Optimal Beliefs, Asset Prices, and the Preference for Skewed Returns

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Introduction

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These three facts emerge in an exchange economy in which people have optimal expectations: trade-off the ex ante benefits of anticipatory utility against the ex post costs of basing investment decisions on biased beliefs

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2. ‘Biases’ to some extent situational

3. Optimal Expectations is an economic model of biases:
    a) cost is ex post cost of decision error
    b) benefit is expected (anticipatory) utility

Main findings

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3. Preference towards skewed assets: biasing up probability of states with lowest probabilities or lowest price 'cheapest' in terms of ex post distortion

4. Price effects: Skewed assets can have lower returns and equity premium can be larger if bad states have low prob

Outline

0.) Introduction and Outline

1.) The investor's problem and the economy

2.) Optimal expectations

3.) Portfolio choice

4.) Asset pricing

5.) Conclusion

1. Tastes and technology

1. Investors solve a two-period portfolio choice problem: invest in period 1, consume only in period 2

2. Uncertainty:
   S states, $\pi_s > 0$ for $s = 1$ to S

3. Assets:
   Complete set of Arrow-Debreu securities, p_s

4. The economy is an exchange economy:
   $\int_i c_s^i di = C_s$ in all states s, returns endogenous

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◇ expected utility flows in the future

⇒ **happier if more optimistic**

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◇ Distorted beliefs distort actions

⇒ **better outcomes if more rational**

3. Optimal beliefs balance these forces

◇ Beliefs maximize well-being: average felicity over states and time

For the Portfolio Choice Problem

	$t = 1$	$t = 2$
felicity in period 1		$\beta \widehat{E}[u(c)]$
felicity in period 2		$\beta u(c)$

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Actions maximize felicity: $\beta \widehat{E}[u(c)]$

Beliefs maximize well-being: $\mathcal{W} = \frac{1}{2} E[\beta \widehat{E}[u(c)] + \beta u(c)]$
subject to:
- agent behavior given these beliefs
- $\widehat{\pi}_s$ are probabilities
- $\widehat{\pi}_s = 0$ if $\pi_s = 0$

Discussion

A. If beliefs are objective, wellbeing = felicity

- ⋄ Only incentive to distort beliefs is anticipatory utility gain

B. Rational expectations are optimal only if

- ⋄ anticipatory utility does enter felicities or
- ⋄ anticipatory utility does not enter well-being $\mathcal{W}$.

C. Frictionless Extreme

D. Payoff: biases are endogenous

- ⋄ biases are small when distort behavior a lot
- ⋄ large when provide the most expected future utility

3. Portfolio choice

1. Two period problem:
   invest in 1, consume in 2; $u() = ln()$

2. Uncertainty:
   S states, $\pi_s > 0$ for $s = 1$ to S

3. S Arrow-Debreu securities:
   Prices p_s

4. $c \geq 0$ in all states

Portfolio choice: $\max_{\{c_s\}} \beta \sum_{s=1}^{S} \widehat{\pi}_s u(c_s)$

subject to $\sum_{s=1}^{S} p_s c_s = 1$

Solution: $c_s^* = \frac{\widehat{\pi}_s}{p_s}$

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Optimal beliefs: $\widehat{\pi}_s^*$ maximize well-being

$$\frac{1}{2} \underbrace{\beta \sum_{s=1}^{S} \widehat{\pi}_s \ln\left(\frac{\widehat{\pi}_s}{p_s}\right)}_{\text{exp felicity at } t=1} + \frac{1}{2} \underbrace{\beta \sum_{s=1}^{S} \pi_s \ln\left(\frac{\widehat{\pi}_s}{p_s}\right)}_{\text{avg felicity at } t=2}$$

subject to $\sum_{s=1}^{S} \widehat{\pi}_s = 1$

A. Optimal beliefs

Proposition There exists a set of optimal subjective probabilities $\{\widehat{\pi}^*\}_{s=1}^{S}$ and $0 < \widehat{\pi}_s^{OE} < 1$ for all s

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The first order conditions for beliefs:

$$\frac{\pi_s}{\widehat{\pi}_s} - \ln \frac{\pi_s}{\widehat{\pi}_s} = \mu - 1 + \ln \frac{p_s}{\pi_s}$$

