0.01	0.00
	(0.02)	(0.06)	(0.07)				(0.01)	(0.03)	(0.02)			
Turkey	-0.57	-1.58	-2.50	0.46	0.43	0.37	-0.39	-0.77	-0.81	0.49	0.37	0.32
	(0.09)	(0.32)	(0.50)				(0.10)	(0.28)	(0.24)			

Sources: International Monetary Fund, International Financial Statistics; OECD Main Economic Indicators; authors' calculations.

Table 6: Test of null-hypothesis of no predictability with Wald statistic

	Australia	Canada	Norway	Sweden	Switzerland
Bootstrap p-value	0.02	0.01	0.07	0.04	0.00

Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations. Notes: We used 10,000 bootstrap samples. The table displays the percentage of those samples that produced Wald statistics larger than the Wald statistic calculated from the data.

Table 7: Out-of-sample forecasting for the NER

(a) RMSPE from panel regression relative to a random walk

	Forecast horizon								
	1Q	2Q	1Y	2Y	3Y	4Y	5Y	6Y	7Y
All Countries	1.01	1.01	1.02	0.98	0.87	0.77	0.65	0.60	0.55
Australia	1.00	1.00	0.99	1.02	0.89	0.73	0.63	0.58	0.60
Canada	1.01	1.02	1.06	1.10	1.02	0.88	0.77	0.74	0.64
Norway	1.01	1.03	1.05	1.04	0.96	0.90	0.77	0.73	0.64
Sweden	1.01	1.03	1.05	0.99	0.81	0.72	0.57	0.52	0.46
Switzerland	1.00	0.99	0.97	0.86	0.73	0.66	0.53	0.42	0.37

Note: The left-hand-side variable is the change in the quarterly average nominal exchange rate at the indicated horizons. The right-hand-side variable is the quarterly real exchange rate (calculated using the quarterly average of the nominal exchange rate and lagged price levels). The sample periods is from 1973Q1 through 2007Q4. We use a training sample of 10 years plus the horizon of the forecast. Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations.

(b) Bootstrap p-values, stationary RER

	Forecast horizon									
	1Q	2Q	1Y	2Y	3Y	4Y	5Y	6Y	7Y	Years 3-7
All Countries	0.71	0.75	0.58	0.20	0.05	0.03	0.01	0.01	0.01	0.00
Australia	0.40	0.35	0.20	0.36	0.11	0.04	0.03	0.03	0.07	0.03
Canada	0.79	0.71	0.83	0.74	0.23	0.10	0.06	0.07	0.05	0.04
Norway	0.83	0.90	0.83	0.39	0.11	0.09	0.05	0.06	0.04	0.03
Sweden	0.83	0.84	0.80	0.22	0.03	0.03	0.01	0.01	0.01	0.01
Switzerland	0.38	0.43	0.33	0.14	0.09	0.08	0.04	0.02	0.02	0.01

(c) RMSPE relative to a random walk, robustness

	Forecast horizon								
	1Q	2Q	1Y	2Y	3Y	4Y	5Y	6Y	7Y
Benchmark	1.01	1.01	1.02	0.98	0.87	0.77	0.65	0.60	0.55
Time t price levels	1.01	1.01	1.02	0.98	0.88	0.78	0.66	0.61	0.56
Sample ends in 2016Q4	1.00	1.00	1.00	0.97	0.92	0.86	0.81	0.77	0.72
Sample ends in 2016Q4 (with Japan)	1.00	1.00	0.99	0.95	0.92	0.89	0.87	0.83	0.77
Unbalanced panel	1.01	1.01	1.02	0.97	0.89	0.83	0.79	0.67	0.60
Country-by-country regressions	1.02	1.03	1.05	1.07	1.02	0.89	0.72	0.66	0.59

Note: The row labeled "Benchmark" is the row labeled "All countries" in panel (a). The other rows are variants of the panel regression considered and described in panel (a) and show the averge mean squared prediction error for all countries. The row labeled "Sample ends in 2016Q4" extends the sample to 2016Q4. The row labeled "Sample ends in 2016Q4 (with Japan)" adds japan to the list of countries in the panel. The row labeled "Unbalanced panel" adds the euro to the panel starting 1999Q1, adds the UK starting in 1993Q1, and adds Japan starting in 1973Q1 and ending in 1994Q4. The row labeled "Country-by-country regressions" does not impose the panel structure. Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations.

(d) Bootstrap p-values, non-stationary RER

	Forecast horizon									
	1Q	2Q	1Y	2Y	3Y	4Y	5Y	6Y	7Y	Years 3-7
All Countries	0.49	0.53	0.40	0.15	0.04	0.02	0.00	0.01	0.01	0.00
Australia	0.33	0.30	0.18	0.33	0.11	0.04	0.03	0.04	0.06	0.02
Canada	0.68	0.58	0.71	0.61	0.24	0.09	0.05	0.06	0.05	0.03
Norway	0.66	0.75	0.65	0.31	0.12	0.10	0.06	0.06	0.05	0.03
Sweden	0.68	0.68	0.64	0.18	0.03	0.02	0.01	0.01	0.01	0.00
Switzerland	0.34	0.38	0.31	0.14	0.08	0.08	0.04	0.03	0.03	0.01

Table 8: Calibrated parameters

Parameter	Value	Model counterpart
σ_M	10.62	Elasticity of money demand
μ	1	Steady state money stock
β	$1.03^{-0.25}$	Steady state interest rate
h	0.65	Habit persistence
σ	1	log utility
ϕ	1	Disutility of labor
γ	0.75	Policy rate smoothing
θ_π	1.5	Taylor principle
ν	6	Intermediate goods firm's markups
ρ_η	0.85	Persistence of interest rate differential
ρ	$\frac{1}{3}$	Substitutability of home and foreign goods
ξ	0.6	Frequency of price adjustment
ϕ_B	0.001	Cost of foreign bond holdings
ν_L	21	Differentiated wage markup
ξ_W	0.65	Frequency of wage adjustment
ω	0.90	Home bias in consumption

Table 9: Model-implied regression results

	$\beta_{i,h}^{NER}$ Horizon (in years)					$\beta_{i,h}^{\pi}$ Horizon (in years)				
	1	3	5	7	10	1	3	5	7	10
Taylor rule:										
Flexible	-0.31	-0.66	-1.03	-1.32	-1.54	-0.03	0.12	0.30	0.41	0.43
Sticky	-0.35	-0.77	-1.19	-1.52	-1.72	0.02	0.22	0.43	0.56	0.59
Money growth rule:										
Flexible	-0.12	-0.31	-0.47	-0.59	-0.67	-0.21	-0.23	-0.27	-0.33	-0.42
Sticky	-0.22	-0.47	-0.64	-0.76	-0.85	-0.09	-0.10	-0.13	-0.19	-0.28
NER targeting rule:										
Flexible	-0.03	-0.03	-0.03	-0.04	-0.05	-0.32	-0.53	-0.72	-0.90	-1.08
Sticky	-0.04	-0.03	-0.04	-0.05	-0.07	-0.17	-0.46	-0.69	-0.88	-1.07

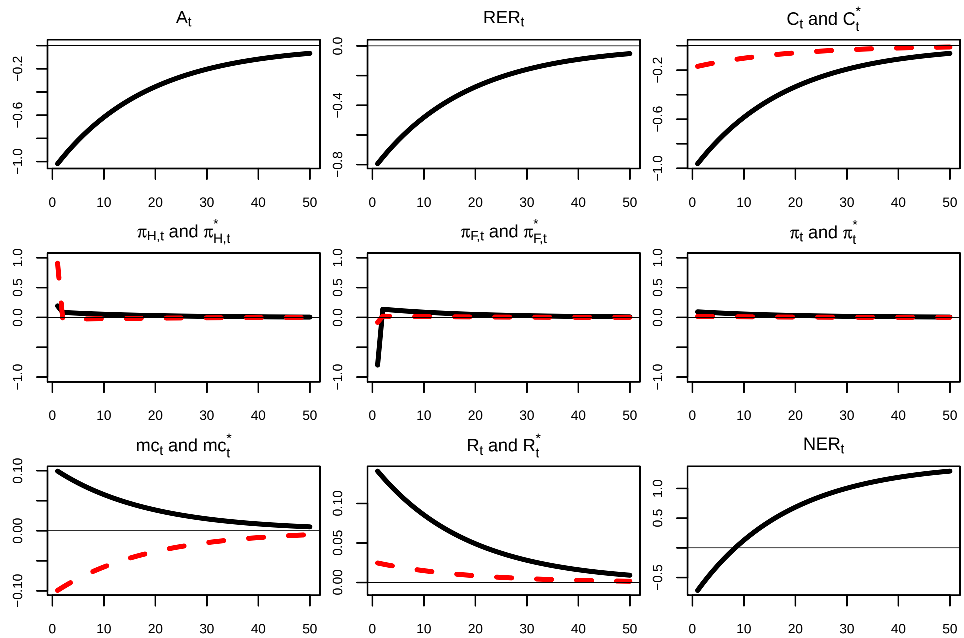
Sources: authors' calculations.

Table 10: Model-implied RMSPE relative to a random walk

	Forecast horizon								
	1Q	2Q	1Y	2Y	3Y	4Y	5Y	6Y	7Y
All Countries	1.01	1.01	1.02	0.98	0.87	0.77	0.65	0.60	0.55
Taylor rule	0.97	0.95	0.92	0.89	0.87	0.85	0.82	0.80	0.80
	(0.01)	(0.02)	(0.04)	(0.07)	(0.10)	(0.12)	(0.14)	(0.16)	(0.18)
Money growth rule	1.00	1.00	1.00	1.01	1.02	1.02	1.01	1.00	0.99
	(0.01)	(0.01)	(0.02)	(0.05)	(0.07)	(0.10)	(0.12)	(0.15)	(0.17)
NER targeting rule	1.01	1.01	1.02	1.03	1.04	1.06	1.07	1.09	1.10
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)	(0.04)	(0.05)	(0.08)

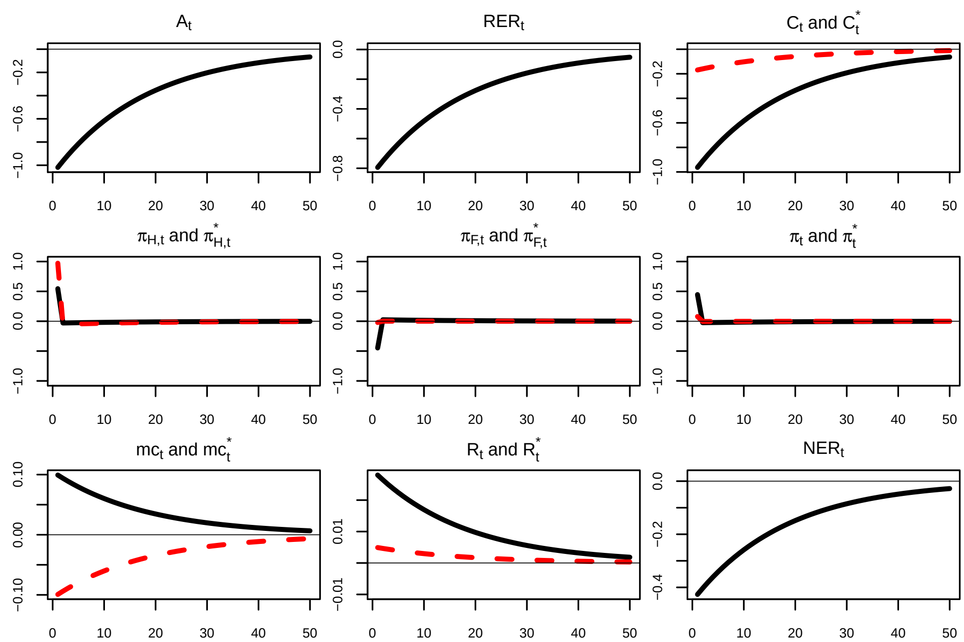
Note: The left-hand-side variable is the change in the quarterly average nominal exchange rate at the indicated horizons. The right-hand-side variable is the quarterly real exchange rate (calculated using the quarterly average of the nominal exchange rate and lagged price levels). The line marked "All Countries" is the same as in Table 7. For model simulations, we simulated 5 countries worth of data for 140 periods each. Sources: Internationl Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations.

