A Note on Nonlinear Measurement of the Consumption Risk of the Stock Market

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This note shows that the results for aggregate data in Parker(2001) are tempered but not reversed by dropping the assumption of joint lognormality of consumption and returns.

The main optimality condition for portfolio choice:

$$E_t\left[m_{t+1}(z_{t+1} - r^f_{t,t+1})\right] = 0 \tag{1}$$

where m_{t+1} is the stochastic discount factor, $r^f_{t,t+1}$ is the risk-free real interest rate in the economy during $t+1$, and z_{t+1} is the return on stocks during $t+1$. Let $r_{t+1} \equiv z_{t+1} - r^f_{t,t+1}$.

The CCAPM assumes the stochastic discount factor is

$$m_{t+1} = \beta\left(C_{t+1}/C_t\right)^{-\gamma}$$

where C is aggregate consumption per capita. Parker (2001) suggests that if consumption is slow to adjust, a superior measure is instead given by using

$$m_{t+S+1} = \beta\left(C_{t+S+1}/C_t\right)^{-\gamma}.$$

for $S > 0$ (see Parker (2001) for the arguments).

The average equity premium is derived from the unconditional expectation of equation (1)

$$E\left[m_{t+S+1}(z_{t+1} - r^f_{t+1})\right] = 0$$

so that

$$E\left[r_{t+1}\right] = -\frac{Cov\left[r_{t+1}, (C_{t+S+1}/C_t)^{-\gamma}\right]}{E\left[(C_{t+S+1}/C_t)^{-\gamma}\right]}.$$

The following graph shows how the right hand side changes with γ and S for chain weighted nondurable consumption per capita and the excess return of the value-weighted return on the NYSE and three-month treasury bills (both from CRISP). For $S = 0$, the best estimate of risk aversion exceeds 75; for $S = 1$, $\gamma \cong 45$; for $S = 2$, $\gamma \cong 30$;for $S = 3$, $\gamma \cong 27$; for $S = 4, 5, 6,$ or 7, the best estiamtes of γ lie in between 20 and 25.

Reference

Parker, Jonathan A., 2001, "The Consumption Risk of the Stock Market," *Brookings Papers on Economic Activity*, 2, 279-348.

Average Excess Return by Risk Aversion and Horizon