

## THE VALDEZ WILDERNESS

The federal government is selling by auction the rights to develop the unexplored oil reserves in the Valdez Wilderness area. The total value of the oil in the Valdez Wilderness is an unknown quantity, which we may denote by  $V$ .

At the Alpha Petroleum Company, the staff geologists' estimate of this unknown value  $V$  is \$10 million. Of course, this is only an imperfect estimate. Past experience with such situations has suggested that the ratio of the actual value of the oil to the geologists' estimate of it may be a Lognormal random variable with mean 1 and standard deviation 0.5. So to simulate this unknown value  $V$ , we could multiply the geologist's \$10 million estimate by a Lognormal random variable with mean 1 and standard deviation 0.5.

Three other oil companies will be bidding against Alpha for the rights to this oil in the Valdez Wilderness. The geologists at each of these companies will be independently generating similarly imperfect estimates of the value  $V$ . So to simulate the estimates of these other companies, we could divide the random variable  $V$  by independent Lognormal random variables that have mean 1 and standard deviation 0.5.

In the auction, each company will submit a bid that depends in some way on its own geologists' estimate of  $V$ . The company that submits the highest bid will pay the amount of its bid and will win the rights to the oil in the Valdez Wilderness. So the winning company's profits will be the actual value  $V$  minus the bid that it submitted in the auction. The losing companies will get zero profits from the auction.

(a) Common practice in other auctions is for each company to compute its bid by multiplying its private estimate by 0.90; that is, each company's bid will be 10% less than its geologists' estimate of the value of oil in the area. Suppose that Alpha and its three competitors all use this 90% strategy to bid in the auction. Develop a simulation model to represent this situation. Using simulation data from at least 400 trials, estimate the expected profit for Alpha Petroleum in this auction, and show the cumulative distribution (or inverse cumulative distribution) for Alpha's profits.

Also, estimate the conditionally expected value of value of the oil in the Valdez Wilderness ( $V$ ) given that Alpha wins the auction in this scenario (with its bid of \$9 million).

(b) Suppose that the three other oil companies use the 90% bidding strategy described above, but Alpha Petroleum is considering other bids. According to your simulation data, what bid would maximize the expected profit for Alpha Petroleum? Show the cumulative distribution for Alpha's profits when it uses this optimal bid.

Also, for this optimal bid, estimate the probability of Alpha winning the auction, and estimate the conditionally expected value of the oil in the Valdez Wilderness ( $V$ ) when Alpha wins the auction.

(c) Find a bidding rule of the form "our bid should be equal to our geologists' estimate multiplied by  $K$ " such that, if the other three bidders were using this bidding rule, then it would also be optimal for Alpha Petroleum to apply the same rule (bidding \$10 million times  $K$ ).

## THE BATES RESTORATION (A)

Russ Gehrig, a construction general contractor, has decided to bid for the contract to do an extensive restoration of the old Bates mansion. The contract will be awarded to the bidder who submits the lowest bid, and the winner will be paid the amount of his bid at the completion of the job. Gehrig estimates that his cost to do the work described in the contract is equally likely to be above or below \$100,000, has probability 1/4 of being below \$86,000, and has probability 1/4 of being above \$120,000.

Gerhig knows that there will be at least one other bidder for the job, but he does not know how many. He has assessed the following probability distribution for the number of other bidders (not including himself):

<u># Other Bidders</u>	<u>Probability</u>
1	0.2
2	0.3
3	0.3
4	0.1
5	0.1

He also figures the each opposing bidder's bid is equally likely to be above or below \$140,000, has probability 1/4 of being below \$120,000, and has probability 1/4 of being above \$180,000.

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## THE BATES RESTORATION (B)

In part (A), we assumed that Gerhig's cost was independent of the other's bids. To scrutinize this assumption (looking for a winner's-curse effect), we could ask Gerhig the following question: "Imagine that you could see the bid of one other contractor for the Bates job, and the bid turned out to be \$180,000 (which is the 3/4-fractile point of your assessed distribution for these bids). If you got this information, would it affect your assessed median for your own cost of doing the job?" Suppose that, in answer to this question, he told us that learning about this relatively high bid would make him a bit more concerned about the possibility of having underestimated the cost, to the extent that he would re-assess his median cost to be about \$105,000.

Revise your model of Gerhig's decision to take account of this information, by making the opposing bids positively correlated with Gerhig's cost of doing the Bates job. In your model, assume that the opposing bids would still be conditionally independent of each other given his cost.

## GLOBAL SPORTS: A MODEL OF GROWTH AND MARKET SIZE

Century Publications is considering a proposal to launch a new magazine called "Global Sports" which would be devoted to international sports, with special features on competitive sports in different foreign countries that are not be familiar to most Americans. The fixed costs of producing the magazine would be \$4 million in the first year, \$3 million in the second year, and then \$2 million per year thereafter. In addition, the variable costs of printing and distributing the magazine would be \$8 per subscriber per year. Subscriptions to the magazine would be sold for \$25 per year.

