

Problems for Chapter 6.

1. Our local heroes are going to the Rose Bowl next month, and we have a licence to sell official "Rose Bowl" sweatshirts. The sweatshirts cost \$10 each to produce, and we will sell them before the Rose Bowl at a price of \$25. At this price, the anticipated demand for our sweatshirts before the Rose Bowl should be a Normal random variable with mean 9000 and standard deviation 2000. After the Rose Bowl, things depend on whether our local heroes win or not. If they win, then we will continue to sell the sweatshirts for \$25, and demand in the month after the game will be a Normal random variable with mean 6000 and standard deviation 2000. If they lose, then we will cut the price to \$12.50 and demand at that price should be a Lognormal random variable with mean 2000 and standard deviation 1000. The probability of our local heroes winning the Rose Bowl game is 0.4. Undemanded sweatshirts will be discarded.

- (a) What production quantity would maximize our expected profit?
- (b) How would our optimal quantity change if we want to maximize our certainty equivalent, when we evaluate risky profits with a risk tolerance of \$50,000?
- (c) Assuming that we want to maximize our expected profit (as in part a), what would be the expected value of perfect information about whether our local heroes will win the Rose Bowl?

2. A decision-maker subjectively assessed the following certainty equivalents:

- (i) "a lottery that pays either \$10,000 or \$0, each with probability 1/2, would be worth \$3800 (for sure) to me."
- (ii) "a lottery that pays either \$5000 or \$0, each with probability 1/2, would be worth \$2200 to me."
- (iii) "a lottery that pays either \$1000 or \$0, each with probability 1/2, would be worth \$420 to me."

- (a) If he has constant risk tolerance, then what risk tolerance indexes would be implied by each of these three statements?
- (b) One of these statements implies a risk-tolerance index that is substantially different from the other two. If the decision-maker reassessed the certainty equivalent for the lottery described in this statement, what certainty equivalent would correspond to the same level of risk tolerance that he expressed in his other two statements?

3. A movie studio is about to release a new film. This studio has a regular policy of spending \$5 million on advertising in local media to promote any new film that they release.

But for some films that are considered to have strong potential, the studio may also invest in a special marketing campaign on national television to increase interest in the film.

The special marketing campaign on national television would cost an extra \$10 million (thus increasing the cost of promoting the film to a total of \$15 million).

The marketing staff at the studio has tried to forecast this new film's potential revenue.

The experts at the studio believe that, if the studio follows its regular marketing policy (without any special national television campaign), the studio's revenue from this film has 50% probability of being greater than \$18 million, 25% probability of being greater than \$30 million, and 25% probability of being less than \$12 million.

Let us assume that this revenue is drawn from a Generalized-Lognormal distribution.

The marketing experts also believe that, whatever the revenue would be under the regular marketing policy, having a special marketing campaign on national television would increase revenue by 50%.

A more complicated strategy has been suggested for this film: to first release it only in the New York metropolitan area (with only the regular local advertising), and then make a decision about national marketing based on the results of the first week in New York.

In previous films that were distributed with the regular policy, the ratio of the first week's revenue in New York divided by the total national revenue has had mean 0.02 and standard deviation 0.01.

So let us assume that the first week's revenue in New York is a random fraction, drawn from a Lognormal distribution with mean 0.02 and standard deviation 0.01, multiplied by the total national revenue that the film would earn with regular the marketing policy.

(a) Make a simulation model to analyze the studio's possible profit under these alternative strategies. Then answer parts (b) and (c) based on a table of output from at least 400 simulations of your model.

(b) Consider first the two simple alternatives (regardless of the NY results) of (b1) distributing the film with regular marketing, and (b2) distributing the film with a special marketing campaign on national television. For each of these two alternatives, estimate the mean and standard deviation of the studio's profit from distributing the film, and show the cumulative risk profile of the studio's profit from distributing the film. (Define profit as revenue minus marketing costs.)

(c) Now consider more complicated strategies of the following form:

"first release the film only in the New York metropolitan area, then spend an extra \$10 million for a special marketing campaign on national television if the first week's revenue in New York is greater than X, but otherwise (if less than X) continue with regular marketing only."

Consider different values of X from \$0 up to \$1 million at intervals of \$0.1 million (that is, consider $X = \$0$, $X = \$0.1$ million, $X = \$0.2$ millions, $X = \$0.3$ million, ..., $X = \$1.0$ million).

Make a table and a chart showing, for each of these strategies, the expected profit and the profit levels that have 10% and 90% cumulative probabilities for the studio from this film.

If the studio wants to maximize its expected profit, which of these strategies should it use?

4. The federal government is selling by auction the rights to develop the unexplored oil reserves in the Valdez Wilderness area. The total value of the oil in the Valdez Wilderness is an unknown quantity, which we may denote by V .

At the Alpha Petroleum Company, the staff geologists' estimate of this unknown value V is \$10 million. Of course, this is only an imperfect estimate. Past experience with such situations has suggested that the ratio of the actual value of the oil to the geologists' estimate of it may be a Lognormal random variable with mean 1 and standard deviation 0.5. So to simulate this unknown value V , we could multiply the geologist's \$10 million estimate by a Lognormal random variable with mean 1 and standard deviation 0.5.

Three other oil companies will be bidding against Alpha for the rights to this oil in the Valdez Wilderness. The geologists at each of these companies will be independently generating similarly imperfect estimates of the value V . So to simulate the estimates of these other companies, we could divide the random variable V by independent Lognormal random variables that have mean 1 and standard deviation 0.5.

In the auction, each company will submit a bid that depends in some way on its own geologists' estimate of V . The company that submits the highest bid will pay the amount of its bid and will win the rights to the oil in the Valdez Wilderness. So the winning company's profits will be the actual value V minus the bid that it submitted in the auction. The losing companies will get zero profits from the auction.

(a) Common practice in other auctions is for each company to compute its bid by multiplying its private estimate by 0.90; that is, each company's bid will be 10% less than its geologists' estimate of the value of oil in the area. Suppose that Alpha and its three competitors all use this 90% strategy to bid in the auction. Develop a simulation model to represent this situation. Using simulation data from at least 400 trials, estimate the expected profit for Alpha Petroleum in this auction, and show the cumulative distribution (or inverse cumulative distribution) for Alpha's profits. Also, estimate the conditionally expected value of value of the oil in the Valdez Wilderness (V) given that Alpha wins the auction in this scenario (with its bid of \$9 million).

(b) Suppose that the three other oil companies use the 90% bidding strategy described above, but Alpha Petroleum is considering other bids. According to your simulation data, what bid would maximize the expected profit for Alpha Petroleum? Show the cumulative distribution for Alpha's profits when it uses this optimal bid. Also, for this optimal bid, estimate the probability of Alpha winning the auction, and estimate the conditionally expected value of the oil in the Valdez Wilderness (V) when Alpha wins the auction.

(c) Find a bidding rule of the form "our bid should be equal to our geologists' estimate multiplied by K " such that, if the other three bidders were using this bidding rule, then it would also be optimal for Alpha Petroleum to apply the same rule (bidding \$10 million times K).