Figure 2: Response to technology shock under Taylor rule



Note: The vertical axis is expressed in percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a $*$.

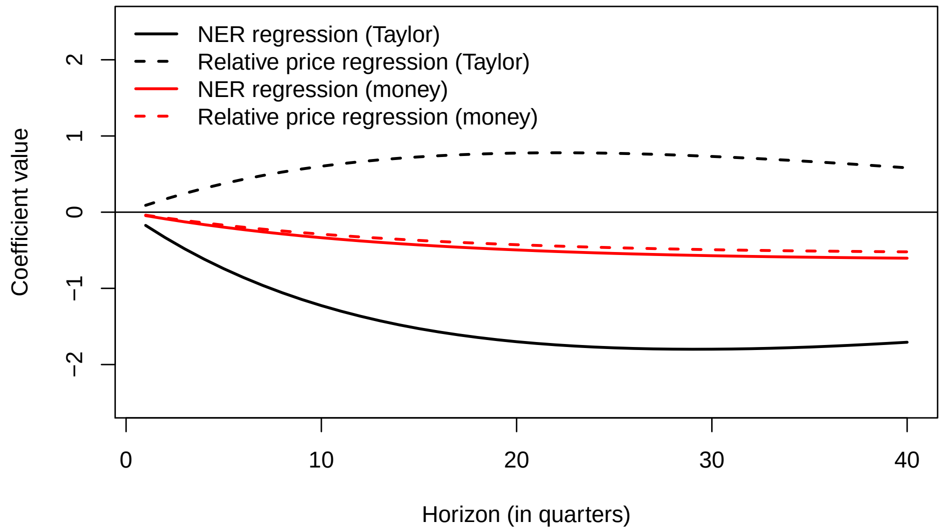
Figure 3: Response to technology shock under money growth rule



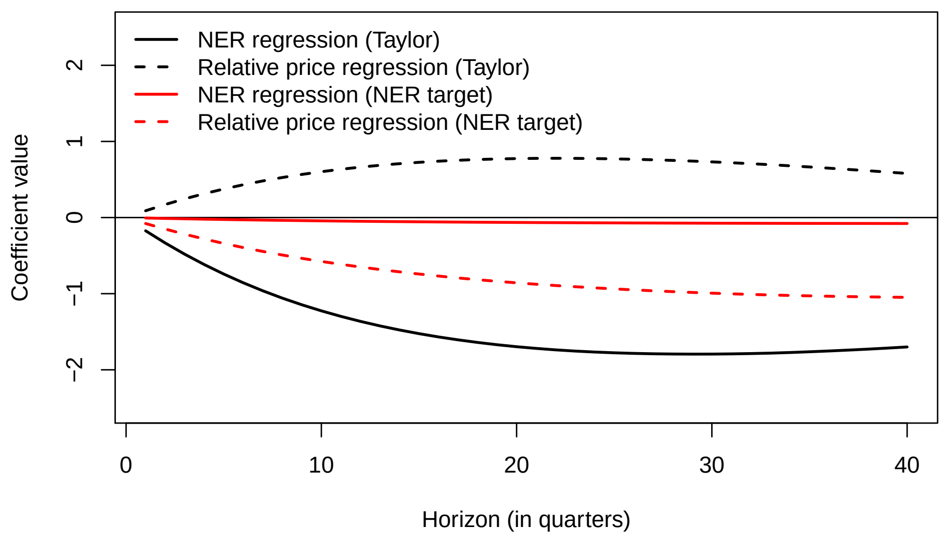
Note: The vertical axis is expressed in percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a $*$.

Figure 4: Implied small-sample values of $\beta_{i,h}^{NER}$ and $\beta_{i,h}^{\pi}$ from small-scale models

(a) Taylor and money growth rules

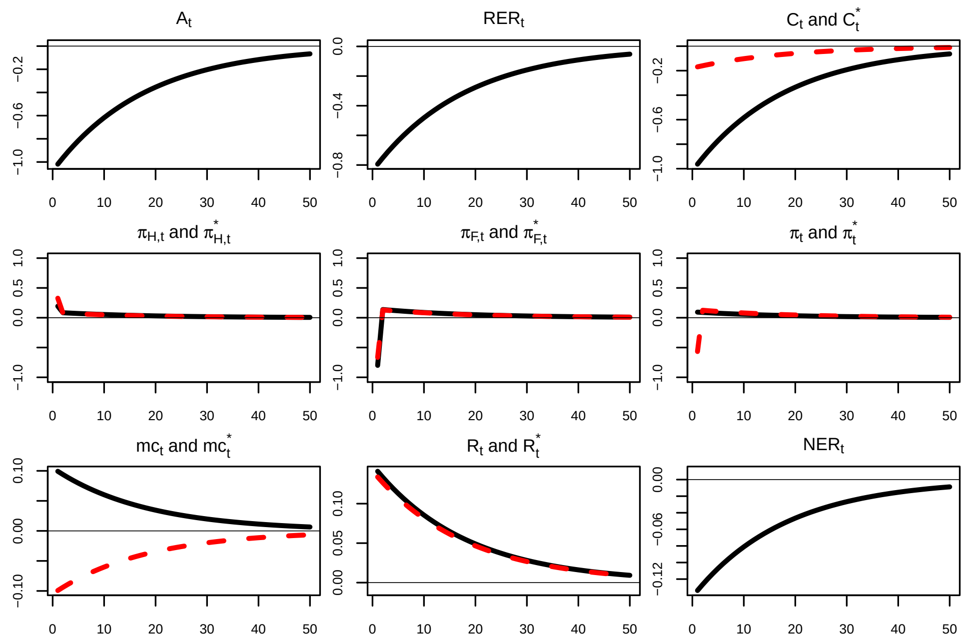


(b) Taylor and NER targeting rules



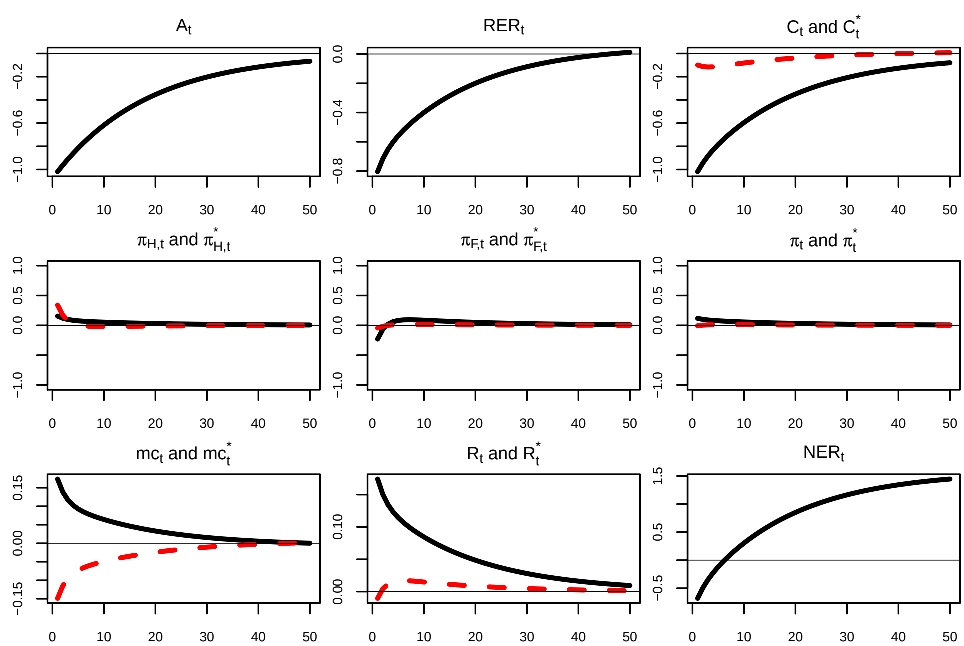
Note: The model-implied values come from our model with no nominal rigidities and only technology shocks.

Figure 5: Response to technology shock under NER targeting rule



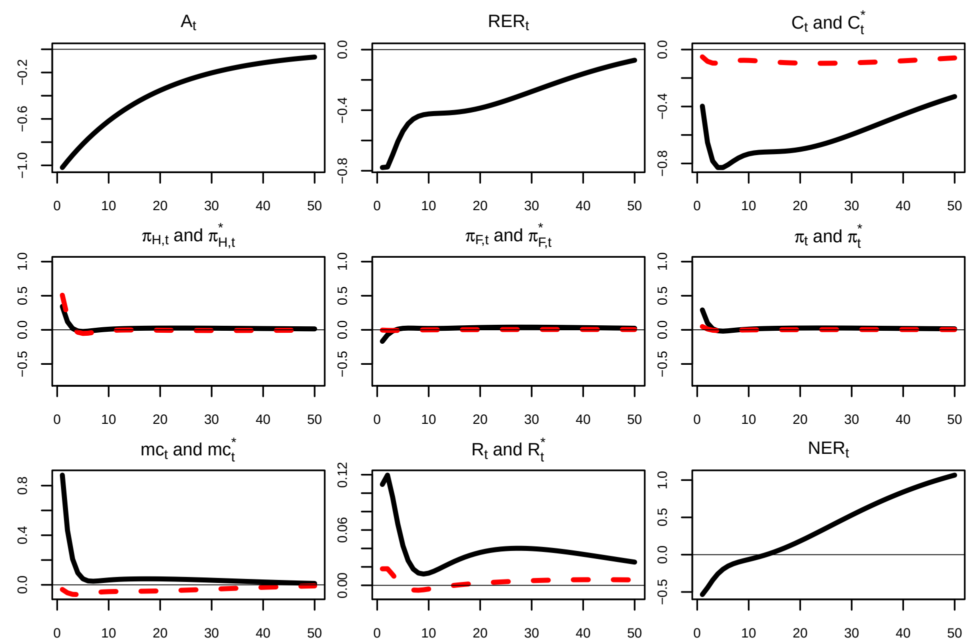
Note: The vertical axis is expressed in percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a $*$.

Figure 6: Response to technology shock under Taylor rule with incomplete markets and sticky prices



Note: The vertical axis is expressed in percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a $*$.

Figure 7: Response to technology shock under Taylor rule with incomplete markets, medium-scale model



Note: The vertical axis is expressed in percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a $*$.

Table 11: Empirical facts about exchange rates

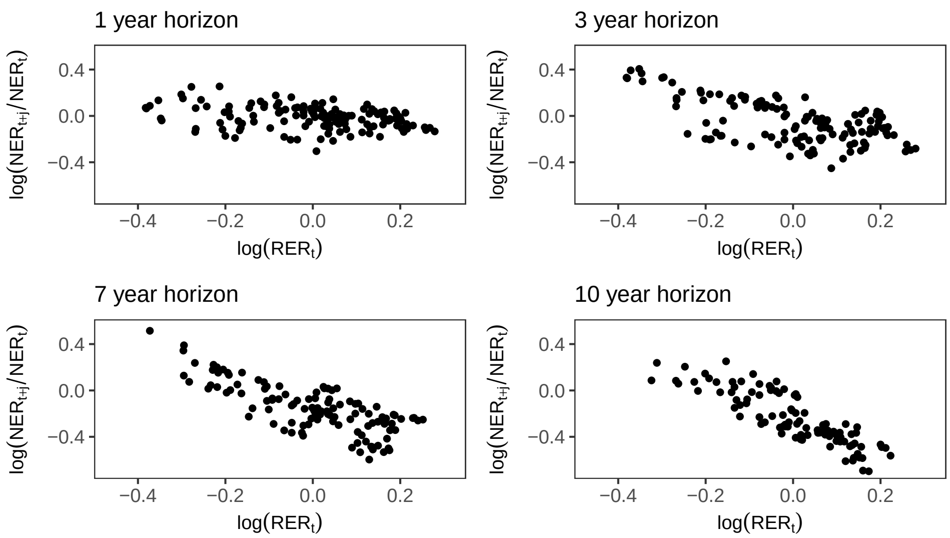
	ρ_{RER}	$\sigma_{\Delta RER}$	$\sigma_{\Delta NER}$	cor$(\Delta RER, \Delta NER)$
Australia	0.971	0.040	0.040	0.968
	(0.848,0.986)	(0.003)	(0.003)	(0.007)
Canada	0.986	0.022	0.022	0.969
	(0.872,0.997)	(0.002)	(0.002)	(0.007)
Norway	0.948	0.043	0.042	0.975
	(0.824,0.972)	(0.002)	(0.002)	(0.005)
Sweden	0.970	0.047	0.048	0.978
	(0.849,0.986)	(0.004)	(0.004)	(0.004)
Switzerland	0.934	0.052	0.052	0.989
	(0.828,0.963)	(0.003)	(0.003)	(0.002)
Nominal rigidities	0.900	0.014	0.012	0.934
Without nominal rigidities	0.859	0.013	0.010	0.693

Note: confidence intervals for ρ_{RER} are constructed from a parametric bootstrap for an AR(1) model of $\log(RER_t)$. We used 10,000 bootstrap draws and report the 0.025% and 0.975% quantiles of the bootstrap distribution of the statistic of interest. Standard errors for $\sigma_{\Delta RER}$ and $\sigma_{\Delta NER}$ are GMM standard errors. Source: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations.