where μ is the multiplier for the constraint $\sum_s \widehat{\pi}_s = 1$

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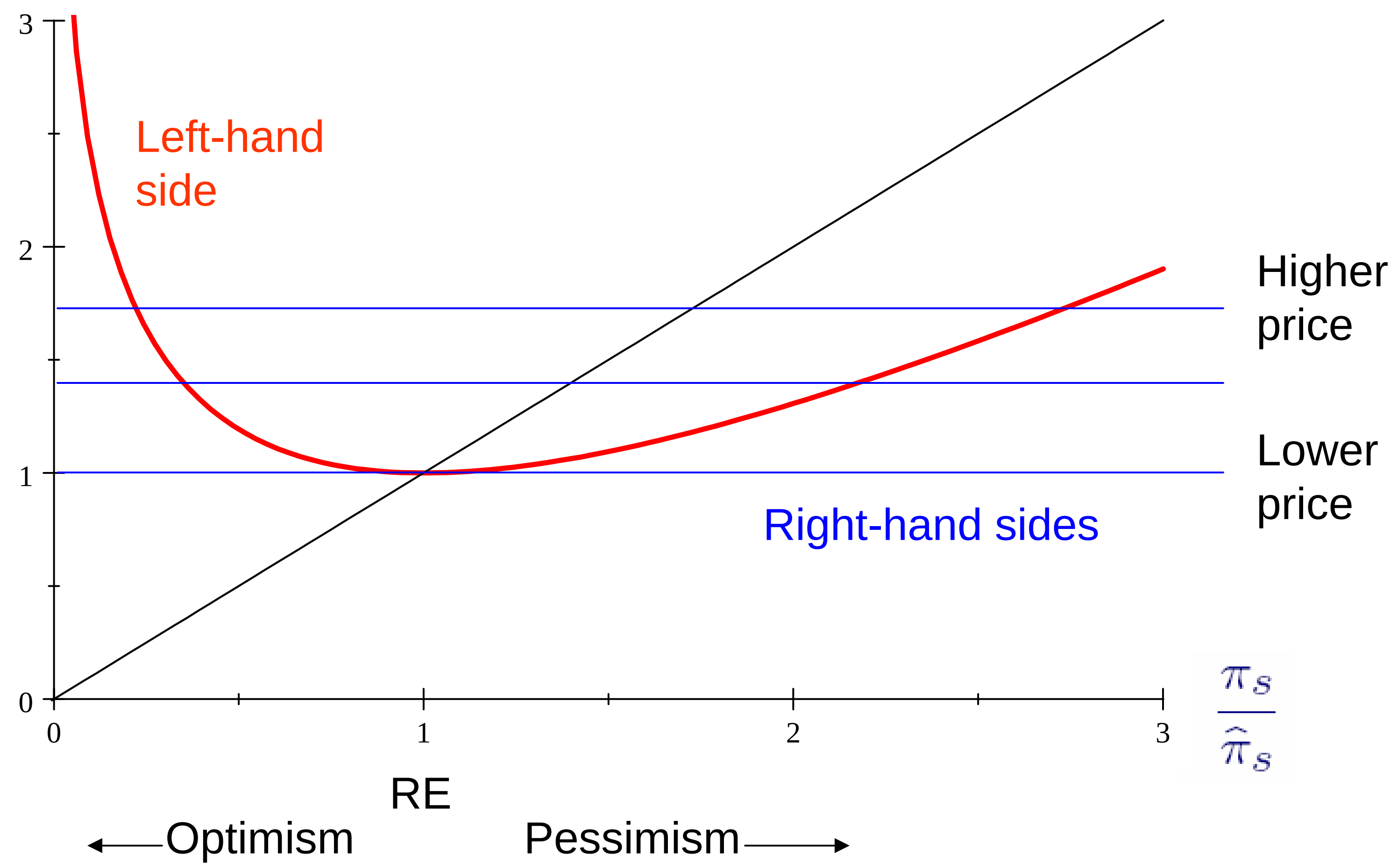

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The second-order conditions for beliefs imply:

$$\widehat{\pi}_s \left[1 - \frac{\pi_t}{\widehat{\pi}_t} \right] \leq \widehat{\pi}_t \left[\frac{\pi_s}{\widehat{\pi}_s} - 1 \right]$$

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$\Rightarrow$ There is at most one state t for which $\widehat{\pi}_t^* > \pi_t$