Senior managers in Century's circulation department have tried to formulate some forecast of the potential for this new magazine, but predicting the number of subscribers for any new publication is always difficult. Survey data has indicated that there may be millions of people in America who have enough serious interest in international sports that they would enjoy this magazine, but this target audience is a dispersed and heterogeneous group, and it is hard to generate any kind of hard data that might indicate how many of them would actually become subscribers.

Whatever the potential market might be, subscriptions in the early years would almost certainly be substantially less than the long-run level. From past experience with other magazines, we can estimate that the logarithmic growth of subscriptions per year (defined in any year  $t$  to be the  $\text{LN}(s_{t+1}/s_t)$  where  $s_t$  is subscriptions in year  $t$ , averaged over the year, and  $s_{t+1}$  is subscriptions in the following year  $t+1$ ) have the following statistics:

Year $t$ (from launch):	1	2	3	4	5	6	7	8	9
$E(\text{LN}(s_{t+1}/s_t))$	.5	.3	.2	.1	0	0	0	0	0
$\text{Stdev}(\text{LN}(s_{t+1}/s_t))$	.3	.2	.15	.07	.04	.04	.04	.04	.04

The circulation managers felt comfortable assuming that growth of "Global Sports" would have statistics similar to these, with subscriptions climbing at first and then leveling off at some plateau level after five years. The question remained, how high would this plateau be. On this important question, there was great uncertainty. It seemed reasonable to guess that circulation in year 5 after launch would be equally likely to be above or below 100,000. Working a bit harder to estimate upper and lower quartile boundaries for year-5 circulation, the circulation managers reached a consensus on 50,000 and 200,000. That is, they assessed a 25% probability of subscriptions in year 5 being above 200,000, and they assessed a 25% probability of subscriptions in year 5 being below 50,000.

(a) The bottom line for Century management is the net present value of profits from the magazine over the 10 years following its launch, using a discount rate of 10% per year. With a simulation model, estimate the expected net present value, and show the cumulative distribution of this net present value, under the assumption that Century will publish this magazine for 10 years.

(b) Managers at Century are considering a strategy in which the critical decision about the continued life of this magazine would be made at the end of the first year. At that time, if we find that our number of subscribers in the first year is below some critical threshold, then we will stop publishing the magazine, thereby avoiding all costs and revenues for the magazine in year 2 and thereafter. On the other hand, if the number of subscribers in the first year is above this critical threshold, then we will make commitments to continue publishing the magazine indefinitely. Our objective is to choose this critical threshold so as to maximize the expected net present value of profits from the magazine over the next 10 years, using a discount rate of 10% per year. Based on a simulation model, estimate the optimal threshold for continuing the magazine beyond the first year. That is, what is the lowest number of subscribers in the first year such that you would recommend continuing to publish the magazine, rather than terminating it? Estimate the expected value and cumulative distribution of the net present value of profits under this optimal strategy.

(c) Can you suggest other termination strategies that Century should consider here?

## A PROBLEM OF LONG-TERM PORTFOLIO MANAGEMENT

A couple have \$60,000 that they want to invest for 10 years (anticipating their young children's future college expenses), and they have identified two mutual funds in which they plan to invest. In each year, the logarithmic rates of return for these funds are Multivariate Normal random variables.

The logarithmic rate of return for Fund 1 each year has mean 0.07 and standard deviation 0.25.

The logarithmic rate of return for Fund 2 each year has mean 0.06 and standard deviation 0.15.

The correlation coefficient between the two funds' logarithmic rates of return each year is 0.65.

The funds' rates of return in any one year are independent of their past and future rates of return.

The couple wants to evaluate the risky returns from their investments ten years from now using a constant risk tolerance of \$50,000.

(a) First let us consider plans in which the couple invests some amount  $\$X$  in Fund 1 now, and invests all the rest of their available cash  $\$60,000 - X$  in Fund 2 now.

To be specific, consider any amount  $\$X$  between \$0 and \$60,000 that is a multiple of \$10,000:

$X=0$ ;  $X=10,000$ ;  $X=20,000$ ;  $X=30,000$ ;  $X=40,000$ ;  $X=50,000$ ; and  $X=60,000$ .

Suppose that, after this initial investment, the couple will hold these investments for ten years, reinvesting all returns in each fund. Based on data from at least 400 simulations of this situation, make a table and a chart showing, for each of these seven possible values of  $X$ :

the expected total value of the couple's portfolio ten years from now,

the low amount that has cumulative probability 0.1 for the value of their portfolio 10 years from now, and the couple's certainty equivalent of their risky portfolio ten years from now (with constant risk tolerance \$50,000).