A Data analysis appendix

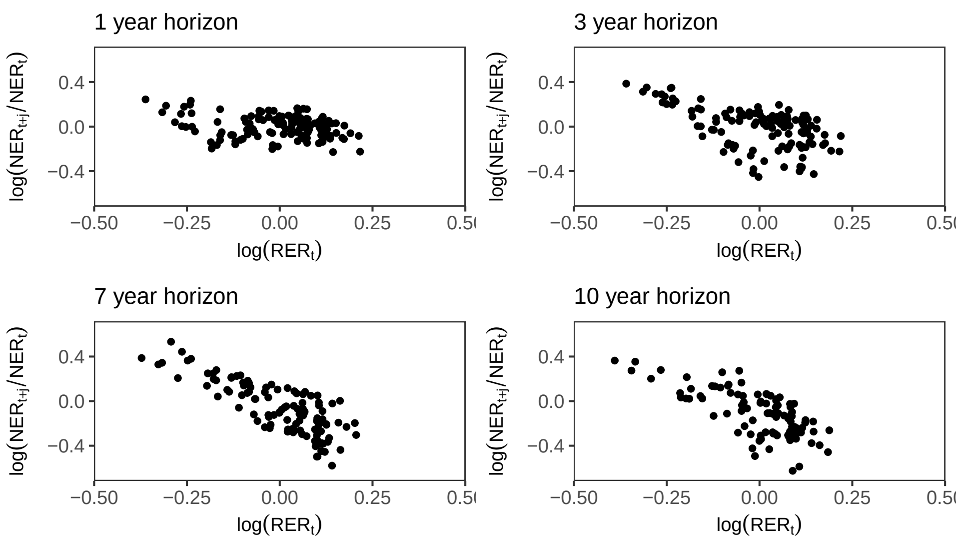
A.1 Scatter plots

Figure 8: Australia: NER and RER data



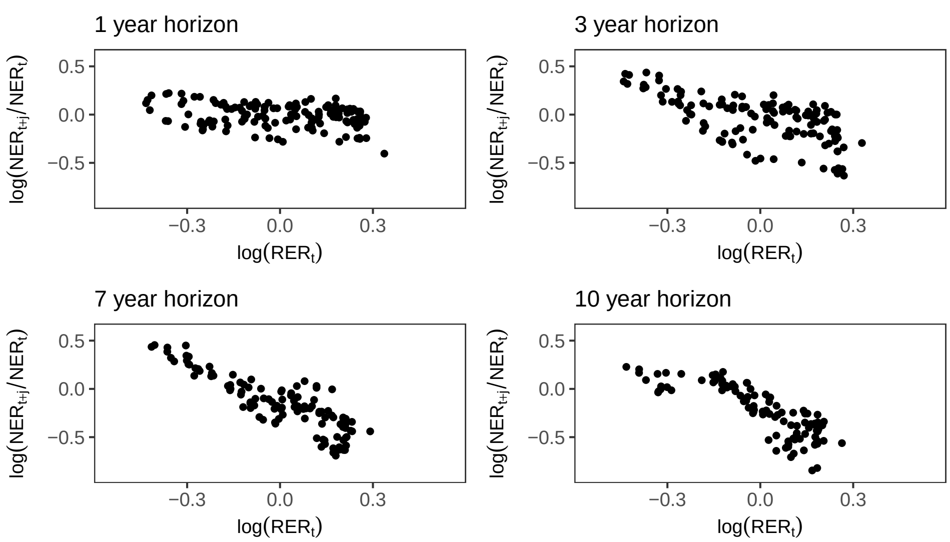
Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations.

Figure 9: Norway: NER and RER data



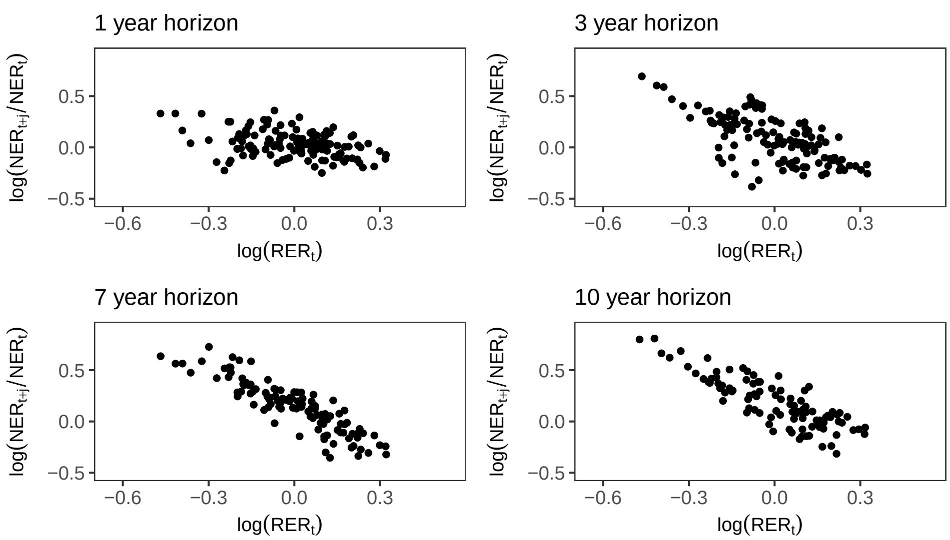
Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations.

Figure 10: Sweden: NER and RER data



Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations.

Figure 11: Switzerland: NER and RER data



Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; authors' calculations.

A.2 Additional regression results

Table 12: NER regression results, sample ending in 2016Q4

	$\beta_{i,h}^{NER}$ Horizon (in years)					R^2 Horizon (in years)				
	1	3	5	7	10	1	3	5	7	10
(a) Flexible, benchmark										
Australia	-0.19	-0.57	-0.94	-1.21	-1.95	0.09	0.28	0.42	0.52	0.83
	(0.09)	(0.15)	(0.15)	(0.18)	(0.14)					
Canada	-0.15	-0.57	-0.98	-1.29	-1.77	0.08	0.32	0.53	0.69	0.93
	(0.07)	(0.18)	(0.19)	(0.11)	(0.08)					
Norway	-0.24	-0.76	-1.26	-1.53	-1.58	0.08	0.27	0.50	0.63	0.62
	(0.11)	(0.14)	(0.19)	(0.25)	(0.23)					
Sweden	-0.20	-0.72	-1.12	-1.29	-1.33	0.09	0.38	0.64	0.77	0.76
	(0.09)	(0.16)	(0.19)	(0.12)	(0.14)					
Switzerland	-0.26	-0.73	-1.16	-1.19	-1.17	0.13	0.36	0.59	0.67	0.64
	(0.10)	(0.16)	(0.19)	(0.16)	(0.13)					
(b) Flexible, other										
Euro Area	-0.22	-0.80	-1.13			0.11	0.54	0.78		
	(0.15)	(0.12)	(0.12)							
Japan	-0.15	-0.63	-0.82			0.06	0.31	0.48		
	(0.09)	(0.18)	(0.16)							
United Kingdom	-0.46	-1.19	-1.61			0.16	0.47	0.61		
	(0.20)	(0.25)	(0.27)							
(c) Fixed										
Hong Kong	0.00	-0.01	-0.02	-0.03	-0.02	0.04	0.30	0.57	0.66	0.50
	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)					
China	-0.07	-0.16	-0.31			0.10	0.11	0.14		
	(0.04)	(0.12)	(0.15)							

Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations. Samples extended to 2016Q4 relative to Table 1.

Table 13: Relative price regression results, sample ending in 2016Q4

	$\beta^{\pi}_{i,h}$ Horizon (in years)					R^2 Horizon (in years)				
	1	3	5	7	10	1	3	5	7	10
(a) Flexible, benchmark										
Australia	0.00	0.02	0.06	0.17	0.34	0.00	0.00	0.02	0.10	0.24
	(0.02)	(0.06)	(0.07)	(0.07)	(0.15)					
Canada	0.01	0.03	0.05	0.10	0.22	0.01	0.01	0.02	0.06	0.17
	(0.01)	(0.03)	(0.05)	(0.08)	(0.15)					
Norway	-0.06	-0.13	-0.10	0.01	0.13	0.09	0.09	0.03	0.00	0.02
	(0.03)	(0.10)	(0.16)	(0.18)	(0.23)					
Sweden	0.02	0.09	0.15	0.17	0.24	0.03	0.09	0.11	0.09	0.09
	(0.02)	(0.09)	(0.15)	(0.17)	(0.24)					
Switzerland	-0.01	0.01	0.07	0.09	0.01	0.01	0.00	0.02	0.02	0.00
	(0.01)	(0.04)	(0.09)	(0.15)	(0.15)					
(b) Flexible, other										
Euro Area	0.00	0.00	0.05			0.00	0.00	0.27		
	(0.01)	(0.02)	(0.02)							
Japan	-0.06	-0.07	-0.03			0.18	0.06	0.01		
	(0.03)	(0.07)	(0.06)							
United Kingdom	-0.01	0.04	0.18			0.00	0.01	0.09		
	(0.03)	(0.09)	(0.11)							
(c) Fixed										
Hong Kong	-0.09	-0.43	-0.88	-1.30	-1.62	0.12	0.35	0.61	0.82	0.98
	(0.05)	(0.15)	(0.14)	(0.08)	(0.05)					
China	-0.04	-0.05	-0.10			0.02	0.01	0.02		
	(0.07)	(0.16)	(0.26)							

Sources: International Monetary Fund, International Financial Statistics; Federal Reserve Board, H.10 Foreign Exchange Rates; OECD Main Economic Indicators; authors' calculations. Samples extended to 2016Q4 relative to Table 2.