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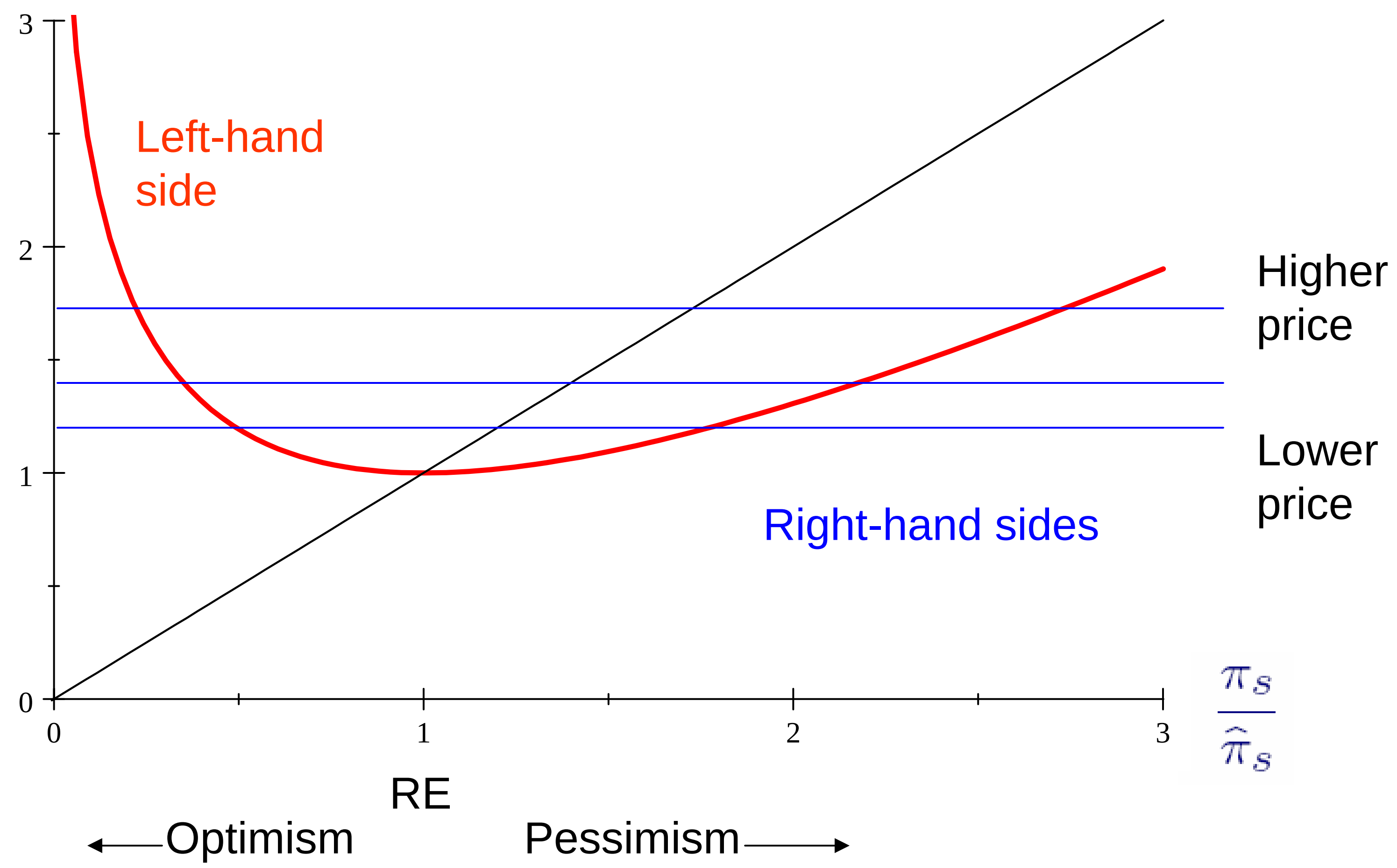
(i) there exists one state, s', such that $\widehat{\pi}_{s'} > \pi_{s'}$, and $\widehat{\pi}_s < \pi_s$ for all $s \neq s'$

(ii) for any two states s and t such that $\widehat{\pi}_s < \pi_s$ and $\widehat{\pi}_t < \pi_t$, $\widehat{\pi}_s/\pi_s < \widehat{\pi}_t/\pi_t$ if and only if $p_s/\pi_s > p_t/\pi_t$ (economywide stochastic discount factors) is larger

First-Order Conditions: $\frac{\pi_s}{\widehat{\pi}_s} - \ln \frac{\pi_s}{\widehat{\pi}_s} = \mu - 1 + \ln \frac{p_s}{\pi_s}$



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Proposition As long as $S > 2$ or $\pi_s \neq 1/2$ or $p_1 \neq p_2$:

(i) If prices are actuarially fair, $p_s/\pi_s = m$ for all s, the investor overestimates the probability of (one of) the state(s) with the lowest probability