(b) Next, let us consider plans in which the couple invests equally in the two funds but also puts some cash into risk-free short-term bonds that pay 4% interest per year (compounded annually). That is, consider plans in which the couple invests some amount  $\$Y$  in Fund 1 now, invests the same amount  $\$Y$  in Fund 2 now, and invests the rest of their available cash  $\$60,000 - 2*Y$  in the 4% bonds.

To be specific, consider any amount  $Y$  between 0 and 30,000 that is a multiple of 5000:

$Y=0$ ;  $Y=5000$ ,  $Y=10,000$ ;  $Y=15,000$ ;  $Y=20,000$ ;  $Y=25,000$ ;  $Y=30,000$ .

Suppose that, after this initial investment, the couple will hold these investments for ten years, reinvesting all returns in each fund.

Make a table and a chart showing, for each of these seven possible values of  $Y$ , the couple's certainty equivalent of their risky portfolio ten years from now (with risk tolerance \$50,000).

(c) In parts (a) and (b) we assumed that all initial investments would be held with reinvestment for ten years. Now let us consider a more complex investment plan, as follows.

The couple initially invests \$30,000 in Fund 1 and another \$30,000 in Fund 2.

At the end of each year thereafter, if the value of their investment in either fund has grown larger than \$30,000 then they will sell the excess over \$30,000 and invest this excess in 4% bonds; but if their investment in the fund is worth less than \$30,000 at the end of the year then they will simply hold this investment into the next year. Thus they will begin each year with \$30,000 or less in each mutual fund, plus a growing balance in safe bonds that pay 4% annual interest.

For this investment plan, estimate the expected total final value of the couple's portfolio in 10 years, and the couple's certainty equivalent of their final portfolio in 10 years (with risk tolerance \$50,000). Also, show the cumulative risk profile for their total final portfolio value 10 years from now. (Ignore any effects of taxes here.)

## GOEING AIRCRAFT: PROJECTING GROWTH OF SALES AND INCOME

In the most recent fiscal year, Goeing Aircraft had net sales equal to \$22.7 billion and had operating income (before depreciation) equal to \$2.3 billion. Over the past 17 years, the annual ratios of operating income to net sales had an average of 0.08, with a standard deviation of 0.02. During the same period, the annual sales growth ratios (current sales divided by previous year's sales) had an average of 1.07 with a standard deviation of 0.16. The logarithmic growth rates of sales (the natural logarithm of the sales growth ratio) had an average of 0.06, with a standard deviation of 0.15.

We want to build a simulation model of sales and operating income over the next 10 years that projects the statistical trends described above and that offers answers to the following questions.

(a) What is the expected value, standard deviation, and cumulative risk profile for the net present value of Goeing's operating income over the next 10 years, computed with a 10% interest rate?

(b) To buy back some of its own stock, Goeing is planning a major new debt issue which will commit Goeing to fixed annual payments over the next 10 years. The chairman of the board wants to have a probability 0.90 that Goeing's annual operating income will never be less than these annual debt payments during any of the next 10 years. What is the largest annual debt payment that Goeing can promise?

(c) Over the past 10 years, only 8 Goeing jets have crashed. But the CEO remarked, "If we ever had a year with 3 or more crashes, then we would have additional costs of \$1 billion that year, and sales would be down the next year by 25% for sure." How might this change our analysis?

Data Used, with Sales and Income in \$millions:

Year	Net Sales	Operating Income
00	8131	691
01	9426	872
02	9788	717
03	9035	574
04	11129	697
05	10354	721
06	13636	969
07	16341	1093
08	15355	710
09	16962	1015
10	20276	1208
11	27595	2208
12	29314	2780
13	30184	3001
14	25438	2716
15	21924	2293
16	19515	1935
17	22681	2345

## EVALUATING A STOCK OPTION

PG Corporation stock is currently selling for \$140 per share. In recent years, annual logarithmic growth rates of PG stock have had an average of 0.18 and a standard deviation of 0.14.

So suppose that the future growth of PG stock value will be a log-Brownian motion with annual log-drift 0.18, and log-volatility of 0.14.

Consider an option to buy one share of PG stock 2 years from now for \$180.

(a) Assuming the statistical probabilities described above, estimate the probability that this option will have a positive value 2 years from now, and estimate the expected monetary value of this option 2 years from now.

(b) The option may be exercised at any time in the next two years, but financial theorists have argued that such options should always be held to the latest exercise date. Assuming the statistical probabilities described above, estimate the probability of the event that PG stock will be below \$180 two years from now but PG stock will have closed above \$180 in at least one of the next 24 months.