B Model Appendix

B.1 Household

The household problem for the representative household in the home country is

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C_{t+j} - h\bar{C}_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} \int_0^1 L_{t+j}(i)^{1+\phi}\, di + \mu \frac{\left(\frac{M_{t+j}}{P_{t+j}}\right)^{1-\sigma_M}}{1-\sigma_M} + \log(\eta_{t+j}) V\left(\frac{B_{H,t+j}}{P_{t+j}}\right) + \log\left(\eta^*_{t+j}\right) V\left(\frac{B_{F,t+j} NER_{t+j}}{P_{t+j}}\right) \right) \tag{B.1}$$

where C_t is consumption, $\bar{C}_t$ is aggregate consumption, $L_t(i)$ is hours worked by member i, $\frac{M_t}{P_t}$ are real money balances. The budget constraint is

$$B_{H,t} + NER_t B_{F,t} + P_t C_t + P_{I,t} I_t + M_t = R_{t-1} B_{H,t-1} + NER_t R^*_{t-1} B_{F,t-1} - \frac{\phi_B}{2}\left(\frac{NER_t B_{F,t}}{P_t}\right)^2 P_t + P_t R_{K,t} K_t + (1+\tau_W) \int_0^1 W_t(i) L_t(i)\, di + T_t + M_{t-1} \tag{B.2}$$

where and $B_{H,t}$ and $B_{F,t}$ are nominal balances of home and foreign bonds, NER_t is the nominal exchange rate quoted as the price of the foreign currency unit, P_t is the price of final goods in the home country, R_t is the nominal interest rate on the home bond and R^*_t is the nominal interest rate on the foreign bond, W_t is the wage rate, R^K_t is the rental rate on capital, K_t, I_t are investment goods, $P_{I,t}$ is the price of investment goods, and T_t are lump-sum profits and taxes. The capital accumulation equation is

$$K_{t+1} = I_t \left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) + (1-\delta) K_t \tag{B.3}$$

The household-wide first-order conditions are

$$\left(C_t - hC_{t-1}\right)^{-\sigma} = \Lambda_t \tag{B.4}$$

$$\Lambda_t = \log(\eta_t) V'\left(\frac{B_{H,t}}{P_t}\right) + \beta R_t E_t \frac{\Lambda_{t+1}}{\pi_{t+1}} \tag{B.5}$$

$$\Lambda_t + \phi_B \left(\frac{NER_t B_{F,t}}{P_t}\right) = \log(\eta^*_t) V'\left(\frac{B_{F,t} NER_t}{P_t}\right) + \beta R^*_t E_t \frac{\Lambda_{t+1}}{\pi_{t+1}} \frac{NER_{t+1}}{NER_t} \tag{B.6}$$

$$\mu \left(\frac{M_t}{P_t}\right)^{-\sigma_M} = \frac{\log(\eta_t)}{R_t} V'\left(\frac{B_{H,t}}{P_t}\right) + \left(\frac{R_t - 1}{R_t}\right) \Lambda_t \tag{B.7}$$

$$\frac{P_{I,t}}{P_t} \Lambda_t = Q_t \left[\left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - \frac{I_t}{I_{t-1}} \phi_K \left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta E_t Q_{t+1} \phi_K \left(\frac{I_{t+1}}{I_t} - 1\right) \frac{I^2_{t+1}}{I^2_t} \tag{B.8}$$

$$Q_t = \beta E_t \left[Q_{t+1}(1-\delta) + \Lambda_{t+1} R_{K,t+1}\right] \tag{B.9}$$

The household problem for the representative household in the foreign country is

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C^*_{t+j} - h\bar{C}^*_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} \int_0^1 L^*_{t+j}(i)^{1+\phi}\, di + \mu \frac{\left(\frac{M^*_{t+j}}{P^*_{t+j}}\right)^{1-\sigma_M}}{1-\sigma_M} + \log(\eta_{t+j}) V\left(\frac{B^*_{H,t+j}}{NER_{t+j} P^*_{t+j}}\right) + \log\left(\eta^*_{t+j}\right) V\left(\frac{B^*_{F,t+j}}{P^*_{t+j}}\right) \right) \tag{B.10}$$

The budget constraint is

$$B^*_{F,t} + NER_t^{-1} B^*_{H,t} + P^*_t C^*_t + P^*_{I,t} I^*_t + M^*_t = R_{t-1} B^*_{H,t-1} NER_t^{-1} + R^*_{t-1} B^*_{F,t-1} - \frac{\phi_B}{2}\left(\frac{B^*_{H,t}}{NER_t P^*_t}\right)^2 P_t + P^*_t R^*_{K,t} K^*_t + (1+\tau_W) \int_0^1 W^*_t(i) L^*_t(i)\, di + T^*_t + M^*_{t-1} \tag{B.11}$$

The capital accumulation equation is

$$K^*_{t+1} = I^*_t \left(1 - \frac{\phi_K}{2}\left(\frac{I^*_t}{I^*_{t-1}} - 1\right)^2\right) + (1-\delta) K^*_t \tag{B.12}$$

The household-wide first-order conditions are

$$\left(C^*_t - hC^*_{t-1}\right)^{-\sigma} = \Lambda^*_t \tag{B.13}$$

$$\Lambda^*_t + \phi_B \left(\frac{B^*_{H,t}}{NER_t P^*_t}\right) = \log(\eta_t) V'\left(\frac{B^*_{H,t}}{NER_t P^*_t}\right) + \beta R_t E_t \frac{\Lambda^*_{t+1}}{\pi^*_{t+1}} \frac{NER_t}{NER_{t+1}} \tag{B.14}$$

$$\Lambda^*_t = \log(\eta^*_t) V'\left(\frac{B^*_{F,t}}{P^*_t}\right) + \beta R^*_t E_t \frac{\Lambda^*_{t+1}}{\pi^*_{t+1}} \tag{B.15}$$

$$\mu \left(\frac{M^*_t}{P^*_t}\right)^{-\sigma_M} = \frac{\log(\eta^*_t)}{R^*_t} V'\left(\frac{B^*_{F,t}}{P^*_t}\right) + \left(\frac{R^*_t - 1}{R^*_t}\right) \Lambda^*_t \tag{B.16}$$

$$\frac{P^*_{I,t}}{P^*_t} \Lambda^*_t = Q^*_t \left[\left(1 - \frac{\phi_K}{2}\left(\frac{I^*_t}{I^*_{t-1}} - 1\right)^2\right) - \frac{I^*_t}{I^*_{t-1}} \phi_K \left(\frac{I^*_t}{I^*_{t-1}} - 1\right)\right] + \beta E_t Q_{t+1} \phi_K \left(\frac{I^*_{t+1}}{I^*_t} - 1\right)\left(\frac{I^*_{t+1}}{I^*_t}\right)^2 \tag{B.17}$$

$$Q_t^* = \beta E_t \left[Q_{t+1}^* (1-\delta) + \Lambda_{t+1}^* R_{K,t+1}^* \right] \tag{B.18}$$

There are similar first-order conditions for the foreign household. Note that we define

$$RER_t = \frac{NER_t P_t^*}{P_t} \tag{B.19}$$

B.2 The labor market

We assume that all of the household members consumer the same amount (perfect consumption insurance). Each household member of is a member of a union that supplies its type of labor, i. Labor is combined via

$$L_t = \left(\int_0^1 L_t(i)^{\frac{\nu_L - 1}{\nu_L}} di \right)^{\frac{\nu_L}{\nu_L - 1}}$$

to produce labor services, which go to the production sector. The aggregator that minimizes the cost of producing labor services is

$$W_t = \left(\int_0^1 W_t(i)^{1-\nu_L} di \right)^{\frac{1}{1-\nu_L}}$$

The demand for a given labor type is

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\nu_L} L_t$$

Unions negotiate their wage with probability $1 - \xi_W$. When they do, they maximize household utility taking demand curves for their labor as given. The first-order condition with respect to the wage is

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\Lambda_{t+j} \frac{1}{P_{t+j}} (1-\nu_L) \left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} (1+\tau_W) + \chi \nu_L \left(\left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} \frac{1}{\tilde{W}_t} \right] = 0$$

where $\tilde{W}_t$ is the chosen wage by a union that can updates its wage. This is simplified to be

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \left[\Lambda_{t+j} \frac{1}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1+\tau_W) \tilde{W}_t - \chi \left(\left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \right] = 0$$

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \left[\Lambda_{t+j} \frac{P_t^{1+\nu_L \phi}}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1+\tau_W) \tilde{w}_t^{1+\nu_L \phi} - \chi \left(\left[\frac{1}{w_{t+j}} \frac{1}{P_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \right] = 0$$

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \left[\Lambda_{t+j} \frac{P_t}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1+\tau_W) \tilde{w}_t^{1+\nu_L \phi} - \chi \left(\left[\frac{1}{w_{t+j}} \frac{P_t}{P_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \right] = 0$$

where $\tilde{w}_t$ is the real wage that is set by unions that optimize. Then, we can write

$$F_{W,t} \tilde{w}_t^{1+\nu_L \phi} = K_{W,t} \tag{B.20}$$

where

$$F_{W,t} = L_t \Lambda_t \frac{\nu_L - 1}{\nu_L} (1+\tau_W) + \beta \xi_W E_t \pi_{t+1}^{-1} \left(\frac{w_{t+1}}{w_t} \pi_{t+1} \right)^{\nu_L} F_{W,t+1} \tag{B.21}$$

and

$$K_{W,t} = \chi L_t^{1+\phi} w_t^{\phi \nu_L} + \beta \xi_W E_t \left(\frac{w_{t+1}}{w_t} \right)^{\nu_L} (\pi_{t+1})^{\nu_L (1+\phi)} K_{W,t+1}. \tag{B.22}$$

Then wages evolve so that

$$W_t = \left((1-\xi_W) \tilde{W}_t^{1-\nu_L} + \xi_W W_{t-1}^{1-\nu_L} \right)^{\frac{1}{1-\nu_L}}$$

which yields

$$w_t = \left((1-\xi_W) \tilde{w}_t^{1-\nu_L} + \xi_W \left(\frac{w_{t-1}}{\pi_t} \right)^{1-\nu_L} \right)^{\frac{1}{1-\nu_L}} \tag{B.23}$$

Note that in the case that $\xi_W = 0$, we have $\tilde{w}_t = w_t$ and

$$\Lambda_t \frac{\nu_L - 1}{\nu_L} (1+\tau_W) w_t = \chi L_t(i)^{\phi} = \chi L_t^{\phi}$$

so that if $\frac{\nu_L - 1}{\nu_L}(1+\tau_W) = 1$ we have the usual intratemporal Euler equation.

In the foreign economy, we have

$$F^*_{W,t}\left(\tilde{w}^*_t\right)^{1+\nu_L\phi} = K^*_{W,t} \tag{B.24}$$

$$F^*_{W,t} = L^*_t \Lambda^*_t \frac{\nu_L - 1}{\nu_L}(1+\tau_W) + \beta\xi_W E_t \left(\pi^*_{t+1}\right)^{-1}\left(\frac{w^*_{t+1}}{w^*_t}\pi^*_{t+1}\right)^{\nu_L} F_{W,t+1} \tag{B.25}$$

$$K^*_{W,t} = \chi\left(L^*_t\right)^{1+\phi}\left(w^*_t\right)^{\phi\nu_L} + \beta\xi_W E_t\left(\frac{w^*_{t+1}}{w^*_t}\right)^{\nu_L}\left(\pi^*_{t+1}\right)^{\nu_L(1+\phi)} K^*_{W,t+1} \tag{B.26}$$

$$w^*_t = \left((1-\xi_W)\left(\tilde{w}^*_t\right)^{1-\nu_L} + \xi_W\left(\frac{w^*_{t-1}}{\pi^*_t}\right)^{1-\nu_L}\right)^{\frac{1}{1-\nu_L}}. \tag{B.27}$$

B.3 Goods aggregators

In each country, perfectly competitive firms aggregate country-specific intermeidate inputs into $Y_{H,t}$, $Y_{F,t}$, $Y^*_{H,t}$, and $Y^*_{F,t}$. These intermediate inputs are used either for the creation of consumption, government purchases, or investment goods so that

$$C_{H,t} + G_{H,t} + I_{H,t} = Y_{H,t} \tag{B.28}$$

$$C_{F,t} + G_{F,t} + I_{F,t} = Y_{F,t} \tag{B.29}$$

$$C^*_{H,t} + G^*_{H,t} + I^*_{H,t} = X^*_{H,t} \tag{B.30}$$

$$C^*_{F,t} + G^*_{F,t} + I^*_{F,t} = X^*_{F,t} \tag{B.31}$$

The values $Y_{H,t}$ and $Y_{F,t}$ are are composites of goods purchased from monopolists by perfectly competitive firms who produce using

$$Y_{H,t} = \left(\int_0^1 X_{H,t}(i)^{\frac{\nu-1}{\nu}}\,di\right)^{\frac{\nu}{\nu-1}}$$

$$Y_{F,t} = \left(\int_0^1 X_{F,t}(i)^{\frac{\nu-1}{\nu}}\,di\right)^{\frac{\nu}{\nu-1}}$$

Demand curves are then of the form

$$X_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t} \tag{B.32}$$

$$X_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\nu} Y_{F,t}. \tag{B.33}$$

The zero profit condition, along with these demand curves, implies

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\nu}\,di\right)^{\frac{1}{1-\nu}} \tag{B.34}$$

Similarly,

$$P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\nu}\,di\right)^{\frac{1}{1-\nu}} \tag{B.35}$$

The foreign country is symmetric. Demand curves are then of the form

$$X^*_{H,t}(i) = \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\nu} X^*_{H,t} \tag{B.36}$$

$$X^*_{F,t}(i) = \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\nu} X^*_{F,t}. \tag{B.37}$$