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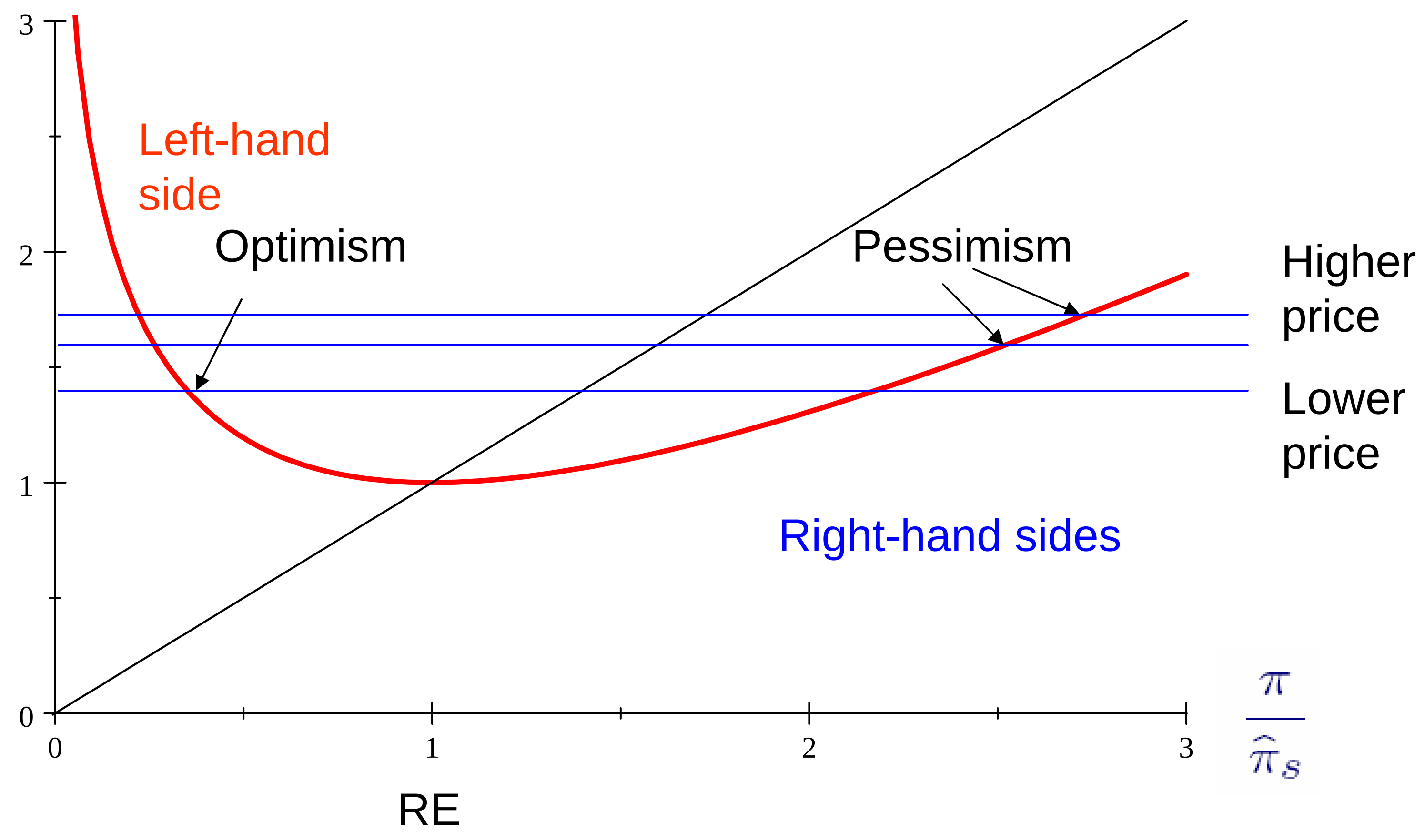

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Remark: Also have some combination and limiting results

