

Bidding Problems

1. Suppose that we are bidding to do a construction job, which is being allocated by a sealed-bid auction. We will win the auction if our bid is lower than any other bidder, in which case our profit will be the amount of our bid minus our cost of doing the job. (If we do not win, then we get zero profit from the auction, of course.)

We have substantial uncertainty about our cost of doing the job. We figure that, if the cost is equally likely to be greater or less than \$4.0 million, has probability 1/4 of being less than \$3.5 million, and has probability 3/4 of being less than \$5.0 million.

We also have uncertainty about the number of bidders. We feel sure that there will be at least one other bidder for the job, and we figure that the expected number of other bidders is 3. So let us suppose that the number of other bidders will be 1 plus a Poisson random variable with mean 2.

(Note: $1 + \text{POISINV}(0.999, 2) = 9$, and so the probability of ten or more opposing bidders under this assumption is negligible, being less than 0.001.)

When we think about the bids that our competitor's might submit, we figure that any single opposing bid is equally likely to be above or below \$7 million, with a probability 1/4 of being below \$5.5 million, and a probability 3/4 of being below \$9 million.

(a) Assume first that the cost and all the opposing bids are independent random variables. Based on a simulation analysis, estimate what optimal bid would maximize our expected profit from participating in this auction? For this bid, what would be our expected profit, and what would be our probability of winning? Make a chart, showing how our expected profit would vary with our bid, for bids from \$4 million to \$18 million.

(b) To check for the winner's curse effect, we should think about a possible relationship between the opposing bids and the actual cost of doing the job. We figure that, if we learned that one randomly-sampled opposing bid was actually equal \$9 million, then we would revise our beliefs about the unknown cost of the job and would say that the cost was equally likely to be above or below \$4.5 million (instead of \$4.0 million). What correlation parameter would be consistent with this assessment?

(c) Revise your model from part (a) by assuming that each opposing bid is related to the cost by the correlation parameter that you computed in part b. Based on a large simulation analysis of this revised model, estimate what optimal bid would maximize our expected profit from participating in this auction? For this bid, what would be our expected profit, and what would be our probability of winning? Make a chart, showing how our expected profit would vary with our bid, for bids from \$4 million to \$18 million.

2. Suppose that we are negotiating to buy the Lepton Company's rights to a process for making a chemical that has great medical potential, which Lepton has recently developed and patented. If the chemical is approved by the FDA as a drug, then its value to us will be a Normal random variable with mean \$120 million and standard deviation \$40 million. (If the chemical is not approved by the FDA, then its value to us will be zero.) Our best guess would be that the probability of FDA approval is 2/3.

But experts at Lepton would have much more accurate information about this probability. So let Q denote the probability of FDA approval as it is assessed by the experts at Lepton. For us, Q is a unknown quantity. Suppose that we have assessed a Beta distribution with mean 2/3 and standard deviation 0.2 for this unknown Q .

Lepton's willingness to sell us the patent will depend on their assessment of Q . If they think that Q is 1/2, then we figure that they would be willing to sell for \$40 million; but if they think that Q is 3/4 they would not sell for less than \$60 million. More generally, we figure that the risk-averse owners of Lepton would be willing to sell us the patent if our bid for the patent is more than $Q \cdot 80$ million, but otherwise they would refuse to sell.

Based on a simulation analysis, estimate our expected profit from submitting a bid of β for Lepton's patent, for values of β that range from \$0 to \$80 million.