The zero profit conditions, along with these demand curves, imply

$$P^*_{H,t} = \left(\int_0^1 P^*_{H,t}(i)^{1-\nu}\,di\right)^{\frac{1}{1-\nu}} \tag{B.38}$$

and

$$P_{F,t}^{*}=\left(\int_{0}^{1}P_{F,t}^{*}\left(i\right)^{1-\nu}di\right)^{\frac{1}{1-\nu}}. \tag{B.39}$$

B.4 Retailers

Final consumption goods, C_t, are created by combining goods from countries H and F ($C_{H,t}$ and $C_{F,t}$) using

$$C_{t}=\left(\omega^{1-\rho}\left(C_{H,t}\right)^{\rho}+\left(1-\omega\right)^{1-\rho}\left(C_{F,t}\right)^{\rho}\right)^{\frac{1}{\rho}} \tag{B.40}$$

Profits are given by

$$P_{t}\left(\omega^{1-\rho}\left(C_{H,t}\right)^{\rho}+\left(1-\omega\right)^{1-\rho}\left(C_{F,t}\right)^{\rho}\right)^{\frac{1}{\rho}}-P_{H,t}C_{H,t}-P_{F,t}C_{F,t}. \tag{B.41}$$

where $P_{H,t}$ is the nominal price of $C_{H,t}$, $P_{F,t}$ is the nominal price of $C_{F,t}$. Demand curves are then

$$C_{H,t}=\left(\frac{P_{H,t}}{P_{t}}\right)^{\frac{1}{\rho-1}}\omega C_{t} \tag{B.42}$$

$$C_{F,t}=\left(\frac{P_{F,t}}{P_{t}}\right)^{\frac{1}{\rho-1}}\left(1-\omega\right)C_{t}. \tag{B.43}$$

There is free entry for retailers, so profits are zero. Substituting demand curves into the profits expression yields

$$P_{t}=\left(\omega P_{H,t}^{\frac{\rho}{\rho-1}}+\left(1-\omega\right)\left(P_{F,t}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{B.44}$$

Government purchases are produced using the same technology so that

$$G_{t}=\left(\omega^{1-\rho}\left(G_{H,t}\right)^{\rho}+\left(1-\omega\right)^{1-\rho}\left(G_{F,t}\right)^{\rho}\right)^{\frac{1}{\rho}} \tag{B.45}$$

This implies demand curves of the form

$$G_{H,t}=\left(\frac{P_{H,t}}{P_{t}}\right)^{\frac{1}{\rho-1}}\omega G_{t} \tag{B.46}$$

and

$$G_{F,t}=\left(\frac{P_{F,t}}{P_{t}}\right)^{\frac{1}{\rho-1}}\left(1-\omega\right)G_{t}. \tag{B.47}$$

In addition, the price of government purchases is the same as the price of the consumption good. Investment goods are produced using the same technology so

Final consumption goods in the foreign country, C_t^*, are created by combining goods for countries H and F ($C_{H,t}^*$ and $C_{F,t}^*$) using

$$C_{t}^{*}=\left(\omega^{1-\rho}\left(C_{F,t}^{*}\right)^{\rho}+\left(1-\omega\right)^{1-\rho}\left(C_{H,t}^{*}\right)^{\rho}\right)^{\frac{1}{\rho}} \tag{B.48}$$

Profits are given by

$$P_{t}^{*}\left(\omega^{1-\rho}\left(C_{F,t}^{*}\right)^{\rho}+\left(1-\omega\right)^{1-\rho}\left(C_{H,t}^{*}\right)^{\rho}\right)^{\frac{1}{\rho}}-P_{F,t}^{*}C_{F,t}^{*}-P_{H,t}^{*}C_{H,t}^{*}. \tag{B.49}$$

where $P_{H,t}^*$ is the nominal price of $C_{H,t}^*$, $P_{F,t}^*$ is the nominal price of $C_{F,t}^*$. Demand curves are given by

$$C_{H,t}^{*}=\left(\frac{P_{H,t}^{*}}{P_{t}^{*}}\right)^{\frac{1}{\rho-1}}\left(1-\omega\right)C_{t}^{*} \tag{B.50}$$

$$C_{F,t}^{*}=\left(\frac{P_{F,t}^{*}}{P_{t}^{*}}\right)^{\frac{1}{\rho-1}}\omega C_{t}^{*} \tag{B.51}$$

The consumer price indexes are given by

$$P_{t}^{*}=\left(\omega\left(P_{F,t}^{*}\right)^{\frac{\rho}{\rho-1}}+\left(1-\omega\right)\left(P_{H,t}^{*}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \tag{B.52}$$

Government purchases are produced using the same technology so that

$$G_{t}^{*}=\left(\omega^{1-\rho}\left(G_{F,t}^{*}\right)^{\rho}+\left(1-\omega\right)^{1-\rho}\left(G_{H,t}^{*}\right)^{\rho}\right)^{\frac{1}{\rho}} \tag{B.53}$$

This implies demand curves of the form

$$G^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_t}\right)^{\frac{1}{\rho-1}} \omega G^*_t \tag{B.54}$$

and

$$G^*_{F,t} = \left(\frac{P^*_{F,t}}{P^*_t}\right)^{\frac{1}{\rho-1}} (1-\omega) G^*_t. \tag{B.55}$$

In addition, the price of government purchases is the same as the price of the consumption good.

B.5 Investment goods

Investment goods, I_t, are created by combining goods from countries H and F ($I_{H,t}$ and $I_{F,t}$) using

$$I_t = \left(\omega_I^{1-\rho} (I_{H,t})^\rho + (1-\omega_I)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1}{\rho}} \tag{B.56}$$

Profits are given by

$$P_{I,t} \left(\omega_I^{1-\rho} (I_{H,t})^\rho + (1-\omega_I)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1}{\rho}} - P_{H,t} I_{H,t} - P_{F,t} I_{F,t}. \tag{B.57}$$

where $P_{H,t}$ is the nominal price of $I_{H,t}$, $P_{F,t}$ is the nominal price of $I_{F,t}$. Note that we are imposing that the price of $I_{H,t}$ and $C_{H,t}$ be the same because they are the same input. Similarly for $I_{F,t}$ and $C_{F,t}$. First-order conditions are

$$P_{I,t} \left(\omega_I^{1-\rho} (I_{H,t})^\rho + (1-\omega_I)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1-\rho}{\rho}} \omega_I^{1-\rho} (I_{H,t})^{\rho-1} = P_{H,t}$$

$$P_{I,t} \left(\omega_I^{1-\rho} (I_{H,t})^\rho + (1-\omega_I)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1-\rho}{\rho}} (1-\omega_I)^{1-\rho} (I_{F,t})^{\rho-1} = P_{F,t}$$

Demand curves are then

$$I_{H,t} = \left(\frac{P_{H,t}}{P_{I,t}}\right)^{\frac{1}{\rho-1}} \omega_I I_t \tag{B.58}$$

$$I_{F,t} = \left(\frac{P_{F,t}}{P_{I,t}}\right)^{\frac{1}{\rho-1}} (1-\omega_I) I_t. \tag{B.59}$$

There is free entry for retailers, so profits are zero. Substituting demand curves into the profits expression yields

$$P_{I,t} = \left(\omega_I P_{H,t}^{\frac{\rho}{\rho-1}} + (1-\omega_I)(P_{F,t})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{B.60}$$

Because we set $\omega = \omega_I$, we have that $P_{I,t} = P_t$. Investment goods in the foreign country, I^*_t, are created by combining goods for countries H and F ($I^*_{H,t}$ and $I^*_{F,t}$) using

$$I^*_t = \left(\omega_I^{1-\rho} \left(I^*_{F,t}\right)^\rho + (1-\omega_I)^{1-\rho} \left(I^*_{H,t}\right)^\rho\right)^{\frac{1}{\rho}} \tag{B.61}$$

Profits are given by

$$P^*_{I,t} \left(\omega_I^{1-\rho} \left(I^*_{F,t}\right)^\rho + (1-\omega_I)^{1-\rho} \left(I^*_{H,t}\right)^\rho\right)^{\frac{1}{\rho}} - P^*_{F,t} I^*_{F,t} - P^*_{H,t} I^*_{H,t}. \tag{B.62}$$

where $P^*_{H,t}$ is the nominal price of $Y^*_{H,t}$, $P^*_{F,t}$ is the nominal price of $Y^*_{F,t}$. Again, we are imposing that the price of $Y^*_{H,t}$ and $I^*_{H,t}$ are the same. Similarly, the price of $Y^*_{F,t}$ and $I^I_{F,t}$ are the same. Demand curves are given by

$$I^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_{I,t}}\right)^{\frac{1}{\rho-1}} (1-\omega_I) I^*_t \tag{B.63}$$

$$I^*_{F,t} = \left(\frac{P^*_{F,t}}{P^*_{I,t}}\right)^{\frac{1}{\rho-1}} \omega_I I^*_t \tag{B.64}$$

The price indexe is given by

$$P^*_{I,t} = \left(\omega_I \left(P^*_{F,t}\right)^{\frac{\rho}{\rho-1}} + (1-\omega_I)\left(P^*_{H,t}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \tag{B.65}$$

Because we set $\omega = \omega_I$, we have that $P_{I,t} = P_t$.

B.6 Bond market clearing

Bonds are in zero net supply so

$$b_{H,t} + b^*_{H,t} = 0 \tag{B.66}$$

$$b_{F,t} + b^*_{F,t} = 0 \tag{B.67}$$

where $b_{H,t} \equiv B_{H,t}/P_t$, $b^*_{H,t} \equiv B^*_{H,t}/P_t$, $b_{F,t} \equiv B_{F,t}/P^*_t$, $b^*_{F,t} \equiv B^*_{F,t}/P^*_t$.

B.7 Monopolists

We introduce price stickiness as a Calvo-style price-setting friction. Monopolists are only able to update their price with probability ξ in each period. If they are not able to update their price, it remains the same as the period before. If monopolist i in the country H can update its price, it chooses $\tilde{P}_{H,t}(i)$ and $\tilde{P}^*_{H,t}(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} \Lambda_{t+j} \left\{ \left(\frac{\tilde{P}_{H,t}(i)}{P_{t+j}} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j} + \left(\frac{NER_{t+j}\tilde{P}^*_{H,t}(i)}{P_{t+j}} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}^*_{H,t}(i)}{P^*_{H,t+j}} \right)^{-\nu} Y^*_{H,t+j} \right\}$$

The FOC with respect to $\tilde{P}_{H,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{\tilde{P}_{H,t}}{P_t} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j} \right] = 0$$

Then we have

$$F_{H,t}\tilde{p}_{H,t} = K_{H,t} \tag{B.68}$$

where we define $F_{H,t}$ and $K_{H,t}$ as recursive sums so that

$$F_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j}$$

$$K_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j}.$$

These can be written as

$$F_{H,t} = \Lambda_t Y_{H,t} + \beta\xi E_t \pi^{-1}_{t+1} \pi^{\nu}_{H,t+1} F_{H,t+1} \tag{B.69}$$

$$K_{H,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y_{H,t} + \beta\xi E_t \pi^{\nu}_{H,t+1} K_{H,t+1} \tag{B.70}$$

where

$$\pi_{H,t} \equiv P_{H,t}/P_{H,t-1} \tag{B.71}$$

The price index for home goods in the home market is given by

$$P_{H,t} = \left((1-\xi)\tilde{P}^{1-\nu}_{H,t} + \xi P^{1-\nu}_{H,t-1} \right)^{\frac{1}{1-\nu}}$$

so that

$$p_{H,t} = \left((1-\xi)\tilde{p}^{1-\nu}_{H,t} + \xi \frac{p^{1-\nu}_{H,t-1}}{\pi^{1-\nu}_t} \right)^{\frac{1}{1-\nu}} \tag{B.72}$$

The FOC with respect to $\tilde{P}^*_{H,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{NER_{t+j}}{NER_t} \frac{NER_t P^*_t}{P_t} \tilde{p}^*_{H,t} \frac{P_t}{P_{t+j}} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu} Y^*_{H,t+j} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu} Y^*_{H,t+j} \right] = 0$$