B. Optimal portfolio choice

Case: actuarially fair prices, $p_s/\pi_s = m$ for all s, $S > 2$

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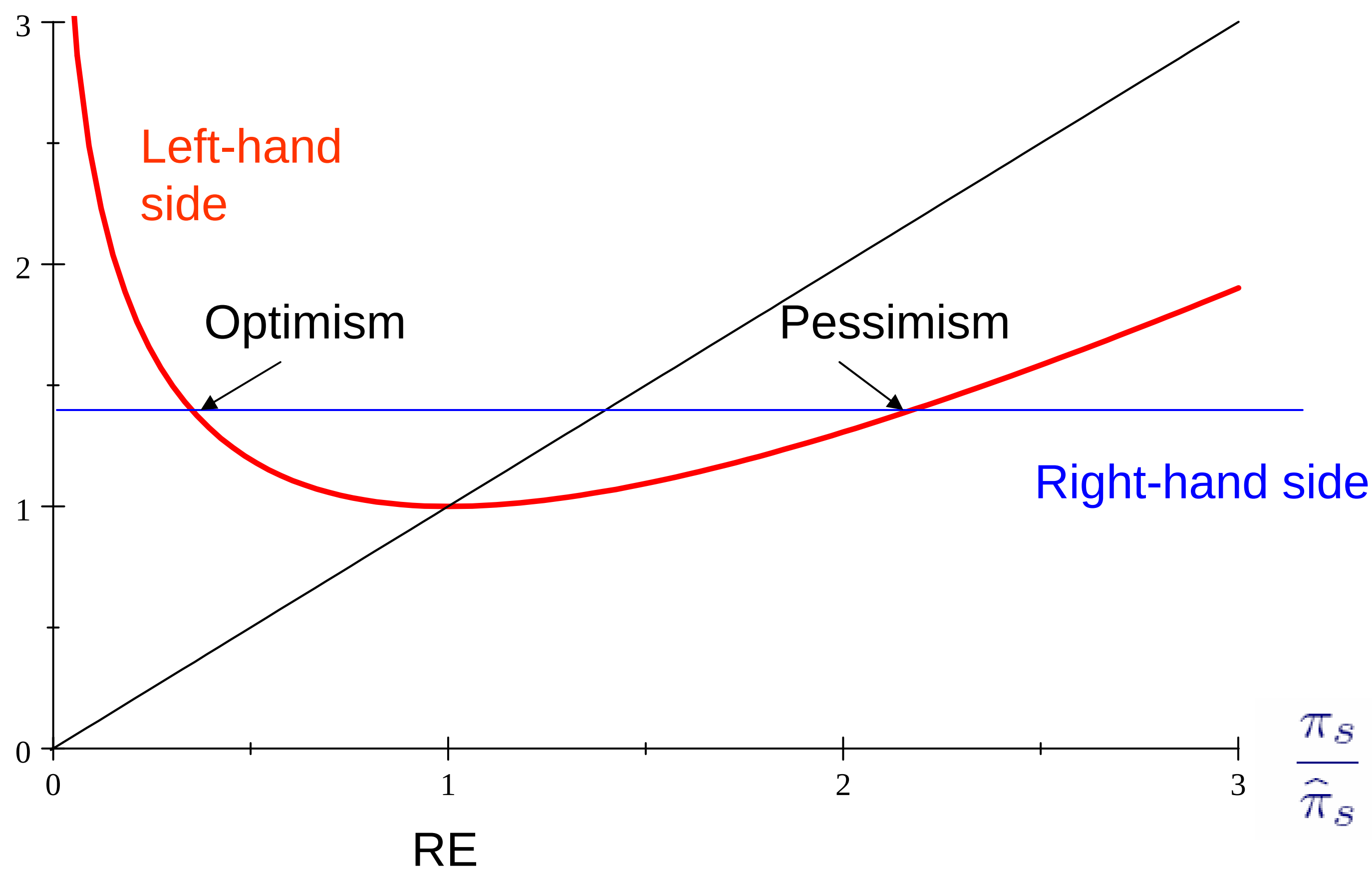

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B. Optimal portfolio choice

Case: actuarially fair prices, $p_s/\pi_s = m$ for all s, $S > 2$

Corollary 1: **(Preference for skewness)** The investor prefers the most skewed Arrow-Debreu assets: the investor buys $\bar{c}$ of the Arrow-Debreu security that pays off with the smallest probability and $\underline{c} < \bar{c}$ of each of the remaining securities

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Corollary 2: **(Two-fund separation)** The investor holds a portfolio consisting of the risk-free asset (an equal amount of all Arrow-Debreu securities) and an additional positive amount of one and only one Arrow-Debreu security

Remark: If $S > 2$ and all probabilities are equal, investor over-invests in the most skewed security (the cheapest), relative to investor with rational expectations

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Implications:

1. Investors well-diversified except for one asset

2. Overinvest in most skewed asset(s)

3. If $\pi_s = \pi$, portfolios can be heterogeneous and idiosyncratically skewed . . .

Aside: Portfolio choice in incomplete markets

Setup: Same two period problem with a general $u()$ satisfying Inada conditions except:

1. Two assets:
   a risk-free asset, return R; a risky asset, return $R + Z$

2. Uncertainty:
   $S > 2$ states, $\pi_s > 0$ for $s = 1$ to S, $Z_s < Z_{s+1}$, $Z_1 < 0 < Z_S$

Portfolio choice: Agent $\max_\alpha \beta \sum_{s=1}^{S} \widehat{\pi}_s u\left(R+\alpha Z_s\right)$

$$\text{FOC:} \quad 0=\sum_{s=1}^{S} \widehat{\pi}_s u'\left(R+\alpha Z_s\right) Z_s \qquad \Rightarrow \alpha^*(\widehat{\pi})$$

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$$\frac{1}{2} \underbrace{\beta \sum_{s=1}^{S} \widehat{\pi}_s u\left(R+\alpha^* Z_s\right)}_{\text{felicity at } t=1}+\quad \frac{1}{2} \beta \underbrace{\sum_{s=1}^{S} \pi_s u\left(R+\alpha^* Z_s\right)}_{\text{`average' utility at } t=2}$$

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$$\text{FOC:} \qquad \underbrace{\frac{\beta}{2}\left(u_S - u_{s'}\right)}_{\text{benefits of anticipation}} = \underbrace{\frac{\beta}{2} \sum_{s=1}^S \pi_s u'\left(R + \alpha^* Z_s\right) Z_s \frac{d\alpha^*}{d\widehat{\pi}_{s'}}}_{\text{costs of changed behavior}}$$

Proposition Excess risk taking due to optimism

(i) (BP 2005) Agents optimistic about states with high payout

if $\alpha^* > 0$, $\sum_{s=1}^{S} (\widehat{\pi}_s - \pi_s)\, u'(R + \alpha^* Z_s)\, Z_s > 0$;

if $\alpha^* < 0$, $\sum_{s=1}^{S} (\widehat{\pi}_s - \pi_s)\, u'(R + \alpha^* Z_s)\, Z_s < 0$.

(ii) (BP 2005) Agents go even more long (short) than agent with rational expectations **or** in the opposite direction

if $E[Z] > 0$, then $\alpha^* > \alpha^{RE} > 0$ or $\alpha^* < 0$;

if $E[Z] < 0$, then $\alpha^* < \alpha^{RE} < 0$ or $\alpha^* > 0$;

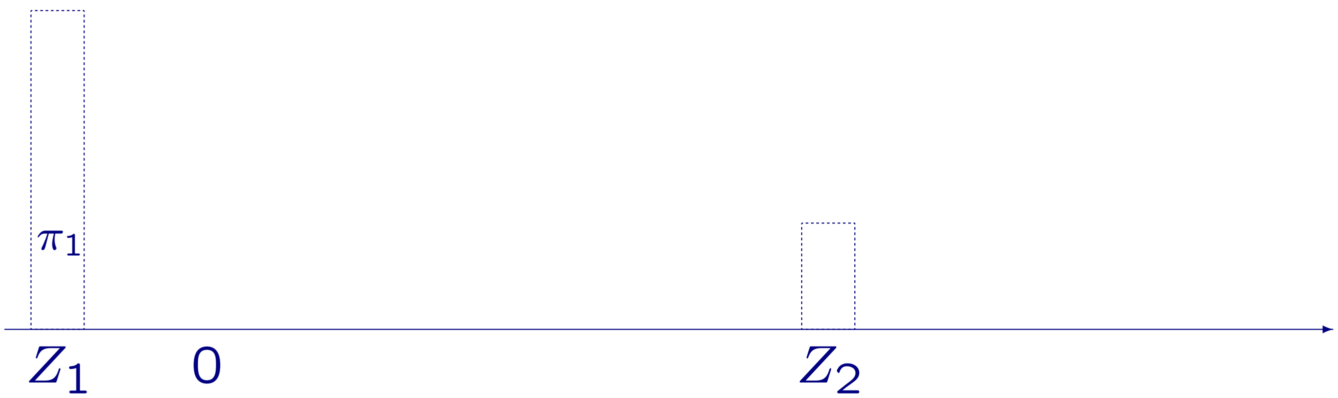
(iii) (Gollier 2005) Agents overestimate extreme probabilities

$\pi_1 > 0$ and $\pi_S > 0$ and $\pi_s = 0$ for $1 < s < S$

Also get preference for skewed returns

Setup:

- ◇ 2 states with payoffs: $Z_1 < 0 < Z_2$,
- ◇ hold variance and mean fixed and $E[Z] < 0$



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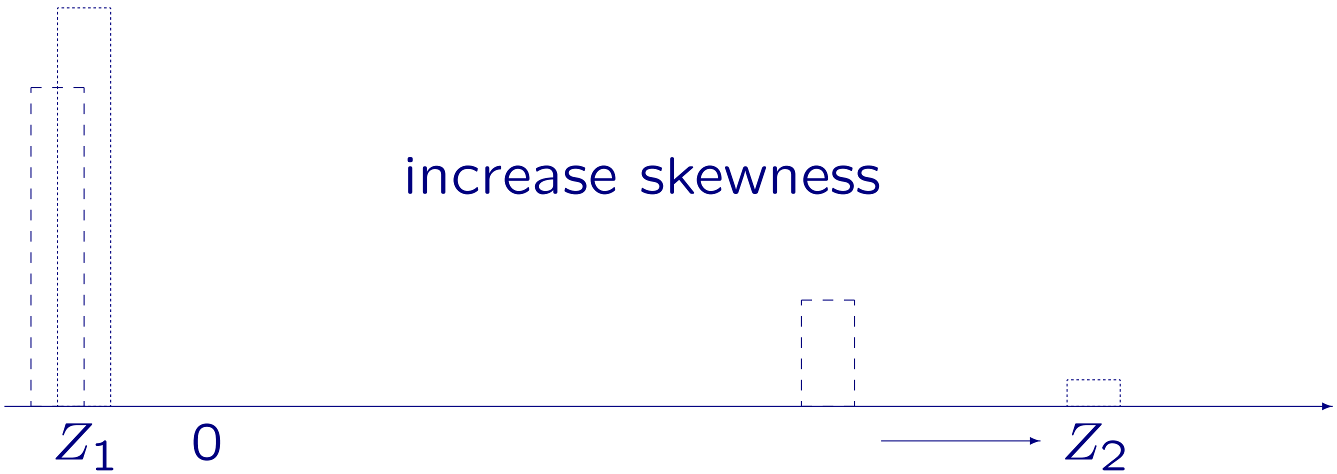