Then we have

$$F^*_{H,t} RER_t \tilde{p}^*_{H,t} = K^*_{H,t} \tag{B.73}$$

where

$$F^*_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{NER_{t+j}}{NER_t} \frac{P_t}{P_{t+j}} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu} Y^*_{H,t+j}$$

$$K^*_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu} Y^*_{H,t+j}.$$

These can be written as

$$F^*_{H,t} = \Lambda_t Y^*_{H,t} + \beta\xi E_t \frac{NER_{t+1}}{NER_t} \pi^{-1}_{t+1} \left(\pi^*_{H,t+1} \right)^{\nu} F^*_{H,t+1} \tag{B.74}$$

$$K^*_{H,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y^*_{H,t} + \beta\xi E_t \left(\pi^*_{H,t+1} \right)^{\nu} K^*_{H,t+1} \tag{B.75}$$

where

$$\pi_{H,t}^{*} \equiv P_{H,t}^{*}/P_{H,t-1}^{*} \tag{B.76}$$

. The price index for home goods in the foreign market is given by

$$P_{H,t}^{*} = \left((1-\xi) \left(\tilde{P}_{H,t}^{*} \right)^{1-\nu} + \xi \left(P_{H,t-1}^{*} \right)^{1-\nu} \right)^{\frac{1}{1-\nu}}$$

so that

$$p_{H,t}^{*} = \left((1-\xi) \left(\tilde{p}_{H,t}^{*} \right)^{1-\nu} + \xi \frac{\left(p_{H,t-1}^{*} \right)^{1-\nu}}{\left(\pi_{t}^{*} \right)^{1-\nu}} \right)^{\frac{1}{1-\nu}} \tag{B.77}$$

The foreign firms are symmetric symmetric. If monopolist i can update its price, it chooses $\tilde{P}_{F,t}^{*}(i)$ and $\tilde{P}_{F,t}(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^{*} \left\{ \left(\frac{\tilde{P}_{F,t}^{*}(i)}{P_{t+j}^{*}} (1+\tau_X) - MC_{t+j}^{*} \right) \left(\frac{\tilde{P}_{F,t}^{*}(i)}{P_{F,t+j}^{*}} \right)^{-\nu} Y_{F,t+j}^{*} + \left(\frac{\tilde{P}_{F,t}(i)}{NER_{t+j} P_{t+j}^{*}} (1+\tau_X) - MC_{t+j}^{*} \right) \left(\frac{\tilde{P}_{F,t}(i)}{P_{F,t+j}} \right)^{-\nu} Y_{F,t+j} \right\}$$

The FOC with respect to $\tilde{P}_{F,t}^{*}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j}^{*} \left[\frac{\tilde{P}_{F,t}^{*}}{P_t^{*}} \frac{P_t^{*}}{P_{t+j}^{*}} \left(\frac{P_{F,t}^{*}}{P_{F,t+j}^{*}} \right)^{-\nu} Y_{F,t+j}^{*} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j}^{*} \left(\frac{P_{F,t}^{*}}{P_{F,t+j}^{*}} \right)^{-\nu} Y_{F,t+j}^{*} \right] = 0.$$

We write this as

$$F_{F,t}^{*} \tilde{p}_{F,t}^{*} = K_{F,t}^{*} \tag{B.78}$$

where

$$F_{F,t}^{*} = \Lambda_t^{*} Y_{F,t}^{*} + \beta\xi E_t \left(\pi_{t+1}^{*} \right)^{-1} \left(\pi_{F,t+1}^{*} \right)^{\nu} F_{F,t+1}^{*} \tag{B.79}$$

and

$$K_{F,t}^{*} = \Lambda_t^{*} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t^{*} Y_{F,t}^{*} + \beta\xi E_t \left(\pi_{F,t+1}^{*} \right)^{\nu} K_{F,t+1}^{*}. \tag{B.80}$$

where

$$\pi_{F,t}^{*} \equiv P_{F,t}^{*}/P_{F,t-1}^{*} \tag{B.81}$$

. The price index implies

$$p_{F,t}^{*} = \left((1-\xi) \left(\tilde{p}_{F,t}^{*} \right)^{1-\nu} + \xi \frac{\left(p_{F,t-1}^{*} \right)^{1-\nu}}{\left(\pi_{t}^{*} \right)^{1-\nu}} \right)^{\frac{1}{1-\nu}} \tag{B.82}$$

The FOC with respect to $\tilde{P}_{F,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j}^{*} \left[\frac{NER_t}{NER_{t+j}} \frac{P_t}{NER_t P_t^{*}} \tilde{p}_{F,t} \frac{P_t^{*}}{P_{t+j}^{*}} \left(\frac{P_{F,t}}{P_{F,t+j}} \right)^{-\nu} Y_{F,t+j} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j}^{*} \left(\frac{P_{F,t}}{P_{F,t+j}} \right)^{-\nu} Y_{F,t+j} \right] = 0$$

We can write this as

$$F_{F,t} \frac{\tilde{p}_{F,t}}{RER_t} = K_{F,t} \tag{B.83}$$

where

$$F_{F,t} = \Lambda_t^{*} Y_{F,t} + \beta\xi E_t \frac{NER_t}{NER_{t+1}} \left(\pi_{t+1}^{*} \right)^{-1} \left(\pi_{F,t+1} \right)^{\nu} F_{F,t+1} \tag{B.84}$$

and

$$K_{F,t} = \Lambda_t^{*} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t^{*} Y_{F,t} + \beta\xi E_t \left(\pi_{F,t+1} \right)^{\nu} K_{F,t+1}. \tag{B.85}$$

where

$$\pi_{F,t} \equiv P_{F,t}/P_{F,t-1} \tag{B.86}$$

The price index implies that

$$p_{F,t} = \left((1-\xi) \left(\tilde{p}_{F,t} \right)^{1-\nu} + \xi \frac{\left(p_{F,t-1} \right)^{1-\nu}}{\left(\pi_t \right)^{1-\nu}} \right)^{\frac{1}{1-\nu}} \tag{B.87}$$

B.8 Marginal cost

Monopolists produce with technology so that

$$X_{H,t}(i) + X_{H,t}^{*}(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha}$$

$$X_{F,t}(i) + X^*_{F,t}(i) = A^*_t K^*_t(i)^\alpha L^*_t(i)^{1-\alpha}$$

where A_t and A^*_t are stochastic processes and, in a slight abuse of notation, the (i) means the amount hired by a particular monopolist. Cost minimization implies

$$R_{K,t} = \alpha MC_t A_t (K_t)^{\alpha-1} (L_t)^{1-\alpha} \tag{B.88}$$

$$\frac{W_t}{P_t} = (1-\alpha) MC_t A_t K^\alpha_t (L_t)^{-\alpha} \tag{B.89}$$

and

$$R^*_{K,t} = \alpha MC^*_t A^*_t (K^*_t)^{\alpha-1} (L^*_t)^{1-\alpha} \tag{B.90}$$

$$\frac{W^*_t}{P^*_t} = (1-\alpha) MC^*_t A^*_t (K^*_t)^\alpha (L^*_t)^{-\alpha}. \tag{B.91}$$

B.9 Aggregation

Aggregating accross firms yields

$$\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t} di + \int_0^1 \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\nu} Y^*_{H,t} di = A_t \int_0^1 K_t(i)^\alpha (L_t(i))^{1-\alpha} di$$

so that

$$d_{H,t} Y_{H,t} + d^*_{H,t} Y^*_{H,t} = A_t K^\alpha_t L^{1-\alpha}_t \tag{B.92}$$

where the last equation follows without the (i)'s because all firms choose the same capital-to-labor ratio from the constant-returns-to-scale production technology and that we have a measure 1 of firms. Similarly

$$d_{F,t} Y_{F,t} + d^*_{F,t} Y^*_{F,t} = A_t (K^*_t)^\alpha (L^*_t)^{1-\alpha} \tag{B.93}$$

Here the dispersion terms can be written recursively as

$$d_{H,t} = (1-\xi) p^\nu_{H,t} (\tilde{p}_{H,t})^{-\nu} + \xi \pi^\nu_{H,t} d_{H,t-1} \tag{B.94}$$

$$d^*_{H,t} = (1-\xi) \left(p^*_{H,t}\right)^\nu \left(\tilde{p}^*_{H,t}\right)^{-\nu} + \xi \left(\pi^*_{H,t}\right)^\nu d^*_{H,t-1}, \tag{B.95}$$

$$d^*_{F,t} = (1-\xi) \left(p^*_{F,t}\right)^\nu \left(\tilde{p}^*_{F,t}\right)^{-\nu} + \xi \left(\pi^*_{F,t}\right)^\nu d^*_{F,t-1}, \tag{B.96}$$

$$d_{F,t} = (1-\xi) (p_{F,t})^\nu (\tilde{p}_{F,t})^{-\nu} + \xi (\pi_{F,t})^\nu d_{F,t-1}. \tag{B.97}$$

B.10 Government

The monetary authority follows a Taylor rule

$$R_t = (R_{t-1})^\gamma \left(R \pi^{\theta_\pi}_t\right)^{1-\gamma} \exp(\epsilon_{R,t}) \text{ where } \theta_\pi > 1 \tag{B.98}$$

or alternatively follows a money growth rule

$$\log\left(\frac{M_t}{M_{t-1}}\right) = \log(x_{M,t}) \tag{B.99}$$

where

$$\log(x_{M,t}) = \rho_{x_M} \log(x_{M,t-1}) + \sigma_{x_M} \epsilon_{x_M,t} \tag{B.100}$$

The fiscal authority balances its budget with lump sum taxes. The foreign monetary authority follows a Taylor rule

$$R^*_t = \left(R^*_{t-1}\right)^\gamma \left(R (\pi^*_t)^{\theta_\pi}\right)^{1-\gamma} \exp\left(\epsilon^*_{R,t}\right) \text{ where } \theta_\pi > 1 \tag{B.101}$$

or alternatively follows a money growth rule

$$\log\left(\frac{M^*_t}{M^*_{t-1}}\right) = \log\left(x^*_{M,t}\right) \tag{B.102}$$

where

$$\log\left(x^*_{M,t}\right) = \rho_{x_M} \log\left(x^*_{M,t-1}\right) + \sigma_{x_M} \epsilon^*_{x_M,t} \tag{B.103}$$

The fiscal authority balances its budget with lump sum taxes. We also consider an NER targeting rule where

$$R_t^* = R\left(NER_t^{-\theta_{NER}}\right)\exp\left(\varepsilon_{R,t}^*\right) \tag{B.104}$$

B.11 Asset-market completeness

When we have asset market completeness, it must be that $\phi_B = 0$. We assume that there is a complete set of Arrow securities. Let s_t denote the state of the world in time t and $s^t = \{s_t, s_{t-1}, \dots\}$. The household in country H prices the Arrow securities that pay off one unit of the H currency in state s_{t+1} so that

$$\frac{Q_t^{s_{t+1}}}{P_t}\Lambda_t = \beta\frac{\Lambda_{t+1}}{P_{t+1}}\Pr\left(s_{t+1}|s^t\right)$$

where $Q_t^{s_{t+1}}$ is the price at time t of the security that pays off at time $t+1$ in state s_{t+1}. The household in country F prices the Arrow securities that pay off one unit of the H currency in state s_{t+1} so that

$$\frac{Q_t^{s_{t+1}}}{NER_t P_t^*}\Lambda_t^* = \beta\frac{\Lambda_{t+1}^*}{NER_{t+1}P_{t+1}^*}\Pr\left(s_{t+1}|s^t\right).$$

Note that it is a global market, so the prices are the same. Then divide

$$\frac{NER_t P_t^*}{P_t}\frac{\Lambda_t}{\Lambda_t^*} = \frac{NER_{t+1}P_{t+1}^*}{P_{t+1}}\frac{\Lambda_{t+1}}{\Lambda_{t+1}^*}$$

$$RER_t\frac{\Lambda_t}{\Lambda_t^*} = RER_{t+1}\frac{\Lambda_{t+1}}{\Lambda_{t+1}^*}$$

Given that this must happen for every date and state,

$$RER_t\frac{\Lambda_t}{\Lambda_t^*} = \kappa \tag{B.105}$$

for all t.

B.12 Equilibrium

When asset markets are incomplete, an equilibrium determines the following 37 endogenous objects: C_t, $C_{H,t}$, $C_{F,t}$, $G_{H,t}$, $G_{F,t}$, Λ_t, L_t, $w_t \equiv \frac{W_t}{P_t}$, $Y_{H,t}$, $Y_{F,t}$, R_t, MC_t, π_t, $m_t \equiv \frac{M_t}{P_t}$, $b_{F,t}$, $b_{H,t}$, K_t, I_t, $I_{H,t}$, $I_{F,t}$, Q_t, $R_{K,t}$, $p_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\tilde{p}_{H,t}$, $F_{H,t}$, $K_{H,t}$, $d_{H,t}$, $\pi_{H,t}$, $\tilde{p}_{F,t}$, $F_{F,t}$, $K_{F,t}$, $d_{F,t}$, $\pi_{F,t}$, $\tilde{w}_t$, $F_{W,t}$, $K_{W,t}$, the 37 star versions, as well as $\Delta NER_t \equiv \frac{NER_t}{NER_{t-1}}$ and RER_t. To determine these 76 variables, we require that the following 76 equations hold: (B.3), (B.4), (B.5), (B.6), (B.7), (B.8), (B.9), (B.12), (B.13), (B.14), (B.15), (B.16), (B.17), (B.18), (B.19), (B.20), (B.21), (B.22), (B.23), (B.24), (B.25), (B.26), (B.27), (B.28), (B.29), (B.30), (B.31), (B.42), (B.43), (B.44), (B.46), (B.47), (B.50), (B.51), (B.52), (B.54), (B.55), (B.58), (B.59), (B.63), (B.64), (B.66), (B.67), (B.68), (B.69), (B.70), (B.71), (B.72), (B.73), (B.74), (B.75), (B.76), (B.77), (B.78), (B.79), (B.80), (B.81), (B.82), (B.83), (B.84), (B.85), (B.86), (B.87), (B.88), (B.89), (B.90), (B.91), (B.92), (B.93), (B.94), (B.95), (B.96), (B.97) along with either (B.98) and (B.101) or (B.99) and (B.102) or (B.98) and (B.104) . Finally, we use the home household budget constraint (B.2). The foreign household budget constraint can be ignored because of Walras' law. If we want complete markets, we use (B.105) instead of (B.2) and set $\phi_B = 0$. In addition, we replace equations (B.6) and (B.14) with the conditions that $b_{H,t} = b_{F,t} = 0$. If we want to exclude capital accumulation, we set $\alpha = 0$ and replace equations (B.3), (B.8), (B.9), and (B.88), by the conditions that $I_t = K_t = Q_t = R_{K,t} = 0$. Similarly, we replace equations (B.12), (B.17), (B.18), and (B.90) by the conditions that $I_t^* = K_t^* = Q_t^* = R_{K,t}^* = 0$.

B.13 Steady State

To determine steady state, we assume that target inflation in both countries is 1. So, $\pi = \pi^* = 1$. The intertemporal Euler equations determine $R = R^* = \beta^{-1}$. We normalized $L = L^* = 1$. From the definition of steady state, $\Delta NER = 1$. We define initial conditions so that $RER = 1$. Firm optimaly and symmetry of the equilibrium, $p_H = p_H^* = p_F = p_F^* = 1$. As a result, $p_I = p_I^* = 1$ and $Q = \Lambda$. Marginal cost is given by

$$MC = \frac{\nu - 1}{\nu}(1 + \tau_X)$$

The rental rate of capital is

$$R_K = \frac{1-\beta(1-\delta)}{\beta}$$

So that

$$\frac{K}{X_H + X_H^*} = \frac{\beta\alpha MC}{1-\beta(1-\delta)}$$

Then

$$\frac{I}{X_H + X_H^*} = \delta \frac{K}{X_H + X_H^*}$$

Since

$$X_H + X_H^* = K^\alpha$$

$$(X_H + X_H^*)^{1-\alpha} = \left(\frac{K}{X_H + X_H^*}\right)^\alpha$$

So that

$$X_H + X_H^* = \left(\frac{\beta\alpha\frac{\nu-1}{\nu}(1+\tau_X)}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} = X_F + X_F^*$$

where the last equality follows by symmetry. With this, we also have K. Demand curves imply

$$Y_H = Y_F \frac{\omega}{1-\omega}$$

and

$$Y_F^* = Y_H^* \frac{\omega}{1-\omega}$$

Similarly,

$$I_H = I_F \frac{\omega_I}{1-\omega_I}$$

and

$$I_F^* = I_H^* \frac{\omega_I}{1-\omega_I}$$

Then

$$Y_H + I_H + Y_H^* + I_H^* = X_H + X_H^*$$

Symmetry and the demand curves imply

$$Y_H + I_H + Y_H \frac{1-\omega}{\omega} + I_H \frac{1-\omega_I}{\omega_I} = X_H + X_H^*$$

$$Y_H \frac{1}{\omega} + I_H \frac{1}{\omega_I} = X_H + X_H^*$$

$$Y + I = X_H + X_H^*$$

So

$$\frac{Y}{K} = \frac{X_H + X_H^*}{K} - \delta$$

which gives us Y. Then,

$$\begin{aligned} Y_F &= (1-\omega)\,Y \\ Y_H &= \omega Y \\ Y_F^* &= \omega Y \\ Y_H^* &= (1-\omega)\,Y \end{aligned}$$

and

$$\begin{aligned} I_F &= (1-\omega_I)\,I \\ I_H &= \omega_I I \\ I_F^* &= \omega_I I \\ I_H^* &= (1-\omega_I)\,I \end{aligned}$$

Given G and G^* this gives us C and C^*, which determine Λ and Λ^*. The values of m and m^* are determined by the money demand equations. The values of w and w^* are determined by

$$w = (1-\alpha)\,MCK^\alpha.$$

Finally, we get χ from

$$\chi = \Lambda \frac{\nu_L - 1}{\nu_L}(1+\tau_W)\,w.$$

B.14 Equilibrium with no capital, flexible prices/wages, complete markets, technology shocks, and a Taylor rule

In addition, we assume that $h = 0$ and $\gamma = 0$ so that there are no state variables. Collect the relevant equations to determine $\hat{C}_t$, $\hat{\Lambda}_t$, $\hat{R}_t$, $\hat{\pi}_t$, $\hat{w}_t$, $\hat{L}_t$, $\hat{C}_{H,t}$, $\hat{C}_{F,t}$, $\hat{p}_{H,t}$, $\hat{p}_{F,t}$, $\widehat{MC}_t$, $\hat{Y}_{H,t}$, $\hat{Y}_{F,t}$, the star versions, and $\Delta\widehat{NER}_t$, $\widehat{RER}_t$. Here $\hat{x}_t \equiv \log(x_t/x)$ is the log deviation from steady state.

$$-\sigma\hat{C}_t = \hat{\Lambda}_t$$

$$-\sigma\hat{C}_t^* = \hat{\Lambda}_t^*$$

$$\hat{\Lambda}_t = \hat{R}_t + E_t\left(\hat{\Lambda}_{t+1} - \hat{\pi}_{t+1}\right)$$

$$\hat{\Lambda}_t^* = \hat{R}_t^* + E_t\left(\hat{\Lambda}_{t+1}^* - \hat{\pi}_{t+1}^*\right)$$

$$\hat{\Lambda}_t + \hat{w}_t = \phi\hat{L}_t$$

$$\hat{\Lambda}_t^* + \hat{w}_t^* = \phi\hat{L}_t^*$$

$$C_H\hat{C}_{H,t} = Y_H\hat{Y}_{H,t}$$

$$C_F\hat{C}_{F,t} = Y_F\hat{Y}_{F,t}$$

$$C_H^*\hat{C}_{H,t}^* = Y_H^*\hat{Y}_{H,t}^*$$

$$C_F^*\hat{C}_{F,t}^* = Y_F^*\hat{Y}_{F,t}^*$$

$$\hat{C}_{H,t} = \frac{1}{\rho-1}\hat{p}_{H,t} + \hat{C}_t$$

$$\hat{C}_{H,t}^* = \frac{1}{\rho-1}\hat{p}_{H,t}^* + \hat{C}_t^*$$

$$\hat{C}_{F,t} = \frac{1}{\rho-1}\hat{p}_{F,t} + \hat{C}_t$$

$$\hat{C}_{F,t}^* = \frac{1}{\rho-1}\hat{p}_{F,t}^* + \hat{C}_t^*$$

$$0 = \omega\hat{p}_{H,t} + (1-\omega)\,\hat{p}_{F,t}$$

$$0 = \omega\hat{p}_{F,t}^* + (1-\omega)\,\hat{p}_{H,t}^*.$$

$$\hat{p}_{H,t} = \widehat{MC}_t$$

$$\hat{p}_{H,t}^* + \widehat{RER}_t = \widehat{MC}_t$$

$$\hat{p}_{F,t} - \widehat{RER}_t = \widehat{MC}_t^*$$

$$\hat{p}_{F,t}^* = \widehat{MC}_t^*$$

$$\hat{w}_t = \widehat{MC}_t + \hat{A}_t$$

$$\hat{w}_t^* = \widehat{MC}_t^* + \hat{A}_t^*$$

$$Y_H\hat{Y}_{H,t} + Y_H^*\hat{Y}_{H,t}^* = \hat{A}_t + \hat{L}_t$$

$$Y_F\hat{Y}_{F,t} + Y_F^*\hat{Y}_{F,t}^* = \hat{A}_t^* + \hat{L}_t^*$$

$$\hat{R}_t = \theta_\pi\hat{\pi}_t$$

$$\hat{R}_t^* = \theta_\pi\hat{\pi}_t^*$$

$$\hat{\Lambda}_t^* - \hat{\Lambda}_t = \widehat{RER}_t$$

$$\widehat{RER}_t - \widehat{RER}_{t-1} = \Delta\widehat{NER}_t + \hat{\pi}_t^* - \hat{\pi}_t$$

First sub out $\hat{Y}_{H,t}$, $\hat{Y}_{H,t}^*$, $\hat{Y}_{F,t}$, and $\hat{Y}_{F,t}^*$ along with $\hat{\Lambda}_t$ and $\hat{\Lambda}_t^*$. Note that the change in NER, inflation and the nominal interest rate only enter 5 equations (they are block recursive). Delete them.

$$-\sigma\hat{C}_t + \hat{w}_t = \phi\hat{L}_t$$

$$-\sigma\hat{C}_t^* + \hat{w}_t^* = \phi\hat{L}_t^*$$

$$\hat{C}_{H,t} = \frac{1}{\rho-1}\hat{p}_{H,t} + \hat{C}_t$$

$$\hat{C}_{H,t}^* = \frac{1}{\rho-1}\hat{p}_{H,t}^* + \hat{C}_t^*$$

$$\hat{C}_{F,t} = \frac{1}{\rho - 1}\hat{p}_{F,t} + \hat{C}_t$$

$$\hat{C}^*_{F,t} = \frac{1}{\rho - 1}\hat{p}^*_{F,t} + \hat{C}^*_t$$

$$0 = \omega\hat{p}_{H,t} + (1-\omega)\,\hat{p}_{F,t}$$

$$0 = \omega\hat{p}^*_{F,t} + (1-\omega)\,\hat{p}^*_{H,t}.$$

$$\hat{p}_{H,t} = \widehat{MC}_t$$

$$\hat{p}^*_{H,t} + \widehat{RER}_t = \widehat{MC}_t$$

$$\hat{p}_{F,t} - \widehat{RER}_t = \widehat{MC}^*_t$$

$$\hat{p}^*_{F,t} = \widehat{MC}^*_t$$

$$\hat{w}_t = \widehat{MC}_t + \hat{A}_t$$

$$\hat{w}^*_t = \widehat{MC}^*_t + \hat{A}^*_t$$

$$C_H\hat{C}_{H,t} + C^*_H\hat{C}^*_{H,t} = \hat{A}_t + \hat{L}_t$$

$$C_F\hat{C}_{F,t} + C^*_F\hat{C}^*_{F,t} = \hat{A}^*_t + \hat{L}^*_t$$

$$-\sigma\hat{C}^*_t + \sigma\hat{C}_t = \widehat{RER}_t$$

Sub out w_t. w^*_t. $C_{H,t}$, $C_{F,t}$, $C^*_{H,t}$, and $C^*_{F,t}$. Also, use that $C_H = \frac{C}{Y}\omega$, $C_F = \frac{C}{Y}(1-\omega)$, $C^*_F = \frac{C}{Y}\omega$, $C^*_H = \frac{C}{Y}(1-\omega)$.