Proposition (BP 2005) An agent with an unbounded utility function holds some of the asset even though its mean payoff is negative if the payoff is sufficiently skewed.

Remark:

Agent goes long for large skewness even though $E[Z] < 0$, since

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4 General Equilibrium

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$\Rightarrow$ prices are fair, $p_s = p$

$\Rightarrow$ portfolios heterogeneous, idiosyncratic consumption risk

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A. Equally probable states and low aggregate risk
- $\Rightarrow$ prices are fair, $p_s = p$
- $\Rightarrow$ portfolios heterogeneous, idiosyncratic consumption risk
- $\Rightarrow$ aggregate risk reduces heterogeneity

B. There exist equilibria with unequally probably states such that
- $\Rightarrow$ more skewed assets have lower returns
- $\Rightarrow$ portfolios heterogeneous, idiosyncratic consumption risk

Definition of Equilibrium

Defintion: An optimal expectations equilibrium is a portfolio allocation $\left\{c_s^i\right\}_{s=1}^{S}$ and beliefs $\left\{\widehat{\pi}_s^i\right\}_{s=1}^{S}$ for each agent i and a price for each asset $\{p_s\}_{s=1}^{S}$ such that:

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1. each agent's portfolio is optimal given his beliefs and prices
2. each agent's beliefs maximize his well-being
3. the market for each asset clears

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Markets clear if $1/S$ investors optimistic about each state

Proposition

For $\pi_s = \pi$, there exists a set of aggregate endowment vectors, including $(C, C, ..., C)$, such that prices are actuarially fair and a fraction $\lambda_{s'}$ of investors buys $\bar{c}$ of Arrow-Debreu security that pays off in state s' and $\underline{c}$ of security for every other state

$\mathbf{C}$ is in this set if $\underline{c} \leq C_s \leq \bar{c}$ and $\sum_{s=1}^{S} C_s = \bar{c} + (S-1)\underline{c}$

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Remark: a fraction $\lambda_s = (C_s - \underline{c})/(\bar{c} - \underline{c})$ of investors buys $\bar{c}$ of the AD security for state s and $\underline{c}$ of other securities

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Remark: unique prices and pattern of beliefs

Remark: aggregate risk decreases heterogeneity in beliefs

B. Unequal probabilities and skewness priced

Initial setup: An economy with

1) $\underline{s}$ low-probability states with endowments C^A and $S - \underline{s}$ high probability states with endowments $C^B < C^A$

2) C^B and C^A such that markets clear with $1/\underline{s}$ investors optimistic about each low-probability state at actuarially fair prices

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Consider lower endowments in the low-probability states => excess demand for the assets in these states and lower expected returns for most skewed assets. Formally . . .

Proposition 6: **(Underperformance of skewed assets)** For a small reduction in $C^A - C^B$ such that p_s does not change for $s > \underline{s}$, p_s increases for $s \leq \underline{s}$ so that

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(i) the securities with the more skewed returns have lower expected returns than in a rational expectations equilibrium

(ii) the securities with the more skewed returns have relatively lower expected returns, $\pi^A/p_s < \pi^B/p_{s'}$ for all $s \leq \underline{s}$ and $s' > \underline{s}$