$$-\sigma\hat{C}_t + \widehat{MC}_t + \hat{A}_t = \phi\hat{L}_t$$

$$-\sigma\hat{C}^*_t + \widehat{MC}^*_t + \hat{A}^*_t = \phi\hat{L}^*_t$$

$$0 = \omega\hat{p}_{H,t} + (1-\omega)\,\hat{p}_{F,t}$$

$$0 = \omega\hat{p}^*_{F,t} + (1-\omega)\,\hat{p}^*_{H,t}.$$

$$\hat{p}_{H,t} = \widehat{MC}_t$$

$$\hat{p}^*_{H,t} + \widehat{RER}_t = \widehat{MC}_t$$

$$\hat{p}_{F,t} - \widehat{RER}_t = \widehat{MC}^*_t$$

$$\hat{p}^*_{F,t} = \widehat{MC}^*_t$$

$$\frac{C}{Y}\omega\left(\frac{1}{\rho-1}\hat{p}_{H,t} + \hat{C}_t\right) + \frac{C}{Y}(1-\omega)\left(\frac{1}{\rho-1}\hat{p}^*_{H,t} + \hat{C}^*_t\right) = \hat{A}_t + \hat{L}_t$$

$$\frac{C}{Y}(1-\omega)\left(\frac{1}{\rho-1}\hat{p}_{F,t} + \hat{C}_t\right) + \frac{C}{Y}\omega\left(\frac{1}{\rho-1}\hat{p}^*_{F,t} + \hat{C}^*_t\right) = \hat{A}^*_t + \hat{L}^*_t$$

$$-\sigma\hat{C}^*_t + \sigma\hat{C}_t = \widehat{RER}_t$$

Sub out $p_{H,t}$, $p^*_{H,t}$, $p_{F,t}$, $p^*_{F,t}$.

$$-\sigma\hat{C}_t + \widehat{MC}_t + \hat{A}_t = \phi\hat{L}_t$$

$$-\sigma\hat{C}^*_t + \widehat{MC}^*_t + \hat{A}^*_t = \phi\hat{L}^*_t$$

$$0 = \omega\widehat{MC}_t + (1-\omega)\left(\widehat{MC}^*_t + \widehat{RER}_t\right)$$

$$0 = \omega\widehat{MC}^*_t + (1-\omega)\left(\widehat{MC}_t - \widehat{RER}_t\right)$$

$$\frac{C}{Y}\omega\left(\frac{1}{\rho-1}\widehat{MC}_t + \hat{C}_t\right) + \frac{C}{Y}(1-\omega)\left(\frac{1}{\rho-1}\left(\widehat{MC}_t - \widehat{RER}_t\right) + \hat{C}^*_t\right) = \hat{A}_t + \hat{L}_t$$

$$\frac{C}{Y}(1-\omega)\left(\frac{1}{\rho-1}\left(\widehat{MC}^*_t + \widehat{RER}_t\right) + \hat{C}_t\right) + \frac{C}{Y}\omega\left(\frac{1}{\rho-1}\widehat{MC}^*_t + \hat{C}^*_t\right) = \hat{A}^*_t + \hat{L}^*_t$$

$$-\sigma\hat{C}^*_t + \sigma\hat{C}_t = \widehat{RER}_t$$

Now strategically subtract equations so as to identify differences, not levels.

$$-\sigma\left(\hat{C}_t - \hat{C}^*_t\right) + \left(\widehat{MC}_t - \widehat{MC}^*_t\right) + \left(\hat{A}_t - \hat{A}^*_t\right) = \phi\left(\hat{L}_t - \hat{L}^*_t\right)$$

$$0 = (2\omega - 1)\left(\widehat{MC}_t - \widehat{MC}^*_t\right) + 2(1-\omega)\,\widehat{RER}_t$$

$$\frac{C}{Y}\frac{1}{\rho-1}\left(\widehat{MC}_t-\widehat{MC}_t^*\right)+\frac{C}{Y}(2\omega-1)\left(\hat{C}_t-\hat{C}_t^*\right)-\frac{C}{Y}2(1-\omega)\frac{1}{\rho-1}\widehat{RER}_t=\left(\hat{A}_t-\hat{A}_t^*\right)+\left(\hat{L}_t-\hat{L}_t^*\right)$$

$$\sigma\left(\hat{C}_t-\hat{C}_t^*\right)=\widehat{RER}_t$$

Now sub out $\hat{C}_t-\hat{C}_t^*$.

$$-\widehat{RER}_t+\left(\widehat{MC}_t-\widehat{MC}_t^*\right)+\left(\hat{A}_t-\hat{A}_t^*\right)=\phi\left(\hat{L}_t-\hat{L}_t^*\right)$$

$$0=(2\omega-1)\left(\widehat{MC}_t-\widehat{MC}_t^*\right)+2(1-\omega)\widehat{RER}_t$$

$$\frac{C}{Y}\frac{1}{\rho-1}\left(\widehat{MC}_t-\widehat{MC}_t^*\right)+\frac{C}{Y}(2\omega-1)\frac{1}{\sigma}\widehat{RER}_t-\frac{C}{Y}2(1-\omega)\frac{1}{\rho-1}\widehat{RER}_t=\left(\hat{A}_t-\hat{A}_t^*\right)+\left(\hat{L}_t-\hat{L}_t^*\right)$$

Now sub out $\widehat{MC}_t-\widehat{MC}_t^*$.

$$-\frac{1}{2\omega-1}\widehat{RER}_t+\left(\hat{A}_t-\hat{A}_t^*\right)=\phi\left(\hat{L}_t-\hat{L}_t^*\right)$$

$$\frac{C}{Y}\left[(2\omega-1)\frac{1}{\sigma}-\frac{2(1-\omega)}{\rho-1}\frac{2\omega}{2\omega-1}\right]\widehat{RER}_t=\left(\hat{A}_t-\hat{A}_t^*\right)+\left(\hat{L}_t-\hat{L}_t^*\right)$$

Finally, sub out $\hat{L}_t-\hat{L}_t^*$

$$(1+\phi)\left(\frac{1}{2\omega-1}+\phi\frac{C}{Y}\left[\frac{2\omega-1}{\sigma}-\frac{2(1-\omega)}{\rho-1}\frac{2\omega}{2\omega-1}\right]\right)^{-1}\left(\hat{A}_t-\hat{A}_t^*\right)=\widehat{RER}_t$$

B.14.1 Overshooting

We just showed that

$$\widehat{RER}_t=\kappa\left(\hat{A}_t-\hat{A}_t^*\right).$$

The intertemporal Euler equations, the Taylor rule, and complete asset markets imply

$$-\widehat{RER}_t=\theta_\pi\left(\hat{\pi}_t-\hat{\pi}_t^*\right)+E_t\left(-\widehat{RER}_{t+1}-\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^*\right)\right).$$

Assuming that the that the A_t and A_t^* are independent AR(1) processes with autocorrelation ρ_A we have:

$$(\rho_A-1)\widehat{RER}_t=\theta_\pi\left(\hat{\pi}_t-\hat{\pi}_t^*\right)-E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^*\right).$$

Solve this forward

$$\left(\hat{\pi}_t-\hat{\pi}_t^*\right)=(\rho_A-1)E_t\sum_{j=0}^{\infty}\frac{1}{\theta_\pi^{1+j}}\widehat{RER}_{t+j}=\frac{\rho_A-1}{\theta_\pi-\rho_A}\widehat{RER}_t.$$

Notice that

$$\left|\frac{\rho_A-1}{\theta_\pi-\rho_A}\right|<1$$

if the Taylor-principle holds. So the inflation differential is less than the RER_t deviation from steady state. This means that, in response to a shock to the RER_t, the relative price level of the two countries moves by less than the movement in the RER. That is, the nominal exchange rate has to move in the *same* direction as the RER. Because the RER is mean reverting but the inflation differential remains the oposit sign as the deviation of the RER_t from its steady state value, the nominal exchange rate moves back toward it initial value, and we get the overshooting pattern.

B.14.2 Regression coefficients

We can write the multi-period change in the RER as

$$\log\left(RER_{t+h}\right)-\log\left(RER_t\right)=\log\left(NER_{t+h}\right)-\log\left(NER_t\right)+\sum_{k=1}^{h}\left(\log\left(\pi_{t+k}^*\right)-\log\left(\pi_{t+k}\right)\right)$$

Take expectations and use our definitions of log-deviations from steady state to get

$$E_t\widehat{RER}_{t+h}-\widehat{RER}_t=E_t\log\left(NER_{t+h}\right)-\log\left(NER_t\right)+\sum_{k=1}^{h}E_t\left(\hat{\pi}_{t+k}^*-\hat{\pi}_{t+k}\right)$$

$$E_t\widehat{RER}_{t+h}-\widehat{RER}_t=E_t\log\left(NER_{t+h}\right)-\log\left(NER_t\right)-\sum_{k=1}^{h}E_t\frac{\rho_A-1}{\theta_\pi-\rho_A}\widehat{RER}_{t+k}$$

$$\left(\rho_A^h - 1\right)\widehat{RER}_t = E_t \log\left(NER_{t+h}\right) - \log\left(NER_t\right) - \frac{\rho_A - 1}{\theta_\pi - \rho_A}\widehat{RER}_t \sum_{k=1}^{h} \rho_A^k$$

$$\left(\rho_A^h - 1\right)\widehat{RER}_t = E_t \log\left(NER_{t+h}\right) - \log\left(NER_t\right) - \frac{\rho_A - 1}{\theta_\pi - \rho_A}\widehat{RER}_t \sum_{k=1}^{h} \rho_A^k$$

$$\left(\rho_A^h - 1\right)\widehat{RER}_t = E_t \log\left(NER_{t+h}\right) - \log\left(NER_t\right) + \frac{\rho_A - \rho_A^{h+1}}{\theta_\pi - \rho_A}\widehat{RER}_t$$

$$-\frac{1 - \rho_A^h}{1 - \rho_A/\theta_\pi}\widehat{RER}_t = E_t \log\left(NER_{t+h}\right) - \log\left(NER_t\right)$$

Define

$$\beta_h^{NER} \equiv -\frac{1 - \rho_A^h}{1 - \rho_A/\theta_\pi}$$

This corresponds to our NER regression. Note that $\beta_h^{NER} < 0$, $\beta_{h+1}^{NER} < \beta_h^{NER}$, and

$$\lim_{h\to\infty} \beta_h^{NER} = -\frac{1}{1 - \rho_A/\theta_\pi} > 1$$

Now let's think about the relative-price regression. Note that

$$\sum_{k=1}^{h} E_t\left(\hat{\pi}_{t+k} - \hat{\pi}^*_{t+k}\right) = \frac{\rho_A - 1}{\theta_\pi - \rho_A}\sum_{k=1}^{h} E_t \widehat{RER}_{t+k}$$

$$-\sum_{k=1}^{h} E_t\left(\hat{\pi}^*_{t+k} - \hat{\pi}_{t+k}\right) = \frac{\rho_A - 1}{\theta_\pi - \rho_A}\widehat{RER}_t \sum_{k=1}^{h} \rho_A^k$$

$$E_t \log\left(\frac{P^*_{t+h}/P^*_t}{P_{t+h}/P_t}\right) = \frac{1 - \rho_A}{\theta_\pi - \rho_A}\frac{\rho_A - \rho_A^{h+1}}{1 - \rho_A}\widehat{RER}_t$$

$$E_t \log\left(\frac{P^*_{t+h}/P^*_t}{P_{t+h}/P_t}\right) = \frac{1 - \rho_A^h}{\theta_\pi/\rho_A - 1} R\hat{E}R_t$$

Define

$$\beta_h^{\pi} \equiv \frac{1 - \rho_A^h}{\theta_\pi/\rho_A - 1}$$

This corresponds to our relative-price regression. Note that $\beta_h^\pi > 0$, $\beta_{h+1}^\pi > \beta_h^\pi$, and

$$\lim_{h\to\infty} \beta_h^\pi = \frac{1}{\theta_\pi/\rho_A - 1}.$$

Now it is apparent that

$$\lim_{h\to\infty}\left(\beta_h^{NER} + \beta_h^\pi\right) = -1$$

and that

$$\beta_h^{NER} = -\beta_h^\pi - \left(1 - \rho_A^h\right)$$

$$\beta_h^{NER} + \beta_h^\pi = -\left(1 - \rho_A^h\right)$$