Discussion

Remark: underdiversification relative to objective, risk endogenously not pooled

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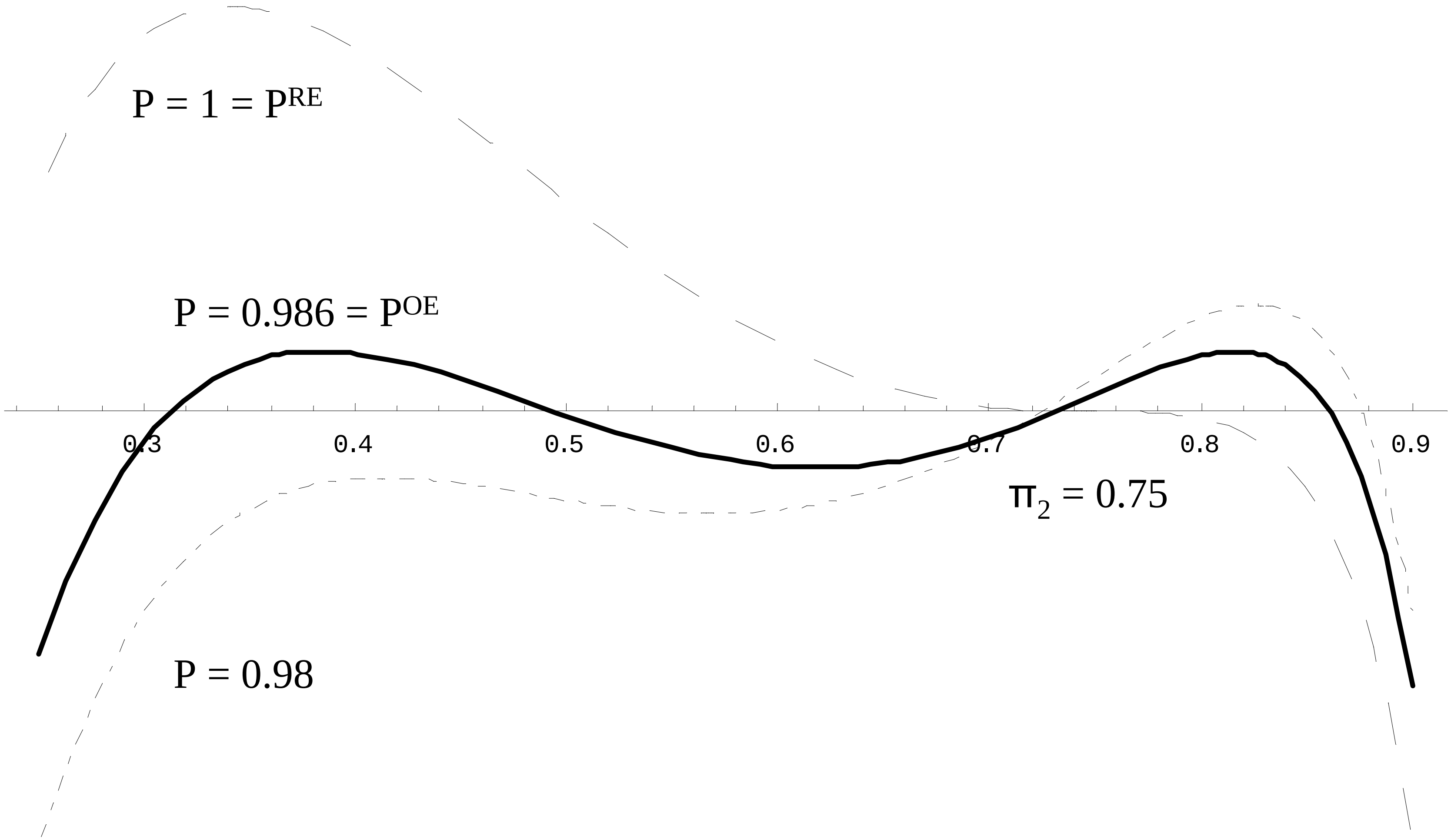
Conjecture: equity premium raised if bad states have low probabilities

Example (BP 2005)

- $S = 2$ and $C_1 = C_2$
- A fixed supply of 'bonds' with normalization $R = 1$
- A risky asset in zero net supply: $1 + Z_s = \frac{1+\varepsilon_s}{P_e} S = 2$
- $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ with $\gamma = 3$
- $\pi_1 = 0.25$, $\pi_2 = 0.75$
- $\varepsilon_1 = -0.6$, $\varepsilon_2 = 0.2$ so $P^{RE} = 1$.

Wellbeing as a Function of the Subjective Probability of a High Payout of the Risky Asset



In this example, as we vary the economic environment, beliefs change . . .

$P^{OE} > P^{RE} = 1$ if payoff is positively skewed (long-shots, IPO)
$P^{OE} < P^{RE} = 1$ if payoff is negatively skewed (stock market).

Conclusion

Rational expectations are sub-optimal:

- ◇ Agents with rational beliefs make the ex post best decisions
- ◇ but agents that care about the future can be happier
  with some optimism
- ◇ **Utility gain/loss determines biases**
- ◇ beliefs are most distorted when costs of errors are small
- ◇ beliefs are most distorted when "dream" benefits are large

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Matches facts on portfolios and asset prices

- beliefs biased towards small probability states and cheap states
- agents diversify except optimistic for one such state
- portfolios heterogeneous; idiosyncratic skewed returns
- low returns on skewed assets